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Vol. CIX.] [Part I.

Journal of the Royal Statistical Society

PART I, 1946

PROCEEDINGS OF THE FIRST ORDINARY MEETING OF THE SESSION, 1945–46, HELD ON NOVEMBER 22ND, 1945, THE PRESIDENT, THE RT. HON. LORD WOOLTON, P.C., C.H., IN THE CHAIR.

Before giving his Inaugural Address the PRESIDENT made a presentation to Miss Catharine Thorburn. He said that in the minutes of the last meeting there was a reference to Miss Thorburn, who had been Assistant Secretary of the Society for more than twenty-five years. They had been years of great difficulty to them all, and years of particular difficulty to the Assistant Secretary of the Society. Twice Miss Thorburn had had to guide the Society in rehabilitation, though fortunately, before the recent war, that word was not used in ordinary conversation. She had had an immense amount of personal trouble in keeping the records of the Society, but always she had been the very willing servant of the Fellows. Some of them accordingly thought they would like to do more than pass a formal vote of thanks to be placed in the Society's official records, and therefore a letter was sent round asking if the Fellows would care to join in a testimonial fund. It must be a gratification to Miss Thorburn to know that it proved not to be a question of whether they would care to join, but of their very great eagerness to join, and therefore he had very great pleasure in presenting to Miss Thorburn on their behalf a cheque for £350. He presented the cheque with the very greatest of goodwill, expressing the thanks of all of them for the devoted service she had rendered to the Society, and all their wishes for her happiness in the future.

MISS THORBURN said that she had been told that all she need say was " Thank you very much," but such a phrase would not express what she felt at receiving this generous token of appreciation, which had been a revelation to her. She had always been acutely aware of her shortcomings, and was very happy at this testimony that she had done a good piece of work. She would warm herself at the recollection for the rest of her life. She was also very glad to be alive to receive this tribute—so much more satisfying than to have it on one's tomb. She would always remember the Society with the greatest affection.

LORD WOOLTON then delivered his Inaugural Address.

SOME ASPECTS OF THE USE OF STATISTICS IN GOVERNMENT, WITH SPECIAL REFERENCE TO THE HUMAN BUDGET

I AM greatly honoured by the position I take to-day as the President of this old, distinguished and learned Society; but my pride is tempered with trepidation. I am the latest of a line of erudite men who have occupied this chair—and I am fearful lest the pride that led me, with inadequate equipment and knowledge, to accept the proffered office should lead to the disappointment of the Society. For I can make no claim to add to your understanding of the science of which you are masters and, for the most part, active practitioners. It is true that a paper I read to the Society thirty-three years ago—which I had prepared under the tuition of my professor, Sir Sydney Chapman—emboldened me to ask you to admit me to your Fellowship. Some years later I was elected to that eclectic circle of the dining club that repaid your speakers with good food and good wine for their learned contributions. I was pleased there to find that men with such high minds should have such good taste and can find an occasion.

These are, however, but poor credentials for submitting a Presidential Address to you, and I am determined, therefore, to leave to others any question of pure statistics, and to submit to you some reflections on the part that I think the statistician will be called upon to play in our future society, and public life. That society has grown out of our past, and without any conscious plan or obvious directive force. If we place ourselves on some intellectual pinnacle and look back on it, we can see that had that society been more ordered in its development, and worked to a man-made and intellectual plan, much suffering and waste might have been avoided, and a higher standard of

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efficiency, both in the realms of commerce and of public affairs, might have been attained by the nation. It may be so. It may be, of course, that such human planning and orderly arrangement would have produced people quite different from those who now inhabit these islands. I am not sure that it would have been to the advantage of the world if that had been so—for, at the moment at any rate, we have reason for some pride in our race—and it may be that the past was but a preparation for the future. Be that as it may—and I offer neither argument nor statistical evidence to support the hypothesis—I have no doubt that there exists to-day a most urgent desire on the part of many people to mould the future—or to have it moulded for them, which is less desirable or encouraging—in a manner that will leave the life of the nation less at the mercy of the blind ravages of chance.

Such a desire seems to me the consequence of an educated social conscience; it is to be applauded if it proceeds to the formulation of plans—and of legislation—that arise from an opinion that is founded on facts. Public opinion that is founded on statistical evidence is destructive of political prejudice; such evidence must, however, be presented in a manner that is capable of being understood by the public; it may, indeed, be necessary to append to its documentation a technical account of precisely how particular results were reached, this for the benefit of expert critics, but the hard discipline of patient thought can achieve simplicity in exposition. Technical methods in statistics are like the scaffolding of a building and the calculations of quantity surveyors and clerks of works. The finished building should be free of scaffolding and not viewed over a foreground of lorries and works hutments. I want to see the science of "Statistics" take its place as the instrument of truth and the servant of reform. Some of its work will destroy prejudices; some will inflame the mind of men against proven injustice.

It was an early illustration of such an idea that led me to the small piece of research that occupied me for two years, and which I submitted to this Society thirty-three years ago. We were hearing then much talk about the capitalist class. The story still persists in political propaganda. I thought, in 1912, that it might be an interesting economic study to find out statistically who these people were who were called the capitalist class and whence they came. It was a personal investigation and not a study of capitalism as a system. I took the Lancashire cotton industry as a sample. The results were revealing. They showed that the people occupying positions as owners or directors of businesses in the cotton trade had, in the large majority, risen to those positions without possessing either initial capital or influence: in the manufacturing industries 76 per cent.; in the weaving industry 63 per cent.; in the spinning industry 73 per cent.; of the mill managers 84 per cent. of the people approached came within this description. These figures show that, in these industries at any rate, there is considerable upward mobility of labour; they demonstrated that proved capacity in the wage-earning class had its chance of the highest rewards, both in position and earning power—that, at any rate in this trade, the "Klassenkampf" of the German economy did not exist and that the capitalist class was one that was constantly being recruited from the body of wage-earners. I have often wished that some more patient student, with more leisure, could carry this sort of research to wider fields, and survey the important industries of the country to-day, including the banks and insurance companies, and give us some factual basis for a better understanding of the formation and structure of our capitalist society. It would be instructive also to trace over a generation of time the history of the leaders in the learned professions and to see how much they have been dependent, or otherwise, on the inheritance of wealth.

On the other hand, some researches carried out in 1904 by Mr. Rowntree on family budgets revealed a standard of living, over a wide area of the community, which quite rightly shocked the complacency of anyone who had any regard either for the health of the nation or the adequacy of the wage level of many of the labouring classes; and I believe the statistics disclosed in his book on Poverty did much to influence the reforms introduced by the Liberal administration that held office in the eight years before the last war. In particular, the published researches into the wages paid in certain sweated trades, led directly to the establishment of Trades Boards, which created the machinery that has stamped out sweated labour in this country—to its profit as well as to its glory.

Anyone looking back over the records of this Society during the latter years of the nineteenth century cannot fail to be impressed by the energy and resource members exercised in striving to get at the facts about prevailing economic and social conditions. Most of the pioneer work was done by private investigators. Many of their enquiries may seem to the more practised statistician of to-day to be somewhat amateurish, but they had little information on which to work, and the public

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had not become inured to the enquiring zeal of the devotees of mass observation, or to the penetration of the official eye that demanded, with legal right, the revelation of their commercial or financial secrets that have now become our common affliction.

Nevertheless, much statistical progress was made in public affairs: the first Census of Production had appeared in 1907; people had become disturbed at the proportion that rent assumed in the cost of living, and Trades Unions' records had thrown some light on the extent of the variation in the figures of employment. A start had been made in providing information that enabled the reading public to form political opinion, and precise information was also of service to Ministers of the Crown, business men and the Trade Union movement in taking an overall view of the changing economic conditions of the country. We were moving, by slow steps, along the road where statistics ceased to be only an academic study and began to play a part in shaping public and official opinion.

Without doubt the exigencies of war—the first world war—hastened this process. Between the wars Government departments began to publish regular information on changes in wage rates, earnings, cost of living, the levels of employment and unemployment, changes in the character of industrial production, imports and exports—in short, we began to express the economic workings of our society in statistical form, and gave those who had the will—and the wisdom—to study these matters the material on which to plan their policies for improvement and expansion.

Perhaps one of the steps that will prove to be most significant was taken by the industrial community through their trade associations. In the 1914–18 war it had been necessary for many business men to make returns of their production to Government departments. After the war they continued, for their own benefit, to compile records on the doings of their trades—they had become more fully organized as trades federations, and abandoned the secrecy that previously they had regarded as essential to the exercise of competition. The iron and steel trade, shipping, the metal trades, the engineering industry, the textile trade, and wholesale and retail houses, all took a part in this new movement. I would recall to their minds the fears with which many of them broke the traditions of a lifetime.

I well remember the efforts I made to persuade the retail trade that it was possible to trust the statisticians of the Bank of England with their most closely guarded secrets of their turnover, and I know that the aggregate figures that are now published monthly are a most valuable asset, both to the trade and to the officers of the Government who seek to estimate the level of consumption.

The Chamber of Shipping gave an important lead in the matter, and has provided an excellent example of the possibility of the co-operation between Government and industry.

In 1932 the President of the Board of Trade (now Viscount Runciman) reminded the President of the Chamber of Shipping that the preparation of an annual estimate of the balance of trade is one of the tasks of the Board, that the shipping industry makes an important contribution to the credit side of the account, and that if this contribution could be accurately assessed it would obviously increase the value of the Board's estimates. He suggested that the Chamber could develop and establish a scheme which would provide reliable estimates of this kind and place the results at the disposal of the Board of Trade.

The Chamber accordingly conducted an enquiry in 1931 and again in 1936, which has been generally admitted to be of very considerable value. The questionnaires used on these occasions were agreed between the statistician of the Chamber and the Statistics Department of the Board of Trade, but great care was taken to ensure that the information obtained from individual firms was kept confidential to the statistician of the Chamber. The work involved in completing the questionnaire was very considerable, but on the second occasion a return of over 90 per cent. of the tonnage owned was received.

The Enquiry justified itself, and in fact the Board of Trade wished for it to be repeated, as they had originally suggested, on an annual basis. The matter will have to be reconsidered when normal trading conditions are restored, but it seems likely that the Enquiry will be continued, particularly in view of the increasing relative importance of the contribution of shipping earnings to our invisible exports.

From 1920 onwards, Dr. Isserlis, the statistician of the Chamber of Shipping, constructed an index of tramp-shipping freights. This was gradually improved in points of detail, and a full account of the index was given in a well-known paper, read by Dr. Isserlis before the Society in 1938.

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It is of some interest to note that the proposer and seconder of the vote of thanks on that occasion were Mr. Macrosty and Mr. Leak of the Board of Trade.

The index itself assumed more than a general interest with the passing of the British Shipping (Assistance) Act of 1935, which provided that " If in the opinion of the Board of Trade, at the end of the subsidy year, the average level of freight rates for the year (expressed as an Index Number representing a percentage) was less than the average for 1929 (represented by 100 per cent.) then the total sum payable by the Treasury for the year will be a sum not exceeding a quarter of a million pounds for every unit per cent., by which the average for the subsidy year is less than the average for 1929, subject to a maximum of £2 million."

Although the Board of Trade did not adopt the Chamber's index officially, there can be no doubt that it was one of the guiding factors in their consideration of the question and in their decision.

The preparation of estimates of national income and expenditure, changes in total consumption, and changes in stocks were undertaken mainly by private persons. As a result of this partnership between the statisticians in Government service, in trade associations, universities and research institutes, the position was beginning to be reached by 1939 when it could be said that the supply of information was comprehensive enough to show the size of the resources of the country and how they were being used. Although much progress had been made, there were many gaps in the kind of information being collected, and those who found themselves in 1939 responsible for the conduct of the country in the rapidly changing conditions of war—when facts were more important than precedents—had very little to help them.

The number of statisticians in Government service was few, and the factual information was rarely in the form that helped in reaching quick decisions. Neither Government departments, trade associations, nor private persons had sufficient detailed information readily at their disposal.

I could give many examples of this. When I was called upon to clothe the Army in May 1939, the War Office had no statistical evidence to assist me. Here there could be no doubt that to guess was to endanger the chance of victory and the security of the State. I had the greatest difficulty in arriving at any figures that would show how many suits of uniform and how many boots were involved. I found that the estimates of the potential loss due to various forms of military reverse—such as attacks on supply dumps by gas, or on troops in action—were little more than guesses, and in those urgent months before the war I called on the services of Professor Greenwood, a past President of this Society, to help me to create a statistical basis on which the operations of this aspect of the war effort could be built. I believe that it was because we established this statistical branch in the War Office—and it was subsequently transferred to the Ministry of Supply when I became Director-General of Equipment and Stores—that this section of our Supply Service never failed, even after the military disaster in France, to meet all the calls that were made upon it. We were operating on a plan that had allowed for the probable deviations of chance, and organization had greatly reduced the need for improvisation. Subsequently the statisticians came to be recognized in the Ministry of Supply as an essential part of the organization: they gave confidence and security to the decisions of the executive, and I am certain that they saved the country very large sums of money.

Again, when I went to the Ministry of Food I found the same lack of statistical data; and again I called on the services of Fellows of this Society. Dr. Heron will perhaps allow me here to pay testimony to the invaluable assistance he rendered to me and to the country in that Ministry. Here indeed was a field for the statistician. We were in a position of much danger. Our resources were very limited; our shipping was subjected to risks, capable of statistical analysis; our needs had to be interpreted in terms of calories and vitamins and other food values rather than in the customary considerations of human appetites; different classes of the people had widely different needs if their health and vigour were to be maintained. Our armies had to be fed according to the needs conditioned by fighting in widely differing climates; our children and nursing mothers had to have food appropriate to their particular needs; and all these variations had to be expressed, finally, in terms which would enable us to determine the nature of our home production and the precise tonnage of our imports of the many items that constituted our programme of purchases overseas.

Finally, a programme of distribution had to be arranged which would secure the unfailing validity of the food coupon in the most remote villages of the country. Even when the enemy's

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air raids had dislocated our transport system, we succeeded in that operation, because we had a statistical basis of facts on which to operate.

I have taken illustrations from my personal experience. The Society will recognize that there were similar calls for precision in the Ministry of Labour for the organization of man-power, in the Ministry of War Transport for the fulfilment of its varying tasks, and indeed in all Departments of State that were concerned with the active operations of war: the Prime Minister created, for his own guidance, a special statistical section of the Cabinet Office that came in the end to act as a statistical clearing-house for the War Cabinet.

I have dwelt at some length on the stages through which we have passed in these last forty years, because I wanted to demonstrate that in affairs of Government we have passed away from the state where political conclusions could only be based on vague impressions to the stage where both opinion and legislative action can be guided by statistical evidence.

For many years now the Treasury has placed such evidence at the disposal of the Chancellor of the Exchequer, and year by year in the days of peace the public were informed on Budget day of the close approximation of the Chancellor's forecast of Income and Expenditure made in the previous budget, to the ascertained facts of our financial operations during the preceding year; and in April 1941 the Chancellor published the first White Paper on "National Income and Expenditure," giving the full account of our operations in this field of Government. The proved competence of the statistician, both in the field of financial forecast and in the operations of Government in war-time, has given much stimulus to those who believe in what, in broad terms, is called National Planning, and in particular to its use in preventing some of the economic waste and personal tragedy that have been an accepted part of our social life in the past.

The development of this science has placed ready to our hand many new instruments for the creation of a fuller life and for the relieving of much physical misfortune. I believe that the science of statistics can be used both for the expansion of our standards of living and for the prevention of those ills that have brought our present economic system into some disrepute in the minds of many people.

Nothing has disturbed our social stability so much as the factor of unemployment. In the past we regarded this as inevitable; in earlier days we left its victims to the mercy of charity; after the last war we established the system of the dole—a thoroughly unscientific arrangement which was based on the inevitability of wide fluctuations in the economic wealth of the country—and prescribed an ambulance service to deal with the casualties, without having much regard for the long-term effect of the treatment on the individual, and none to the prevention of the cause of the casualties.

This is the most urgent social problem that confronts the State, and I have been led by my belief in the use of the statistical method of ordering those operations of our economic life that are within the control and competence of Government, to believe that we can solve this problem, at any rate to some large measure. It was this belief that led me, as Minister of Reconstruction, to present to Parliament, with the full agreement of all parties in the Cabinet, in May 1944, the White Paper on Employment Policy (Command Paper 6527). It stated that "the Government accept as one of their primary aims and responsibilities the maintenance of a high and stable level of employment after the war." It gave a detailed account of the methods that could be used to secure this aim. This paper has been widely read by the people of this country; it has been published by Governments overseas; it has been the subject of much discussion in the United States of America and throughout the Empire, and I believe it remains the standing challenge to our ability to create an economy in this country that will influence the opinion and the actions of every country in the world. I have brought it to your notice because I am convinced that, for its success, it depends not only on the goodwill of Ministers, but upon the co-operation of the statisticians and the business community. Intentions are not enough: we must have knowledge of facts as they are and capacity to deduce future trends from these facts. No Government could be expected to carry out such a policy without having access to such information as it may need. It would be like asking a doctor to be responsible for getting a patient back to health but denying him the right to ascertain the patient's temperature, to check the pulse, test the condition of his heart, or to ask questions about his past health.

I realize that industrialists are more than a little tired of the trouble and the expense of filling in forms for Government Departments. I realize, too, that many of the questions asked may in the

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past have savoured of inquisitiveness. Here the skill of the statistician is needed. It is useless and aggravating to call for information in more detail than is necessary to give proper guidance to action, and the mass of statistics that accumulate both in business concerns and in Government offices are a dangerous reflection on the practical competence of the statistician. It is stated that when the great economic blitz hit Detroit, Mr. Ford, going round his works, asked what went on in a large six-storey building; he was told it was the statistical department. He replied, according to rumour, "Tell them they are fired." His manager expostulated, and Mr. Ford replied, "When they've all gone we'll start a new department that will produce only the statistics that our managers can use." Here is a lesson that Government statisticians might well remember if they are to secure the support of the industrial community.

But I beg that such support will be forthcoming, and that industrialists will join with Government in this task. We want a human budget, as well as a financial budget, each year. To achieve this, the White Paper on Employment Policy stated that the principal class of statistics (in addition to those available before the war) which must be obtained for the efficient operation of an employment policy were:

(*a*) Statistics of employment and unemployment, including quarterly or monthly statement, of present and prospective employment in the main industries and areas in the country, based on returns from employers.

(*b*) Regular information relating to savings, projected capital expenditure by public authorities, and, as far as possible, by private industry.

(*c*) An annual census of production showing the structure of the main groups of industries in the preceding year, including, *inter alia*, details of the quantity and value of output, stocks and work in progress.

(*d*) Monthly figures of production, consumption and stocks and, if possible, figures of orders on hand, based on sample returns obtained periodically throughout the year from large firms, trade associations, and public institutions.

(*e*) Annual and quarterly estimates of foreign capital movements and balance of foreign payments.

The monthly estimates of employment, together with the information forthcoming from the administration of the unemployment insurance scheme, would permit a continuous picture to be available of changes in employment by industries and regions. The monthly returns of production, supported by the more detailed information forthcoming from a census once a year, would show corresponding changes in production by industries. The information on balance of payments would show how the excess of imports over exports was being met by income from overseas investments and by receipt from shipping earnings.

Particular importance is attached in the White Paper on Employment Policy to the need for getting regular information about actual capital expenditure and about proposed programmes for capital expenditure. This is a new field in which statisticians have to work, and it entails obtaining information about capital expenditure plans, not only from Government departments, local authorities, and public utilities, but also from private industry, where before the war there were very marked fluctuations in capital expenditure from year to year.

On the basis of this regular information about employment, production, consumption of goods, alance of payments and investment, it should be possible to continue to prepare—and possibly compile for shorter periods than hitherto—estimates of national income and expenditure. If, as stated in the White Paper, steps are to be taken to maintain total demand for goods and services at a high level, it will be necessary to have these figures in order to indicate the extent to which general demand is being maintained: these are the figures that indicate the trend of employment.

The whole of this work of collecting information for employment policy is one in which the close partnership built up before the war between statisticians in the Government service and their professional colleagues in trade associations, in the Universities and research institutes will have to be maintained. It has been aptly said that collecting information to guide employment policy in time of peace will be an experiment in social accounting with the statisticians assuming the rôle of accountants. So much depends on this collaboration that I feel no reluctance in drawing the attention of this Society to its importance.

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Those responsible for the execution of Employment Policy in practice may be reminded of the lines from the opening stanza of Robert Bridges' "The Testament of Beauty":—

"Our stability is but balance, and conduct lies in masterful administration of the unforeseen."

The unforeseen is often predictable, if we are willing to undertake the hard and detailed work of examining causes and interpreting present facts—and this is work that members of this Society are competent to initiate and to guide and control.

No aspect of our social and economic life is of such importance to our future stability as a nation as this old and dangerous problem of unemployment, and I have therefore ventured to give to you a presidential address, bare of statistics, but I hope full in its expression of my belief in the importance of the statistician in helping to direct and encourage a masterful administration in this field of Government.

The HONORARY SECRETARY said that it was the custom on these occasions for the vote of thanks to be moved by the President's immediate predecessor in office and to be seconded by another Past President. Dr. Snow, unfortunately, was unable to be present, owing to illness, and he had asked that his speech in proposing the vote of thanks might be read.

DR. E. C. SNOW (read by Professor A. Bradford Hill): It is my pleasant duty, as immediate Past President, to propose a vote of thanks to the new President for his Address. Until the last occasion, the Presidential Address has not been the subject of discussion. On the last occasion a change in the traditional procedure was made and a long and critical discussion resulted. On this occasion we are returning to the traditional practice, and neither proposer nor seconder will accordingly attempt any critical discussion of the stimulating Address Lord Woolton has presented to us.

It was only natural that, when I knew of the subject of the Address, my mind went back to the first occasion when I met Mr. Marquis. It was in 1916 or 1917, and it was at a fairly early stage of the very first action which had even been taken in this country to regulate and plan industry. In view of the great development of industrial regulation which has followed, and seems likely to continue, it may not be out of place to refer to the first occasion of all from which so much has developed. It was in 1915, when the War Office was very much concerned about the subject of sandbags, things which had never been needed in previous wars, and for which no provision had been made. The sudden development of trench warfare created an overwhelming demand for sandbags. The Contracts Department at the War Office, who were at that time only concerned with the product they wanted to buy and not with the raw materials and/or processing of them, found that the price was going up by leaps and bounds, and were greatly concerned over a type of problem they had never previously had to face. The only possible power of dictating to industry at that time was through an Army Council Order, and the Director of Army Contracts was anxious to have an Order which would regulate the buying of raw materials in the jute industry and control its price. Economic reasoning was not, however, a strong point of the Secretary of State for War at the time—Lord Kitchener—and action was postponed. Prices continued to soar, but later a graph was produced as some additional evidence to convince the Secretary of State that urgent action was needed. This graph showed prices of raw jute, jute yarn, and of the manufactured cloth or hessian. It was drawn on such scales that for the pre-war period the three lines generally coincided, with many small fluctuations. After the development of the demand for sandbags, the three lines on the graph began to diverge widely. That of raw jute showed an upward incline of about 15°, that of the yarn an upward incline of about 45°, and that of the cloth an upward incline of more than 60°. This graph did what plain arguments had failed to do, and the Secretary of State for War was persuaded to agree to the issue of an Army Council Order regulating many of the activities of the jute industry.

Extensions of this rapidly followed, and it was not long before a Raw Materials Section was formed in the Army Contracts Department of the War Office. A number of men who afterwards acquired fame in various walks of life came on to its staff. I call to mind Sir Adrian Boult and Mr. Philip Guedalla, as well as Lord Woolton. Lord Woolton's pet job, I remember, was to produce and get the public to wear a standard boot. I believe he had not previously had any active association with industry, and this wartime boot scheme was, I suppose, the small acorn from which

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has grown the massive oak which now covers Lord Woolton's vast interests in industry. When he was first approached to be President of the Society, he was a Minister of State. He agreed, and at the time of election to the Presidency it was, I believe, the first case in which an active Minister had been President. Before he had performed any Presidential functions, however, the electors, by implication, expressed the opinion that he could not hold the two Offices in conjunction, and so Lord Woolton becomes a President—like a number of distinguished people in the past—as ex-Minister. But this fact does not appear to have added to Lord Woolton's leisure, and, not unnaturally, a number of important industrial organizations wish to have the benefit of his advice and judgment. However, we look forward to his presiding over these monthly general meetings, and also to having his advice at the regular Council Meetings on the matters of importance to statisticians on which the Society is at present engaged.

Lord Woolton's Address dealt with the statistical needs of a planned economy. A few remarks on the general subject of planning may not, therefore, be out of place. I suppose nobody now exists in this country who can seriously advocate that there should be no economic planning, and the question is "How far should it go?" How much scope should be left for opportunism; for seizing an opening not previously foreseen? Views on this must necessarily be varied, but my own predeliction would be to leave substantial scope for the operation of chance. Perhaps my feelings on this point are influenced by the fact that though I have led a life almost wholly unplanned, I have nevertheless suffered chronically from over-employment. I believe that those who try to plan to the last detail overlook the important part which chance plays, and must continue to play, in human affairs. This subject, I remember, was well dealt with in a chapter in a book by the famous French philosopher, Poincaré, some time before the last War. Before plans can be put into operation time elapses, and during this time all sorts of things may happen which may interfere with the successful development of the plans. I find a comparison between the existing British and American points of view in this matter illuminating, and I may mention a recent example of this difference in outlook. In the arrangements for Missions of industrialists to visit Germany, we have laid our plans apparently to the last detail, even to the extent of ascertaining the denomination of the Minister of Religion who should be called upon if a member of the Mission unfortunately becomes a casualty while in Germany. All this planning, however, has taken time, and now the advent of winter is introducing a physical handicap and causing further delay, and Missions which were contemplated months ago have not yet left. The Americans, however, sent their Missions over with little pre-planning. They were apparently told to get on with the job, and worked out their own plans as best they could. No doubt they met with many difficulties, but they seem to have been able to deal with them on the spot, and have finished some of the jobs before we have really started. In military affairs, too, it happens that it was an American General who, by seizing an opportunity not included in his plans, was able to make progress which appreciably shortened the war. On the other hand, as readers of Mr. Churchill's book on the Dardanelles Campaign will remember, it was a British junior commander who, having achieved an objective planned for him, rested on his laurels and, failing to see an opportunity presented to him, missed the chance of making that campaign a success which would have had a vital influence on the future of the world.

I, therefore, am one of those who would not advocate planning to the last detail, or sticking to plans whatever may befall. I have seen many cases in my life where a quick judgment and seizing an opportunity has led to happy development, and I think it would be a misfortune if, by laying undue stress on planning, our faculties for seeing and seizing an opportunity were blunted. The history of research, as well as the history of business, affords many illustrations of the part which chance may play. I remember before the 1914 war, when London first had a high-powered organiser as its Educational Officer who planned everything in minute detail, and who was not particularly popular with some people engaged on research work, a cartoon appeared in a small paper depicting a dejected looking Newton making his weekly call at 10.30 on Monday to report that he was sorry he had no progress yet to announce in connection with his investigations into gravitation, but pleading that, nevertheless, his subsistence allowance should be continued. The inspiration of genius must continue to be one of the most potent factors in human progress, and all efforts to plan the activities of geniuses will fail.

I suppose that we shall follow the traditional British course in this matter. After having erred on the side of under-planning, we shall err on the side of over-planning, from which we shall, in

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due course, swing back again to some intermediate stage. Personally, I hope that the process will not be a long one.

Our President, in discussing only statistics required in planning has naturally not disclosed his own views on the policy behind planning. Perhaps some time during his term of Office we may be fortunate enough to have a paper which will enable the Society to enter upon a wide discussion of the subject, and I am sure that, if we do, his views will be listened to with great interest. He was in a position during the greater part of the war to see that statisticians were called upon to play a full part in the solution of the many problems which presented themselves, and it was fortunate that one so fully convinced of the need for skilled statisticians held such important offices at critical periods. It might well have been otherwise.

The Society has, accordingly, already cause for gratitude to Lord Woolton, and that gratitude will no doubt be further increased during his two years of Presidency. I have, therefore, very much pleasure in proposing a vote of thanks to him and I will ask Professor Greenwood to second it.

PROFESSOR MAJOR GREENWOOD, in seconding, said: As Dr. Snow has remarked, during the 111 years of life of our Society several men who in the old-fashioned phraseology have been His Majesty's or Her Majesty's confidential servants have honoured the Society by filling the presidential chair. In my own short recollection I have sat under three "confidential servants"—Lord George Hamilton, Sir Herbert (now Viscount) Samuel, and Lord Kennet. There is only one of our former Presidents who occupied a position in the Cabinet Council in times approximating in gravity to those during which our new President has held office, and he was the very first President, the third Marquess of Lansdowne, who became President of this Society in 1834. It is possible that our memory of who exactly he was may be a little dim. He was the Marquess of Lansdowne, who brought the young Tom Macaulay into Parliament for one of his pocket boroughs, but those of us who sat for a schools certificate in history may know that 28 years before he became our President—he was rather young, even by the standard of those days, being only twenty-six—he became Chancellor of the Exchequer in the War Cabinet of 1806, which had to face a position very nearly as gloomy as the Cabinet of 1940, and did not face it perhaps quite as successfully.

I do not know whether the Marquess delivered a presidential address, and, alas, Mr. Macrosty is not here to tell us. If he did, and if he was a man of normal gratitude, I should think he spoke rather kindly of statisticians. His great great grandfather, Sir William Petty, was quite an eminent statistician. Sir William Petty's daughter and heiress married into the Fitzmaurice family, and our Lord Lansdowne's father, the Earl of Shelburne, an unpopular person, who was for a brief while Prime Minister, said of this ancestress, "She was a very ugly woman who brought into our family what degree of sense may have appeared in it and whatever wealth is likely to remain in it." His son, our first President, had certain solid grounds for speaking favourably of statisticians, but whatever he did say, I am confident that he did not say that the War Cabinet's efforts of 1806 were largely furthered by statisticians official or unofficial, and it is reserved to our new President to pay statisticians that compliment. He has paid it sincerely, and we are sincerely grateful as representatives of statisticians in this country for the compliment which has been paid, and we honour those of our colleagues who have enabled him to pay it.

We also, I think, are conscious of our responsibility as the representative body of statisticians, because the higher and more responsible the tasks of statisticians the more important is it that they should be equal to those tasks. On the technical side, the Society has had the subject under consideration for some little time. Our Council appointed a special committee, and after considering the report of that committee submitted to a general meeting of this Society proposals the object of which was that the Society should accept responsibility for prescribing the training and for testing by examination the qualifications of those persons who wished to enter the profession of statistician and were willing to be guided by us. The proposals were approved by the very large majority of the Fellows present, and I hope and believe that they will be brought into operation in the very near future, possibly not in the precise form in which they were submitted, but in a way which will substantially fulfil our objects.

We as Fellows of this Society, like all sensible men, have no illusions as to the perfection of prescribed education and examination. We are perfectly well aware that any system of that kind cannot exclude from a profession a good many persons who are quite unfit to exercise that profession and that it will exclude some persons eminently qualified to exercise it. We all know this.

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One of the greatest classical scholars in this country, according to good judges, of whom I am not one, the late Professor Housman, did not get a third or fourth class in Oxford " greats ". He was ploughed neck and crop. One of the greatest mathematicians—again according to competent judges—in any age, Ramanujan, failed hopelessly in the matriculation examination in London University. Most medical men over the age of thirty will tell you that in the *Medical Register* there are a good many persons with many medical diplomas and degrees who are extremely poor doctors, and that, on the other hand, there are some ladies and gentlemen who found the greatest difficulty in satisfying the examiners that they were worthy of the humblest qualifications but became extremely wise and competent doctors. We all know this. It is not knowledge confined to the Fellows of this Society, it is the knowledge of all sensible persons.

But, after all, we are statisticians, and we must reason statistically. Supposing all barriers to entry to the *Medical Register* by training and examination were thrown down, the advantage to the public of admitting a few men of genius with a psychological difficulty in passing examinations would not compensate, statistically speaking, for the inrush of impudent pretenders; for the very good reason that in the profession of medicine, or of statistics, or of law, technical competence is a matter that the public is not very well able to judge. We need no examinations for the profession of comedian because the public is a perfectly competent judge whether it is amused or not. But when it comes to the question of medical treatment or statistical treatment, the ability to judge (as in the case of Lord Kitchener, mentioned by Dr. Snow) is by no means a universal possession even of very intelligent people.

That, I think, is the case for the action that the Society has already taken. But, as our new President has tactfully reminded us—and his anecdote of Mr. Ford is very relevant—something more than technical knowledge and skill is required to make a wise statistician. What that quality is, is a matter of definition. In the medical profession I think we should say it was something called clinical instinct; in research, a flair, a power such as Farr had of seeing important truths hidden behind tables. As one gets older one uses simpler words, and I should be rather inclined to say it was commonsense. We cannot test that by examination, but even here our Society can do good work, and we are helped to do that by the very wise rule we have followed for so many years of electing as President a gentleman who may not be a professional statistician, but whose primary interest has been statistical, and alternatively a gentleman who, like our President today, may have done very competent statistical work but whose main interest in life is not primarily statistical. It seems to me that we can help there. Every Fellow in this Society, of course, knows Todhunter's theory of probability by heart, and may recall Todhunter's judgment of another expert, that he seemed really to have thought all data valuable, however imperfect, provided they were garnished with a sufficient number of signs of integration. I have sometimes thought that some official and unofficial statisticians are of opinion that a report is always valuable provided it is garnished with a sufficiently large number of tables. It is part of our duty as a Society to dispel that illusion. In the past many gentlemen have come to us with elaborate statistical plans and have left the meeting possibly sadder but certainly wiser men than they entered it. I believe that has been equally due to technical criticism and to the sort of criticism that gentlemen of wider experience of life can give. Therefore to the gratitude we already owe to our new President for a stimulating address we will give him a vote of thanks warmer still in anticipation of what he is going to do for us in the next two years. I have the greatest pleasure in seconding this vote of thanks.

The vote of thanks was put to the meeting and carried unanimously.

LORD WOOLTON thanked Dr. Snow *in absentia* and Professor Greenwood for their words, and said that if he could be of any service to the Society during the time they retained him as President he would regard it as a pleasure as well as a privilege.

As a result of the ballot taken during the meeting, the candidates named below were elected Fellows of the Society:—

Ivor Charles, Capt.
Joseph Rock Cooper.
Stella Vivian Cunliffe.
Leslie John Gollop.
Frederick Horace Hanks.
Peter John Hilton, B.A.

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Maurice William John Dobeyn.
John Parker Gilbert Dowman.
Herbert Goodier.
Ronald James Rivers Halliday.
James Douglas Michelsen.
Harold Ernest Osborn.
Robert Lewis Plackett.
Maurice Henri Quenouille.
Ernest Edward Taylor.
Andrew Tessler.
David Wakefield.
Andrew Morris Walker.
D. A. V. Allen.
George Douglas Amery, M.C., M.A.
David James Bishop.
Victor Bloch, Dr. jur.
Eric John Metius Buckatzch, M.A.
A. M. Burdon-Cooper, Major.
Charles Frederick Carter.
William Leslie Carter.
Thomas Chinloy.
Leslie Edwin Creed.
Kalyan Kumar Datta, B.Sc.
Raymond Daw, F.I.A.
Herbert William George Deeks.
Johannes Bernardus Dirk Derksen.
William John Dixon.
Henry William Durant.
Henry Algernon Thomas Dyke-Hart.
Kenneth George Forecast.
Reginald Walter Howell.
Aldred Charles Hutchins.
Edward Francis Jackson.
Geoffrey Harcourt Jowett.
Frederick Arthur Lewis, F.I.A.
Jack Edward Mantell.
Joan Grace Marley, B.A.
George William Mayne, Major.
Thomas Scruton Meadows.
Harold Moore.
Geoffrey Miles Morant.
Kesavan Raghavan Nair.
Charles Philip Guy Neat.
Kenneth Owen, Major.
John Francis George Perceval, Major.
Frederick Bernard Pooley.
Eric Tressler Purslow.
Ram Mohan Ray.
Thomas Hewitson Seymour.
Cedric Austen Bardell Smith.
Leon Solomon.
Michael Henry Spieler.
Edward Thomas Steer.
Ian Sutherland.
George Kenneth Vincent, B.Sc.
James Wallace, M.A.
Sidney Weinberg.
Bernard Lewis Welch, Ph.D.
Clive Bertram Williams.

Corporate Representatives.

Albert Bernard Hollowood, *representing the* Council of Industrial Design.
Leonard Godfrey Jennings, *representing the* Timber Development Association, Ltd.
Rosi M. Klaar, *representing the* Manchester Oil Refinery, Ltd.
Mrs. Elinor Orr, *representing the* Industrial Welfare Society, Inc.
Felix Winter, *representing the* Economist newspaper.

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A Review of Recent Statistical Developments in Sampling and Sampling Surveys

By F. Yates, Sc.D.

[Read before the Royal Statistical Society, January 23rd, 1946, the President, the Rt. Hon. Lord Woolton, P.C., C.H., in the Chair]

1. *Introduction*

Sampling is not a new subject. My object in bringing it up for discussion to-night is twofold. Firstly, with the increase in economic planning, and the development of the social sciences, the need for economic and social censuses and surveys has greatly increased. Such surveys generally require some form of sampling for their efficient and speedy execution. Secondly, development of new statistical methods in agriculture and biology has led to developments in sampling theory which are relevant to all branches of sampling. While it is broadly true that no really new methods of selecting representative samples have been introduced in recent years, the theory underlying the various methods is now much better understood, and practical procedures are available for estimating the sampling errors of complicated as well as simple sampling methods.

Although I have devoted considerable space to discussion of methods of estimating sampling errors, I do not wish to imply that such estimates of error are always required before a critical analysis of the results of a survey can be undertaken—we may often be satisfied, from previous experience, or from the general behaviour of the results themselves, that adequate accuracy has been achieved. On the other hand, comparison of the efficiency of different sampling methods, and intelligent planning of future surveys, can only be made by detailed analysis of the various components of variation to which the material is subject, and a study of their effects on sampling errors.

I have not attempted to discuss this latter problem in detail in the present paper. What has been attempted is a comprehensive summary of the various sampling methods that are commonly employed, and their inter-relations, together with an outline of the appropriate methods of arriving at estimates of the quantities under survey, and of determining the errors to which these estimates are subject.

While the parts of the paper dealing with the estimation of sampling errors are necessarily somewhat technical, the remainder of the paper has, I hope, been written in reasonably non-technical language. Sampling, after all, is largely a matter of common-sense, and the common-sense approach has often resulted in more rapid progress in technique than has the more purely mathematical approach. What has been lacking in the purely common-sense approach is a method of estimating the efficiency of different sampling procedures. A synthesis is therefore required. It is this synthesis I have attempted in the present paper.

2. *Position in* 1924

In May 1924 the Bureau of the International Institute of Statistics appointed a commission for the purpose of studying the application of the representative method in statistics. The members of the commission were Professor Arthur Bowley, Professor Corrado Gini, Mr. Adolf Jensen, M. Lucien March, Professor Verrijn Stuart, and Professor Frantz Žižek. The commission presented its report in 1925 (Jensen, 1926). Appended to the report were a paper by Jensen on "The Representative Method in Practice," and a paper by Bowley on "The Measurement of the Precision Attained in Sampling," together with the two shorter papers by Stuart and March. The report and the attached papers give a good picture of the sampling methods then in use in economic and social surveys and in the analysis of census material, and of the statistical theory on which these methods were based.

The conclusions of the report are embodied in a set of resolutions which were adopted by the International Institute of Statistics. These are as follows:

The International Institute of Statistics

Considering that it is necessary in many cases to draw general conclusions based upon partial investigations owing to the impossibility of procuring a complete statistical material;

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Considering that even in such cases where complete material is available it may be sufficient to work up a portion of the material, provided that this working up is done in a rational manner; and

Considering that the saving of labour, time and money which is possible by limiting the investigation to a portion of the material will often make it possible to make a much more extensive use of the information at hand and to enter far more deeply into the subject than is possible by a working up of the whole material;

I. With reference to the Resolution passed at the Session at Berlin in 1903, again calls attention to the very considerable advantages which can be obtained by applying the Representative Method under the following conditions:—

The results of a partial investigation should only be generalized provided that the sample used is in its nature sufficiently representative of the totality. In such respects the sample may be selected in different ways; the following two main cases, however, are to be distinguished:

(*A*) *Random Selection.* A number of units are selected in such a way that exact equality of chance of inclusion is the dominant rule. Then precision is related to the number included which should be large enough to render insignificant accidental deviations;

(*B*) *Purposive Selection.* A number of groups of units are selected which together yield nearly the same characteristics as the totality. In order to have any knowledge of the precision of the estimate it is necessary that sufficient groups should be included to allow the variation between the characteristics of the groups to be measured. But since the precision often depends to a great extent on the discretion used in making the selection, the following controls are recommended:—

1. The selection on the same principle should be made twice or more; after their comparison, the samples can be merged. (This recommendation is also applicable to the Random Selection);
2. In repeated observations, the relation between the part and the whole should from time to time be examined more minutely;

II. Recommends that the investigation should be so arranged wherever possible, as to allow of a mathematical statement of the precision of the results, and that with these results should be given an indication of the extent of the error to which they are liable;

III. Repeats the wish expressed in the Resolution of 1903, that in the report on the results of every representative investigation an explicit account in detail of the method of selecting the sample adopted should be given.

Two main features will strike the present-day reader: first, the considerable prominence given to the method of purposive selection, and second, the lack of any very clear conception of the possibility, except by the selection of units wholly at random, or by the inadequate procedure of sub-dividing the sample into two or more parts, of so designing sampling enquiries that the sampling errors should be capable of exact estimation from the results of the enquiry itself.

3. *Developments in biological and agricultural research*

At about this time a number of new developments were taking place in the statistical methods used in biological and agricultural research. Those connected with the design and analysis of agricultural field experiments have been of particular importance to sampling theory. In such experiments each treatment is repeated only a few times, and consequently the then customary procedure of estimating the error of each treatment mean from deviations of the yields of the individual plots from that mean led to very imprecise results. Moreover, such estimates were in any case usually invalidated by the fact that each replicate of the experiment was arranged in a compact block of plots on the ground, so as to eliminate fertility differences as far as possible.

The introduction of the analysis of variance technique by R. A. Fisher gave a convenient arithmetical procedure for pooling estimates of error from different treatments, and simultaneously eliminating variation due to blocks, or other features of the layout. Consideration of the theoretical basis of the analysis of variance led in turn to the introduction of the principle of

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randomization, thereby ensuring that the estimates of error should be valid whatever the nature of the variations in fertility.

The first published application of the analysis of variance (Fisher and Mackenzie, 1923) is of considerable interest, as it shows how the ideas developed. The potato experiment there analysed contained several systematic features of arrangement, which were not discussed or fully taken account of in the analysis.

Fuller consideration of the problems raised by the analysis of this and similar experiments led to a rapid refinement of the technique, and by 1926 the essentials of good experimental design and analysis were fully realized (Fisher, 1926). The necessity of certain elements of randomization had been recognized, and the analysis of variance, as then developed, provided a method of pooling of the estimates of error from the different treatments, of eliminating components of variability which did not affect the treatment comparisons, and of furnishing separate estimates of error for treatment comparisons which, because of features of the design, were of differing accuracy. Parallel development of the t and z tests enabled these estimates of error to be used as a basis for exact tests of significance.

The first steps in the application of the new technique of analysis to sampling problems were taken by Clapham (1929). Clapham was concerned with the problem of estimating the yields of experimental plots of cereals from a number of small sampling units cut from each plot. He used the analysis of variance to calculate the sampling errors to which sampling units of various types were subject. The method was subsequently tried out in practice by Clapham (1931), and its efficiency was further examined by the present author (Yates and Zacopanay, 1935). A similar technique was developed for potatoes by Wishart and Clapham (1929).

Clapham's work led the way in the development of sampling techniques applicable to many agricultural and biological problems, such as the estimation of the growth rate and chemical composition of crops, the degree of infestation of the soil and crops with diseases and insect pests, the bacterial content of soils and liquids, the sampling of heterogeneous materials for chemical analysis. In these investigations the sampling problems were approached from a new angle, and for the first time those concerned with drawing up sampling schemes made a general practice of estimating sampling errors from the results of their observations, both to ascertain whether the sampling actually undertaken was adequate for the purpose in hand, and to increase the efficiency of future sampling of the same type of material. Trial sampling schemes of various kinds were also tested out, to determine the most suitable type of sampling unit.

The principles on which this work was based are very simple. They can be summarized in the following terms (Yates, 1935):

> (1) If bias is to be avoided, the selection of the samples must be determined by some process uninfluenced by the qualities of the objects sampled and free from any element of choice on the part of the observer.
>
> (2) If a valid estimate of sampling error is to be available, each batch of material must be so sampled that two or more sampling units are obtained from it. These sampling units must be a random selection from the whole aggregate of sampling units that can be taken from the batch of material, and all the sampling units in the aggregate must be of approximately the same size and pattern, and must together comprise the whole of the batch of material.

The importance of the first condition was well known to those who drew up the report to the International Institute of Statistics in 1924. It was the methodology implicit in the second condition which constituted the new advance. Realization of the functions of strict processes of randomization in agricultural field experiments had led to a corresponding realization of its importance in providing a valid estimate of error in sampling. "At random" no longer meant "haphazard." Again, the analysis of variance, by making possible the pooling of estimates of error and the separation of components of error which were not homogeneous, enabled the number of independent sampling units taken from each batch of the material to be reduced to a small number, and so permitted the use of relatively complicated sampling schemes, often involving sampling in two or more stages.

In most of these early applications the material sampled was of such a nature that it could be divided into sampling units of approximately the same size and shape, and consequently formally

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simple sampling schemes could be adopted. When, however, the methods began to be applied to such problems as crop estimation, the estimation of timber resources, and surveys of economic conditions and practices on farms, where the natural units of the population under survey are of widely differing size, new difficulties arose, which were by no means fully resolved at the outbreak of the war (see, for example, Cochran (1939)). In particular, methods were required for dealing with differences in variability of different parts of the material, and of handling sampling errors of material in which the sampling units were of widely differing size.

During the war intensive use of sampling in economic and social surveys and in problems arising in operational research, such as bomb distributions, amount of damage in blitzed towns, etc., has led to a further study of method both here and in America, and many of the above difficulties have been resolved, though there has, as yet, been no proper codification of the new methods, similar to that which has existed for some years for methods suitable for agricultural and biological material. I have therefore attempted, in the succeeding sections of this paper, to give an outline of these developments and their applications to the problems arising in area sampling, and in social and economic surveys.

4. *Summary of sampling methods which provide estimates of sampling errors*

It will first be profitable to make a short list of the various sampling methods which are in common use in one or other of the various fields to which sampling is applied, and which are capable, in virtue of the random elements in the selection process, of furnishing exact estimates of sampling error.

All forms of sampling involve some choice of sampling unit. The sampling units chosen may be natural units of the material to be sampled, such as the individuals of a human population, or natural aggregates of such individuals, such as households, or they may be formed by arbitrary sub-division of the material, such as the areas formed by grid squares on a map. Sampling units need not necessarily all be of the same size, though if there is marked variation in size, estimates based on ratios or percentages are usually required, and this complicates the estimation of sampling errors.

(*a*) *Random sampling* (*without restrictions*)

This is the simplest form of sampling, often referred to, but rarely used in practice, since its place is taken by some form of quasi-random sampling (see Section 10).

In random sampling selection from the whole population of sampling units into which the material is divided is made by some strictly random process, such as numbering all the units and selecting the requisite number of numbers at random by drawing lots or by the aid of a table of random numbers.

(*b*) *Stratified sampling* (*random sampling from groups*)

In stratified sampling the whole of the material to be sampled is divided into groups or strata. The same proportion is then selected from each stratum by some process of strict random selection within each stratum.*

If the strata are so chosen that each forms a relatively homogeneous group, the accuracy of the sample will be considerably increased, since each stratum is represented in the correct proportion in the sample.

It is worth noting that if the total number of units falling in each stratum is already known, as is frequently the case from previous census material, there is no need to divide the whole population into strata before selecting the sample. A stratified sample can be constructed by selecting at random from the whole population, classifying the sampling units into the strata as they are selected, and rejecting any further units falling in a given stratum as soon as the quota for that stratum is obtained.

(*c*) *Sub-sampling*

Sampling may be performed in two or more stages, the whole population being divided into a number of large sampling units, each of which contains a number of smaller sampling units. A

* The term " stratified sampling " is also used to cover the case in which different proportions are taken from the various strata. This case is described in Section 9, under the term " variable sampling fraction." (See Mr. Kendall's contribution to the discussion.)

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sample of the large sampling units may then be taken, and from each of the large sampling units so chosen a proportion of the smaller sampling units may be selected. Thus, for example, if a sample of all the inhabitants of a country is required, the country may first be divided into large sampling units consisting of towns and rural areas. A random selection from these towns and rural areas may then be made, and from each of the towns and rural areas selected a further random selection of individuals may be made.

Stratification and other sampling devices may be and often are used in conjunction with sub-sampling at any or all of the stages of the sampling.

(*d*) *Stratification for two or more factors*

Stratification can be carried out simultaneously for two or more factors. Thus, we might classify a population according to income group and also according to age and sex. The number of sub-strata will then be equal to the number of cells in the two or more way table, corresponding to the factors for which the stratification is carried out. A random selection of equal proportions from each of these sub-strata will be exactly equivalent to ordinary stratified sampling with the sub-stratum as the unit of stratification.

If the number of cells is large, the sampling procedure is likely to become very involved. Moreover, it frequently happens that although the marginal totals of the numbers of sampling units in the two or more way table of the factors are known from previous census material, the numbers in the separate cells are unknown. These difficulties can be overcome by constructing a sample which is so adjusted that the proportions in the sample agree with the marginal totals of each factor separately. Such a sample can be called a sample stratified for two or more factors without control of sub-strata. The same process of selection can be used as was suggested for constructing an ordinary stratified sample without actually sub-dividing the population. The number of rejections at the end of the process is larger the larger the number of factors for which stratification is required.

(*e*) *Balancing*

Control of a quantitative character—*e.g.*, income—may be obtained by stratification in groups corresponding to ranges of values of the quantitative character, but if the number of units to be included in the sample is small, and if control of other (non-quantitative) characters is also desired, it may be simpler to select a sample which is balanced for the quantitative factor. In a balanced sample the mean value of the balanced factor in the sample is equal to the mean of the factor in the whole population.

It is important, in selecting a balanced sample, that the process of selection is such that, apart from the restriction imposed by the balancing requirements (and any other restrictions due to stratification, etc., for other factors), the selection is equivalent to random selection.* This is the essential difference between a balanced sample and a purposively selected sample.

A procedure analogous to that suggested for the construction of a stratified sample without actually dividing the whole population into strata will effect this. A random sample is first selected (stratified for other factors if required). Further members are then selected by the same random process, the first member being compared with the first member of the original sample, the second with the second member and so on, the new member being substituted for the original member if balance is thereby improved.

Such a procedure is apt to become tedious if balance for a number of factors is attempted. In such cases the simultaneous choice of a number of alternative members from pairs selected as above will facilitate the process, though the exact conditions required for strict equivalence to a random sample have not yet been specified.

5. *Methods of forming estimates and calculating sampling errors*

All the sampling methods set out in Section 4 are governed by the condition that the chance of inclusion of any sampling unit or sub-unit in the sample is the same. Consequently, the

* There does not appear to be any procedure which will give exact equivalence to a restricted random sample whatever the form of the parent distribution: even if such a sample could be obtained estimates derived from it would in fact be subject to some elements of bias, though such bias is not likely to be of importance in practice.

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estimate of the mean of any quantity (or proportion of any characteristic) in the parent population may be obtained directly by taking the mean of the corresponding quantity in the sample data. Similarly, estimates of the totals of the parent population may be formed by multiplying the sample totals by the reciprocal of the sampling fraction.

The estimation of the sampling errors to which these estimates are subject depends on the sampling procedure which has been followed. The estimation of the errors of qualitative data differs from that of quantitative data in that the variability of the proportion possessing a given characteristic in a random sample follows the binomial law of error, and therefore depends only on the proportion in the universe and the number in the sample. Consequently the standard deviation of a sampling unit does not require estimation. If, however, the sampling units are made up of numbers of individuals, and the proportion of individuals possessing a given characteristic is of interest, correlation between individuals in the same sampling unit will invalidate estimates of sampling error derived from the binomial distribution, and the standard deviation of a sampling unit must be estimated in the same manner as for quantitative data, scoring the individuals 1 if they possess the characteristic and 0 if they do not.

(*a*) *Random sampling* (*no restrictions*)

The standard deviation of a single sampling unit can be calculated in the ordinary manner and used to estimate the sampling error of the mean or total as required.

If y_r is the value of a variate y in unit r of the sample, $\bar{y}$ the mean and $S(y)$ the sum of y for all units of the sample, $\bar{Y}$ and $\Sigma(Y)$ the estimates of the mean and the total of the population, n the number in the sample, N the number in the population, f the sampling fraction, s the estimated standard deviation of a sampling unit, and S.E. denotes the estimated sampling standard error, we have

$$f = n/N$$
$$\bar{Y} = \bar{y}$$
$$\Sigma(Y) = \frac{1}{f}S(y)$$
$$s^2 = \frac{S(y_r - y)^2}{n-1} = \frac{S(y^2) - yS(y)}{n-1}$$
$$\text{S.E. of } \bar{Y} = s\sqrt{\frac{1-f}{n}}$$
$$\text{S.E. of } \Sigma(Y) = \frac{ns}{f}\sqrt{\frac{1-f}{n}}$$

The factor $\sqrt{1-f}$ occurs in the estimates of the sampling standard errors because the population sampled is finite. This factor should not be included when testing the difference of the means of two sampled populations to see whether, for example, they are subject to different causal agents.

(*b*) *Stratified sampling*

In estimating the sampling error of a stratified sample the variability between the different strata must be eliminated from the estimate of the variance of a single sampling unit. As pointed out above, this can be effected by the analysis of variance procedure. If t is the number of strata, the analysis of variance can be set out as follows:

	Degrees of freedom	Sum of squares	Mean square
Between strata	$t-1$	A	
Within strata	$n-t$	B	s_t^2
Total	$n-1$	C	s^2

If S, S' and S'' indicate summation over the whole sample, a single (s'th) stratum, and the whole group of strata respectively, the between strata and total sums of squares are given by the formulæ:

$$A = S''\{\bar{y}_s S'(y_s)\} - \bar{y}S(y)$$
$$C = S(y^2) - \bar{y}S(y)$$

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The sum of squares B within strata is then obtained by subtraction. It will be seen that the sum of squares within strata is the sum of the squares of the deviations from the strata means, and the total sum of squares is the sum of the squares of the deviations from the general mean.

The mean squares can then be obtained by division of the sums of squares by the corresponding degrees of freedom. The formulæ given for the standard errors of a random sample without restrictions hold with the substitution of s_i for s. The increase in efficiency due to the use of a stratified sample instead of a fully random sample is given by the ratio s^2/s_i^2.

If the variability of the different strata is very different, it is best to calculate the standard deviation of a single sampling unit, and thence the standard errors of the strata means and totals, for each stratum separately. The standard error of the estimate of the total of the whole population will then be given by the square root of the sum of the squares of the standard errors of the estimates of the totals of the separate strata. The differences between the means of different strata can likewise be tested by means of their separate standard errors (omitting, if inappropriate, the factor $\sqrt{1-f}$).

If the numbers of sampling units in some or all of the strata are too small for estimation of separate standard errors, the pooled estimate given by the ordinary analysis of variance will still give an estimate of error applicable to the mean or total of the whole population, even when the variability of the different strata is different. If the numbers in the different strata are unequal, some small adjustment is theoretically necessary, as can be seen by making separate estimates, but this can usually be ignored.

A useful method of dealing with variation in sampling error when the number of units sampled in each stratum is small, is the construction of an error graph. In such a graph the estimated standard deviation (of a single sampling unit) is plotted against the estimated mean of the corresponding stratum, or other suitable statistic. A curve can be fitted to the points so obtained (either graphically or by some more exact method) so as to give an improved estimate of the standard deviation associated with any value of the mean. This procedure is also of value when sampling from batches of material, if the number of sampling units taken from each batch is too small for precise determination of the sampling error. It was followed, for example, in the Wireworm Survey (Yates and Finney, 1942) in which 20 sampling units were taken from each field, and in which there was strong association between the sampling variability and the degree of infestation.

(*c*) *Sub-sampling*

In the case of sub-sampling there will be a separate sampling error corresponding to each stage of the sampling. As a simple example we may take the case of a two-stage sampling process, stratified in the first stage, in which the sampling units of the first stage (from which the sub-samples are drawn) are all of the same size.

If there are h sub-units in each main unit of which k are included in the sub-sample, the analysis of variance will be as follows:

		Degrees of freedom	Sum of squares	Mean square
Between main sampling unit means	Between strata	$t-1$	A	
	Within strata	$n-t$	B	s_i^2
	Total	$n-1$	C	
Sub-units	Between main units	$n-1$	kC	
	Within main units	$n(k-1)$	D	s_u^2
	Total	$nk-1$	E	

A complication arises in that each main unit contains k sampled sub-units. The numerical quantities entering into the first part of the analysis will therefore consist of the means (or totals) of k sub-units. Totals are generally more convenient for actual analysis, but the analysis of variance is best set out in terms of means, in which case all sums of squares in the first part derived from the totals must be divided by k^2. The total sum of squares of the first part of the analysis will enter into the second part as " between main units," but must here be multiplied by k to make it comparable with the other sums of squares in this part. (An alternative procedure is to divide all sums of squares of totals in the first part by k instead of k^2 so that both parts are directly comparable.)

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The variability contributed by the second stage of sampling will be included in the overall estimate provided by the first part of the analysis. Consequently if the sampling fraction at the first stage is small, the sampling error can be derived directly from s_i^2 as in ordinary stratified sampling, and the second part of the analysis will then only be required if the efficiency of the sampling procedure requires review. The full expression for the sampling standard error of the estimate of the mean of all sub-units in the population is

$$\sqrt{\frac{s_i^2}{n}(1-f_1)+\frac{s_u^2}{nk}f_1(1-f_2)}$$

where f_1 and f_2 are the sampling fractions of the first and second stage respectively.

Knowing the values of s_i^2 and s_u^2, the effect on the sampling error of changes in the intensity of sampling at each stage can be evaluated. If the number of sub-sampling units taken from each main unit is altered from k to k', s_u^2 will remain unaltered, but s_i^2 will asume a new value given (except for errors of estimation) by

$$s_i'^2 = s_i^2 + \left(\frac{1}{k'}-\frac{1}{k}\right)s_u^2$$

The effect of varying the number of sub-sampling units in each main unit, or the number of main units in the whole sample, or combinations of the two, on the accuracy of the sampling process can thus be evaluated.

(*d*) *Stratification for two or more factors*

The analysis of variance procedure used in a stratified sample can be followed. If the sample is stratified for two or more factors without control of sub-strata, the variability due to the separate factors can only be eliminated by fitting constants for these factors in the manner developed for the analysis of variance of multiple classifications with unequal numbers in the different classes (Yates, 1934).*

If there is no control of sub-strata it is difficult to estimate differences in variability between the different strata, especially if there are changes in variability in the various strata of more than one factor. Nevertheless, an overall analysis of variance will give an estimate of error which can be applied to the means and totals for the whole population.

(*e*) *Balancing*

In order to make a proper estimate of the sampling error of a balanced sample, it is necessary to use the analytical procedure known as the analysis of covariance. This is exactly analogous to the analysis of variance procedure which would be used if the sample were not balanced.

The first step is to calculate the sums of squares for the control variate as well as for the main variate. At the same time a third column is inserted in the analysis of variance table in which are entered the sums of products of the control and main variate which correspond to the sums of squares of the control and main variate respectively in the other two columns. Thus, the total sum of products will be given by the formula $S(x-\bar{x})(y-\bar{y})$, corresponding to the two sums of squares $S(x-\bar{x})^2$ and $S(y-\bar{y})^2$. In the formulæ given above squares are therefore replaced by products, and products of means and corresponding totals by products of means of the control variate and totals of the main variate (or vice versa).

Thus in a balanced stratified sample the sums of squares and products would appear as follows:

	Degrees of freedom	Sx^2	Sxy	Sy^2
Between strata	$t-1$	A'	A''	A
Within strata	$n-t$	B'	B''	B
Total	$n-1$	C'	C''	C

where, for example, $A'' = S''\{\bar{x}_s S'(y_s)\} - xS(y)$.

A corrected within strata mean square is now obtained from the formula

$$s_i'^2 = \frac{B - B''^2/B'}{n-t-1}$$

All the sampling errors are calculated using this corrected mean square.

* As I indicated at the meeting, this problem requires further investigation. I hope to publish a note on the matter shortly.

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It may be noted that $B - B''^2/B'$ or $B - bB''$ is the sum of squares of deviations about the regression line (or in the case of a stratified sample a series of parallel regression lines) of the main variate on the control variate, the regression coefficient being given by

$$b = B''/B'$$

In the calculation of s_i^2 the number of degrees of freedom is reduced by one to allow for the estimation of the regression coefficient from the data. The reason why it is unnecessary to estimate the regression coefficient when estimating the population mean or total, is because balance automatically ensures that the adjustment for the regression is zero, whatever the value of the coefficient.

When two or more variates are balanced, the same general procedure holds, but it is now necessary to estimate the sum of squares of deviations from the values given by a partial regression equation. This involves the solution of simultaneous linear equations, the procedure for which is described by Fisher in *Statistical Methods for Research Workers* and elsewhere.

6. *Methods of improving the accuracy of a sample enquiry by adjustment of the results*

In all the methods of sampling outlined above the estimates of the population means and totals are obtained directly from the sample means and totals. In certain circumstances, however, when additional facts about the whole population are known, it is possible to provide more accurate estimates by adjustment of the sample means in the light of these facts. In most types of sampling enquiry such adjustment would not be justified: it is usually better to use the additional information when taking the sample so that the above simple rules of estimation can be followed, rather than to make adjustments involving additional calculations after taking the sample. Nevertheless cases do arise where such adjustments are of value, either because the imposition of additional restrictions would be impossible, or would be more trouble than making the adjustments, or because more accurate estimates are required from material already collected. Furthermore, study of the principles involved helps in the appreciation of the underlying principles governing all sampling.

(*a*) *Adjustment of a random sample to give the effect of a stratified sample*

If the members of a random sample are classified into different strata and the proportions $p_1, p_2, p_3, \ldots$ of the whole population falling in these strata are known, the population mean can be estimated from the means $\bar{y}_1, \bar{y}_2, \bar{y}_3, \ldots$ of the different strata derived from the sample by the formula

$$\bar{Y} = p_1\bar{y}_1 + p_2\bar{y}_2 + p_3\bar{y}_3 + \ldots$$

It can be shown that, provided the number of units falling in each stratum is not too small, the difference in accuracy between such an adjusted estimate and an estimate obtained from a stratified sample containing the same total number of sampling units will be very small. The sampling error appropriate to such an adjusted estimate can be calculated as if the sample were a stratified sample. To avoid minor complications it is best to estimate the error of the mean of each stratum separately, and calculate the error of the final estimate from these errors by the formula

$$\sqrt{p_1^2 s_1^2 \frac{1-f_1}{n_1} + p_2^2 s_2^2 \frac{1-f_2}{n_2} + \ldots}$$

where the symbols are those used in Section 5(*a*) and the suffixes refer to the different strata.

(*b*) *Use of covariance to give the effect of a balanced sample*

The estimate of the population mean can be adjusted for one or more control factors which have not been balanced in the sample, by the use of a regression. The regression can be calculated in the manner outlined above for balanced samples. In the case of a single control factor the adjusted estimate of the mean will be

$$\bar{Y} = \bar{y} + b(\bar{X} - x)$$

when $\bar{X}$ is the population mean of the control variate.

The error of this adjusted estimate will be the same as the error of the estimate from a balanced

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sample of the same size and type, except for a slight increase due to error in the estimation of b, which, however, is usually small enough to be ignored.

7. *Use of ratios or percentages*

It frequently happens that the value of more than one quantitative variate is recorded for each sampling unit. Often, also, the ratio between a pair of such variates is considerably less variable than are the separate variates. Such ratios may be of interest in themselves, or their average values may provide a basis for the estimation of the population total of one of the variates, when the total of the other variate is known accurately.

Thus, for example, the yield per acre of a field is likely to be less variable than either the total yield or the acreage of a field, since fields vary considerably in size. If therefore in a crop estimation scheme the yields of a random sample of fields are determined, and the total acreage under the crop is known from previous returns, the total yield can best be estimated from the formula:

$$\text{total yield} = \text{estimated mean yield per acre} \times \text{total acreage}.$$

The method of estimating the mean yield per acre requires some consideration. If the mean of the yields per acre of all the sample fields is taken, the estimate of the total yield will be subject to bias when yield per acre is associated with size of field. This bias can be avoided by weighting the estimates of the mean yields per acre by the acreages of the fields from which they were obtained. Such a weighted estimate is exactly equivalent to sum of the yields of all the sampled fields divided by the sum of the acreages of all the sampled fields.

Although the estimates based on the weighted means of the yields per acre are free from bias, they are not necessarily the most accurate that can be obtained. If the yields per acre, as determined by the sampling procedure, are equally variable for small and large fields, the unweighted mean will give the most accurate estimate of the mean yield per acre, though one that will be biased if there is association between yield per acre and size of field.

In practical sampling procedures we are therefore frequently confronted with the possibility of using various alternative estimates, some of which will be certainly unbiased, but which may be of lower accuracy than other estimates, which will themselves only be unbiased if certain conditions hold. Whether these conditions do in fact hold can usually be tested by means of the sampling data. In certain cases we may even be prepared to accept some risk of bias in return for reduction in the variability of the results. This is especially the case when the sampling is repeated at intervals and changes are of more importance than absolute magnitudes.

In the Survey of Fertilizer Practice (Yates, Boyd and Mathison, 1944), for example, information was obtained from a sample of farms on the amount and composition of fertilizer applied per acre to different crops, one field being selected at random on each farm from all the fields under each given crop. In the analysis these rates of application were weighted by the total area of the given crop on the farm. This procedure eliminates all bias due to farmers who grow large acreages of the crop applying fertilizers at different rates (on the average) from farmers who grow small acreages—such differences certainly exist—but it will not eliminate bias arising from differences of treatment of small and large fields on the same farm. To do this each rate should be weighted by a quantity nx, where x is the acreage of the sampled field and n is the number of fields on the farm.

The estimation of the sampling error of an estimate based on percentages or ratios presents certain special problems. In the case of the unweighted mean of the ratios

$$\bar{p} = \frac{1}{n} S\left(\frac{y}{x}\right)$$

the variability of the mean may be calculated in the ordinary manner by carrying out the appropriate analysis of variance on the individual ratios, though even in this case complications are apt to arise, as for instance when a stratified sample of fields has been taken but the area figures are only available for the whole of the country.

The calculation of the sampling error of a weighted mean of the ratios

$$\bar{p}_w = \frac{S(y)}{S(x)}$$

is more difficult. It might at first sight be thought that the estimate of error could be calculated by the ordinary rules applicable to weighted observations. This procedure, however, will be

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found to give an estimate of error which may be seriously biased if the weights x are not inversely proportional to the sampling variances of the separate ratios.

An estimate of error which is unbiased whatever the law of variation in the sampling variances * can be obtained from the unweighted sum of squares of the deviations from the line $y = p_w x$. This sum of squares is given by

$$Q = S(y - \bar{p}_w x)^2 = S(y^2) - 2\bar{p}_w S(xy) + \bar{p}_w^2 S(x^2).$$

Since the fitting is not "least square," the rule as to degrees of freedom does not hold exactly, but for practical purposes Q may be taken as based on $n - 1$ degrees of freedom. The standard error of the ratio $\bar{p}_w$ is therefore

$$\frac{1}{S(x)}\sqrt{\frac{nQ(1-f)}{n-1}}$$

It will be recognized that utilization of percentages in the manner outlined above is only appropriate when additional information is available on one of the variates, or when the value of the ratio itself is of interest. If the population values of both variates are estimated from the sample, then each variate should be treated separately. If the acreage as well as the yield per acre is estimated from the same sample of fields, for example, the total yield of the crop should be estimated directly from the total yield of all the sampled fields. Use of a weighted estimate of the yield per acre is then in fact exactly equivalent to the direct estimate based on the total yield of all the sampled fields, since

$$\bar{p}_w \frac{1}{f} S(x) = \frac{S(y)}{S(x)} \frac{1}{f} S(x) = \frac{1}{f} S(y)$$

It should be further recognized that adjustment of the sample mean of y to allow for differences between the sample and population means of x can be carried out by means of a regression of y on x in the manner of Section 6(*b*). This will prove a more accurate method of adjustment if the mean ratio of y to x does not remain substantially constant over the whole range—*i.e.*, if the regression line does not pass near the origin.

8. *Treatment of sampling units containing different numbers of individuals*

A commonly occurring problem, which throws further light on sampling problems arising in the use of ratios or percentages, is the treatment of surveys in which the sampling units contain different numbers of similar individuals. Surveys of human populations in which households are taken as the sampling units are a case of this type. In such surveys percentages of the population falling in given categories, or quantitative estimates per head of population, are usually required, rather than estimates of totals for the whole population. The problem is therefore essentially one of ratios.

As an example we may take the case of a nutrition survey covering a sample of households in which some quantitative assessment of the degree of adequacy of the nutrition of each individual in the household is made. The results for all the members of one household are clearly likely to be correlated, and account must therefore be taken of the fact that the sampling is by households and not by individuals.

Considering, for simplicity, households of one, two and three members, and denoting some quantitative measure for members of such households by y_{1r}, y_{2rs}, y_{3rs}, where r denotes the household and s the member, we may write

$$\begin{aligned} y_{1r} &= a_1 + u_{1r} + v_{1r} \\ y_{2rs} &= a_2 + u_{2r} + v_{2rs} \\ y_{3rs} &= a_3 + u_{3r} + v_{3rs} \end{aligned}$$

where a_1, a_2, a_3 are the means for families of one, two and three in the population (assumed large), u_{1r}, u_{2r} and u_{3r} are quantities whose means are zero and which vary from household to household, and v_{1r}, v_{2rs}, v_{3rs} are similar quantities which vary from individual to individual. (In the case of

* Cochran (1942) recognized the existence, but did not bring out the importance, of an unbiased estimate of error of this type in discussing the adjustment of samples by linear regression. He did not give an unbiased estimate applicable to ratios.

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families of one, u and v cannot be separated: both have been included to preserve formal symmetry.)

Let the variance of u_1 be U_1, etc., and let the numbers of households of one, two and three in the sample be n_1, n_2, n_3, with $n = n_1 + 2n_2 + 3n_3$. The analysis of variance will be

		Degrees of freedom	Expectation of mean square
Between means of households	Between households of different sizes ...	2	—
	Between households of one	$n_1 - 1$	$U_1 + V_1$
	,, ,, two	$n_2 - 1$	$U_2 + \frac{1}{2}V_2$
	,, ,, three...	$n_3 - 1$	$U_3 + \frac{1}{3}V_3$
Within households	Within households of two	n_2	V_2
	,, ,, three	$2n_3$	V_3

This analysis is similar to that obtained in two stage sampling (Section 5 (*c*)).

The estimates of the population means for the different household sizes will be given by the corresponding sample means, $\bar{y}_1$, $\bar{y}_2$, $\bar{y}_3$, which will have sampling standard errors of

$$\sqrt{\frac{1}{n_1}(U_1 + V_1)}, \sqrt{\frac{1}{n_2}(U_2 + \tfrac{1}{2}V_2)}, \sqrt{\frac{1}{n_3}(U_3 + \tfrac{1}{3}V_3)}$$

If the proportions p_1, p_2, p_3 of the different households in the population are known, the estimate of the general population mean for all households will be

$$\frac{p_1\bar{y}_1 + 2p_2\bar{y}_2 + 3p_3\bar{y}_3}{p_1 + 2p_2 + 3p_3}$$

The sampling standard error of this quantity is

$$\frac{1}{p_1 + 2p_2 + 3p_3}\sqrt{\frac{p_1^2}{n_1}(U_1 + V_1) + \frac{4p_2^2}{n_2}(U_2 + \tfrac{1}{2}V_2) + \frac{9p_3^2}{n_3}(U_3 + \tfrac{1}{3}V_3)}$$

or since $n_1 : n_2 : n_3 = p_1 : p_2 : p_3$ approximately, the standard error is approximately

$$\sqrt{\frac{p_1(U_1 + V_1) + 4p_2(U_2 + \tfrac{1}{2}V_2) + 9p_3(U_3 + \tfrac{1}{3}V_3)}{n(p_1 + 2p_2 + 3p_3)}}$$

If a random sample of n individuals had been taken the sampling standard error would have been

$$\sqrt{\frac{p_1(U_1 + V_1) + 2p_2(U_2 + V_2) + 3p_3(U_3 + V_3)}{n(p_1 + 2p_2 + 3p_3)}}$$

Comparison of these two standard errors will give the loss of accuracy due to taking households as sampling units.

If the proportions of the different households in the population are not known, the sampling standard error given above, with n_1, n_2, n_3, substituted for p_1, p_2, p_3, is still applicable if the estimate is regarded as the estimate of the mean of a set of households containing the same proportions of households of different sizes as the actual sample. If it is regarded as an estimate of the mean of the population from which the sample is drawn, allowance for errors due to differences between the proportions in the sample and in the population must be made. For the determination of this latter error the formula already given in Section 7 is appropriate. If $\bar{y}_{2r}$ is the mean of the rth family of two, etc., and $\bar{Y}$ is the estimate of the mean of the population, so that

$$\bar{Y} = \frac{n_1\bar{y}_1 + 2n_2\bar{y}_2 + 3n_3\bar{y}_3}{n_1 + 2n_2 + 3n_3}$$

the expression for Q becomes

$$Q = S_1(y_{1r} - \bar{y}_1)^2 + 4S_2(\bar{y}_{2r} - \bar{y}_2)^2 + 9S_3(\bar{y}_{3r} - \bar{y}_3)^2 + n_1(\bar{y}_1 - Y)^2 + 4n_2(\bar{y}_2 - \bar{Y})^2 + 9n_3(\bar{y}_3 - \bar{Y})^2$$

If the adequacy of nutrition varies considerably between households of different size, the distinction between these two errors is of importance. If we wish to compare the adequacy of nutrition of two towns, for example, we can either compare the actual means, recognizing that

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part (perhaps the major part) of the observed difference will be due to differences in the proportions of the different sizes of household in the two towns, or we can standardize the proportions for the two towns, calculating adjusted means for these standard proportions. In the latter case the first of the above errors is appropriate, in the former the second.

The contrast between the overall sum of squares between households derived from the standard analysis of variance on individuals and the quantity Q is worth noting. In the standard analysis of variance the sum of squares derived from the means of households of size 2, for example, is multiplied by 2, and therefore the contributions to the total sum of squares are weighted in proportion to the size of household. In the quantity Q, on the other hand, they are weighted in proportion to the square of the size of household. The standard analysis will give the most accurate estimate of the sampling error if the variance of the household means is inversely proportional to size of household, but the estimate will be seriously biased if some other variance relationship holds. The quantity Q will give an estimate of sampling error which is unbiased, whatever the variance relationship.

9. *Use of a variable sampling fraction*

In economic material, as mentioned in Section 3, the units often vary very greatly in size, the variability amongst units of similar size being usually closely related to size. In such cases, for a given amount of sampling, a considerably more accurate estimate will in general be obtained if a greater proportion of the more variable units is taken. Thus, for example, if we wish to ascertain by sampling methods the number of workers employed at a given time in an industry, it will pay to obtain returns from a greater proportion of the big firms than of the small firms. The practical method of doing this is to stratify the firms into size groups, using larger sampling fractions for the groups containing the bigger firms.

In principle this idea is not new. It was recognized, for example, by Neyman (1934), but Neyman recommended that a preliminary sample should be taken in order to determine the variability of the different groups, whereas in practice the variation in the variability from group to group can often be roughly foretold from the nature of the material, so that reasonably efficient sampling fractions can be chosen in advance of any sampling.

To obtain the greatest precision with a given total number of sampling units, the sampling fractions must be proportional to the standard deviations within the different groups. Thus, if the standard deviations are $\sigma_1, \sigma_2, \sigma_3, \ldots$ the sampling fractions $f_1, f_2, f_3, \ldots$ will be given by

$$f_1/\sigma_1 = f_2/\sigma_2 = f_3/\sigma_3 \ldots$$

If, in a finite population, this equation results in any sampling fraction having a value greater than unity, then the whole of that group must be included in the sample.

No great accuracy in choice of sampling fractions is required. If the sampling fractions actually used are roughly proportional to the ideal values, almost full efficiency will be attained. In the common case in which the standard deviations are approximately proportional to the means of the different size groups, the sampling fractions will be given by the formula

$$f_1/m_1 = f_2/m_2 = f_3/m_3.$$

where $m_1, m_2, m_3, \ldots$ are prior estimates of these means.

If a variable sampling fraction is used, it is of course necessary to weight the sample totals for the different groups in proportion to the reciprocals of the sampling fractions. In other words, the total of the population must be estimated from the sample totals of the different groups by the formula

$$\Sigma(Y) = S(y_1)/f_1 + S(y_2)/f_2 + S(y_3)/f_3 + \ldots$$

The mean of the population can then be estimated from the estimated total $\Sigma(Y)$.

Where the material to be sampled consists of a small number of very large units with high variability, together with a much larger number of small units subject to considerably lesser variability, use of a variable sampling fraction will give very considerable gains in precision. The results of the National Farm Survey are an example of material of this type. The survey covered all holdings in England and Wales over five acres. In order to obtain a summary of the results

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over the whole country (with sub-divisions for counties, types of farming, etc.), the holdings were divided into size groups, and sampling fractions were chosen as follows:

Size group (acres)	Average size (acres)	No. of holdings	Sampling fraction (per cent.)	No. of holdings in sample
5– 25	12	101,450	5	5,072
25–100	55	111,360	10	11,136
100–300	165	65,210	25	16,302
300–700	413	11,150	50	5,575
over 700	1,035	1,430	100	1,430
		290,600	(13·6)	39,515

To obtain adequate representation of the smaller size groups for purposes where contrasts between farms of different sizes were required, and to give additional accuracy to estimates involving numbers of farms (regardless of size), the sampling fractions for these groups were relatively larger than those which would give the most accurate estimates of the areas of land falling in different categories.

Some calculations of the gain in efficiency resulting from the use of a variable sampling fraction in the National Farm Survey showed that the sampling variance for measures of the percentage of farm land possessing given attributes was one third that which would have been obtained from a sample containing the same total number of farms with equal proportions taken from each of the size groups. On the other hand, stratification into size groups would not in itself have resulted in any great reduction in error, since the proportions of the different size groups possessing such attributes did not differ substantially.

Another case in which the use of a variable sampling fraction results in large gains of efficiency is where some such quantity as crop acreage is being estimated, and the crop in question occupies very different proportions of the total land area in different districts. Here more intense sampling of the areas with high proportions of the crop will be advantageous. The large gain in efficiency obtained by Mahalanobis (1944) in the estimation of the area of jute as a result of minimising a cost function appears to have been mainly due to the fact that the cost function was such that a variable sampling fraction was determined.

10. *Quasi-random sampling*

In practice, at least in economic and social studies, it is rare for a sample to be selected by strict random selection. Generally some form of selection from a list, such as taking every tenth name in the list, or other form of systematic selection, is used. If the list is arranged in substantially random order—*e.g.*, if it is alphabetical—a quasi-random sample can be treated as if it were a random sample. In general, however, quasi-random sampling automatically results in some form of stratification. This, if every tenth house is taken from an electoral register which is arranged by wards and streets, one tenth of all the houses in each ward will automatically be taken. Consequently, the sample will be stratified by wards. Further elements of stratification will also occur to a greater or less extent. For example, in the long streets the number of houses taken will be very approximately one-tenth of the whole.

In calculating the sampling errors of material of this type it is usually sufficient to eliminate the effects of the more important divisions, such as wards, ignoring the effects of the smaller sub-divisions, such as streets, it being recognized that the sampling error so obtained is likely to be a slight over-estimate.

Attempts are sometimes made to overcome difficulties of this kind by taking two random starting points for each street and then selecting houses at equal intervals. Thus, instead of taking every tenth house, we might select two numbers at random between 1 and 20, say 12 and 15, and take two groups of houses from the street, namely 12, 32, 52, etc., . . . and 15, 35, 55, etc. . . . Differences between the means of the groups will provide a formally correct estimate of the sampling error. The additional trouble, both of selection and of computation, is such, however, that this procedure is not of great practical value.

11. *Systematic sampling*

Area sampling presents special problems. The classical case is the sampling of a field under an agricultural crop to determine the yield or other characteristics of the crop. Inasmuch as the

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fertility may vary over different parts of the field, more accurate results will clearly be obtained if the units of the sample are distributed more or less evenly over the field. The simplest way of ensuring this, while satisfying the conditions required for a valid estimate of error, is to divide the field into blocks, sub-divide each block into small areas, and to select two or more small areas at random from each block. This constitutes an ordinary stratified sample, with the blocks as strata and the small areas as sampling units; fertility differences between the different blocks will be eliminated from the sampling error. A modification of this procedure is to use complex sampling units, each unit being made up of a set of small areas arranged in some pattern which ensures a reasonably even spread over the block. This will eliminate a good deal of the variation in fertility within blocks. Provided condition (2) of Section 3 is satisfied a valid estimate of sampling error is still possible.

In many types of area sampling the location of the sampled areas on parallel lines is desirable. In sampling agricultural crops, for instance, the rows may have to be followed; in sampling a forest area it may be convenient to follow a fixed compass bearing from a point located on a base line. Sampling is then often two-stage, the chosen lines constituting the first stage, and the sampling of the lines the second.

Considering the first stage, the area may be divided into a set of blocks bounded by lines parallel to the direction of sampling, and two lines may be selected at random from each block. This, however, will not lead to regular spacing of the lines, and in extreme cases four contiguous lines may be selected. It is clear that an even spacing of the lines will in general give a more accurate representation of the area. Even spacing has the additional advantages that location of the lines is simpler, and that the construction of an approximate map of the area, if this is required, is facilitated, since there will be no blank patches such as tend to occur with random location. Such even spacing may be termed *systematic sampling*. There are certain possibilities of bias which must not be overlooked, but which are usually of little practical importance in the types of area sampling for which systematic sampling is most suitable.

The same considerations apply to the second stage, the sampling of the selected lines. If evenly spaced units are taken on these lines, and the lines themselves are evenly spaced, a systematic grid pattern of sampling units will result.

The relative merits of systematic and random sampling of the grid and line type have been the subject of lengthy controversy, particularly in the United States, in connection with the sampling of forest areas. I have recently been making an investigation of the problem designed to ascertain:

(1) The gain in accuracy obtained by the use of systematic samples instead of samples randomly located in pairs within blocks.

(2) What methods, if any, are available to make an approximate estimate of the sampling errors involved in systematic sampling.

This investigation is not fully complete, but certain tentative conclusions may be set out here. First, as regards gain in accuracy, this will naturally depend very much on the nature of the material. If the chief source of variation is of a random nature, then clearly the gain in accuracy of systematic over random samples will be small. On the other hand, if the variation is of a continuous type, the gain may be very considerable.

The gain in accuracy due to systematic sampling with lines spaced at a distance A compared with random sampling with pairs of lines randomly located in blocks of width $2A$ can be looked on as made up of two parts, (*a*) that due to a reduction in block size from width $2A$ to A, (*b*) that due to the location of the line at the centre of a block of width A instead of at random in such a block. The gain in accuracy due to (*a*) in any particular type of material can therefore be assessed if the variances within blocks of width A and $2A$ are known. The assessment of the part (*b*) of the gain is much more difficult, since we become involved in certain properties in the variation of the sampled material which are very difficult to determine.

The variance in blocks of width A can with a certain amount of ingenuity be determined from the results of random sampling in which random pairs of samples are taken from blocks of width $2A$, provided the locations of the sampled lines are known. Thus it is possible, in any particular case of random sampling, to make an estimate of the part (*a*) of the gain that would have resulted from the use of systematic sampling.

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If a systematic sample has been taken, on the other hand, we cannot, from the internal evidence of the sample itself, determine either the gain in accuracy over a random sample or the actual accuracy of the systematic sample. Nevertheless an upper limit to the sampling error can always be obtained from the differences of consecutive pairs of systematically located units, but such an estimate, with continuously varying material, may be much above the true value.

To make an estimate of the degree to which the continuous features in the variation of the material reduce the sampling error, some supplementary observations are necessary. One method which appears to give satisfactory results is to take systematic samples at four times the normal density—*i.e.*, at a spacing of $\frac{1}{4}A$—over part of the material. From these observations what may be called "systematic sub-samples with spacing A, adjusted for end conditions," covering blocks of moderate width—say $4A$—can be constructed.* There will be four such sub-samples per block, and the variances within blocks of sub-samples separated by $\frac{1}{4}A$ and of those separated by $\frac{1}{2}A$ can be calculated. These variances can then be used as the basis of an estimate of the sampling variance of systematic samples with spacing A.

An alternative method of which the possibilities have not yet been fully investigated is to take a second systematic sample which has a slightly different spacing to the original sample, so that information is available on the differences between lines at all spacings.

These methods, however, require further investigation before they can be confidently advocated for use in practice. All that can be said at present is that although a systematic sample will unquestionably give more accurate information than a random sample on material which varies in a more or less continuous manner, its accuracy cannot be assessed without supplementary sampling: from the internal evidence of the sample itself only an upper limit can be assigned to the sampling error, and the greater the advantage of systematic over random sampling, the more widely will this upper limit be separated from the true value. Nevertheless, I feel certain that in extensive area surveys systematic line or grid sampling is preferable to random sampling. The additional work required to investigate the accuracy attained will be more than compensated for by the saving in work in the survey itself.

12. *Applications in social surveys*

With the development of the social sciences and the adoption of a planned economy, surveys should become of increasing importance. The use of sampling in social surveys is not new, and in some respects the methods are relatively simple. For most types of survey covering a single town or rural area the population can easily be divided into relatively small sampling units, either separate individuals or households, and there is usually no difficulty in obtaining a quasi-random sample of such a population. The only points of difficulty in the estimation of sampling errors arise from variation in size of sampling unit when households are used, and the quasi-random nature of most samples.

If a small sample of the population of a whole country is required, a further difficulty is encountered. A sample selected in equal proportions from all the towns and rural areas in the country would be so scattered that the amount of travelling involved in order to carry out interviews would be quite excessive. Frequently, therefore, sampling has to be limited to certain towns and rural areas, even when a sample representative of the whole country is required.

For certain purposes the selection of towns in which opportunities of survey are particularly favourable more than outweighs the objection that such a sample cannot be regarded as fully representative of the whole country. Provided this lack of representativeness is clearly recognized, no harm will ensue. Moreover, if surveys have been carried out in a number of towns and rural areas which are widely contrasted, and there is found to be little difference between the results of the different towns and areas, it may be assumed that the results will be reasonably representative for the whole country.

The simplest method of dealing with the problem of securing a fully representative sample is to take a random sample of towns and rural areas, stratifying as far as possible into geographical regions, and by such simple characteristics as size and degree of industrialization. There does, however, seem to be a strong case for attempting the construction of some form of balanced

* If the observed values are numbered consecutively, the systematic sub-sample totals will be of the form: $\frac{1}{2}y_1 + y_5 + y_9 + y_{13} + \frac{1}{2}y_{17}$, $\frac{1}{2}y_2 + y_6 + y_{10} + y_{14} + \frac{1}{2}y_{18}$, $\frac{1}{2}y_3 + y_7 + y_{11} + y_{15} + \frac{1}{2}y_{19}$, $y_4 + y_8 + y_{12} + y_{16}$.

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sample in which various control factors are used. As far as I am aware, the possibilities of effecting this have never been thoroughly investigated. In this connection the ingenious method followed in constructing a sample of farming districts for economic surveys in the United States is worth consideration (Hagood and Bernert, 1945). In order to obtain a sample which was reasonably balanced for a number of control factors, while avoiding the disadvantages of purposive selection, an index consisting of a linear function of these control factors was formed, and the districts were then stratified on the basis of the values of this index, random selection of one district from each stratum being made. In certain cases an auxiliary index was also used in forming the strata.

Unfortunately, emphasis on economy of interviewers' time has only too often led to the abandonment of the principle of random or quasi-random selection. In surveys of public opinion the procedure known as " quota " sampling in the United States, whereby a sample is made up by selection of the requisite quota of individuals in various income groups, etc., by the interviewers themselves, has become popular.

If such selection were otherwise random, it would of course be equivalent to a stratified sample, and would be quite satisfactory, but the actual methods of selection adopted are by no means random. Thus Box and Thomas (1944) give the following description of one of the methods followed by the Wartime Social Survey:

> " Where income group is used to control the sample, information is sought from the police, or other officials with local knowledge, as to the districts in which families belonging to different income groups are most frequently to be found. In towns where there are regional investigators this information has already been collected and areas marked off on street maps. The investigator then goes to the different types of district and selects streets where she expects to find families in the different groups. The houses to be visited in these streets are selected according to some previously determined plan—*e.g.*, every tenth house or from lists of random numbers. If the household does not belong to the expected income group, it is classified in the group to which it does belong, and further calls are made in other streets until the required quota of each group is completed."

It is clear that such a method may very easily introduce serious biases. It is also quite impossible to assess the accuracy of the results from the internal evidence of the observations themselves.

On this point Deming, in the course of a valuable review of the principles governing the conduct of sampling surveys (Deming, 1945), has written:

> " Unless biases can be removed satisfactorily a method of collection that appears to be cheap is too often cheap only in the sense of providing a lot of schedules per dollar, but may actually be very costly when measured in the amount of useful information per dollar or the damage done through misinformation."

That the quota method, in spite of objections that can be raised to it, has given satisfaction for certain types of work is due primarily, I think, to the fact that it has been mainly used for opinion surveys where there is in general little check on the accuracy of the results actually obtained and where, in any case, no very precise results are required. Many opinion surveys are also repeated from time to time, and changes of opinion are the points of chief interest. If the sample is selected in the same manner on each occasion, any bias will affect the results on all occasions more or less equally, and consequently trends of opinion will be truly reflected in the results.

When, however, quota sampling is used for more serious work on which administrative decisions have to be based, and which require proper quantitative estimates of the various characteristics under survey, its weakness becomes apparent. There is also the further important objection that the quota method involves a considerable loosening of control over the interviewers. This often has a bad effect on the selection of the sample—for example, interviewers may tend to choose houses or districts from which it is easy to obtain answers.

Even apart from questions of bias, it is doubtful whether the use of an elaborate quota system is likely to produce results which are appreciably more accurate than some simple form of quasi-random selection. Particularly in the case of qualitative characters, the gain due to stratification is much less than is commonly supposed. If, for example, a population is divided into five equal

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groups of which 70, 60, 50, 40 and 30 per cent. respectively give a positive answer to a certain question, a fully random sample will only need to be 8 per cent. larger (for the same accuracy) than a stratified sample. Compared with a quasi-random sample from an electoral register arranged by wards, which will automatically be partially stratified by social class and income group, the gain of a stratified sample is likely to be much less. If all the percentages are small, or in the neighbourhood of 100 per cent., the gain is in any case quite trivial: with percentages of 10, $7\frac{1}{2}$, 5, $2\frac{1}{2}$ and 0 per cent. for example, the gain of a stratified over a fully random sample is 0·7 per cent.

The doubts and difficulties to which the use of the quota method by the Wartime Social Survey has given rise are well illustrated by the lengthy discussion on this point in the paper referred to above (Box and Thomas, 1945). It is there stated:

> "In inquiries relating to the whole adult civilian population, the population is stratified by sex and by occupation, as well as by region and by urban and rural areas. Stratification by sex presents no difficulty, but the division of the population into occupation groups and allotting the appropriate number of interviews to each group is a matter of some concern. The lack of any up-to-date information on the proportions of the population following different occupations is a serious drawback to the Survey. Also such classifications as have been made at different times are not altogether satisfactory for social survey purposes."

The authors do not appear to recognize that this particular problem would not have arisen at all had they been content to use some method of random sampling. While a quasi-random sample over the whole population of a town is undoubtedly the simplest, a sample in which certain districts only are taken, these being a sample of all districts, with a sub-sample of houses in each selected district, would probably be nearly as accurate, and would substantially cut down travelling time, which is the main argument in favour of the method used at present.

It was these considerations that led to the decision, in the social surveys undertaken by the Ministry of Home Security, to use a quasi-random sample over the whole of each surveyed town, with scrupulous attention to such details as "calling back." These surveys were carried out in order to determine the reactions of populations of raided towns to air raids, and involved the evaluation of such quantities as amount of time lost from work, the amount of evacuation, etc., and it was of the utmost importance that the reliability of the results should not be called in question on statistical grounds.

In conclusion I would like to emphasize that in many social surveys the sampling problems involved constitute only a minor difficulty, compared with the problem of obtaining reliable information on the subject under enquiry. This involves careful drafting of the questionnaire, the inclusion only of questions for which it is reasonably certain that the questionees know the relevant facts, and are prepared to give truthful answers, and a high degree of skill and tact on the part of the field workers. These points were well brought out in the paper on the Wartime Social Survey and the subsequent discussion.

13. *The planning of sampling enquiries*

The efficient planning of a sampling enquiry requires knowledge of the different components of variability of the material, and also a knowledge of the relative costs of collecting the information with different types of sampling unit.

The ideal method of studying variability is to have available complete information on an adequate and representative body of material. We can then make a thorough analysis of the different components of variability, and from the results of this analysis calculate the sampling errors to be expected in different sampling procedures. Alternatively, trial sampling schemes of various kinds can be tried out, and the errors to which they are subject estimated in the ordinary manner. If this is done, however, some supplementary study of the variability will usually throw light on the reasons why one method is more accurate than another, and may suggest further improvements.

The importance of having available an adequate and representative body of material for study must, however, be emphasized. Individual batches of material often possess features of variability which specially favour one type of sampling unit, but which are not repeated in other batches of similar material. Thus the procedure of harvesting part of a single field under an agricultural crop in small units, and reaching conclusions as to the best type of sampling unit from the performance of various units on this one small area, cannot be regarded as sound.

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In practice, however, complete information of this kind is frequently not available. Often it would be quite impracticable to collect it. It is not sufficiently realized that critical analysis of the results of any properly executed sampling enquiry will not only enable the accuracy of the survey itself to be determined, but will also usually throw considerable light on whether future enquiries on the same type of material can be more efficiently planned. To give a few simple examples: the gain due to stratification can be evaluated, whether the original enquiry was itself stratified or not, provided the individual units can be assigned to the strata under consideration; the gain due to the use of a variable sampling fraction can be similarly estimated; the best ratio between intensity of sampling at two different stages can be assessed in the light of the sampling errors at the two stages and the relative costs of sampling at the two stages.

It is not possible, however, to assess the effect of all possible changes in sampling procedure from the results of a single enquiry: thus it is impossible to determine the variability within households from a random sample of individuals unless the sampling is so intense that an adequate number of pairs of individuals from within households are obtained. Though frequently a small amount of supplementary sampling, which can be carried out at the same time as the main enquiry, will provide vital information on variability that would otherwise be lacking.

The need for thorough studies of the efficiency of different sampling procedures on different types of material is great, and I hope that many more workers will be persuaded to undertake them in future. All such studies demand a considerable amount of numerical work, but the burden of computation can be lightened by the use of mechanical aids, in particular punched-card methods.

One reason why such studies are often neglected is that once a survey has been completed the question of whether it could have been carried out more efficiently is historical as far as that survey is concerned. Often, too, the person in charge of a series of surveys makes a sufficient study of the results of the first one or two surveys to satisfy himself of the adequacy of the technique, and to introduce any necessary improvements, but he fails to publish his conclusions, and other workers in the same field have to go over the same ground again, or follow blindly in his footsteps. Is it too much to hope that the results of such enquiries will be published more frequently in future?

The full value of any study of efficiency, and indeed of the magnitude of the sampling errors, is realized when a new survey on the same type of material is undertaken. Had it not been that a fairly thorough investigation of the sampling errors of the 1938–9 Census of Woodlands had been made,* it would have been impossible to plan the 1942 Census with any confidence. (For a brief description of this Census see Yates, (1943).) As it was, it was possible to give a firm assurance that the survey, if properly carried out in the manner planned, would give results of the required accuracy.

References

Bowley, A. L. (1926), *Bull. Int. Inst. Stat.*, **22**, 440.
Box, H., and Thomas, G. (1944), *J. Roy. Stat. Soc.*, **107**, 151.
Clapham, A. R. (1929), *J. Agric. Sci.*, **19**, 214.
Clapham, A. R. (1931), *ibid.*, **21**, 366.
Cochran, W. G. (1939), *J. Amer. Stat. Ass.*, **34**, 492.
Cochran, W. G. (1942), *ibid.*, **37**, 199.
Deming, W. E. (1945), *ibid.*, **40**, 307.
Fisher, R. A., and Mackenzie, W. A. (1923), *J. Agric. Sci.*, **13**, 311.
Fisher, R. A. (1926), *J. Min. Agric.*, **33**, 503.
Hagood, M. J., and Bernert, E. J. (1945), *J. Amer. Stat. Ass.*, **40**, 330.
Jensen, A. (1926), *Bull. Int. Inst. Stat.*, **22**, 359, 381.
Mahalanobis, P. C. (1944), *Phil. Trans. Roy. Soc. Lond.*, **231**, 329.
March, L. (1926), *Bull. Int. Inst. Stat.*, **22**, 440.
Neyman, J. (1934), *J. Roy. Stat. Soc.*, **97**, 558.
Stuart, C. A. V. (1926), *Bull. Int. Inst. Stat.*, **22**, 440.
Wishart, J., and Clapham, A. R. (1929), *J. Agric. Sci.*, **19**, 600.
Yates, F., and Zacopanay, I. (1935), *ibid.*, **25**, 545.
Yates, F. (1934), *J. Amer. Stat. Ass.*, **29**, 51.
Yates, F. (1935), *Ann. Eug.*, **6**, 202.
Yates, F., and Finney, D. J. (1942), *Ann. App. Biol.*, **29**, 156.
Yates, F. (1943), *Roy. Soc. Arts*, 1.
Yates, F., Boyd, D. A., and Mathison, I. (1944), *Emp. J. Exp. Agric.*, **12**, 163.

* But, I have to confess, not published!

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Discussion on Dr. Yates's Paper

Professor R. A. Fisher: I beg to propose a vote of thanks to Dr. Yates, who has put before us a discussion of quite the most important question in practical statistics; this has been of exceptional value, because it is directly based on his own extensive experiences. When, early last year, I had the opportunity to visit India, I saw in the Statistical Institute of Calcutta an organization built for and actively and, I hope, permanently engaged upon sampling surveys on a very large scale, and I was impressed not only by the evident value of such surveys, more especially in a country such as India, where statistical data are often sadly faulty, but by the close co-ordination of highly trained mathematicians with the practical business of obtaining sociological, economic, and agricultural results. Incidentally, all statisticians will welcome the fact that last year the Royal Society honoured Professor Mahalanobis with their Fellowship, for the Statistical Institute is essentially his creation. As Dr. Yates has pointed out, the subject presents numerous detailed, intricate and often difficult problems of interest to the pure mathematician. When sampling units differ in size and in variability between and within strata a number of devices, sometimes analogous to simple weighting, sometimes to regression, have been found servicable, and these might be thought by a mathematical statistician often to supply a complete solution of a given problem. The solution is, I submit, almost always complete only in relation to data following certain prescribed characteristics, and those characteristics are recognizable by experience. It is therefore in my opinion of the utmost importance in the development of this branch of research as a tool of practical administration, that there should be continuity of sampling studies at the few centres capable of carrying them out effectively. I sincerely hope that the body of experience that Dr. Yates has gathered at Rothamsted will be utilized—exploited, if that term is preferred—by a continuance of such studies in the same institution, so that there shall always be a body of persons with practical experience of the types of material coming under survey.

Dr. D. V. Glass: I have pleasure in seconding the vote of thanks to Dr. Yates for his most valuable paper. My own interest in the paper lies primarily in its application to social surveys. I anticipate that there will be a good deal of discussion on that aspect, but perhaps I may help to provoke argument by suggesting that Dr. Yates is a little hard on quota sampling. It is not quite true to say that the only reason this method is still used is because it is applied largely to surveys of opinions, a field in which the validity of the results cannot be checked. There are, after all, the checks offered by elections, and in the United States, in spite of the attack on the method in the report on the Congressional investigation of the Gallop Poll, the unadjusted figures of that poll did yield results very close to the actual results of the Presidential election. Very close results were also obtained at previous Presidential elections.

One point is that once the boundaries of agriculture are transcended and sampling techniques applied to the social field, the questions of administration and costs become increasingly important. The application of quasi-random sampling to problems of individual characteristics and opinions necessarily involves, as Dr. Yates points out, scrupulous attention to results where contact has not been possible the first time. This very greatly adds to the work of interviewers and to the costs of any enquiry. That is the real reason why quota sampling continues to be used. There is no doubt, however, that Dr. Yates's general comments on quota sampling are appropriate, and that the method is only a last resort and should be avoided as far as possible. In particular, quota sampling does not permit sufficient control of the interviewers.

Dr. Yates is also undoubtedly right in saying that for many social surveys (that is, local surveys), sampling problems are relatively unimportant. But the problems become very important once attempts are made to obtain national estimates. Yet it is just here that there would seem to be the greatest scope for sample surveys in the future. Within my own field, for example—that of population—probably the only way of obtaining up-to-date information on such problems as internal migration, housing, and the labour force, is by sample surveys, while the combination of sample and complete enumeration would make our censuses more fruitful and allow the results to be published much more quickly than has hitherto been the case. Here, however, the sampling problem itself becomes very important. Even in drawing a sample from the universe of a complete enumeration, it was found necessary in the United States to apply rigorous techniques in order to avoid bias in selecting households from enumerators' schedules.

I wonder, however, whether in this country we are really made for the large-scale applications of sampling. In the United States, where there has been a very rapid development in recent years in the application of sampling techniques to social problems, this development has been made possible by close collaboration between experts in sampling theory and the Government departments wishing to make use of sample surveys. The use of inappropriate methods has been minimized by a central scrutiny of all sample surveys planned for or by Government departments. And at the same time a great deal of practical work has been done to make possible the application of more reliable methods —for example, the work necessary as a basis for the new area-sampling developments.

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The situation is unfortunately very different here. Although most of the pioneer work in sampling theory has come from this country, there is much less collaboration between sampling experts and Government departments, and it is likely that many of the sample surveys done for the Government in the field of social questions would not bear really close scrutiny. I can only hope that Dr. Yates's paper today, and the discussion which follows it, will, by drawing attention to this side of the question, encourage closer collaboration and scrutiny in the future. And I feel that it would be a most valuable act on the part of the Society if they recommended the establishment of a governmental committee for scrutinizing sample surveys or, failing that, set up their own committee to deal unofficially with this question. Now that Dr. Yates has produced a codification of sampling techniques, an opportunity is afforded for ensuring that the expansion of sampling in the social field is done on appropriate and valid lines.

The vote of thanks was then put to the meeting and carried unanimously.

MR. E. G. REEVE said that he thought everyone working on sampling problems would be interested in Dr. Yates's communication, which set out so clearly the conditions to which sampling of maximum efficiency must conform. Good sampling was essential to a social survey, and in the Government's social survey organization the sampling problem was one which confronted them at the beginning of every investigation. It was necessary to make as much use as possible of previous knowledge about the constitution of the parent population, and after drawing the sample to examine its efficiency in the light of the methods which Dr. Yates had suggested. They had been able to collect less general information suitable for these purposes than might have been expected in view of the number of surveys they had done. This was for two reasons: (1) the abnormal and changing distribution of the population under wartime conditions, and (2) the direction of most, if not all, of the surveys towards obtaining information which was urgently required for practical purposes, which involved the grouping of the sampling units and the partialling out of the variables in a manner not the most suitable for fundamental research. For example, a sample might be distributed in three or four groups of income in order that the members of the sample should fall simultaneously into social, as well as into economic groups. This did not lend itself, without the expenditure of much extra time and effort, either to a precise estimate of the linearity of the regressions or to the calculation of the regressions themselves as defined by the appropriate normal equations.

These were some of the reasons why it had been necessary for them to test their sampling in a much more rough-and-ready fashion than Dr. Yates had shown to be desirable. Some of these checks nevertheless did raise a reasonably strong presumption concerning the efficiency of the sampling. For instance, they drew a sample of about 2,000 people chosen by a method involving the quota system in such a way that the ultimate selection was made by interviewers using their judgment in making their choices as random as possible, taking partly people in houses and partly people in work places. In factories this involved choosing workers, for example, from every other machine or from every two or three machines in some systematic way. This sample yielded a frequency distribution as follows: 6, 21, 48, 20, 4, 1 per cent., the "one" referred to people for whom they had not an assessment, because they declined to answer or because, for some other reason, an answer was not obtained. A second sample directed at the same parent population, and of about the same size, was drawn for another purpose; this was done by a method involving the selection of cards, the cards being chosen from a file at regular intervals. This method yielded a frequency distribution comparing with the former as follows: 7, 20, 45, 22, 5, 1 per cent.

In the case of the first sample they were able to make a further comparison, this time in terms of age distribution. The comparison was made between one of their samples and the returns of the Registrar-General. Their own sample in terms of age was 23, 35, 42 per cent., and the Registrar-General's was: 23, 32, 45 per cent.

They had a third comparison, but he would not go into further details at the moment. It was by such methods that they had been accustomed to validate, as far as they could, the sampling procedure they had adopted. In a survey studying the needs of old people, the sample was stratified into eleven civil defence regions and four sizes of towns. Sixty-five towns were used to draw the sample and in each town the dwellings were drawn at random from the local authority's rating lists. Usually on surveys they had found that to get interviewers to call back at all frequently to pick up information from people not at home the first time imposed a considerable psychological strain upon the interviewers and was apt to cause some embarrassment to the informants. The analysis of variance was finding increasingly wide use and Dr. Yates's paper would facilitate the application of this technique to social research. He had drawn attention to the features of variability offered by individual batches of material. There was reason to suppose that the social continuum was rich in undetected variables, and the methods he had described might be of use in discovering them or in confirming or refuting their supposed existence. At the Social Survey they very much welcomed the suggestions for getting these methods into regular practice as far as possible, and they would be very much pleased to make available to Dr. Yates or to any other investigators who were interested in the problem any of their data.

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PROFESSOR R. G. D. ALLEN said that in a paper of this kind it was grossly unfair to criticize the author for sins of omission. It was, of course, inevitable that the reader remembered developments in sampling theory and practice other than those considered by Dr. Yates as he ran through the paragraphs. What the speaker had to say was partly illustrative of some of Dr. Yates's points and partly supplementary to his account.

His main object was to describe in some detail a particular sub-sampling design actually in use in the United States. This design involved area sampling, not with the grid system, as for agricultural or forest problems, but with ordinary administrative areas and for economic and social surveys. The design was devised by the Bureau of the Census (United States Department of Commerce), and the theoretical aspects had been the subject of a paper by Hansen and Hurwitz of the staff of the Bureau.*

The basic problem was to obtain a monthly analysis of the United States labour force from a sample restricted by budgetary considerations to less than 0·1 per cent. of the population, and to field operations from a small number of centres. The analysis was to show agricultural employed, non-agricultural employed, unemployed, and non-workers, each divided by sex and (for some purposes) by age groups. A two-stage sub-sampling design, with stratification at each stage, was selected as follows:

(1) In the primary sample about 2,000 groups of counties, generally single or in adjacent pairs, provided the sampling units. These were stratified into 68 strata, according to various factors, and from each stratum a random selection of one unit was made, with probability of selection proportionate to size (population in 1940 census). The sampling ratio varied considerably, being 100 per cent. in some strata, with large cities, such as New York and Chicago, automatically included.

(2) In the sub-sample each primary stratum and the corresponding primary unit selected was divided by areas into sub-strata. From each sub-stratum within the primary unit selected a random sample of households was taken according to a rather complex formula. The object was to take a fixed number of households from each primary unit, distributed over the sub-strata according to the proportions found in the primary stratum as a whole (and not in the primary unit selected). On the basis of 1940 populations, this fixed sampling ratios in the sub-strata of the selected primary unit and these ratios were applied in each month's survey. Since changes in the population distribution had taken place, the actual number of households taken diverged somewhat from the original fixed number and the changes showed up in the sample.

(3) The estimate used for the required national totals (*e.g.*, of unemployed) was the total from the sample, weighted only in the primary strata by the relevant sampling ratios. The estimate could be improved by estimating ratios (*e.g.*, per cent. unemployed to total population) and applying them to the U.S. population known from other sources. Cf. Section 7 of Dr. Yates's paper.

This design was similar to, but more complicated than, that described by Dr. Yates in Section 5 (*c*). Some new theoretical points emerged. As compared with the simpler and more usual sub-sampling systems, this design involved larger primary sampling units, of greatly varying size; selection of primary sampling units by probability proportionate to size; and stratification at the sub-sampling as well as at the primary sampling stage. Hansen and Hurwitz showed that, in the actual population they used, each of these three refinements reduced considerably the mean square error of estimate. Without sub-stratification, their estimates were unbiased as compared with biased estimates, with greater mean square errors, often proposed in simpler sub-sampling schemes. With sub-stratification, however, the estimates used were biased, but Hansen and Hurwitz claimed that the bias was not great and more than compensated by reduction in the mean square error. There was, in his view, not enough in Dr. Yates's paper about such questions of the use of biased estimates from samples.

The sub-sampling, area-sampling design described was in use in the monthly survey of the United States Labour Force. It was, clearly, of much wider applicability. In the first place, it could be (and was being) used for other current surveys in the United States—*e.g.*, for consumer requirements or retail sales—and could probably be used with equal success in other countries. Secondly, although the conditions under which this sub-sampling system was employed were particularly favourable, conditions nearly as good were to be expected in other fields. Hansen and Hurwitz showed, in effect, that there was a gain in efficiency in the use of their design if: (*a*) the correlation of the character investigated within a primary sampling unit tended to decrease, but less than proportionately, as the size of the sampling unit increased—*i.e.*, larger sampling units were more heterogeneous—and (*b*) the variance of the character within a primary sampling unit tended to increase, but less than proportionately, as the size of the sampling unit increased. These conditions

* Hansen, Morris H. and Hurwitz, William N., (1943) *Ann. Math. Stat.*, **14**, 333. See also *A New Sample of the Population*, U.S. Department of Commerce, Sept. 1944, by the same authors. Similar sampling problems have been investigated in the U.S. Department of Agriculture.

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must be quite commonly satisfied, as Dr. Yates, he thought, would agree. Much of what Dr. Yates said in Section 9 of his paper applied to the system now described. In fact, not only did the sampling fractions vary as recommended by Dr. Yates, but they reached 100 per cent. in some strata containing large and diverse metropolitan areas.

On the problems and limitations of stratification Dr. Yates is very illuminating, and it is interesting to compare his results with those reached fifteen years ago by Prof. Bowley in the *New Survey of London Life and Labour*.* The House Sample in this survey involved stratification in two ways. First, the method of selecting households in a given borough was that described by Dr. Yates as quasi-random (Section 10), by streets in alphabetical order only, not in wards. Prof. Bowley dismissed the gains as slight, and Dr. Yates would agree. Secondly, for the whole London area the stratification was by boroughs, in which the sampling ratios were intended to be constant, but in the end showed considerable variation. Perhaps the most interesting comparisons were on the question of the effect of variations in the sampling ratios (Section 9). Prof. Bowley commented that the gain from stratification "is nearly neutralized by some variation in the sampling fraction." † This followed since his cases generally showed little correlation between the sampling ratio and the size and variability of the character in the different boroughs.

Mr. André Gabor said that the author had certainly earned the lasting gratitude of all who, in one way or another, were currently concerned with sampling problems in their daily work. He congratulated him especially on the lucidity of his presentation of recent advances in the theory and technique of sampling, a lucidity which was not always found in the papers of those whose contributions to these advances were comparable to the share Dr. Yates could claim for himself.

He had no criticism to offer, but would be grateful if Dr. Yates could give a few further words of guidance to those who were unfortunate enough to be constantly confronted with a problem which was the bugbear of statisticians, the sampling of small finite groups with a considerable dispersion of each of the variates studied. The problem was common to many sociological and economic enquiries. It was, for instance, present in all its complexity in the study of the financial results of type groups of farms. It was well known that in this country, mainly owing to variations of physical characteristics, many distinct type groups of farms were found which consisted of no more than a few hundred ultimate units.

For the purpose of enquiries on the national scale, it was of course possible to treat these groups as strata or to use some of their characteristics as balancing controls. But there was also a desire to study these small groups in isolation, and this was where the difficulties arose.

They had some idea of the variances involved, and found that the sampling fractions required for a random or stratified random sample necessary to give results of reasonable precision might reach 80 per cent. or more. But in this field attempts to obtain as much as a 50 per cent. sample were almost certainly doomed, since the investigations required active co-operation, and this was seldom forthcoming from more than one-third of the farmers approached.

The problem mentioned was typical of many others. In the discussion of the *War-time Social Survey* the familiar warnings against replacement of recalcitrant or otherwise unsuitable sampling units by the field-workers were once again issued, and the answer was along similarly familiar lines. It was pointed out that such replacements were inevitable in certain instances, and that it seemed unlikely that they introduced any bias of consequence. The same warnings and the same answers were heard in 1924, when Mr. Hilton read his famous paper *Enquiry by Sample*, and it seemed that in this particular respect the otherwise considerable progress of sampling theory since 1924 could offer little more than renewed exhortations. The speaker would be grateful if Dr. Yates could further enlighten them on this point.

In particular, he appreciated his comments on two questions, based on passages in his paper. The first of these referred to the technique of balancing. Would it be correct to say that the method was equivalent to the selection of two or more complete sets of members for each stratum or cell, with a subsequent application of purposive selection in respect of the balanced factor, with the restriction that the first member must be selected from the set consisting of the first members of each complement, and so on? If this interpretation were correct, the number of such replacements might become considerable, and if the group sampled happened to be small, so that the minimum number of units required for the ultimate sample represented a relatively high sampling fraction, it might involve a situation in which each or almost each unit of the original group came up for purposive selection within one set or another. Would it be possible to give some practical guidance as to the limits of applicability of the method, although, as stated in the paper, the exact conditions required for strict equivalence to a random sample had not yet been specified?

His second question originated in the remark Dr. Yates appended to the use of the correction factor "square root of one minus f," as mentioned under "Random Sampling." Dr. Yates pointed out that this factor should not be included when testing the significance of the difference of the means

* Volume III, Appendix IV. The volume appeared in 1932, but the sampling work dates back to 1928-30.
† *Op. cit.*, pp. 445–6.

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of two populations. It was clear enough that the *t*-test, for instance, was not strictly justified if the standard errors were so adjusted, although Schumacher and Chapman recommended this procedure in connection with samples of finite populations in their book on *Sampling Methods in Forestry and Range Management*. But did Dr. Yates refer in the passage mentioned to infinite populations only, or did he mean that in the case of finite populations the indications of the *t*-test were valid if the sampling standard errors unadjusted were employed, or did the remark refer to other tests of significance?

MR. A. P. ZENTLER said he wished to make some comments on a rather special aspect of the Social Surveys field, a field which, as Dr. Yates pointed out, is becoming increasingly important. He referred to market research as undertaken *by* commercial firms *for* commercial firms. Statisticians who had been engaged in this type of market research viewed with considerable concern the quality of the work done by the average market research firm. Dr. Yates criticized some of the sampling methods used by the War-time Social Survey, He, the speaker, assured him that good or bad, these methods were considerably in advance of commercial market research.

It would not be an exaggeration to say that by and large market research firms were blissfully unaware of the fact that the very foundation of sample investigation was probability theory. Inasmuch as such firms were aware of the probabilistic relationship between the " statistics " of the sample and those of the population from which the sample is derived, they were very many years behind the times in their methods. Even the resolutions adopted by the National Institute of Statistics in 1924—which Dr. Yates naturally considered out-dated—would seem new, strange and " much too theoretical " to the practical market research man. There was in commercial market research a definite myth of the large sample, no appreciation whatever of small sample theory of tests of significance, etc. To talk to a market research manager about the " efficiency " of a sample in the Fisherian sense would be simply ridiculous. There was very definite neglect of the principles of randomization and no realization of the fact that unless this element of randomness were present it was not possible to talk of margins of error.

To take, perhaps, the worst example of this kind, one of the very popular methods of obtaining information on the behaviour of consumers, was that of the consumer panel. The speaker thought that far too much use was made of this technique in cases in which safer methods could be employed. The obvious disadvantage of the consumer panel was that once you had selected your sample, you must stick to it for a considerable period of time, so that if your selection of persons for the panel were not random, all your weekly and monthly results over a year or more would be biased. This was not the case in other types of market research, where the sample was changed every time an investigation was carried out. However, for the purpose of obtaining *quantitative* information on the actual day-to-day use of a product consumed regularly, no technique which could replace the consumer panel had—as far as he knew—yet been evolved.

The point he wished to emphasize was that in the majority of cases these panels were nowhere near a random sample. The randomness was lost in two ways: (1) by some of the randomly chosen individuals not being at home when the investigator called to recruit him or her for the panel and, (2) by some of those selected at random refusing to join the panel. Such individuals were replaced very often by the nearest available and willing individual, thus introducing obvious bias. He knew of cases where not less than 50 per cent. of the randomly selected housewives were replaced in this way, and yet the firm concerned went merrily ahead and produced results which, it maintained, were representative of the general public, and which, they assured their clients, were within certain definite limits of error.

He thought the main difficulty was that market research in industry, instead of being conducted from the point of view of sampling technique and interpretation of results, by the statistical expert, was entrusted to the know-all " practical business man." Also, he felt, the same shoddy workmanship was present in a number of cases where the statistical aspects of market research were handled by economists. The latters' deficiency in probabilistic statistics was nothing new to this Society. The problem of adequate training in statistics for graduates in economics was, he understood, already under consideration. The reason why he raised the question of commercial market research today was his belief that it behoved the Royal Statistical Society to try to prevent interested parties from misusing the science of statistics. He also thought that market research was a problem of national importance, as it would doubtless play an important part in the re-organization of industry and in the expansion of exports. Good market research would be invaluable, bad market research disastrous.

A topical example was that of the newly formed British Export Trade Research Organization, better known as BETRO, whose success or failure would make a lot of difference to this country's export drive. He had recommended through appropriate channels that a fully qualified statistician be appointed (if only in a consulting capacity) to the staff of that research organization with a view to supervising the statistical side of this work. He did not know if his advice would be followed, but the efforts, however great, of isolated individuals in this matter of commercial-market research were obviously inadequate.

The problem he raised was, he thought, important and urgent enough for the Royal Statistical

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Society to take action in the national interest. He would be very glad if the Council would consider the possibility of the Society appointing a Committee to investigate whether market research as carried out by the average firm in this country was based on sound statistical theory. He was confident that if the necessary assurances in respect of " anonymity " were given, market research firms would be willing to put the necessary material at their disposal; but whether or not the enquiry he suggested succeeded, it would have served at least one very valuable purpose—that of drawing the attention of innocent industrialists, who pay large sums of money for market research, to the fact that this new marketing tool was not without considerable dangers. This, in itself, would compel commercial undertakings to look more closely into their methods, and, after all, that was all that was wanted.

Mr. W. L. SEMPLE asked if Dr. Yates considered that recent national surveys employing stratified sampling had rather avoided an important issue by assuming that the strata had coincided with administrative or geographical areas which might bear little relation to the distribution of the items being investigated? Some rapid method of establishing the location and extent of such strata was required before any real increase in accuracy over the random method could be expected. What did Dr. Yates consider the most efficient method of approaching this problem when no accurate independent evidence was available?

Mr. M. G. KENDALL said that he had not supposed that in stratified sampling one had necessarily to select the same proportion from each stratum. Dr. Yates spoke of a random selection of equal proportions from each of the sub-strata. The only use he had seen of the term " stratification " did not restrict the sampling to equality of proportion in that way.

Mr. Kendall's next point was more important. Dr. Yates said at the beginning of his Section 5 that all the sampling methods set out in Section 4 were governed by the property that the chances of inclusion of sampling units were the same. The speaker could not see that that was true of balanced sampling. If the chance of any member being chosen was the same then all samples were equally probable, which conflicted with the requirement that only balanced samples were chosen. The matter could be looked at in this way: suppose a population consisted of 1,000 members, each of which had a value of — 1, and one member with a value of 1,000. The mean was zero. By taking two samples, comparing them in pairs and rejecting one member, one would never include the member with the value of 1,000; one would always take those which had a value of — 1. In that case, although admittedly it was an extreme one, certain members of the sample would never be chosen at all. He would have expected that the balanced sample would be very far from random. The same considerations applied to any J-shaped population. He thought that the proposed use of the covariance technique broke down on this point. The fundamental difficulty was really this: one drew a sample and compared a control factor with some known properties of the population. If the control agreed with the population then there was some presumption that the sample was representative in other respects; but if it did not, one then had to correct the factor in which one was interested by reference to the control factor. Bowley once tried to do this with a certain amount of success, but the attempts of Gini and Galvani were a failure. One had necessarily to calculate one's correlations from the sample, and if it was likely to be biased in respect of one characteristic then one's regressions or correlations or covariances might be biased. What was to be done to correct the figures to get a true correlation Mr. Kendall did not know, but he hoped that Dr. Yates would continue the work on which he was engaged and throw yet more light upon the subject.

Mr. K. R. NAIR said he had not many comments to make on Dr. Yates's illuminating paper. As Professor Fisher pointed out, in India, thanks to the enthusiasm and pioneering efforts of Professor Mahalanobis, much work has been done on sample surveys. Dr. Yates had already referred to a paper published by Professor Mahalanobis in the *Phil. Trans.* of the Royal Society. If any of those present had the patience to go through this very long paper they would come across a reference to some remarkable work of Hubback (later Sir John Hubback, the first Governor of Orissa) undertaken as early as 1923 when, as a revenue official, he conducted crop-cutting experiments, using the random sampling technique, to determine the yield of paddy in Bihar and Orissa. Probably these were the earliest crop-cutting experiments based on the principle of random sampling conducted anywhere in the world. Hubback's methods were used with complete success by Deshmukh (later Sir C. D. Deshmukh, the present Governor of the Reserve Bank of India and President of the Indian Statistical Institute in the current year) in the Central Provinces in 1928, 1929, and 1930. Since 1938 extensive random sampling experiments for estimating the out-turn of a number of important crops like jute, paddy, wheat, grain and sugar cane had been carried out by the Indian Statistical Institute in Bengal, Bihar and the United Provinces. Those interested might refer to a report on the Bihar Survey published by Professor Mahalanobis in *Sankhya* (Vol. 7 (1) August 1945).

Turning next to one or two specific points in Dr. Yates's paper, Mr. Nair found that in Section 7 the method of sampling for crop yield that Dr. Yates had in mind differed somewhat from what

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had been employed in India. He seemed to have an entire field as his sampling unit and possibly the yield records for these fields were directly obtained from the farmers at the end of the harvest season. In India such a procedure would not work owing, among other things, to the illiteracy of the majority of the Indian farming population. They decided in India to cut only a small portion of a randomly selected field, preferably two independent cuts, at points randomly located within the field. Much discussion had naturally centred round the question of size and shape of cut. The official size in India was $\frac{1}{20}$th of an acre. Hubback demonstrated that cuts of much smaller size (he actually used $\frac{1}{3200}$ of an acre shaped like an equi-lateral triangle) were likely to give better results. Professor Mahalanobis had also more or less come to the same general conclusion.

In Section 11 Dr. Yates said: "In many types of area sampling the location of the sample areas on parallel lines is desirable." It was remarkable that Hubback followed the very same procedure in 1925 to estimate crop acreage. His plan was to "make the sampler march from centre to centre across country as nearly as he conveniently could in a straight line."

Lastly, Mr. Nair noted with immense satisfaction the remarkable agreement between the views expressed in the concluding section of Dr. Yates's paper and those held and, might he say, practised, in India by Professor Mahalanobis and his co-workers for over a decade. He had no doubt that Dr. Yates's paper would be read with great interest by his (the speaker's) former colleagues in India.

The following comments were received in writing.

DR. H. O. HARTLEY: My contribution is concerned with Dr. Yates's point made on p. 28. Here he discusses the inadequacy of "quota sampling" when this is practised in a not strictly random manner. In the case of sampling households, which is mentioned, there is usually little difficulty in selecting a random sample of house *numbers* from a street list; indeed, the procedure described in the paper by the War-time Social Survey (and quoted) appears to make provision for *this* part of the correct sampling procedure. The practical difficulty arises when the interviewer finds on calling that there is nobody in. Thus a large proportion of the households originally selected drop out of the random sample at the first call. Now, it is well known that if these "nobody in" households are left out, the remainder of the original random sample is seriously biased. For instance, there will be an unduly low proportion of housewives with part or full-time work, an unduly high proportion of young children, an unduly low proportion of "queuers", and many other misrepresentations. A classical example of this bias is mentioned by Professor Bradford Hill in his book, *Principles of Medical Statistics*. The example is the Ministry of Health report on the influenza pandemic of 1918. In this enquiry houses which were found closed at the time of the visit had to be ignored.

Now let us consider Dr. Yates's remedy. It is, I think, indicated on p. 29—"scrupulous attention to calling back." His interviewers call again and again at the "nobody in" houses. Now, I understand that the interviewers of the War-time Social Survey have, in some surveys, made second and third calls, and recovered some, but by no means all, households of the original random sample. It appears that a very large number of calls must be made to interview approximately all households in the old random sample. Indeed, there are households where there is never anybody in at the respectable hours during which Civil Servants do their interviewing, particularly in war-time! Households like these are bound to be included in the original random sample. If these householders *must* be interviewed, are these unfortunate victims of randomization to be shadowed and tracked down at night by special officials?

We should like to hear more about this from Dr. Yates. Whatever the success of the method, it is bound to be laborious. I should like to indicate, therefore, an alternative which is decidedly cheaper, and would like Dr. Yates's opinion whether it will qualify as a quasi-random sample.

The method is simply to eliminate the bias by introducing another set of strata. All households may be classified into groups according to the length of daytime period (the time of interviewing) during which an adult may be found at home. There are the households where there is always somebody in, which we may call the "100 per cent. at homes," and there will be, on the other hand, households where only during 10 per cent. of the daytime somebody is in, the "10 per cent. at homes," there will be "90 per cent. at homes," "80 per cent. at homes," and so on. Now, a rather larger random or quasi-random sample of households is selected as before, but now only *one* call is made at each house of the sample, all "not in" at the call drop out of the sample. There are definite rules of chance for this game of hide and seek, just as accurate as Dr. Yates's laws of randomness! Of the "100 per cent. at homes" none of the random sample is being lost, of the (say) "40 per cent. at homes" we lose in the average 60 per cent. of the original frequency in the random sample, but—and this is the point—the remaining 40 per cent. can now be regarded as a quasi-random sample *within the stratum* of the "40 per cent. at homes." All the households in the "40 per cent. at home" group had approximately an equal chance of being in this sample.

Now, in order to utilize this theoretical device, important additional information is required—we must be able to classify all actually interviewed households as to whether they are "100 per cent. at homes," "90 per cent. at homes" and so on. In practice, therefore, the interviewers would have to add to their questionnaire an appropriate question as to how often it occurs that there is nobody

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at home. This is certainly an awkward question to answer, but a question to which the answer should be of similar reliability to the answers which the sampling enquiry attempts to analyse. If this is done, all interviewed households can be classified in their appropriate " at home " stratum. We now have a quasi-random sample within each stratum, but this sample is not properly stratified. We therefore eliminate the bias by some such formula as is given by Dr. Yates on p. 20. The formula for $\bar{Y}$ gives the unbiassed estimate of the population mean, similar formulæ apply to other statistics. The true stratum frequencies p are estimated from the sampled households. They are, in fact, directly proportional to the sampled stratum frequencies and inversely proportional to the " per cent. at home " figures. The computational labour involved can be reduced considerably by introducing weight-figures corresponding to each household (*e.g.*, $\frac{1}{4}$ for the 40 per cent. at home," $\frac{1}{10}$ for the " 100 per cent. at home ") and adding these weights instead of counting households in each stratum cell.

Details of this scheme were given to the War-time Social Survey, but I understand that, owing to pressure of work, an opportunity of trying it has, as yet, not arisen.

There are assumptions and difficulties, in particular the " never at home " households still require some special control. However, the scheme is put forward in the spirit of co-operation for critical discussion.

MR. F. J. ANSCOMBE and MR. M. H. QUENOUILLE: Dr. Yates recently invited us to examine the problem raised by him at the end of Section 4(*e*) of his paper, of how to draw a balanced sample from a population. Our findings, though not conclusive for practical purposes, may be of interest.

We consider the following problem. From a single-variate population with known mean m it is desired to draw a sample $x_1, x_2 \ldots x_n$ of size n satisfying the condition

$$\frac{1}{n}(x_1 + x_2 + \ldots + x_n) = m \quad \ldots \ldots \ldots \quad (1)$$

but otherwise random; *i.e.*, we wish to draw a random member from the population of all random samples of size n from the parent population which happen to satisfy condition (1).

We take first the following method, which is much the same as the more flexible of the methods proposed by Dr. Yates.

Method A. Take a random sample of size n, and write it down in a row. If it does not satisfy condition (1), take a second sample of size n, and write it down underneath. Consider the 2^n samples formed by taking one member from each column. If none of these satisfies (1), take a third sample and consider the 3^n samples formed by taking one member from each column. Continue in this way until a sample satisfying (1) has been found. If more than one possible sample can be taken at the same time, select one of them at random.

By considering examples (given below) in which samples of size 2 are drawn from populations which take only two or three different values, we have shewn that the repeated application of method A leads to the various possible balanced samples satisfying condition (1) being chosen with incorrect frequencies. Thus Method A is not an exact procedure.

It seems unlikely that any slight modification of Method A will alter the situation, and in an endeavour to find an exact method two quite different methods have been considered, as follows:

Method B. Start sampling and write the observations down in a line. Stop as soon as the last n observations satisfy the balancing condition (1).

Method C. Sample as in Method B, until a sample of size n can be picked out satisfying (1). If there is more than one possible sample, select one of them at random. (After N values have been drawn from the parent population, ${}^{N}C_{n}$ samples of size n can be picked out.)

Method B is clearly much more expensive than Method A in the number of items needed to be sampled from the parent population in order to find a balanced sample, while Method C is less expensive than Method A. Neither of these methods appears to be exact, and it is tempting to conclude that the only exact method of reaching balanced samples with correct frequencies is to draw samples of n repeatedly until a complete balanced sample is encountered—which as a practical procedure is out of the question.

It must be emphasized, however, that the following examples cast no light on the efficiency of the three methods considered in giving balanced samples in almost the correct proportions when the sample size n is fairly large (as it always will be in survey work). It seems intuitively quite likely (though we have proved nothing about it) that all three methods would be accurate enough for ordinary use, with n large and the parent distribution continuous. Further, it would be rash to draw any conclusion from the examples below as to the relative accuracy of the three methods.

Example 1. Consider a population with three possible values, 0, 1, and 2, of which the probabilities are ε, $1-2\varepsilon$, ε, respectively, where ε is small; and suppose we wish to draw balanced samples of size 2. The balancing condition is

$$x_1 + x_2 = 2 \quad \ldots \ldots \ldots \quad (2)$$

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There are only two possible samples satisfying (2), namely (1, 1) and (2, 0) (or (0, 2)). They should occur with frequencies in the ratio $(1 - 2\varepsilon)^2$ to $2\varepsilon^2$, so that the chance of a (2, 0) is

$$\frac{2\varepsilon^2}{1 - 4\varepsilon + 6\varepsilon^2} = 2\varepsilon^2(1 + 4\varepsilon + 0(\varepsilon^2)).$$

But Method A gives the result (2, 0) with a frequency greater than

$$2\varepsilon^2(1 + 8\varepsilon + 0(\varepsilon^2)).$$

(This is in fact the chance of getting a (2, 0) without simultaneously getting a (1, 1).) The frequency of (2, 0)'s given by Method B is

$$\frac{4\varepsilon^2(1 - \varepsilon)}{1 - 3\varepsilon + 4\varepsilon^2}$$

and by Method C is

$$6\varepsilon^2\left(1 - \frac{\varepsilon^2}{3(1 - \varepsilon)^2}\right).$$

Thus when ε is small Method B gives about twice, and Method C three times, the correct number of (2, 0)'s, and Method A also gives too many, though less strikingly.

It may be noted that, apart from the question of the frequencies of differently constituted balanced samples, Method B will tend to give samples that are not arranged in random order within themselves, but which have the members with greater probability first, and the rarer members afterwards.

Example 2. If we consider the drawing of a balanced stratified sample, an easier example can be adduced to shew the incorrectness of Methods A and B (Method C needs modification before it can be applied to stratified sampling, and in the present instance will be identical with Method A). Consider drawing a sample of size 2, of which one member x_1 is drawn from a population which takes the values 0, 1 with chances q, p, and the other member x_2 from a population which takes 0, 1 with chances p, q—*i.e.*, the same chances reversed ($p + q = 1$). To balance for overall population mean we must have

$$x_1 + x_2 = 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

There are two possible samples, (0, 1) and (1, 0), and the frequency with which the 1 should come from the first population is

$$\frac{p^2}{p^2 + q^2}$$

Using Method A, with the first column containing members of the first population and the second column members of the second population, we get, instead of the above frequency, the following one:

$$\frac{(1 + q)p^2}{p^2 + q^2 + pq}$$

By Method B, sampling from the two populations alternately, we get the same frequency as by Method A.

Apart from the problem of how to draw a balanced sample, we may ask whether a balanced sample is a desirable thing to use anyway. Mr. M. G. Kendall, in his remarks at the meeting, referred to the condition that "the chance of inclusion of any sampling unit or sub-unit in the sample is the same" (at the beginning of section 5 of Dr. Yates's paper), and showed that with a J-shaped population the highest members could not possibly ever be included in a balanced sample of given size, as there were no members sufficiently small to counterbalance them.

Suppose $f(x)$ is the frequency function of the population (assumed homogeneous), and let $f_n(x)$ denote the frequency function of the sum of n independent members of the population. Then the frequency function of any member of a sample of size n balanced by condition (1) is proportional to

$$f(x)f_{n-1}(nm - x).$$

If we suppose the parent population to be normal with variance σ^2, we find that a member of the balanced sample is distributed in a normal distribution with the same mean and variance $\frac{n-1}{n}\sigma^2$. If we suppose the parent population to have the J-shaped distribution $e^{-x}dx (0 \leqslant x < \infty)$, with unit variance, we find that a member of the balanced sample has a distribution which approaches the parent distribution as n increases, and has variance $\frac{n-1}{n+1}$. It would seem, therefore, that if n is fairly large not much trouble will arise from the non-representativeness of the balanced sample.

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FLIGHT-LIEUTENANT W. R. BUCKLAND: I must express regret that I was unable to attend when Dr. Yates presented his paper. His expressed aim was to give a comprehensive summary of the various methods of sampling that are commonly employed, their interactions, derived estimates and sampling errors. For those who are returning to serious statistical work after an interruption of five years this paper is of the utmost value in the enormous task of the re-orientation of our knowledge: as one of these people, I wish to record my own grateful thanks to Dr. Yates.

With reference to the principle of Random Selection laid down by the 1925 report to the International Institute of Statistics, it may be of interest to set down some of the points on a situation which arose in Bomber Command. An air raid on an " area " target before the advent of the atomic bomb could be regarded as a continuous process in a finite period of time. It is a well-known fact that when operating at night, each bomber aircraft carried a high-powered photographic flash with which it could obtain a photograph of the approximate point of impact of its bomb load. In theory, therefore, we have a finite population of photographs governed by the number of aircraft proceeding to the target area. In practice, however, there are various reasons which operate to reduce the number of available photographs. Allowing for various forms of technical failure due either to the flash, or the camera, and the aircraft who do not traverse the target area, we arrive at a net figure which is broadly composed of :

A. Aircraft, the position of which can be identified from the ground detail registered on the night photograph.

B. Aircraft, the position of which are not immediately identifiable because the photographic negative has become covered with a miscellaneous collection of light tracks emanating from fires and other light sources on the ground.

Therefore, for the purposes of Raid Assessment, can the aircraft under "A" be regarded as a random sample, or a sample which is sufficiently representative to permit the use of techniques associated with random sampling?

Because of the tremendous build-up of fire sources of light which was the usual result from the type of attack we are considering, most of the photographs showing plottable ground detail would be from the earlier stages of the attack. Thus they constituted a biased sample, and posed a problem not unlike the Time Correlation problem cited by Professor E. S. Pearson in his paper on " Sampling Problems in Industry " (*J.R.S.S., Suppl.*, Vol. I, No. 2, p. 108). The problem was how to correct this bias in the nucleus of intelligence material, which was itself of a very high order of value, and none of which could therefore be disregarded.

The fundamental work here was done by Squadron-Leader B. Babington-Smith, a Fellow of this Society. He evolved a method of plotting photographs from group " B " above by using the intersections of fire-track patterns. For control points he used the fire patterns which appear on those photographs which showed a combination of plottable ground detail and fire-tracks. This enabled the time structure of the sample to be rectified. We might call the result a quasi-stratified sample, for the reason that none of the information under group "A" could be ignored.

Turning now to Dr. Yates's remarks on the increased need for social census *and surveys* and that *surveys* generally require some form of sampling for their efficient and speedy execution, it is surely a matter for regret that the announcements of the new sample survey into the structure of families should have been almost universally labelled as a " Census." In fact, our greatest national newspaper headed its news column, " Sample Family Census," which, in accordance with the usual English practice, is a contradiction in terms. Already we have had various appeals to the public to play their part and complete the forms (cp. Roy Harrod in a B.B.C. talk on January 18th). One danger is that people who have no children, or who are not particularly addicted to children, will just not bother even to render a nil return, and thereby introduce bias in favour of people with children or people who like children.

From several comments which I have seen in the press or heard while travelling in public vehicles, there is a great need for some immediate and simple explanation of the mechanics of sample surveys. Any confusion of terminology which might react against the proper use of sample surveys by:

(*a*) Antagonizing public opinion; or

(*b*) Discouraging official bodies if anything goes not according to plan because of the effects of (*a*),

is surely most undesirable at this stage. The problem would also appear to have a quasi-legalistic setting, in that in this country a Census is compulsory, whereas a Sample Survey is voluntary. This distinction, when set in the present institutional background, might well have an adverse reaction upon a sample survey if it is persistently, but erroneously, called a Census.

DR. YATES said he preferred not to make an immediate reply to the many interesting points and suggestions which had been put forward and for which he wished to thank the contributors, but to avail himself of the opportunity of replying in detail in the *Supplement*.

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He would now merely mention the problem of quota sampling, which Dr. Glass had discussed at some length. Dr. Glass had put forward an able plea for the method, but his subsequent description of the types of sampling arising in social investigations in which he was interested, had considerably weakened this plea, since Dr. Yates was sure Dr. Glass would agree that quota sampling was entirely unsuitable for these investigations. At the same time he wanted to make it clear that quota sampling was not, in his opinion, always unsatisfactory: there were fields in which it provided a suitable method.

Dr. Yates subsequently wrote as follows:—

The contributors to the discussion, and those who have subsequently sent written contributions have raised many points, which I can only touch on briefly here. I should like to take this opportunity, however, of expressing my appreciation of the great interest which has been shown in the subject, and the kind way in which the paper has been received.

I am in entire agreement with Professor Fisher's remarks on the importance of experience in dealing with the sampling problems arising in different fields of enquiry. I also fully agree with his proposed solution—that it should be the task of certain institutions to act as vehicles for preserving the continuity of this experience. If this is to be effected, not only must the institutions themselves be set up, but in addition the posts in them must be made sufficiently attractive and have sufficient security of tenure to enable a nucleus of able and reasonably permanent staff to be maintained.

In conversation after the meeting Professor Fisher raised another point in connection with bias, which is, I think, worthy of mention. As I pointed out in the paper, we are often confronted with the choice of using an estimate which is certainly unbiased, but which is of low accuracy, or of using an alternative estimate of higher accuracy which is subject to certain elements of bias. In making comparisons between different parts of the sample, bias, provided it is substantially constant, is of little importance, and consequently for this purpose we may justifiably use the biased but more accurate estimate. On the other hand in making an overall estimate we are in general anxious to secure that this estimate shall be unbiased, but equally do not usually require that it shall be of high precision. Consequently for this purpose we can satisfactorily use the unbiased estimate, even though it is of lower accuracy.

I have already dealt with Dr. Glass's main point concerning quota sampling, and I am glad to see that he supports my view that quota sampling does not permit adequate control of interviewers. I am fully in agreement with his comments about the need for greater use of sampling methods by government departments. Improvement can probably best be effected by steady work in increasing the efficiency of sampling surveys, and by more adequate training in sampling theory and practice of those engaged in statistics in government departments, so that they have a better realization of the potentialities of well-planned sampling enquiries, and are able to see that enquiries carried out by their own departments are properly planned and executed.

I am not sure that I have grasped the import of all Mr. Reeve's remarks. Here I would only say, firstly, that if some form of random sampling is adopted, previous knowledge of the constitution of the parent population is not of great importance; secondly, that comparisons of frequency distributions derived from the parent population and the sample, though a useful indication of the existence of absence of bias, must be treated with some reserve; and thirdly, that the function of the analysis of variance, as outlined in my paper, was not the detection of previously undetected variables but the estimation of error.

The interesting developments in the United States reported by Hansen and Hurwitz to which Professor Allen has drawn attention are a further illustration of the rapid progress being made in that country in the application of sampling techniques to social and economic problems. I am also grateful to him for the details he has provided of Professor Bowley's pioneer work. Professor Allen expressed disappointment that I did not discuss more fully the use of biased estimates from samples. I did devote considerable space to the discussion of the biases that are likely to be of practical importance, but I must plead guilty to a deliberate sin of omission of any discussion of the type of bias referred to in Hansen's and Hurwitz's paper. It is true, for example, that if the total yield of a crop is estimated by multiplying the weighted mean yield per acre in the sample (p_w in my notation) by the total acreage of the crop in the country, the estimate will be slightly biased in the sense that the mean of a set of such estimates from different samples will not tend exactly to the true total yield as the number of sets is increased. Such biases, however, are almost always trivial in comparison with sampling errors, and become of progressively less importance as the size of the sample is increased, whereas biases of the type discussed in the paper are independent of the sample size, and therefore become of progressively greater importance as the size of the sample is increased.

Apart from this, as Professor Allen points out, Hansen and Hurwitz are merely applying the principles of stratification, sub-sampling, and the use of the variable sampling fraction, that are outlined in the present paper. The main interest in their paper, I think, lies in the fact that they have evaluated and reported the actual gains obtained by the application of these principles to a particular type of material. Indeed I feel that their elaborate generalised mathematical statements tend to obscure rather than clarify their numerical results.

Mr. Gabor's difficulty in the study of the financial results of type groups of farms does not seem

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to be primarily one of sampling. If the type groups initially chosen are so small that a sample of 80% or more of each group is required to provide the necessary accuracy then there can be little point in sampling at all, though the question does arise whether it might not be better to use larger type groups, and sample less intensively within each group. In any case, if only one third of the farmers approached are of the type that will give the information required, it would appear that no study on these lines can be regarded as representative of conditions operating on all farms within the group. It cannot be too strongly emphasised that, if the necessary information cannot be obtained for certain elements of a population under investigation, then no amount of statistical ingenuity will provide us with information which is certainly representative of the whole population. Other evidence may indicate how far the excluded part of the population is similar to the remainder, and it is sometimes possible to make some allowance for lack of similarity, *e.g.*, by the use of regression on a control variate, but the basis of such an allowance must always be somewhat arbitrary and speculative, and its determination is not really a sampling problem.

As I indicated in my paper, the question of balancing is still under investigation, but the method of substitution there proposed should, I think, work satisfactorily even with high sampling fractions, provided the number of units in the sample is not too small.

The reason why the correction factor $\sqrt{1-f}$ should not be included when testing the significance of the difference of the means of samples from two groups, is that in such a case we usually wish to test whether there are any underlying causes which affect all the individuals of one group and not the other, *i.e.*, whether both groups can be regarded as random samples from the same (hypothetical) infinite population.

I am interested in Mr. Zentler's remarks on market research. It is not a field with which I am personally acquainted, but if the situation is as he states, then there is clearly room for improvement.

Mr. Semple is, I am afraid, asking for the impossible. If the population to be sampled cannot be divided into reasonably uniform strata according to some already known classification, then it is necessary to accept the higher variability which lack of such strata implies.

In answer to Mr. Kendall's first point, I did not wish to imply that the term stratification should be limited to the case in which an equal fraction is taken from each stratum; indeed I dealt with the more general case in the section on the variable sampling fraction.

I fully agree with Mr. Kendall's general conclusion on balanced sampling, which is confirmed by Mr. Anscombe and Mr. Quenouille's written contribution. The latter contribution demonstrates that balanced sampling cannot be more than an approximate process, which will not be satisfactory when applied to very extreme types of distribution. In Mr. Kendall's example, however, in which 1,000 members have a value — 1 and one member has a value of 1,000, no method of sampling can be satisfactory. Such populations can only be dealt with by prior stratification and use of the variable sampling fraction.

Mr. Nair has mentioned a number of practical problems that arise in crop estimation, and I am interested to hear that experience in India is similar to that in this country and elsewhere.

Dr. Hartley's method of dealing with the difficulty of "calling back" in social surveys is an ingenious one, but I am not sure whether it would be very satisfactory in practice. It has, moreover, the objection that the information for houses which are usually empty is of relatively lower precision than that for those which are usually occupied. I have been told that it is not really very difficult in the majority of cases to find out from neighbours, etc., when the occupant of a certain house is likely to be in. But once any calling back at non-random times is resorted to Dr. Hartley's method breaks down.

An alternative method of reducing the total amount of calling back is to sub-sample the houses in which say more than one call back is required, selecting say 50 per cent. of such houses for further calls, subsequently doubling the contribution of this 50 per cent. to the total sample. There are, of course, always likely to be a few houses from which it is impossible to obtain the required information, either because of failure to meet the occupant, or for other reasons, and such lack of information must be accepted. In any case, Dr. Hartley's suggestion would not get over this difficulty.

I am not sure that I agree with F./Lt. Buckland's suggestion on the use of the words "census" and "survey." The term census, I believe, originally connoted the simple counting of human populations, and has since been extended to cover the assessment of quantities of various kinds, *e.g.*, census of production, census of woodlands. The term survey, usually connotes some more general type of investigation, though such investigations of course often involve enumeration and quantitative assessment. Sampling may be used in either type of investigation.

I do not think that the difficulty of ensuring the co-operation of the public in sampling censuses and surveys can be solved by any change of nomenclature. Indeed, there is no valid reason why if the furnishing of information is compulsory in a complete census, the furnishing of the same information should not also be made compulsory for a similar sample census. If the recent "Family Census" has suffered in any degree from refusals to furnish information, a large share of the blame must be attached to certain irresponsible sections of the Press, which conducted what can only be regarded as a scurrilous campaign against the census. Had the census been compulsory such a campaign would not have been possible.

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As a result of the ballot taken during the meeting, the candidates named below were elected Fellows of the Society:—

Owen John Beilby.
Jack Edric Blundell, M.Com.
Kathleen Sarah Bodkin.
Pleasance Clara Bray.
Eric Arthur Cheeseman.
Roland David Clarke, F.I.A.
John Farquhar Conn, D.Sc., M.Inst.N.A.
Leslie Bennet Craigie Cunningham.
Thomas Leonard Drinkwater.
Philip Derek Jessel Druiff.
Archibald William John Dymond.
Johan Hendrik Enters.
Zahurul Hassan Sharib, M.A., Ph.D., LL.B.
Philip Heber Hull, B.A.
William Robert Brough Hynd.
James Ronald Illingworth, B.Sc.
William Joseph Jennett.
Popatlal Dahyabhai Kora.
John Longden.
Philip Lyle.
James Frank Lyne.
Byomkes Majumdar, B.Sc.
John Isaac Mason, M.A.
S. T. Merani, Ph.D.
George Morgenstern, B.Sc.
Corneles Albert Lloyd Myburgh.
John Thomas Armstrong Polwart.
Devarakonda Venkata Rajalakshman.
Joseph Rose.
Osmond M. Royes.
Joseph Safkin.
You Poh Seng, B.Sc.
Dorothy Ruth Shanahan.
Nina Shooter, B.Sc.
Jan Sittig.
David Hammond Smith.
Gwyn Owen Stephens.
George Tomkins.
Sydney Walter Twine.
T. V. Viswanathan, M.A., B.Sc.
John Percy Hartley Walton, B.Sc.
Eric William Watson.
John Harold West.
William Idris Williams.

Corporate representatives.

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Cumulative Sampling: a Speculation as to What Happens in Copying Manuscripts

By G. Udny Yule, F.R.S.

1. In his work on the palæography of the *De Imitatione Christi* (2) Mgr. Puyol discussed a number of the manuscript and early printed texts on the basis of the variant readings recorded in the volume entitled *Variantes* (1) issued in the same year. The texts considered, listed on pp. vi and vii of the *Variantes* and on pp. 266, 267 of the *Paléographie*, number 67 in all and, leaving aside a group of 11 hybrids, can be divided into a Class A of 22 texts of Italian origin, all copied in Italy, and a Class B of 34 texts of Transalpine origin.* The texts of the respective classes present a certain family likeness to each other, and Puyol lists (2, pp. 80–107) 364 pairs of distinctive readings by which the class of a text may be distinguished. Of these only a single pair is drawn from Lib. I, so that the conclusion as to two classes hardly applies to that book: but there are 118 pairs of distinctive readings for Lib. II, 208 for Lib. III, and 37 for Lib. IV.

2. The problem obviously arises which of these two Classes, the Italian or the Transalpine, better termed Teutonic for an English reader, the more closely represents the primitive text, the original prototype, if such a prototype exists or has ever existed. For Lib. I special argument is necessary (2, pp. 274 *et seq.*), as the texts can only be analysed into five Genera and an isolated (Italian) Species. Puyol finally concludes that this isolated species (containing only the two MSS. *Aronensis* and *Ravennatensis*) most closely represents the prototype. But this special argument does not present any statistical interest.

3. When he turns to the remaining books, II, III and IV, however, which present the division into two classes, the argument, oddly enough, does become statistical, though the author does not recognize the fact. He has no shadow of doubt, moreover, as to the correctness of his argument and his conclusion: indeed, he seems to think the conclusion intuitively obvious. The matter being important, I had better give his own words. After a short introductory statement to the effect that the division of the texts of Lib. II into two Classes A and B absolves him from the necessity of proceeding step by step from the particular to the general, as he has had to do in the preceding argument about Lib. I, he continues as follows (2, p. 293):—

> Les Variantes des deux Classes A et B sont assez nombreuses, pour permettre d'établir une comparaison décisive. On peut, du premier coup, reconnaître quelle est, des deux Classes, l'archétype de l'autre.
>
> C'est la Classe A. Les textes Transalpins proviennent des textes Italiens.
>
> On s'arrêtera à la démonstration de ce fait capital, parce que cette démonstration servira aux livres III et IV, qui, à peu de chose près, sont identiques, dans leurs éléments, au livre II.
>
> *La Classe A est antérieure à la Classe B.*
>
> Si les corrections qui ont déterminé les Différences entre la Classe A et la Classe B ont été opérées sur un texte Italien, il doit en résulter qu'on ne doit pas rencontrer trace de ces corrections dans les textes Italiens. La raison en est que les textes Italiens, étant supposés antérieurs, ne peuvent reproduire des textes qui, selon l'hypothèse, sont postérieurs. Si, par exemple, les textes Italiens 1, 16 donnent la leçon, *in domino*, et si, postérieurement, les textes

* These are the numbers from *Paléographie*, p. 266. The numbers in lists on p. 79 of the same work are 22, 32, 9, but *p*5, *s*5, *x*2 and *x*4 have been omitted. In line 5 of the Transalpine list on the same page there are two MSS. listed as *q*1: the Paris MS. should be *q*2. Such errors and discrepancies are painfully frequent.

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Transalpins ont introduit en correction, *in deo*, il en résulte que cette dernière leçon ne peut se rencontrer dans les textes Italiens qu'à l'état d'anomalie.

Je dis anomalie. Car il peut bien arriver que des copistes, par distraction ou autrement, écrivent un mot pour un autre, surtout quand il s'agit de synonymes. . . .

The theorem is in brief: "if A represents the more primitive class of texts more A-readings will be found in B-texts than B-readings in A-texts," and examples are then given to show that this is in fact the case.* Puyol's theorem, if true, is evidently important, for it is of general application; and it is a statistical theorem.

4. But is his argument valid? It seems to me to make two tacit assumptions, the first of which is certainly wrong, and the second highly unlikely. In the first place, primitive character of text is treated as equivalent to earliness of date of the MS., whereas there is obviously no necessary connection between the two. A late MS. may be merely a late copy of a primitive text; date and primitiveness must be kept carefully distinct. In the second place, it appears to be assumed that after the first Italian text had been exported over the Alps, to be edited into a Teutonic text, all copying of texts in Italy abruptly ceased. For if not, what would be likely to happen? Surely after the first text had arrived in "Transalpinia" and been duly edited, as assumed, others would follow it. A return trade would then start from Transalpinia to Italy, into which a number of the new Teutonic texts would be imported. A copyist in an Italian monastery might then have before him simultaneously an Italian text and a Teutonic text, and there is no apparent reason why he should not have introduced any number of Teutonic readings into the new text that he was making. Puyol's argument seems to break down at once.

5. Let us consider the case quite generally, but make our basic assumptions more definite, for the argument seems to me to be rendered obscure by Puyol's failing to present any clear scheme of what he supposes to happen. Suppose that a primitive A-text develops within its own closed area, the A-enclave, producing in the ordinary way more or less varying versions by the carelessness of copyists or the whims of editors. A rather abnormal version of this text, due to slovenly copying or to frequent amendments by a critical editor, then escapes immediately after production into another enclave separated from the first by a wall that is normally impassable, and there develops a series of B-texts. If, after the period of separate development necessary for the creation of two distinct classes, the dividing wall between the A-enclave and the B-enclave is broken down, what will happen? Surely the answer is so dependent on the conditions existing that no definite or confident reply can be given. Indeed, the one thing that is clear is that happenings will be in no way dependent on the relative ages or primitiveness of the respective texts. If traffic is symmetrical, A-copies passing into the B-enclave and B-copies into the A-enclave in approximately equal numbers, and if other conditions are about the same, then "penetration" of A-texts by B-readings and of B-texts by A-readings will be about the same. But if conditions are not symmetrical, if A-copyists have a stronger prejudice against B-readings than B-copyists have against A-readings, or if more A-texts are imported into the B-enclave than vice versa, this symmetry will be upset. Moreover, some B-readings in some A-texts may not represent "penetration" at all. Some B-readings may well have existed in some A-texts before that ancestral B-text was exported to its new enclave. Quite clearly Puyol's theorem is not valid. There is no reason to suppose the phenomenon observed indicates the more primitive character of the A-text.

6. Indeed, a prolonged and detailed study of the pairs of critical readings led me very definitely to the conclusion that the Teutonic, not the Italian, family of texts represents the more primitive form; † a conclusion in itself the more probable as it is consistent with the traditional authorship of Thomas à Kempis. This study, moreover, brought me gradually to an entirely different

* The examples seem to me to require checking: I have some difficulty in reconciling some of the tables with the data in the *Variantes* volume. Moreover, the author does not, as he says he does on (2) p. 294, direct his examination "uniquement sur les Variantes qui constituent la division des deux Classes A et B." In the little table immediately following this statement only two of the four instances (the first and last) occur in the list of distinctive readings for Lib. II on pp. 81 *et seq.* In the table at the top of p. 296 eight of the thirteen instances are not in the list of distinctive readings. In what follows I have, however, assumed that the general conclusion may be trusted: that you do, in fact, find more A-readings in B-texts than B-readings in A-texts. Any statistician will see a difficulty looming ahead, and will want to know where the line was drawn between an A-text and a hybrid or a B-text and a hybrid. Puyol implies that the families are "natural" families.

† I hope to publish this evidence later.

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conception of the possible mode of creation of two distinct families of texts in the MSS. of a given work.

7. Puyol's view of the mode of formation of the Teutonic text implies that the changes made from what he regards as the primitive text were careful and conscious changes made by a thoughtful editor. But writers on textual criticism seem to be generally agreed that scholarly and critical editors of this kind were rare, and that copyists were in the main stupid and careless. A study of the distinctive A and B readings tabulated by Puyol amply confirms this opinion. Many changes suggest mere whim or possibly momentary inadvertence. Others suggest complete insensitiveness to style, the change made wrecking that assonance of two consecutive phrase-endings which is so characteristic of the *Imitatio*, or destroying the carefully constructed symmetry of a sentence. Other differences, again, are quite clear examples of carelessness—instances of what is termed *haplography*, for example, where the eye of the copyist has slipped on from one occurrence of a word to a second occurrence of the same word a little farther on, and he has omitted everything between, or of *dittography*, that is, repetition of a word or words from the preceding or following context. A large element of chance must enter into all such changes. We might regard the MS. to be copied as presenting a series of danger-points for the copyist, at some of which he is much more likely to make a change than at others. And at such danger-points, moreover, two copyists may differ considerably in their reactions, and the one may be much more likely to make a change than the other. At points 1, 2, 3, . . . the chance of the first making given alterations may be p_1, p_2, p_3 . . ., of the second making the same alterations $2p_1$, $2p_2$, $2p_3$ . . ., or the ratios of the chances may vary from point to point. If the copyist makes a series of *successive* copies, copy 1 from the prototype, copy 2 from copy 1, copy 3 from copy 2, and so on, the copy of the rth generation will tend to get more and more full of changes as r increases, the most probable alterations occurring mainly in the earlier copies, the less likely changes increasingly frequent in the later ones.

8. And I think we may go a little farther than this, and regard the changed readings as probably more stable—less liable to further change—than the primitive text. That this is certainly so in some cases is obvious: where haplography has led to an error, that same error cannot be committed again at the same point, for the text is now left with only a single occurrence of the dangerous word. But in all cases there must be some cause to suggest a change or bring about an alteration, and in so far as the particular copyist is a normal specimen of his class, the change made will in general tend to be passed by others, and so to be more stable: it would be going too far to say the change would be *approved* by others, for it is unlikely that conscious approval would come into the matter at all. In particular, the readings surviving after a prolonged series of successive copyings, such as is supposed above, may well be conspicuously more stable than the primitive text, by that stage worn down to a dead level of dullness with little left to irritate the stupid or entrap the careless, just as a beautiful and varied mountainous country is at last worn down to the dullness of a peneplain. The degraded text has then practically reached its final form, and further copyings will have little effect. A number of such final forms of the text that have passed through different lines of copyists will nevertheless tend to be very closely alike, for, on the average, one line of copyists is likely to resemble another, and the changes made in each line will tend to be changes conditioned by much the same psychological characteristics. If we sort out the changes that are common to all or to the great majority of the degraded texts, these will be the most probable changes, and may give some indication of the psychology in question, but special experiments made to throw light on the subject might yield very interesting results.

9. The reader no doubt by this time sees the conclusion to which my argument tends. Apart from genuine editorial emendations or some form of accident or carelessness which leads, say, to loss of part of the text, and so to the formation of special genera or species, there will always be a tendency to the creation of two fundamental classes: (1) a Class A fairly closely adhering to the prototype, the creation of exceptionally careful copyists and even so unlikely to be represented by any MS. removed by more than one or two generations (of copying) from the prototype itself; (2) the other, Z, the final degradation-product. The members of Z will present a closer family resemblance to each other than the members of A, for in Z practically all the unstable readings of A will have been eliminated, whereas copies of the prototype will have been made by copyists of varying degrees of carefulness, and it may well be a problem where the line is to be drawn that is to separate a bad copy from Class A and place it in the class of hybrids. Manuscripts of the A-class will present far more Z-readings than texts of the Z-class will present A-readings, and so on the

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conceptions here developed Puyol's Class A of Italian texts should clearly be labelled Z, not A: it represents, not the primitive text, but its ultimate degradation.

10. It may help to clarify the nature and illustrate the consequences of my hypothesis if we consider a parallel in artificial chance in which the conditions are greatly simplified. Let us suppose a board to be provided with n numbered squares or cells, and a disk, black on one side and white on the other, to be placed in each cell, white side uppermost. The experimenter then goes over all the cells in order, one by one, and for each cell determines by some chance-process (the toss of a coin, the throw of a die, or the use of a table of random numbers) whether or no he will turn the disk over so as to show the black side uppermost. For the greatest possible simplicity we will suppose the probability of the decisive chance-event to be the same throughout, say p. When the first *traverse* of the board, as I shall term it, has been completed, a certain number of the cells, say m_1, will show black uppermost, corresponding to a new reading, while the rest still show white. The experimenter now proceeds to a second traverse of the board, but this time he need pay attention only to the cells still showing white disks, for (again to simplify conditions to the greatest possible extent) we will assume that a disk once turned over can neither be turned back again to white, nor turned to any colour other than black; *i.e.*, that new readings are not merely more stable than primitive readings, but absolutely stable and not subject to further chance and change. On completion of the second traverse a further number of disks, say m_2, making a total of $t_2 = m_1 + m_2$, will show black, this result corresponding to a copy of the second generation with t_2 new readings. So the game continues, the number of disks showing black after the rth traverse being say

$$t_r = m_1 + m_2 + m_3 + \ldots + m_{r-1} + m_r \quad \ldots\ldots\ldots \quad (1)$$

Here the increments m tend to decrease as r increases, at first rapidly, and then more slowly as t_r approaches the limit n, the number of cells. In this stage the value of m_r may obviously be zero on occasion. It is t_r rather than m_r which interests us most, for t_r corresponds to the total number of altered readings in the rth successive copy of the manuscript: it is t_r which conditions the appearance of the board after each traverse. The problem is to find the expected values of the mean and variance of t_r in this process of what may be termed "cumulative sampling."

11. Let p and q, as above, be the chances of success and failure of the decisive event, n the number of cells. The solution of our problem is rather complicated if we tackle it directly, but the answers can be written down at once if we look at matters from the standpoint, not of t_r, but of its complement $n - t_r$, the number of cells in which the disks have not been moved after the rth traverse and still show white. The chance of the disk in any one cell not being moved in r traverses is q^r, hence the expected number of undisturbed disks is $q^r n$, or of disks that have been turned over $(1 - q^r)n$, that is

$$\text{expected mean } (t_r) = (1 - q^r)n \quad \ldots\ldots\ldots \quad (2)$$

For the variance of $n - t_r$, which is identical with the variance of t_r, we similarly have at once

$$\text{expected variance } (t_r) = q^r(1 - q^r)n \quad \ldots\ldots\ldots \quad (3)$$

the distribution being a binomial distribution with constants q^r, $1 - q^r$, and n. It may be as well to emphasize the highly simplified conditions assumed. In terms of the statistical "game" the assumptions are: (1) The chance p of the success of the decisive event is the same on every occasion; (2) a disk once turned over cannot be turned back again, nor can it be altered to any colour other than black. In terms of the copying of manuscripts, to which the experiment offers an analogy, it is assumed that (1) the chance of a reading being altered does not vary indefinitely from point to point of the text, but has a finite value p at n points and is zero at all other points, so that we have only two classes of loci in the text, alterable and unalterable; (2) when a reading has been altered, the new reading is unalterable, instead of being merely rather more stable than the primitive reading.

12. In Table 1 are given the expected mean values of t_r/n; *i.e.*, $1 - q^r$, for certain values of q and values of r from 1 to 10 by units, and thence by steps of 5 units to $r = 50$. When p is as small as 0·05, and q consequently 0·95, t_r rises comparatively slowly, and even after ten traverses is no more than 0·4013; by the fiftieth traverse the total of disks turned is a little over 92 per cent. of the whole—but it may be doubted whether in the process of copying manuscripts anything like the fiftieth "generation" would ever be reached. When $q = 0{\cdot}9$ matters proceed a little faster. After the tenth traverse over 65 per cent. of the disks will, on the average, have been turned, and a

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TABLE 1

Giving the expected mean values of t_r/n for certain values of q. Decimal points have been omitted; read 0500 *as* 0·0500 *and so on. A dash means "greater than* 0·99995*"*

Traverse number r	Value of q					
	0·95	0·9	0·8	0·7	0·6	0·5
1	0500	1000	2000	3000	4000	5000
2	0975	1900	3600	5100	6400	7500
3	1426	2710	4880	6570	7840	8750
4	1855	3439	5904	7599	8704	9375
5	2262	4095	6723	8319	9222	9687
6	2649	4686	7379	8824	9533	9844
7	3017	5217	7903	9176	9720	9922
8	3366	5695	8322	9424	9832	9961
9	3698	6126	8658	9596	9899	9980
10	4013	6513	8926	9718	9940	9990
15	5367	7941	9648	9953	9995	—
20	6415	8784	9885	9992	9999	—
25	7226	9282	9962	9999	—	—
30	7854	9576	9988	—	—	—
35	8339	9750	9996	—	—	—
40	8715	9852	9999	—	—	—
45	9006	9913	—	—	—	—
50	9231	9948	—	—	—	—

figure of over 95 per cent. will have been attained by the thirtieth traverse. In the next column, for $q = 0{\cdot}8$, the change is even more marked: t_r has reached 67 per cent. after the fifth traverse, against 65 per cent. after the tenth traverse in the preceding column, and a figure over 95 per cent. is attained by the fifteenth traverse. Passing on to the last column, for $p = q = 0{\cdot}5$, half the disks on the average will be turned after the first traverse, 75 per cent. after the second, and a figure of over 95 per cent. is attained after the fifth. With a copyist of a degree of carelessness represented by the small figure $p = 0{\cdot}05$, his first copy will presumably be classed as Class A, seeing that it has only 5 per cent. of Z-readings, but even his second (successive) copy may be classed as hybrid.

TABLE 2

Giving the expected values of the standard deviation of t_r, divided by $\sqrt{n}$, for certain values of q. Decimal points have been omitted; read 2179 *as* 0·2179 *and so on*

Traverse number r	Value of q					
	0·95	0·9	0·8	0·7	0·6	0·5
1	2179	3000	4000	4583	4899	5000
2	2966	3923	4800	4999	4800	4330
3	3497	4445	4999	4747	4115	3307
4	3887	4750	4918	4271	3359	2421
5	4184	4917	4694	3739	2678	1740
6	4413	4990	4398	3222	2109	1240
7	4590	4995	4071	2749	1650	0880
8	4725	4951	3787	2331	1285	0624
9	4827	4872	3409	1968	0999	0441
10	4902	4766	3096	1657	0775	0312
15	4987	4044	1842	0687	0217	0055
20	4796	3268	1068	0282	0060	0010
25	4477	2581	0613	0116	0017	0002
30	4106	2015	0352	0047	0005	0000
35	3722	1562	0201	0019	0001	0000
40	3347	1207	0115	0008	0000	0000
45	2993	0865	0066	0003	0000	0000
50	2665	0716	0038	0001	0000	0000

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After that *all* his copies will be classed as hybrids: he is hardly likely to produce a chain of successive copies as long as 40 or 50 that will yield a Z-text with 95 per cent. or more of new readings. If no such chain of successive copies is likely to be longer than 5 to 10 generations—and surely much larger figures are improbable?—the copyist must have a degree of carelessness represented by $p = 0{\cdot}3$ to $0{\cdot}5$ or even more ($q = 0{\cdot}7$ to $0{\cdot}5$ or less) in order to produce a Z-text, taking 5 per cent. as the permissible limit of the proportion of A-readings that would separate a hybrid text from a Z-text. But copyists of this degree of carelessness are incapable of producing an A-text at all: even their first copies are likely to show 30 to 50 per cent. of Z-readings, and a copy with less than 5 per cent. can hardly occur even as an extreme fluke. A-texts and Z-texts are therefore likely to be produced by different copyists. In a chain of copies terminating in Z-texts, careless copyists ($q = 0{\cdot}7$ or less) must have intervened somewhere if the chain is not to become of improbable length. We have thus learnt some practical lessons even from this table of theoretical means.

13. In Table 2 are given, for the same values of q and the traverse-number, the values of the expected standard deviations of t_r divided by $\sqrt{n}$; *i.e.*, $\sqrt{q^r(1-q^r)}$. I have tabulated this figure rather than the variance, as it indicates the order of magnitude of the range of scatter round the mean in Table 1. The course of the standard deviation with increasing r is very interesting. In the first column we see that the s.d. attains a maximum somewhere in the neighbourhood of $r = 15$; in the second column the maximum is at $r = 7$; in the third at $r = 3$; in the fourth ($q = 0{\cdot}7$) at $r = 2$. In the column $q = 0{\cdot}6$ the maximum is at $r = 1$, though the figure for $r = 2$ is not much less, and in the following column for $q = 0{\cdot}5$ the standard deviation for $r = 1$ is quite considerably greater than the following figure for $r = 2$. In fact, we have

$$\text{exp. var. } (t_{r+1}) > \text{exp. var. } (t_r)$$

if

$$q^{r+1}(1-q^{r+1}) > q^r(1-q^r):$$

that is if

$$q^r - q^{r+2} > 1 - q,$$

or

$$q^r(1+q) > 1,$$

or

$$r < -\frac{\log(1+q)}{\log q} \qquad \ldots\ldots\ldots\ldots \quad (4)$$

Taking $q = 0{\cdot}95$, we find this condition to give $r < 13{\cdot}02$, or if $r = 13$, the variance after the next traverse is slightly increased: calculation of the actual variances shows, in fact, that the standard deviation of t_{14} is greater than that of t_{13}, but only in the fifth place of decimals. It is evident from the run of the figures that there must be a critical value of q between $0{\cdot}7$ and $0{\cdot}6$ which will bring the maximum just to $r = 1$; that is to say, make the variance at $r = 1$ identical with the variance at $r = 2$. The condition is

$$q(1-q) = q^2(1-q^2)$$

which leads to the quadratic

$$q^2 + q = 1,$$

and this gives the solution $q = 0{\cdot}618034$. It may be remarked that, while the variance is small when q approaches either limit, unity or zero, the first case is not of much practical importance, for it can only occur when p is *very* small—a good deal smaller than our lowest value tabulated—and r is unity or not much greater than unity. But the second case will occur whenever r becomes sufficiently large, whatever the value of q. The coefficients of $\sqrt{n}$ in Table 2 are all less than $0{\cdot}1$ at the bottom of every column after the first, and in the last two columns less than $0{\cdot}00005$.

14. *Experiment in Cumulative Sampling*, I. It seemed to me that it would be of interest to carry out an experiment, or more than one experiment, to illustrate the preceding theoretical conclusions and to throw further light on the form of the frequency distributions of t_r when a number of successive samplings have been completed up to any stage desired—any assigned value of r. In the first instance I decided to make $p = q = 0{\cdot}5$, $n = 100$; to complete each sampling up to $r = 5$; and to make 100 samplings up to this stage. The Random Sampling Numbers of Kendall and Babington Smith were used, digits 0 to 4 denoting success of the decisive event and digits 5 to 9 a failure. Digits in the four columns of the first block of each thousand were held to apply to the first traverse, of the second block to the second traverse, and so on; consequently only the first five blocks of each thousand were used, and the hundred samplings were each made from a

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TABLE 3

Showing for a cumulative sampling experiment, $p = q = 0{\cdot}5$, $n = 100$, *number of samplings* $N = 100$, *the frequencies of given values of* t, *at each of the first five traverses—e.g., after the first traverse three samplings gave only* 39 *disks turned, but one gave* 60: *the expected number* 50 *occurred only four times, but* 51 *and* 52 *each occurred* 12 *times*

Traverse number									
1		2		3		4		5	
t	f_t	t	f_t	t	f_t	t	f_t	t	f_t
60	1	84	1	94	2	100	—	100	5
59	2	83	2	93	3	99	2	99	10
58	4	82	1	92	3	98	4	98	23
57	2	81	1	91	6	97	7	97	22
56	2	80	4	90	11	96	10	96	17
55	3	79	6	89	5	95	12	95	13
54	5	78	9	88	7	94	14	94	4
53	8	77	6	87	13	93	18	93	3
52	12	76	12	86	17	92	12	92	3
51	12	75	8	85	9	91	9	—	—
50	4	74	15	84	7	90	4	—	—
49	6	73	6	83	7	89	5	—	—
48	8	72	4	82	4	88	1	—	—
47	4	71	6	81	2	87	—	—	—
46	5	70	5	80	2	86	2	—	—
45	3	69	5	79	—	—	—	—	—
44	5	68	1	78	—	—	—	—	—
43	4	67	2	77	2	—	—	—	—
42	4	66	1	—	—	—	—	—	—
41	2	65	—	—	—	—	—	—	—
40	1	64	4	—	—	—	—	—	—
39	3	63	1	—	—	—	—	—	—
Total	100	Total	100	Total	100	Total	100	Total	100

different table. Table 3 gives the resulting frequency distributions. After the first traverse the expected value of t is 50, but it will be seen that the actual values ranged from 39, which occurred three times, up to 60, which occurred only once. After the second traverse the expected value of t is 75, but observed numbers ranged from 63, just clear of the maximum for the preceding distribution, up to 84. For the third traverse the range is 77 to 94, against an expectation of 87·5; for the fourth a range of 86 to 99, against an expectation of 93·75; and for the fifth a range of 92 to 100, against an expectation of 96·875. If we were dealing with copying manuscripts instead of with this game, no one, I think, would be willing to accept copies with 39 altered readings out of a maximum of 100 as anything but hybrids, and these are the lowest figures occurring in the very first generation of copies.

15. Table 4 gives the means and standard deviations, expected and observed. Looking at the means first, we see that all five observed values are below expectation, but too much importance

TABLE 4

Showing the means and standard deviations of the distributions of Table 3, *compared with the expected values*

Traverse number r	Means		Standard deviations	
	Expected	Observed	Expected	Observed
1	50	49·61	5·00	4·95
2	75	74·27	4·33	4·38
3	87·5	86·71	3·31	3·45
4	93·75	93·47	2·42	2·65
5	96·875	96·76	1·74	1·82

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must not be attached to this, for the series of observed means is evidently a correlated series. Suppose the number observed at the first traverse is 40 instead of 50; then the expected increment at the second traverse will be $0{\cdot}5 \times 60$ or the expected value of t_2 40 + 30 = 70, instead of 75—again below expectation. In these circumstances I hesitate to give any decided judgment, but on the whole the agreement between expectation and observation does not look to me bad, in spite of the magnitude of the differences at the second and third traverses. Turning to the standard deviations, the first observed figure is a little below expectation, all the others above, but all differences are very moderate. We again have to deal with a correlated series, for the scatter of t_r will be dependent on the scatter of t_{r-1}, but the agreement looks to me quite good.

16. *Experiment in Cumulative Sampling*, II. For the second experiment I took $p = 0{\cdot}1$, $q = 0{\cdot}9$, $n = 100$, but made 100 samplings up to $r = 10$ instead of $r = 5$. Zeros in the tables of sampling numbers were taken to denote success of the decisive event, otherwise the work was carried out as before, with the exception that all ten blocks of each thousand-table had to be used. Table 5, Parts I and II, and Table 6 give the results in the same form as before. The differences from the preceding example are quite striking. Even in Table 5 it is clear to the eye that the scatter does not steadily and rapidly decrease, as in Table 3, but increases from the first to the third or fourth traverse, after which there is no very obvious alteration. The observed standard deviations, given in Table 6, actually reach a maximum at traverse 4, and there is a fluctuating fall from that point to traverse 10, but the fall is not sufficiently emphatic to stand out in Table 5. The figures for expected values show that the maximum ought to be at traverse 7, with 6 and 8 pretty close to the same figure. All the observed standard deviations are below expectation, the difference being greatest, 0·65, for traverse 10. As regards the means, only the figure for the third traverse is above expectation, all the others being below, but the differences are more moderate than in Table 4. It may be noted that, if we assume that only a copy of a manuscript with not

TABLE 5

Showing for a cumulative sampling experiment, $p = 0{\cdot}1$, $q = 0{\cdot}9$, $n = 100$, number of samplings $N = 100$, the frequencies of given values of t_r (cf. heading of Table 3) for each of the first ten traverses

Part I. Traverses 1 to 5

Traverse number									
1		2		3		4		5	
t	f_t	t	f_t	t	f_t	t	f_t	t	f_t
22	—	29	1	37	1	44	3	52	1
21	—	28	—	36	2	43	2	51	—
20	—	27	—	35	2	42	1	50	1
19	—	26	2	34	2	41	1	49	—
18	1	25	5	33	3	40	4	48	4
17	—	24	3	32	4	39	7	47	5
16	1	23	3	31	6	38	4	46	5
15	2	22	9	30	9	37	7	45	3
14	5	21	12	29	9	36	8	44	8
13	7	20	9	28	10	35	16	43	7
12	9	19	11	27	8	34	9	42	10
11	18	18	11	26	12	33	4	41	10
10	13	17	9	25	7	32	8	40	11
9	16	16	6	24	5	31	6	39	7
8	12	15	7	23	6	30	4	38	1
7	10	14	4	22	5	29	6	37	7
6	5	13	5	21	3	28	3	36	7
5	3	12	2	20	3	27	2	35	4
4	3	11	1	19	1	26	3	34	4
3	—	10	—	18	2	25	—	33	3
2	—	9	—	17	—	24	1	32	1
1	—	8	—	16	—	23	—	31	1
—	—	7	—	15	—	22	1	30	—
Total	100	Total	100	Total	100	Total	100	Total	100

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Part II. Traverses 6 to 10

Traverse number									
6		7		8		9		10	
t	f_t	t	f_t	t	f_t	t	f_t	t	f_t
58	1	62	1	68	2	71	1	74	3
57	—	61	1	67	—	70	2	73	2
56	—	60	2	66	2	69	2	72	1
55	3	59	4	65	3	68	2	71	1
54	3	58	5	64	—	67	4	70	5
53	5	57	6	63	2	66	5	69	6
52	3	56	1	62	2	65	5	68	7
51	4	55	9	61	7	64	5	67	9
50	6	54	8	60	5	63	12	66	15
49	6	53	7	59	10	62	7	65	7
48	7	52	9	58	6	61	6	64	9
47	10	51	5	57	12	60	8	63	9
46	9	50	11	56	9	59	14	62	9
45	13	49	5	55	10	58	11	61	4
44	6	48	9	54	10	57	4	60	6
43	8	47	7	53	5	56	4	59	1
42	5	46	6	52	6	55	4	58	2
41	3	45	2	51	1	54	1	57	—
40	3	44	1	50	5	53	1	56	2
39	2	43	—	49	1	52	—	55	1
38	1	42	—	48	1	51	—	54	—
37	1	41	1	47	—	50	1	53	—
36	—	40	—	46	—	49	—	52	1
35	—	39	—	45	—	48	1	51	—
34	1	38	—	44	1	47	—	50	—
Total	100	Total	100	Total	100	Total	100	Total	100

TABLE 6

Showing the means and standard deviations of the distributions of Table 5, compared with the expected values

Traverse number	Means		Standard deviations	
	Expected	Observed	Expected	Observed
1	10	9·73	3	2·74
2	19	18·94	3·92	3·60
3	27·1	27·20	4·44	4·13
4	34·39	34·29	4·75	4·48
5	40·95	40·77	4·92	4·36
6	46·86	46·57	4·99	4·41
7	52·17	51·95	5·00	4·34
8	56·95	56·75	4·95	4·33
9	61·26	61·05	4·87	4·26
10	65·13	64·98	4·77	4·12

more than five new readings out of the 100 is to be classed as A, not hybrid, Table 5, Part I, shows that six of 100 copies made direct from the original by a copyist with carelessness equivalent to $p = 0{\cdot}1$ might perhaps be so classed, if he happened to behave exactly as in this experiment; a pretty poor performance and just a chance at that.

I admit that the ideas developed in §§ 7–13 are highly conjectural, but think they deserve consideration.

References

1 Puyol, P. E. (1898). *Variantes du Livre De Imitatione Christi.* Paris, Retaux.
2 Puyol, P. E. (1898). *Paléographie, Classement, Généalogie du Livre De Imitatione Christi.* Paris, Retaux.
3 Kendall, M. G., and B. Babington Smith (1939). *Tables of Random Sampling Numbers.* Cambridge University Press.

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The Elementary Statistics of Majority Voting

By Professor L. S. Penrose

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1. *Introduction*

The arithmetic of voting has received surprisingly little attention from either politicians or sociologists, except from the purely actuarial point of view. Yet there are a few very simple and obvious statistical rules which can apply to decisions made by majority vote, whether the vote is actually taken or only implied potentially, and which deserve to be better understood. The equitable representation of political opinion is partly a question of statistical probability and, since stable and representative government, both on the national and federal levels, is probably essential to the well-being of mankind, sooner or later these matters must be properly investigated. In the present communication, some suggestions are made in the hope of stimulating further discussion of the subject.

2. *Decisions made by majority vote*

(a) *The power of a single vote*

The most elementary case is that in which a committee makes a decision by majority vote in favour of, or against, a given proposal. This method of procedure is almost universal in civilized human communities and may be said to be a basic democratic principle approved, for example, by all members of the United Nations. The fact that, in practice, a great many decisions are taken without formal voting does not alter the essential arithmetical basis. Typical cases where decisions are taken without actual voting are: (i) when a spokesman acts on behalf of the majority of a committee, (ii) when a majority acquiesces in a decision with the knowledge that a vote would not alter the result, and (iii) when agreement is a mere formality to confirm a decision already reached. The numbers obtained in actual voting, especially when voting takes place by a show of hands, are often influenced by the desire of voters to be on the winning side, so that the majority appears to be larger than it really is.

In a committee of three people one member will obtain the decision of his choice—that is to say, he will be on the winning side in 75 per cent. of the votings, if the other two members vote in a random manner. In a committee of five, the chance that one member will obtain the decision he wishes will be 11/16. In general, the power of the individual vote can be measured by the amount by which his chance of being on the winning side exceeds one half. The power, thus defined, is the same as half the likelihood of a situation in which an individual vote can be decisive—that is to say, a situation in which the remaining votes are equally divided upon the issue at stake. The general formula for the probability of equal division of n random votes, where n is an even number, approaches $\sqrt{2/n\pi}$ when n is large. It follows that the power of the individual vote is inversely proportional to the square root of the number of people in the committee. Thus the individual vote has just half as much power (in the sense here defined) in a committee of 400 as it has in a committee of twenty.

(b) *The power of a bloc vote*

The individual may feel flattered by the knowledge that his voting power does not diminish as rapidly as he might have expected, when the size of the electorate increases, but a phenomenon of more practical importance to him concerns the power of a bloc vote. If a committee or electorate

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consists of two sections, a "resolute" bloc and an "indifferent" random voting group,* a small "resolute" group of people, who always vote together can exercise a surprisingly powerful control over the whole committee. Thus, three resolute votes can control a committee of twenty-three to the same extent that one vote can control a committee of three. Furthermore, a bloc of twenty-three could control, again to the same extent, an electorate of over 1,000, and a bloc of 1,000 could likewise control the decisions of an indifferent electorate of over 2,000,000. These blocs have about a 75 per cent. chance of carrying the decision in their respective electorates, but, by increasing the size of the resolute bloc, any specified degree of control can be obtained. Blocs three times as great as those mentioned would carry the decisions they desired in nearly 96 per cent. of the situations encountered. A bloc of size $\sqrt{n}$ always can carry 84 per cent. of the decisions, when the indifferent group has n voters, and a bloc of size $\sqrt[x]{n}$ can carry $\frac{1}{2}(1 + a)$ decisions where a is the area under the normal probability curve as given in the usual tables. (See Table 1.)

TABLE 1.

Indifferent population	Percentages of decisions controlled by resolute blocs		
	84·1%	97·7%	99·9%
	Population of resolute bloc		
25	5	10	15
100	10	20	30
10,000	100	200	300
1,000,000	1,000	2,000	3,000
100,000,000	10,000	20,000	30,000
2,500,000,000	50,000	100,000	150,000

The important statistical fact which emerges from this discussion is the very high degree of control exercised by a comparatively small resolute group when the indifferent population is very numerous. The same degrees of control are obtained in two populations by blocs proportional in sizes to the square roots of the respective population numbers.

It is obvious that the successful control of voting by a small minority depends on the indifference of the remainder of the electorate. If the remainder breaks up into blocs of dimensions similar to those of the original resolute bloc, the power of this original bloc is lessened; the situation reverts approximately to that in which each bloc can be treated as an individual, and the population then behaves like a small committee with each bloc acting as one member.

(c) *Hierarchies*

The controlling power of an individual or of a small bloc can be increased by setting up a hierarchical structure such as is commonly found in authoritarian and semi-authoritarian systems—*e.g.*, in military, ecclesiastical and industrial organizations. The necessary condition for such increased control by a few is that the decision made by any sub-group shall be, at least temporarily, binding on all the members of that sub-group, so that each sub-group forms a bloc in relation to a larger population. Such a sub-group can, in its turn, be controlled by other smaller sub-groups in the same manner and, ultimately, a very small resolute bloc can control the decisions of millions of indifferent voters.

In many forms of hierarchical control, notably in armed forces, votes are not taken, and control is exercised by the potential majority. To obtain the most efficient results from hierarchical control, the basic population must be prevented from forming blocs of any kind and must be kept in a state of indifference. The recent German dictatorship, for example, was astute enough to realize the importance of breaking up even the apparently most harmless clubs and societies and of keeping the population, so far as was possible, ignorant of political issues.

* These terms "resolute" and "indifferent" were suggested by G. Beall.

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3. *Representation*

(a) *Groups of different sizes*

Once a decision has been made by majority vote, the action taken in consequence of this decision may be said to be in accordance with the wishes of all those voting in favour of it. Nevertheless, the same result could have been achieved if only the excess of voters, who constitute "edge" or the majority by which the vote was carried, had been in favour of the resolution and the remainder of the committee had been indifferent to the issue and had "paired" their votes. Thus, we can only be certain that a majority decision represents the views of the controlling "edge" of the voters. For example, if sixty people voted in favour of and forty against a resolution, only twenty people (those referred to here as the "edge") certainly attained the decision of their choice, because the remaining eighty votes could have been paired indifferently. If this principle is accepted, it follows that the degree of representativeness of a decision depends upon the magnitude of the "edge." In the same committee this quantity will vary each time a vote is taken, but over a long period it will obtain an average value which is determined, among other things, by the square root of the number of voters. When a committee of 100 makes a decision by majority vote, this decision is likely to represent the opinions of half (and not one quarter, as might have been expected) as many people as would a decision made by a committee of 400. This result is an illustration of a familiar statistical principle—namely, that the significance of any observation depends upon the square root of the number of cases upon which it is based.

The argument can be carried a step further, for, in most forms of government, a population elects one or more spokesmen to act on its behalf. The views of such a spokesman are likely to be representative of a larger proportion of the people in a numerically small electorate than in a numerically large electorate. If, for example, a spokesman represents the considered views of ten out of a committee of 100, he will represent the views of twenty, or only 5 per cent. of a committee of 400, when all conditions other than those relating to the size of the committee remain constant. Consequently, governments representing relatively small groups of people (*e.g.*, small nations) are likely to be more representative, in the sense that they are likely to act in accordance with the views of larger proportions of their populations, than governments representing relatively large groups of people. Conversely, small groups of people or small nations are less easily controlled by resolute fractions or minorities than are large groups or nations. If democracy is defined as a condition in which the government tends to carry out the wishes of the people, small nations, by virtue of their smallness, are likely to be more democratic than large nations.

A specially constructed hierarchical system of government, which concentrates power of decision into the hands of a very small bloc, can lead to a situation where the spokesmen for a nation are representative only of a very small proportion of the people's opinions. The proportion of the electorate represented by the opinions of the bloc which governs by hierarchical control becomes naturally smaller as the electorate becomes more extensive.

(b) *Groups of spokesmen*

By electing a spokesman, who is to take his place in an assembly of spokesmen units, a group reduces itself to a resolute bloc of one unit. Should a set of electorates, all of the same size, contribute one member each to an assembly, the situation is reduced to that of a committee of spokesmen in which the power of each member can be duly estimated from the size of the committee. In such a case each member of the original set of electorates is symmetrically and fairly represented in the committee. Important inequalities arise, however, when the contributing electorates differ in size from one another. For example, in a federal assembly of nations, would it be equitable for two nations of the relative sizes of China and Switzerland each to have one vote? On the other hand, would it be any fairer if the greater one had 100 times as many votes as the lesser, as would result from allotting voting power or membership in the assembly on a strictly "per capita" basis? The answer seems to be that the number of votes (or members) which each nation contributes to an assembly of spokesmen should be proportional to the number of people whose opinions each spokesman probably represents. The number of people represented by the spokesman of each electorate has been shown to be proportional to the square root of the number of people who can vote. A nation of 400 million people should, therefore, have ten times as many votes (or members) on an international assembly as a nation of 4 million people.

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4. *Suggestion for votes in a world assembly*

Ideally, every responsible human being should have equal power in a world assembly. This situation, however, can only arise in an assembly of nations of equal sizes, each enfranchized to the same degree. To award nations voting powers proportional to their populations would give the spokesmen for large nations too much power in relation to the numbers of their sponsors. On the other hand, if, as at present arranged, large and small nations have equal voting powers, the spokesmen for small nations are felt to be too significant, and artificial rules about the meanings of votes and vetoes have to be constructed to redress the balance.

Let us examine some of the consequences of basing the voting power in a world assembly upon the square root of the number of persons in each nation's electorate. In the first place, advantage would go to the nation with an extensive franchise. The necessity for some general rule to exclude children, the grossly illiterate and the insane from voting in every country would naturally follow. Secondly, though a nation could gain in voting power, if it grew or absorbed other nations, it would have to grow to four times its size in order to double its voting strength. Conversely, if a nation divided into four equally large independent administrations, which had common friendly interests, the new group of states would possess double the voting power of the old one, and it would be twice as representative of the people.

TABLE 2

Nation	Approximate potential electorate (pre-war figures in millions)	Voting power, *i.e.*, square root of potential electorate (in thousands)
(a) *Members of United Nations*		
U.K.	30·0	5·5
Canada	7·0	2·7
South Africa	5·7	2·4
Australia	4·0	2·0
New Zealand	1·0	1·0
India	235·0	15·3
U.S.A.	91·0	9·5
U.S.S.R.	71·0	8·5
Ukraine	13·9	3·7
White Russia	2·7	1·6
France	27·0	5·2
China	299·0	17·3
Belgium	4·4	2·1
Czechoslovakia	9·8	3·1
Denmark	2·4	1·6
Greece	4·1	2·0
Netherlands	8·0	2·5
Norway	1·9	1·4
Poland	20·0	4·5
Yugoslavia	9·3	3·1
Iraq	2·0	1·4
Iran	6·7	2·6
Syria	2·4	1·5
Abyssinia	7·7	2·8
Egypt	9·3	3·1
Liberia	1·3	1·1
Mexico	10·9	3·3
Guatemala	1·1	1·0
Cuba	2·5	1·6
Haiti	1·5	1·2
Argentina	7·0	2·6
Bolivia	2·0	1·4
Brazil	27·0	5·2
Chile	2·5	1·6
Columbia	5·7	2·4
Ecuador	1·3	1·1
Peru	3·7	1·9
Uruguay	1·3	1·1
Venezuela	2·0	1·4

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TABLE 2—*continued*

Nation	Approximate potential electorate (pre-war figures in millions)	Voting power, *i.e.*, square root of potential electorate (in thousands)
(b) *Other Nations*		
Austria	4·4	2·1
Bulgaria	4·0	2·0
Finland	2·4	1·6
Germany	42·0	6·5
Hungary	5·8	2·4
Italy	28·0	5·3
Portugal	4·0	2·0
Rumania	12·0	3·5
Spain	14·5	3·8
Sweden	4·1	2·0
Switzerland	2·7	1·6
Turkey	9·3	3·1
Afghanistan	4·1	2·0
Japan	56·0	7·5
Malaya	2·9	1·7
Arabia	1·0	1·0
Nepal	3·8	1·9
Siam	7·7	2·8
Tibet	2·0	1·4
Yemen	2·4	1·5
Morocco	3·4	1·8
Eire	2·0	1·4
Ceylon	3·5	1·9
Sudan	4·0	2·0
British West Indies	1·1	1·0
Total	1,162·2	202·1

Clearly, a lower limit to the size of a nation is required to stop indefinite subdivision. This limit would be determined by the maximum number of seats or votes thought suitable in the assembly. One vote per million electors would imply less than 2,000 for the maximal size of a hypothetical world assembly of 2,000 nations. The political world as at present constituted, with its predominance of large nations, would not require more than about 200 seats in an assembly built up as suggested here. Assuming that all participant nations were equally franchized, say up to two-thirds of their populations, we should get a world assembly something like that indicated in Table 2. Nations with electorates less than 1 million are excluded. The appropriate voting powers, based upon the *per capita* system and the square roots of the electorates, can be compared.

Each nation represented on the world assembly would, theoretically, be required to furnish a credible list of voters. Fortunately, errors in making up such lists will be less significant in large than in small electorates on the system here described.

5. *Summary*

The application of some elementary statistical principles to the problem of representative government leads to the conclusion that small electorates are less susceptible to control by resolute blocs and are likely to obtain more representative governments than large electorates. If factors other than numerical size are constant, the spokesman for any electorate represents a section of that electorate proportional to the square root of the total number of electors.

The suggestion is made that the voting power of each nation in a world assembly should be proportional to the square root of the number of people on each nation's voting list.

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Regulations of the Research and Industrial Applications Sections

The detailed regulations for the conduct of the Research Section and the Industrial Applications Section of the Society have now been finally approved by the Council, and the full text is printed below for the information of our readers.

Research Section

Organization

1. The Section shall be constituted and carried on subject to the Bye-Laws of the Society, and in accordance with these Regulations, made and subject to amendment by the Council of the Society. The Section shall be concerned with research into the theory of statistics and statistical methods and with the development of their applications.

Membership

2. Any Fellow of the Society and any other person who is approved by the Section Committee may become a member of the Section on making written application to the Section Secretary.

3. A Fellow of the Society shall not be required to make any payment for membership of the Section, but every other member shall pay an annual subscription of ten shillings, which shall be due in advance on January 1st, provided that any person who becomes a member and pays his initial annual subscription in the last quarter of any year shall not be required to pay any subscription in respect of the following year. Subscriptions shall be payable direct to the Hon. Treasurer of the Royal Statistical Society, 4, Portugal Street, W.C. 2.

4. If any member of the Section has not paid his annual subscription by June 30th, the Secretary of the Royal Statistical Society shall advise him and the Section Secretary in writing of the default, and if the arrears are not paid up by September 30th, the defaulter shall cease to be a member.

5. Payment of an annual subscription in respect of membership of the Section shall not confer on the member any of the privileges of Fellowship of the Society.

6. Members who are not in arrear with their annual subscriptions may withdraw from membership by giving written notice to the Section Secretary.

The Section Committee

7. Subject to these Regulations, all the affairs of the Section shall be directed by a Section Committee consisting of two Fellows nominated by the Council of the Society and ten members elected by the Section.

8. (*a*) Three of the ten members of the Section Committee elected by the Section shall retire annually and shall not be eligible for re-election for the period of one year. The Committee shall decide on the members to retire, and in doing so shall have regard to seniority on the Committee and degree of attendance.

(*b*) The Section Committee shall nominate members for election to the Committee for the following session, and these nominations shall be circulated to all members at least a month in advance of the date of the last ordinary meeting of the Section in the current session. Members of the Section may, if they so desire, nominate further candidates (being members of the Section) for election, in which case a ballot shall be taken at the meeting. Candidates so nominated must be proposed and seconded by members of the Section and nominations must be received in writing by the Secretary of the Section at least a week before the date of the meeting.

9. The Section Committee shall elect a Chairman and a Secretary from among its members. These two officers shall retire after a year of office but are to be eligible for re-election.

10. The Section Committee shall, *inter alia*, arrange periodical meetings of the Section for the reading of papers and discussions or for practical demonstrations.

11. The Section Committee shall send to one of the Honorary Secretaries of the Society (i) not later than November 1st in each year a detailed estimate of the total expenditure of the Section in the ensuing year with a request for funds to meet such expenditure, and (ii) as soon as possible after the close of every calendar year a detailed statement of the expenditure of the Section in the past year (with vouchers) and a report on the activities of the Section.

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12. The Section Committee shall, subject to direction from the Council, be entrusted with the editorial arrangements for the *Supplement* to the Society's *Journal*, provided that one of the editors of the *Supplement* shall be an Honorary Secretary of the Society.

Industrial Applications Section

Organization

1. The Section shall be constituted and carried on subject to the Bye-Laws of the Society, and in accordance with these Regulations, made and subject to amendment by the Council of the Society. The Section shall be concerned with the application of statistical technique to industrial research, development and manufacture, including inspection.

Membership

2. The members of the Section shall be the members of the Local Groups formed in accordance with these Bye-Laws.

3. Any Fellow of the Society and any other person who is approved by the relative Local Group Committee or, before the formation of a Local Group Committee, by the Section Committee may become a member of a Local Group on making written application to the Local Group Secretary or to the Section Secretary.

4. A Fellow of the Society shall not be required to make any payment for membership of a Local Group, but every other member of a Local Group shall pay an annual subscription of ten shillings, which shall be due in advance on January 1st, provided that any person who becomes a member of a Local Group and pays his initial annual subscription in the last quarter of any year shall not be required to pay any subscription in respect of the following year. Subscriptions shall be payable direct to the Honorary Treasurer of the Royal Statistical Society, 4, Portugal Street, W.C.2.

5. If any member of a Local Group has not paid his annual subscription by June 30th, the Secretary of the Royal Statistical Society shall advise him and the Local Group Secretary in writing of the default, and if the arrears are not paid up by September 30th, the defaulter shall cease to be a member.

6. Payment of an annual subscription in respect of membership of a Local Group shall not confer on the member any of the privileges of Fellowship of the Society.

7. Members who are not in arrear with their annual subscriptions may withdraw from membership by giving written notice to the relative Local Group Secretary.

The Local Group Committee

8. The members of each Local Group shall, not later than October 31st of each year, appoint a Local Group Committee consisting of at least four members and including a Chairman and a Secretary. At least one of the members shall retire each year, and not be eligible for re-election during the forthcoming session, and no member may serve continuously for more than four years.

9. Every Local Group Committee shall, subject to the direction of the Section Committee, arrange for periodical meetings of the Local Group for the reading of papers, discussions or demonstrations. The Session of meetings shall be from July 1st to June 30th.

10. Every Local Group Committee shall, not later than a date to be fixed by the Section Committee, nominate each Session two of its members, one of whom at least shall be an officer of the Local Group Committee, to be members of the Section Committee.

11. Every Local Group Committee shall send to the Section Secretary, not later than October 31st, an estimate of its probable expenditure during the ensuing year, and, not later than January 31st, send to the Section Secretary a statement of its expenditure (with vouchers) and a report of its activities during the past year, together with some account of the discussions which have taken place.

12. The Local Group Committee shall send to the Section Committee a copy of any paper read before the Group which it considers worthy of being filed in the Society's Library, and may recommend that any of such papers shall be published by the Society.

13. Every Local Group Secretary shall send to one of the Honorary Secretaries of the Society, and to the Section Secretary, a copy of every notice issued to the members of the Local Group.

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THE SECTION COMMITTEE.

14. Subject to these Regulations, all the affairs of the Local Groups shall be directed by the Section Committee, consisting of two or more Fellows nominated by the Council of the Society, and members nominated by each Local Group Committee, provided, however, that the Fellows so nominated by the Council of the Society shall not form a majority of the Section Committee.

15. The Section Committee shall elect a Chairman and a Secretary from among its members.

16. The Section Committee may form or approve the formation of a Local Group in any district in which twenty or more persons signify their willingness to become members of such Local Group.

17. The Section Committee shall consider the estimates and statements of expenditure and the reports supplied by the Local Group Committees, and shall send to one of the Honorary Secretaries of the Society (i) not later than November 30th a detailed estimate of the total expenditure of the Section in the ensuing year with a request for funds to meet such expenditure, and (ii) not later than February 28th a detailed statement of the expenditure of the Section in the past year (with vouchers), a report on the activities of the Section and some account of the discussions arranged by, and papers read before, each Local Group. The Section Committee may recommend that any of such papers shall be published by the Society or filed in its Library for reference.

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REVIEWS OF STATISTICAL AND ECONOMIC BOOKS

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1.—*Commercial Policy in the Post-War World. A Report by the Economic and Financial Committees of the League of Nations.* Princeton University Press, Princeton, N.J. 1945. 9″ × 6″. 124 pp.

In the period between the two World Wars, the Economic and Financial Committees of the League of Nations paid particular attention to the problem of reducing obstacles to international trade and abolishing all forms of discrimination in this sphere. To-day the policy of raising standards of living and achieving full employment through the expansion of international trade is on the point of receiving serious consideration by many Governments. Should the policy be widely adopted, the Committees believe it may be useful to offer this Report, containing conclusions drawn from their own experience together with suggestions based on it, as a guide to those who will have the task of drawing up the programme of international action which will naturally follow.

After the first World War the desire for a freer multilateral trading system was not sufficiently deep-rooted to survive the storms of depression which began to spread across the world in 1930, and with the breakdown in international co-operation each nation reacted to the instinct for self-preservation by turning to economic nationalism. To-day the desire for expansion in international trade is more firmly bedded in the popular demand for higher standards of living and higher and more stable levels of employment, and its ability to weather future storms is accordingly enhanced.

The Report shows how international action can remove many of the factors impeding the adoption of liberal commercial policies. It emphasizes the fact that the trade restrictions are largely conceived for the benefit of producers with quite inadequate regard for consumers' interests. Governments, however, are beginning to think more about the well-being of the consumer, and at the same time the latter is being educated to realize the interdependence between prosperous world trade and higher standards of living. Circumstances are therefore becoming increasingly favourable for the prospective International Conference on Trade and Employment, at which, according to the White Paper Cmd. 6709, it is hoped to set up an International Trade Organization to provide "... permanent machinery for international collaboration in matters affecting international commerce, with a view to continuous consultation, the provision of expert advice, the formulation of agreed policies, procedures and plans, and to the development of agreed rules of conduct in regard to matters affecting international trade."

After the transition period, the Committees strongly recommend the removal of all quantitative restrictions and exchange controls, which prevent national prices from keeping in step with world prices, which lend themselves to arbitrary methods of controlling foreign trade, and which, over a long period, cannot be reconciled with the principle of most-favoured-nation treatment. Furthermore, the quantitative controls should not be replaced by high tariffs, a course which would only foster retaliation and the growth of vested interests, and if, in existing tariffs, specific duties are replaced by *ad valorem* and negotiable rates, the way will be open for some of the stronger countries to take the lead in concluding bilateral or multilateral trade agreements which should, as far as possible, embody an unconditional most-favoured-nation clause. An annex of 38 pages gives extracts from a League document expounding the doctrine of the Economic Committee relating to most-favoured-nation treatment, a fact which serves to indicate the importance the Committees attach to securing the insertion of this clause in trade agreements.

Much can also be done to facilitate the conduct of commercial operations and the conclusion of commercial treaties by standardizing and simplifying customs practices, tariff classifications and nomenclature, and by codifying most-favoured-nation treatment. No doubt the International Trade Organization will be able to take up these subjects for further study from the advanced point to which the League Committees have developed them. T. M. R.

2.—*The Exploitation of Foreign Labour by Germany.* Studies and Reports, Series C, No. 25. International Labour Office, Montreal, 1945. 9″ × 6″. v + 286 pp. 6*s*.

The Exploitation of Foreign Labour by Germany is a study of that new phenomenon of the war, the gigantic compulsory movement of workers from the conquered countries to the victor's homeland. This Report bears the usual marks of competence and thoroughness which characterize most I.L.O. publications. Mr. J. H. E. Fried, who prepared it, evidently spared no effort in obtaining detailed information on the labour conditions of foreign workers in Germany.

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The Report does not claim to have exhausted the subject, but "attempts merely to set forth the evidence at present available." German official publications, as well as German newspapers and periodicals, form the bulk of the sources of information; due account is taken of the tendency to present the foreign workers' "régime" in a favourable light.

It should be said at this point that the Report seriously endeavours and to a large extent succeeds in keeping an objective tone; it does not take the horrified attitude it could so easily have adopted. This, in the reviewer's opinion, is one of the outstanding qualities of the book, the more remarkable if one thinks of what Mr. Fried's personal feelings (as an American and a member of the I.L.O.) must have been when wading through the evidence. This objective attitude of the Report has two merits. Firstly, it makes the reader feel that this is no mere anti-Nazi propaganda, and secondly, by its very soberness, it drives home more effectively than any emotional description the abject misery of a foreign worker's life in Germany.

The Exploitation of Foreign Labour by Germany is subdivided into sixteen chapters and eight appendices, which together embrace all the main aspects of the subject. Questions of policy, administrative problems, the recruitment, living conditions, remuneration, provisions for dependents, conditions of work, social insurance of foreign workers, as well as many other points are exhaustively treated. Indeed, the wealth of detail is such that the reader may sometimes feel that more use might have been made of small print and footnotes. It is invidious to choose special points for detailed comment from this mass of information. Yet some aspects of the Report arouse one's interest more particularly. To begin with, there is the knotty problem which must have faced the German authorities, of how to reconcile the man-power hunger of their war industries with the obvious dangers of importing into Germany millions of young men who were enemy aliens and potentially a very great threat to the régime. Secondly, how was one to decide between lowering the morale and health of these workers as a safeguard against an organized uprising, and keeping them at a high level of efficiency in the interest of production. One feels that, in spite of the Nazi sneerings at the democracies' endless deliberations, a considerable amount of consultation must have taken place and much effort must have been spent by the German authorities on issues of this kind.

The conditions of work, the combination of threats, material rewards and other inducements to raise the productivity of foreign workers make fascinating reading for the student of industrial psychology. It would be interesting to know whether the very excessive hours of work imposed by the Nazis on aliens were due to their desire to keep foreign workers down for fear of disturbances, or to the lack of confidence of the political authorities in the findings of psychologists. A third alternative is that the now generally accepted ideas on productivity and excessive working hours were found not to apply to the psychological *milieu* of slave workers. There is probably a wealth of information on psychological testing of workers in the archives of the German Ministry of Labour, and, unpleasant as it is to use information obtained at the price of great suffering, every effort should be made to obtain as many data as possible. Some of the methods used by the Germans are a vindication of what has come to be known as "the Hawthorne experiment"; even the ruthless Nazi tyrants recognized the importance of morale to productivity, and serious attempts were made to promote community and team spirit amongst foreign workers.

In the last chapter ("The Legacy of the System") of the Report we are told that in their more optimistic moods the Nazis hoped that this introduction of foreign workers (many of whom were uneducated and backward people) to the wonders of German industry would instil into these "inferior specimens of humanity" a boundless admiration for the master-race and a strong desire to emulate them and co-operate with them. In this, the Report tells us, the Nazis failed entirely. "All the indications are that these men and women, and indeed children too, have emerged from their experiences with hatred against the system of bondage under which they lived and worked and against those responsible for such a system."

A. P. Z.

3.—*Radio Advertising for Retailers.* By C. H. Sandage. Harvard University Press. 1945. $8\frac{1}{2}'' \times 5\frac{1}{2}$. xv + 280 pp. $4.00; or Oxford University Press, *22s. 6d.*

From studies of famine conditions in Europe and of our own shortage of consumer goods, it is curious to turn our eyes westwards, and to observe that at the height of the recent conflict, a professor at Harvard University, assisted by an advisory committee of four other professors, was making a study of the extent to which American retailers could increase their sales by using radio advertising.

This book is the report of such a study. It is several removes from being of great interest in England, since if the selling of goods becomes a problem again here, no advertising broadcasting is available, and if advertising broadcasting becomes available again it will be only from Continental stations with a wide coverage unsuitable for the local requirements of retailers. Moreover, the study consisted of little more than the canvassing of a large number of opinions from retailers who had used radio advertising, and the conclusions are open to the usual objections to this method of study.

Nevertheless the book contains some points of interest. In the first place, it is probably news to most people in this country that small towns in the States have broadcasting stations of their

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own, and that these stations are widely used by local retail shops to advertise their wares. This method of advertising is considered particularly effective for disposing of gluts of perishable commodities, an application which even a Socialist state might find useful.

It is also interesting to hear that broadcasting science divides music into "popular, hillbilly, western, religious, familiar, classical, semi-classical and corn," and that dramatized readings of the Bible in serial form are found to combine effectiveness as an advertising medium with their obvious cultural and religious advantages.

In view of this use of the Bible it is nice to find, in a series of searching questions the advertiser is recommended to ask himself, that the somewhat cynical: "Have my commercials been written in terms of the self-interest of my customers?" is softened by: "Have commercials been checked for truthfulness and sincerity?" On the other hand, it is difficult for anyone this side of the Atlantic to approve of the question "Have I auditioned the radio program to make sure of the quality of its rendition?"

A. M. L.

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STATISTICAL NOTES

(1) British Official Statistics

During the six months October 1945 to March 1946 there was a rise in general wholesale prices as shown by the Board of Trade index-number of nearly 1·4 per cent. There was a very slight decline in the level of prices of food and tobacco (about 0·1 per cent.) and a rise in that of industrial materials and manufactures of 2·1 per cent. There have been several changes in the price of wheat, to a considerable extent resulting from fluctuations in the controlled extraction rate of flour. The extraction rate in March was again raised to 85 per cent., with a consequent rise in the price paid by millers. Apart from seasonal changes, the only other alteration of importance was a fall (6 per cent.) in the controlled price of wet fish in December. With the exception of a slight fall in the period June to November 1943, the group index-number for meat and fish has remained stationary at 121·8 since January 1943. As regards industrial materials and manufactures, the most important change was an increase of 8·0 per cent. in the iron and steel group in January 1946, caused by the withdrawal of subsidies since December 1945. For some items the increase has been very considerable. Pig iron has advanced from 12½ to 18 per cent. and marked bars 25 per cent. There was a reduction in February of from 10 to 12 per cent. in the prices of chemical wood pulp, and a reduction in March of the average price of raw rubber of about 6½ per cent. At the end of March 1946 general wholesale prices as registered by the index-number showed an increase of 75 per cent. as compared with prices in August 1939, and there were similar increases in the groups covering articles of food and tobacco and those covering industrial materials and manufactures.

The following table gives the index-numbers for the period from September 1945 to March 1946.

(Averages for the year 1930 = 100)

Date	Total food	Total not food	All articles	Basic materials (excluding fuel)	Intermediate products	Manufactured articles	Building materials
September 1945	158·3	175·3	169·6	188·4	182·5	160·8	157·3
October „	158·1	175·1	169·4	188·6	181·8	160·7	157·8
November „	158·3	174·9	169·4	188·4	181·8	160·8	157·8
December „	157·6	175·2	169·3	188·4	183·5	161·0	157·8
January 1946	157·4	179·3	171·9	187·3	189·5	168·0	161·3
February „	157·4	178·8	171·6	184·5	190·1	168·1	162·5
March „	158·2	178·9	171·9	184·6	190·2	168·1	162·5
March 1945	*156·4*	*173·5*	*167·8*	*186·6*	*179·6*	*161·0*	*156·0*
August 1939	*90·4*	*102·2*	*98·1*	*94·5*	*104·0*	*108·7*	*104·1*
Percentage increase in March 1946 over—							
December 1945	0·4	2·1	1·5	2·0 *	3·5	4·2	2·9
March „	1·2	3·1	2·4	1·1 *	5·9	4·4	4·2
August 1939	75·1	75·0	75·1	95·4	82·6	54·6	56·1

* Decrease.

As regards the twelve months ended December 1945, the principal changes in price were an increase in that of coal of about 8·8 per cent., of pig iron from 10½ to 12 per cent., of zinc 21½ per cent., and of English lead 19 per cent. There were decreases of 22½ per cent. in the prices of aluminium, of 11½ per cent. in that of motor spirit and of 19 per cent. for fuel and burning oil. Lubricating oils fell in price nearly 30 per cent.

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The figures of certain other index-numbers for wholesale prices are given below, and also those of the United States Bureau of Labor for similar prices.

Date	Board of Trade (1930 = 100)	*Economist* (1927 = 100)	*Statist* (1866–77 = 100)	*The Times* (1913 = 100)	U.S. Bureau of Labor (1926 = 100)*
September 1945	169·6	119·5	162·7	188·6	105·0
October „	169·4	119·6	162·7	188·6	105·4
November „	169·4	119·6	162·7	187·8	106·3
December „	169·3	119·7	163·2	188·1	106·8
January 1946	171·9	122·4	165·6	192·3	106·8
February „	171·6	122·6	166·2	193·0	107·1
March „	171·9	122·9	166·6	193·9	108·3
March 1945	*167·8*	*119·0*	*162·1*	*186·1*	*105·1*
August 1939	*98·1*	*70·3*	*90·4*	*114·5*	*74·8*
Percentage increase in March 1946 over—					
March 1945	2·4	3·3	2·8	4·2	3·0
August 1939	75·1	74·8	84·3	69·3	44·8

* Mean of weekly figures.

There was hardly any change in the *cost of living* of working-class families during the six months ended April 1st, 1946, according to the index-numbers prepared by the Ministry of Labour and National Service. The stationariness of most of the articles of food is due of course to the action of subsidies, and the presence of " utility " clothing tends to prevent any rise in articles of dress.

The following table shows the index-number for the six months ending April 1st, 1946, according to the various groups of articles into which it is divided.

(Prices at July 1914 = 100)

Date	Food	Rent and rates	Clothing	Fuel and light	Other items	Total
October 1st, 1945	169	166	345	276	292	203
November „	169	166	345	276	292	203
December „	169	166	345	275	292	203
January 1st, 1946	169	166	345	275	293	203
February „	169	166	345	275	293	203
March „	169	166	345	275	293	203
April „	169	166	345	275	293	203
April 1st, 1945	*168*	*164*	*345–350*	*266*	*291*	*202*
September 1st, 1939	*138*	*162*	*205–210*	*180–185*	*180*	*152*
Percentage increase at April 1946 over—						
April 1945	0·6	1·2	0·7 *	4·0	0·7	0·5
September 1939	22	2–3	66	51	64	31

* Decrease.

Since the war, food prices covered by the index-number have advanced only 22 per cent., and the cost of living by 31 per cent. Articles of clothing, fuel and light and miscellaneous items have advanced 66 per cent., 51 per cent. and 64 per cent. respectively, and it is a question of importance whether most of these items are at present " weighted " heavily enough in the index.

The number of insured unemployed on the employment registers of the Ministry of Labour in Great Britain has continued to rise steadily since October 1945, when it amounted to

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233,326. At March 11th, 1946, the number had increased to 371,916, a rise of 138,590. The men and boys had increased by 107,595 and of women and girls by 30,995. The total number on the registers at March 11th, 1946, included 70,563 married women, an increase of 24,699 since October 15th, 1945, and 24,581 ex-service persons who had had no employment since leaving the service, an increase of 20,099 over the same period. Of the total of 371,916 there were 299,010 (187,413 men and 102,645 women) who had been unemployed for more than two weeks and 70,544 (42,052 men and 21,241 women) who had been unemployed for less than that period. Of the total of 9,186 uninsured persons on the registers at March 11th there were 4,079 boys and girls under 16 who had not yet entered industry. It is not possible to state exactly the proportion unemployed, but it is probably not in excess of 2½ per cent.

The table below gives the numbers on the registers of the employment offices in Great Britain at the date of ascertainment in each month from October 1945 to March 1946.

Date	Men and boys	Women and girls	Total (insured unemployed)	Total (insured and uninsured)
October 15th, 1945	131,832	101,494	233,326	245,810
November 12th, ,, ...	150,994	115,467	266,461	278,158
December 10th, ,, ...	163,307	121,449	284,756	293,295
January 14th, 1946	199,097	130,357	329,454	344,834
February 11th, ,, ...	224,027	131,551	355,578	367,910
March 11th, ,, ...	239,427	132,489	371,916 *	381,102 *

* These figures exclude 24,020 persons on the registers who had been classified as unsuitable for ordinary employment (22,296 insured and 1,724 uninsured persons).

A material change in the *distribution of medium incomes* is recorded in the White Paper on National Income and Expenditure 1938–45 (Cmd. 6784). In 1938 the estimated number of individuals (counting a married couple as one) assessed to Income Tax with gross incomes between £250 and £500 per annum was 1,745,000. In 1944 the figure was 5,200,000, an almost threefold increase. In the £500–1,000 range the numbers rose from 500,000 to 1,400,000, or by about 180 per cent., and in the £1,000–2,000 group from 195,000 to 520,000, or by 166 per cent. The number of persons earning less than £250 is not estimated, but their higher earnings would presumably account for the bulk of the increase in the first group, while the payment of war bonuses and higher salaries and wages would give rise to transfers from one group to another. Entirely new entrants, especially women, into income-producing occupations would be another factor in the increase.

In addition to these figures of gross incomes, a separate estimate is made of the distribution of net incomes.

(Thousands)

Range of income	Numbers before Tax		Numbers after Tax	
	1938	1944	1938	1944
£250– 500	1,745	5,200	1,820	5,050
£500–1,000	500	1,400	450	830
£1,000–2,000	195	520	155	186

The distribution of net incomes after deduction of tax gives some idea how the rise in taxation during the war years affected these medium income groups. The £250–500 range naturally shows the least relative change owing to the operation of abatements and allowances. In the other two groups, however, the pressure of taxation leads to a very much smaller increase in net than in gross incomes. Owing to movement between groups, the numbers before and after tax cannot be exactly compared, but it will be seen that whereas there were in 1944 1,400,000 gross incomes of £500–1,000 there were only 830,000 net incomes of the same amount, and in the upper group (£1,000–2,000) only 186,000 net incomes against 520,000 gross.

The total number of individuals with *net* incomes after tax from £150 upwards is estimated at

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7,000,000 in 1938 and 13,500,000 in 1944–5, and of these the three medium groups £250–2,000 accounted for about 34·6 per cent. in 1938, and 45 per cent. in 1944. Of the remainder 64·3 and 54·8 per cent. in the same two years were represented by net incomes of £150–250 per annum, only a quite small fraction of the total having incomes over the £2,000 limit, viz., 75,000 out of 7,000,000 in 1938 and 34,000 out of 13,500,000 in 1944.

The tendency for the numbers in the medium income groups to increase even before the war can be seen by comparing the 1938 figures given above for gross incomes with estimates relating to the year 1919–20 supplied by the Inland Revenue to the Colwyn Committee on National Debt and Taxation, 1927 (Cmd. 2800). These showed that there were in that year some 963,000 incomes in the £250–500 group (including some entirely relieved of taxation by allowances, etc.), a figure which had increased to 1,745,000 in 1938, or by 80 per cent. The numbers in the £500–1,000 group rose from 330,500 in 1919–20 to 500,000 in 1938, or by about 51 per cent., while the upper group showed a nearly corresponding growth from 133,000 to 195,000. So far as these figures are comparable, it would seem that the total number of persons in these three income groups increased from 1,377,000 in 1919–20 to 2,440,000 in 1938 and to 7,120,000 in 1944. The latter figures are no doubt exceptional, and due to war conditions, though some substantial portion of the increase may be expected to be maintained.

The annual *index-number of agricultural prices* calculated by the Ministry of Agriculture for 1945 shows an increase of 4 points over the previous year, the figure being 173½ as against 169½ in 1944. As compared with the pre-war period of 1937–39, the increase is from 90½ to 173½, or a total rise over six years of 91·7 per cent. The changes in the last five years are shown below in the three broad groups of commodities which make up the index-number. Allowance is made for Government subsidies, including payments in respect of the acreage under wheat and potatoes based on the estimated quantities entering into sale. These two latter payments were equivalent to 4*s.* 8*d.* per cwt. for wheat and 40*s.* 8*d.* per ton for potatoes, figures slightly higher than those of the previous year.

(Base 1927–29 = 100)

	1941	1942	1943	1944	1945
Cereals and farm crops	155	187½	181	180½	184
Live stock and live stock products	137	150	151½	158½	164
Fruit, vegetables and glasshouse produce ...	228	186	204	204	193½
General index	150½	161½	163½	169½	173½

The level of prices in the cereal and farm crops group is higher than in the livestock group, but in the last two years the latter has risen rather more than the former. The changes in the individual items are not very noticeable, though the index for milk, the principal item in the live-stock group, moved up to 192, as against 187 in 1944 and 178 in 1943.

(2) Other Statistics

Over the trade year February 1945 to January 1946 the total amount of retail sales, according to the statistics prepared by the Bank of England and published in the *Board of Trade Journal*, showed an increase in value over the previous twelve months of 7½ per cent. Articles of food and perishables increased 3·9 per cent. and those of non-food merchandise 13·2 per cent. The increase in sales was greatest in the furniture and hardware departments (over 40 per cent.), and least in the boot and shoe department (2·6 per cent.) and in grocery and provisions (1·2 per cent.). The index-number (average daily sales in 1942 = 100) for the year 1945 was 112, compared with 105 in 1944.

In his annual figures of *Statistics of Failures in the United Kingdom and Ireland* Mr. Richard Seyd gives a total of 329 for the year 1945 (294 Bankruptcies and 35 Deeds of Arrangement, includ-

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ing Scottish and Trust Deeds). This compares with a total of 319 in 1944. Of the 1945 total 36 (29 Bankruptcies and 7 Deeds of Arrangement) were in the financial, wholesale, and manufacturing trades, 99 (96 Bankruptcies and 3 Deeds of Arrangement) in the professional classes, etc., and 194 (169 Bankruptcies and 25 Deeds of Arrangement) in the retail trades and working classes. Of these 21 were in Eire (2 wholesale, 5 professional, and 14 retail). 1,109 Limited Companies were wound up voluntarily during the year (982 English, 87 Scottish, 14 Northern Ireland, and 26 Eire); 68 (3 Eire) were on account of liabilities, of which 41 were in the financial, wholesale, and manufacturing trades, 1 in the professional classes, etc., and 26 in the retail trades. The number of failures of private persons was 55 in 1945 compared with 50 in 1944. In the "retail trades" category farmers again showed the highest number of failures (55 compared with 28 last year); clerks and commercial travellers again came second with 22 (compared with 21 last year), and builders were again third, with 13 (compared with 16 last year).

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CURRENT NOTES

An important new organization, the Scottish Statistical Research Bureau, has come into being under the joint auspices of the four Scottish Universities and the Faculty of Actuaries. The purpose of the Bureau is to bring together those who have accumulated material which requires investigation and analysis and experts who can treat such material statistically. As is well known, valuable data are constantly being collected in the course, for example, of hospital practice, but often, for one reason or another, remain unanalysed so that the facts that lie behind them are not brought to light. The first investigation, already in progress, is concerned with data collected by the ulcer clinic of Glasgow Western Infirmary. At present the Scottish Universities and the Faculty of Actuaries are the only members of the Bureau. Later, however, it is intended that other organizations interested in statistical research of various kinds shall be admitted to membership. A central committee representative of the members will be responsible for the administration and functioning of the Bureau. The first chairman is Mr. J. G. Kyd, Registrar-General for Scotland and President of the Faculty of Actuaries. The secretary is Mr. E. Waugh, F.F.A., Faculty of Actuaries, St. Andrew Square, Edinburgh.

An Institute of Statistics has been established in the State College of North Carolina. It embodies two divisions: the Department of Experimental Statistics at Raleigh and the Graduate Division of Mathematical Statistics at Chapel Hill, which, in combination, provide facilities for students for the degrees of M.S. and Ph.D. in Statistics. The purpose of the Institute is (1) to establish a teaching, research, consulting and service centre for statistical work, (2) to promote the advantageous use of efficient statistical methods in research problems and other diverse fields of application, and (3) to advance the theory of statistics through the discovery of new mathematical techniques and ideas of wide applicability. The Director of the Institute is Miss Gertrude Cox; the staff at Raleigh includes Mr. W. G. Cochran, a Fellow of this Society, formerly at Rothamsted, while Professor Harold Hotelling is shortly to join the staff at Chapel Hill.

Fellows who share the enthusiasm, if not the profound knowledge, of the late George Wood for statistics of our national game will be interested in the recent founding of a Society of Cricket Statisticians. The main object of this Society, according to *The Times* of March 27th, is the preparation of figures relating to all first-class cricketers playing in England since 1870. Subsidiary objects include the formation of a central library and an exchange section for "Cricketaria." The chairman is Mr. Anthony Weigall and the honorary secretary Mr. S. Canynge Caple, Egloshayle Cottage, Tintagel, Cornwall, to whom applications for membership should be sent. The entrance fee is 5*s*. with a similar annual subscription. Membership is limited to cricket statisticians and collectors of "cricketaria" but presumably the definition of the former may cover a wider field than would appear on the surface. Much of the impetus to the early work on probability arose from games of chance and, as Sir William Elderton has shown, theoretical problems of general interest may be approached through the game of skill. Perhaps, therefore, he who is primarily interested in the gamma function or a Dirichlet integral may share the field of play with one who is fascinated by the number of centuries scored by Jack Hobbs in 1925, even though one views it from the deep and one from mid-wicket.

Two Sarah Frances Hutchinson Cowles Fellowships for women will be awarded by the University of Chicago for the academic year 1947–48 on nomination by the Cowles Commission for Research in Economics. Applicants must be students of outstanding promise, preparing for the degree of master or doctor in the field of social sciences and statistics, preferably in quantitative economics or mathematical statistics. The Fellowships amount to $1000 each, but may be supplemented by an additional grant of $500 if the work of the Fellowship holder lies within the Cowles Commission's field of interest. Holders will be expected to be in residence at the University of Chicago. Applications and supporting documents must be filed before March 1, 1947. Application forms and further particulars may be secured from the Cowles Commission for Research in Economics, The University of Chicago, Chicago 37, Illinois, U.S.A.

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OBITUARY

Miss Eleanor Rathbone, M.P.

Miss Eleanor Rathbone was a Fellow of the Society, and I used to meet her in discussions at the Liverpool Statistical Society nearly forty years ago. In those days she was a student of social conditions and a patient research worker. She was fortunate in being born into one of the great Liverpool families of the Victorian Era: her parents were not only wealthy, but they had a strong tradition of social obligation and of public work. Conditions in Liverpool in those days certainly provided an adequate field for anyone interested in the morbid anatomy of the social order. Great wealth, and poverty of a degree now happily unknown, were to be found within a few yards of one another; dock labour was entirely casual and grossly overcrowded; the condition of the children in the poor parts of the city was deplorable.

This was the environment in which Eleanor Rathbone began her work first of research, and then of education and propaganda. She struggled hard against her nature; she was determined not to be a sentimentalist; maybe she put too high a value in those days on the cold logic of argument, forgetting that sentiment is no unworthy weapon in the hands of the social reformer. For long years her progress was slow because she took so little trouble to lead those who did not at once accept her valuations. As the years went by maybe she changed a little in her approach; but certainly the steadfast devotion and complete lack of self-interest that she showed in her public work claimed for her views a wider and more sympathetic understanding.

Her early pioneer work on Family Budgets is now little known; her investigation into housing conditions was of local importance; her administrative work for the families of soldiers and sailors in the last war was a piece of public work that was of more moment than appeared at the time. Consider this work, with its detailed enquiry into the financial conditions of women whose husbands were serving in the Forces, and consider it against the background of all Eleanor Rathbone's previous investigations into conditions of poverty: it is clear that there arose, from the evidence, an overwhelming case for some system of payment to the mothers of children varying accordingly to the need of the family; the larger the family the greater the poverty, was the frequent conclusion of such investigations as were made by Eleanor Rathbone in this work—and it was from that time that she stepped out into national public life, happily and properly sitting for a University Seat in Parliament, and devoted herself to the task of compelling public attention to the plans she adumbrated for Family Allowances.

This is no effort at a complete valuation of the life of Eleanor Rathbone: I have just taken one aspect of it to show how it developed through the good practice of scientific method, from enquiry to conclusion, from facts to conviction—and then from conviction to advocacy and propaganda. The nation has benefited greatly by her life-work, and students of social science will for a long time, I hope, regard her life and method as encouraging examples. The Family Allowance Act is a national memorial to her, and she must be very happy about it.

Woolton.

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STATISTICAL AND ECONOMIC ARTICLES IN RECENT PERIODICALS

UNITED KINGDOM—

Advancement of Science—

November 1945—Science in Education: *A. E. E. McKenzie.*

April 1946—Conference on Scientific Research and Industrial Planning: Fundamental research in relation to the community; Planned research; Economic aspects of research; The human factor. Victory Congress of L'Association Française pour l'Avancement des Sciences: *J. Verne.* The place of science in town and country planning: *L. Dudley Stamp.*

Annals of Eugenics—

November 1945—Some orthogonal properties of the 4×4 and 6×6 Latin squares: *D. J. Finney.* Stored products, and the insects infesting them as examples of the logarithmic series: *J. L. Harrison.* A system of confounding for factors with more than two alternatives, giving completely orthogonal cubes and higher powers: *R. A. Fisher.* The fractional replication of factorial arrangements: *D. J. Finney.*

April 1946—Orthogonal partitions of the 5×5 Latin squares: *D. J. Finney.* New Zealand blood-type distributions: preliminary investigations: *R. F. Paddock*, with a note by *G. M. Morant.* Genetic theories of *Rh* blood types: *V. R. Khanolkar* and *L. D. Sanghvi.* The inheritance of allelomorphs of the *Rh* gene in fifty families: *Majory McFarlane.* Filial and fraternal correlations in successive generations: *R. G. Stanton.* A further note on the sib-pair linkage method: *L. S. Penrose.* Crossing-over in a pedigree containing hæmophilic and colour-blind individuals: *W. J. B. Riddell.* An investigation of 171 possible linkage relationships in man: *H. W. Kloepfer.*

The Banker—

December 1945—Post-war trade with Europe: *Service Étranger.* The Giro system: *T. Garside.* The Export Credits Department: *C. J. Shimmins.*

Bankers' Magazine—

February 1946—Banking profits in 1945. A plea for the use of a more scientific basis in the calculation of bank charges: *J. C. Stickler.*

March 1946—The Public Corporation. Bank Balance Sheets.

Biometrika—

November 1945—On the use of matrices in certain population mathematics: *P. H. Leslie.* On a method of estimating frequencies: *J. B. S. Haldane.* The mathematics of a population composed of k stationary strata each recruited from the stratum below and supported at the lowest level by a uniform annual number of entrants: *H. L. Seal.* The treatment of ties in ranking problems: *M. G. Kendall.* The probability integral of the mean deviation. Papers by *H. J. Godwin* and *H. O. Hartley*, editorial note by *E. S. Pearson*, and tables.

Economica—

November 1945—The economic organization of a P.O.W. camp: *R. A. Radford.* Competition in retail trade: *W. Arthur Lewis.* Recent contributions to general equilibrium economics: *J. R. Hicks.* Income tax and incentive to effort: *H. S. Booker.* A note on the elasticity of expectations: *L. M. Lachmann.*

Economic Journal—

December 1945—The problem of exports: *D. H. Robertson.* An outline of economic conditions in France under the German occupation: *L. Baudin.* Some considerations on stability conditions, employment and real wage rates: *A. C. Pigou.* Professor Hayek on economic planning and political liberty: *E. F. M. Durbin.* Public finance and changes in the value of money: *Colin Clark.* Fiscal policy and propensity to consume: *R. S. Williams.*

Eugenics Review—

January 1946—The relation between eye-colour and defective colour-vision: *Cyril Burt.* Gregory King and the population of England and Wales at the end of the seventeenth century: *D. V. Glass.*

Institute of Bankers, Journal—

October 1945—The economic consequences of the war: *Sir Charles Lidbury.*

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UNITED KINGDOM—*Contd.*

Oxford Institute of Statistics, Bulletin—

November 3, 1945—Reflections on the Budget: *T. Balogh.* The changes in income tax: *J. L. Nicholson.* The economic situation in the Netherlands in Autumn 1945: *J. Tinbergen.*

November 24, 1945—War-time changes in industrial employment: *K. Forchheimer.* The problem of inflation in Continental Europe: *J. Steindl.* Food consumption in North-Western and Western Europe, 1943–45: *K. Mandelbaum.*

December 15, 1945—Proper nutrition at low cost: *T. Schulz.* Man-power in the reconversion period in U.S.A. and U.K.: *F. A. Burchardt.*

January, 1946—The Anglo-American financial agreement: *Sir Hubert Henderson.* The relation between birth rates and economic activity in Sweden, 1920–44: *Hannes Hyrenius.*

February, 1946—Income and household expenditure of working-class families with children. Part I. Income and expenditure on non-food items: *T. Schulz.* Employment and output in the building trades: *S. Moos.* The Swiss economy in the transition period: *O. Angehrn.*

March, 1946—Income and household expenditure of working-class families with children. Part II. Outlay on food and nutrition: *T. Schulz.* Readjustment problems in Sweden: *E. Lundberg.* Czechoslovak industry after nationalization: *J. Goldman.*

Manchester Statistical Society, Transactions—

Session 1944–45—The shape and size of the exports merchanting section of the cotton industry: *J. Kenyon.* The reconstruction of the health services: *C. Brown Metcalfe.* The work of the financial journalist: *R. H. Fry.* Mobilization of man-power in Great Britain for the Second Great War: *Sir Godfrey H. Ince.*

Public Administration—Spring Number, 1946—Mobilization of man-power: *Sir Godfrey H. Ince.* County War Agricultural Executive Committee: *Sir Donald E. Vandepeer.* Note on the origins of British Military Government in North-West Europe: *D. R. Woodman.*

Review of Economic Studies—

Vol. XII, (1), No. 31. Some aspects of the theory of futures trading: *Gerda Blau.* Labour for the war industries: the experience of Coventry: *A. Shenfield* and *P. Sargant Florence.* Regional differences of productivity in European agriculture: *A. Daniel.* The interrelations of shifts in demand: *D. H. Robertson, J. R. Hicks* and *O. Lange.*

AUSTRALIA—

Economic Record—

December 1945—The transition to a peace economy: *H. Burton.* The first four reports of the Rural Reconstruction Commission: *A. B. Ritchie.* The growth of New Zealand's general government debt: *E. P. Neale.* The future of Federal aid: *G. L. Wood.* A "made to measure" tariff: *W. J. Rose.* Full Employment: the British, Canadian and Australian White Papers. *D. H. Merry* and *G. R. Bruns.*

INDIA—

Indian Journal of Economics—

January 1946—Conference papers read at the 28th conference of the Indian Economic Association, Lahore, January 1946. (Whole number.)

Sankhyā—

Vol. 7, Part 2, 1945—The logical inversion of the notion of the random variable: *R. A. Fisher.* The individual sampling distribution of the maximum, the minimum and any intermediate of the p-statistics on the null-hypothesis: *S. N. Roy.* Distribution of certain frequency constants in samples from non-normal populations: *K. C. Cheriyan.* Statistics and social values: *N. C. Mehta.* Statistics in the service of planning: *R. K. Mukherjee.* Differential rates of natural increase: *N. G. D. Joarder.* Cost of living indices at Cawnpore: *S. P. Saksena.* The Cawnpore mill-worker, his family and his expenditure: *J. K. Pande.* Agricultural co-operation in Bengal and rest of British India (1918–19 to 1938–39): *T. Ghose* and *H. Sinha.* On the totality of the solutions for the symmetrical incomplete block designs: $\lambda = 2$, $k = 5$ or 6: *Q. M. Husain.*

UNITED STATES—

American Academy of Political and Social Science, Annals—

November 1945—Building the future city. (Whole number.)

January 1946—Essential human rights. (Whole number.)

March 1946—Controlling group prejudice. (Whole number.)

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UNITED STATES—*Contd.*

American Economic Review—

December 1945—The interest rate and investment in a dynamic economy: *F. A. Lutz.* Is a rise in the interest rates desirable or inevitable?: *L. H. Seltzer.* The concept of economic surplus: *K. E. Boulding.* Annual wage guarantee plans: *Rita Ricardo.* British policy and a world economy: *E. M. Bernstein.* The theory of economic behaviour: *Leonid Hurwicz.*

American Statistical Association, Journal—

December 1945—Statistical quality control in war production: *Holbrook Working.* Statistical estimation of simultaneous economic relations: *Tjalling Koopmans.* The relationship between price change and price level for common stocks: *Zenon Szatrowski.* Apportionment of representatives in Congress on the basis of special populations: *Morris B. Ullman.* The square root method and its use in correlation and regression: *Paul S. Dwyer.* Management statistics in war contract renegotiation: *Norman C. Parkin.*

American Statistical Association, Biometrics Bulletin—

December 1945—Some applications of statistical methods to fishery problems: *William E. Ricker.*

February 1946—Field sampling for the estimation of wireworm populations: *D. J. Finney.*

Annals of Mathematical Statistics—

September 1945—Choice of one among several statistical hypotheses: *Ralph J. Brookner.* A two-sample test for a linear hypothesis whose power is independent of the variance: *Charles Stein.* Compact computation of the inverse of a matrix: *Frederick V. Waugh* and *Paul S. Dwyer.* Multiple matching and runs by the symbolic method: *Irving Kaplansky* and *John Riordan.* On the power functions for the E^2-test and the T^2-test: *P. L. Hsu.* Some generalizations of the theory of cumulative sums of random variables: *Abraham Wald.* On the design of experiments for weighing and making other types of measurements: *K. Kishen.* Note on the law of large numbers and "fair" games: *W. Feller.* A note on rank, multicollinearity and multiple regression: *Gerhard Tintner.* Note on the distribution of the serial correlation coefficient: *William G. Madow.*

December 1945—On the normal approximation of the binomial distribution: *W. Feller.* The variance of the measure of a two-dimensional random set: *J. Bronowski* and *J. Neyman.* On the measure of a random set II: *H. E. Robbins.* Sampling from a changing population: *Reinhold Baer.* Testing the homogeneity of Poisson: *Paul G. Hoel.* Some combinatorial formulas on mathematical expectation: *L. C. Hsu.* On the constituent items of the reduction and the remainder in the method of Least Squares: *S. Vajda.*

Econometrica—

October 1945—Stability of multiple markets: The Hicks conditions: *Lloyd A. Metzler.* A theory of general short-run equilibrium: *Donald M. Fort.* Multiplier effects of a balanced budget: *Trygve Haavelmo.* Measuring marginal utility by reactions to risk: *William Vickery.* Liquidity preference of large manufacturing corporations (1921–1939): *Avram Kisselgoff.*

Harvard Business Review—

Winter Number 1946—Financing the Internation Bank: *Henry C. Wallich.* Small business: A community problem: *Charles C. Abbott.* Crisis in the domestic shipping industry: *Paul F. Lawler.*

Spring Number 1946—Geographical aspects of a minimum wage: *John A. Van.* Use of newspapers by national advertisers: *Neil H. Borden.* World rubber problems: *Klaus E. Knorr.*

Journal of Experimental Education—

December 1945—The measurement of teaching ability. (Whole number.)

Journal of Political Economy—

December 1945—The theory of optimum population for a closed economy: *Manuel Gottlieb.* Deficiencies in the gross-national-product concept as a national measure: *Joseph Mayer.*

February 1946—Some reflections on a theory of labor-management relations: *Frederick H. Harbison.* Effectiveness of factory labor: South–North comparison: *Richard A. Lester.*

Quarterly Journal of Economics—

November 1945—Problems of European reconstruction: *Nehemiah Robinson.* The criterion of maximum profits in the theory of investment: *Friedrich A. Lutz.* Trade-union government: a formal analysis: *Joseph Shister.* Post-war exchange-rate parities: *Morris E. Garnsey.* The old-age reserve fund is not "illusory": *George B. Robinson.* The old-age reserve fund is not "illusory": *George B. Robinson.* Exports, imports, domestic output, and employment: *W. W. Leontief.*

February 1946—Price regulation in the paper industry: *J. A. Guthrie.* Labor and the recovery program, 1933: *Irving Bernstein.* International payments in national income: *Pao-San Ou.*

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UNITED STATES—*Contd.*

Review of Economic Statistics—

November 1945—Price control and control of inflation: *E. M. H. Lloyd.* World prosperity and the British balance of payments: *Randall Hinshaw* and *Lloyd A. Metzler.* 1944–1945 programs for postwar social security and medical care: *Edwin E. Witte.* Postwar national income: *J. Mayer, H. G. Moulton, M. Jacobstein, L. H. Kimmel, R. S. Tucker, E. E. Hagen, L. H. Bean.*

February 1946—Relationship between consumers' expenditures, savings, and disposable income: *W. S. Woytinsky.* The postwar demand for United States exports: *J. Hans Adler.* Postwar economic policies: *Howard S. Ellis.*

EGYPT—

L'Égypte Contemporaine—

March 1945—Economic relationship between the United States and Egypt: *J. Landis.* Le financement de l'industrie en Égypte: *A. Eman.* De l'utilité du libre échange dans les relations économiques d'après guerre: *Ahmed Moussa.* Value of agricultural products and other commodities pertaining to agriculture for the years 1937–1942: *Mahmound Anis.*

MEXICO—

Estadística—

December 1945—Renta nacional; su significado y medición: *Loreto M. Domínguez.* Un programa de estadística de agricultura: *H. H. Schutz.* Pagos internacionales de los Estados Unidos durante el período de guerra: Bureau of Foreign and Domestic Commerce. Note on a general expression of the means: *Milton da Silva Rodrigues.* Instituto Nacional de Estadística de Colombia: *Antonio Garcia.* Attributes of wage earner families in relation to size: *Harold F. Greenway.* La composición familiar de asalariados en el Distrito Federal de Venezuela; ajustamiento de los datos de observación: *Eric Michalup.* La estadística como profesión: National Roster of Scientific and Specialized Personnel. Aspectos de método estadístico: *W. Edwards Deming.* Responsabilidades demográficas una oficina de Registro Civil: *W. Thurber Fales.*

SWEDEN—

Ekonomisk Tidskrift—

December 1945—Inflationsproblemet i Finland: *Klaus Waris.* Järnvägarnas godstariffsystem och dess återverkningar på konkurrensen mellan Järnvägar och Bilar: *Arne Sjoberg.* Det Teoretiska studiet av transportvässendets problem: *N. E. Ehrnrooth.*

INTERNATIONAL—

International Labour Review—

October 1945—Migration of indigenous workers in the Belgian Congo: *Pierre de Briey.* The planning of public investment in Australia: *Richard I. Downing.*

November 1945—The maintenance of full employment after the transition period: a comparison of the problem in the United States and the United Kingdom: *M. Kalecki.* Social policy in China: *T. K. Djang.*

December 1945—Full employment in the transition period. Foreign labour in the United States during the war: *Julia Henderson.* The experience of the Women's Employment Board in Australia: *Judge Alfred W. Foster.* The inflation of the cost of living and wages in Greece during the German occupation: *S. Agapitides.*

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LIST OF ADDITIONS TO THE LIBRARY

Since the issue of Parts III–IV, 1945, the Society has received the publications enumerated below.

I.—OFFICIAL PUBLICATIONS

(a) United Kingdom.

Agriculture and Fisheries, Ministry of. Report of the Committee on Higher Agricultural Education in England and Wales. London: H.M.S.O., 1946. Cmd. 6728. 9½″. 86 pp. 1*s.* 3*d.*

Agriculture and Fisheries, Ministry of, and Department of Agriculture for Scotland. Report of the Wool Marketing Committee. London : H.M.S.O., 1946. 9½″. 23 pp. 4*d.*

Colonial Office.

Labour conditions in East Africa. Report by Major St. J. Orde Browne. London : H.M.S.O., 1946. (Colonial No. 193.) 9½″. 94 pp. Map. 2*s.*

Report and proceedings of the Cocoa Research Conference held at the Colonial Office, May–June 1945. London : H.M.S.O., 1945. (Colonial 192.) 9½″. 168 pp. 3*s.*

Food, Ministry of. Report of the Conference on the Post-war Loaf. London: H.M.S.O., 1945. 9½″. 24 pp. 4*d.*

Fuel and Power, Ministry of.

Bristol and Somerset coalfield regional survey report. London : H.M.S.O., 1946. 9½″. 84 pp. Maps. 1*s.* 6*d.*

South Wales coalfield (including Pembrokeshire) regional survey report. London : H.M.S.O., 1946. 9½″. 218 pp. Maps. 4*s.*

The Gas industry : report of the committee of enquiry. London : H.M.S.O., 1946. 9½″. 6 + 57 pp. Maps. 2*s.*

Health, Ministry of.

Hospital survey. The hospital services of the South-western area. London: H.M.S.O., 1945. 13½″. 208 pp. 10*s.*

National Health Service Bill : summary of the proposed new service. London : H.M.S.O., 1946. Cmd. 6761. 9½″. 18 pp. 3*d.*

Reports on Public Health and Medical Subjects No. 93. A report on the readjustment in civil life of soldiers discharged from the army on account of neurosis, by E. Guttman and E. L. Thomas. London : H.M.S.O., 1946. 9½″. 72 pp. 1*s.* 3*d.*

Medical Research Council. Special Report Series No. 253. Tables of representative values of foods commonly used in tropical countries, by B. S. Platt. London : H.M.S.O., 1945. 9½″. 41 pp. 9*d.*

National Insurance, Ministry of. National Insurance Bill, 1946. Report by the Government Actuary on the financial provisions of the Bill. London : H.M.S.O., 1946. Cmd. 6730. 9½″. 32 pp. 6*d.*

Overseas Trade, Department of. Review of commercial conditions : British West Indies, Aug.–Sept. 1945. 48 pp. 1*s.* Eire, Sept. 1945. 23 pp. 6*d.* India, Aug. 1945. 60 pp. 1*s.* Union of South Africa, July 1945. 34 pp. 6*d.* London : H.M.S.O., 1945–6. 8¼″. 4 vols.

Scotland, Department of Health for. The provision of houses for owner-occupation in Scotland. Report by the Scottish Housing Advisory Committee. Edinburgh: H.M.S.O., 1946. Cmd. 6741. 9½″. iv + 74 pp. 1*s.* 3*d.*

Trade, Board of.

A central institute of management : the report of a committee appointed by the President of the Board of Trade in November 1945 to formulate detailed proposals for setting up a central institute for all questions connected with management. London : H.M.S.O., 1946. 9½″. 15 pp. 3*d.*

Report of the Census of Distribution Committee. London : H.M.S.O., 1946. Cmd. 6764. 9½″. v + 28pp. 6*d.*

Report of the Working Committee on China Clay. London : H.M.S.O., 1946. Cmd. 6748. 9½″. 14 pp. 3*d.*

Shipping movement at United Kingdom ports in each of the years 1938 to 1945. London : H.M.S.O., 1946. 9½″. 13 pp. 3*d.*

Treasury.

Specific agreements regarding settlement for Lend Lease, Reciprocal Aid, surplus war property and claims. London : H.M.S.O., 1946. Cmd. 6778. 9½″. 38 pp. 6*d.*

Statistical material presented during the Washington negotiations. London : H.M.S.O., 1945. 9½″. 15 pp. 3*d.*

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(b) International and Others

International Labour Office.

Safety in dock work (Studies and Reports, F. 2nd section, 10). Montreal (London: King & Staples), 1945. 9″. v + 225 pp. 6*s*.

Social security for seafarers. (Studies and Reports, M. 19). Montreal (London: King & Staples), 1945. 9″. vi + 264 pp. 6*s*.

League of Nations.

Economic, Financial and Transit Department.

Economic demography of eastern and southern Europe, by Wilbert E. Moore, of the Office of Population Research, Princeton University. (1945, II. A. 9.) Geneva (London: Allen & Unwin), 1945. 9″. 299 pp.

Industrialization and foreign trade. (1945, II. A. 10.) Geneva, 1945. 9″. 167 pp.

Mozambique.

Repartição Técnica de Estatistica. Censo da população em 1940. IV. População indigena. População por grupos de idades segundo o sexo e a ocupação. vi + 99 pp. V. População indigena. Fecundidade. vi + 229 pp. Lourenco Marques, 1944–5. 10½″. 2 vols.

Norway.

Statistisk Sentralbyrå. Statistisk-økonomisk utsyn over krigsårene. Oslo, 1945. 10½″. 307 pp.

Switzerland.

Eidgenössisches Statistisches Amt. Eidgenössische Volkszählung 1 Dezember 1941. Bd. 7. Kanton Bern. (S. Q. der S. Heft 153) Bern, 1945. 11¾″. 324 pp.

United States of America.

Overall Economic Effects Division. The United States Strategic Bombing Survey. The effects of strategic bombing on the German war economy. Washington: Overall Economic Effects Division, Oct. 31, 1945. 10¼″. ix + 286 pp.

Yugoslavia.

Statistique Générale d'État. Résultats définitifs du recensement de la population du 31 mars 1931. Livre IV. Population de fait suivant la profession principale. Sarajevo, 1940. 11″. xiv + 341 + 1 pp.

II.—AUTHORS AND MISCELLANEOUS.

Aitken (*A. C.*). Statistical mathematics. 4th ed. Edinburgh and London: Oliver and Boyd, 1945. 7¼″. vii + 153 pp. 5*s*.

Barrett (*G. E.*) and *Pridmore* (*A. W.*). A trial sampling-survey of domestic-supply conditions. (Technical report K/T119). London: Brit. Electric and Allied Industries Research Assoc., 1946. 11″. 11 pp. 3*s*. 6*d*.

Baykov (*Alexander*). The development of the Soviet economic system: an essay on the experience of planning in the U.S.S.R. (National Institute of Economic and Social Research, Economic and Social Studies V.). Cambridge University Press, 1946. 8½″. xv + 514 pp. 30*s*.

British Association for the Advancement of Science. Mathematical tables, Vol. 1. Circular and hyperbolic functions, exponential and sine and cosine integrals, factorial functions and allied functions, hermitian probability functions. Prepared for the Committee for the Calculation of Mathematical Tables. 2nd ed. Cambridge University Press, 1946. 11″. xi + 72 pp. 10*s*.

British Iron and Steel Federation. Statistics of the iron and steel industry of the United Kingdom (for the years 1939–44). London: B.I.S.F., 1945. 9½″. viii + 57 pp.

Columbia University, Statistical Research Group. Sequential analysis of statistical data: applications. Prepared by the Statistical Research Group, Columbia University, for the Applied Mathematics Panel, National Defense Research Committee, Office of Scientific Research and Development. New York: Columbia University Press, 1946. (London: Oxford University Press.) 11″. [303] pp. 42*s*.

Friedman (*Milton*) and *Kuznets* (*Simon*). Income from independent professional practice. (Publications of the National Bureau of Economic Research, 45.) New York, 1945. 9″. xiii + 599 pp. $4.00.

Gabor (*André*). Sampling problems of the Farm Management Survey and costing enquiries. Sutton Bonington, Loughborough: Midland Agricultural College, 1946. 13″. 22 pp. (Presented by the Author.)

Ghose (*Tariniprosad*). Economic survey of destitutes. Calcutta: J. N. Chatterji, Advertising and Sales Promotion Co., 1945. 8½″. 20 pp. R. 1.

Glover (*James W.*), editor. Tables of applied mathematics in finance, insurance, statistics, edited by James W. Glover. Compound interest functions and logarithms of compound interest functions, life assurance and disability insurance functions, probability and statistical functions, and seven place logarithms of numbers from 1 to 100,000. Ann Arbor, Mich.: George Wahr, 1930. 9″. xiii + 678 pp.

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Authors and Miscellaneous—*Contd.*

Grundy (*Fred*) and *Titmuss* (*Richard M.*). Report on Luton, prepared for the Borough Council by Fred Grundy . . . and Richard M. Titmuss . . . in collaboration with many others. Luton: Gibbs, Bamforth & Co., 1945. 11¼″. 139 pp. Maps. 21*s.* cloth; 12*s.* 6*d.* paper.

Harrod (*R. F.*). A page of British folly. London: Macmillan, 1946. 7½″. iv + 60 pp. 1*s.* 6*d.*

Landry (*Adolphe*). Traité de démographie, composé avec la collaboration de Henri Bunle . . . Pierre Depoid . . . Michel Huber . . . Alfred Sauvy. Paris: Payot, 1945. 9″. 651 pp. 35*f.* (Presented by Monsieur R. Varin.)

Levy (*Hyman*) and *Preidel* (*E. E.*). Elementary statistics. London: Nelson, 1945. 7″. vii + 184 pp. 5*s.*

Linder (*Arthur*). Statistische Methoden für Naturwissenschafter, Mediziner, und Ingenieure. Basel: Verlag Birkhäuser, 1945. 9½″. 150 pp. 18.50 *f.* bound; 15.50*f.* paper.

Lotka (*Alfred J.*). The law of evolution as a maximal principle. (*Human Biology*, 17, 1945, pp. 167–194.) 9½″. (Presented by the Author.)

Mandelbaum (*K.*) and *Schenider* (*J. R. L.*). The industrialization of backward areas. (Institute of Statistics Monograph 2.) Oxford: Blackwell, 1945. 8½″. viii + 111 pp. 10*s.* 6*d.*

Pember and Boyle. British government securities in the twentieth century. London: Pember and Boyle, 1945. 8¼″ × 13″. 85 pp. Folded tables.

Saunders (*C. T.*). Man-power distribution 1939–45: some international comparisons. Manchester Statistical Society, 1946. 8½″. 41 pp.

Scottish Council for Research in Education. The Scottish Council for Research in Education, its aims and activities. (Publications xxiv.) Bickley, Kent: University of London Press, 1945. 8½″. 45 pp. 1*s.*

Smith (*James G.*) and *Duncan* (*Acheson J.*). Elementary statistics and applications. Fundamentals of the theory of statistics. New York and London: McGraw-Hill, 1944. 8¼″. x + 720 pp.

—— Sampling statistics and applications. Fundamentals of the theory of statistics. New York and London: McGraw-Hill, 1945. 8¼″. xii + 498 pp.

Steindl (*Joseph*). Small and big business: economic problems of the size of firms. (Institute of Statistics, Monograph 1.) Oxford: Blackwell, 1945. 8½″. v + 66 pp. 7*s.* 6*d.* cloth; 5*s.* paper.

Vakil (*C. N.*) and *Anjaria* (*J. J.*). The future of the rupee. Bombay: Popular Book Co., 1945. 8½″. 4 + 49 pp. *Rs.* 1.4.

Wheeler (*Ferdinand C.*), editor. The technique of marketing research, prepared by the Committee on Marketing Research Technique of the American Marketing Association. Ferdinand C. Wheeler, Chairman. New York and London: McGraw-Hill, 1937. 8¼″. xviii + 427 pp.

Wigglesworth (*Frederick*). The money of democracy. London: St. George Book Co., 1945. 7¼″. 127 pp. 6*s.* (Presented by the Author.)

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REVENUE OF THE UNITED KINGDOM

Net Produce in Quarters of 1945, *and in Financial Years ended March* 31, 1945–46, 1944–45, 1943–44, 1942–43

(000's omitted)

QUARTERS ended	March 31, 1945	June 30, 1945	Sept. 30, 1945	Dec. 31, 1945	Total for Calendar Year 1944
	£	£	£	£	£
Customs	145,119	141,223	136,615	147,142	570,099
Excise	121,100	129,200	134,800	138,700	523,800
Stamps and Estate etc. Duties	37,447	32,809	35,297	37,701	143,254
Other Inland Revenue Duties	597	96	25	35	753
Post Office	29,050	27,750	28,400	29,950	115,150
Wireless Licences	1,550	930	860	1,610	3,950
National Defence Contributions	8,677	7,768	9,975	8,165	34,585
Excess Profits Tax	104,424	80,000	128,683	120,525	439,632
	447,964	425,776	474,655	483,828	1,831,223
Income Tax and Surtax	776,804	201,204	257,772	197,048	1,432,828
	1,224,768	626,980	732,427	680,876	3,264,051
Motor Vehicles Duties	23,140	4,550	4,543	2,771	35,004
Crown Lands	260	260	210	250	980
Interest on Sundry Loans	2,305	2,694	2,198	2,495	9,692
Miscellaneous Receipts	26,120	5,037	40,402	11,356	82,915
	1,276,593	639,521	779,780	697,748	3,392,642

YEARS ended March 31	1945–46	1944–45	1945–46 (compared with 1944–45)		Corresponding Years	
			Increase	Decrease	1943–44	1942–43
	£	£	£	£	£	£
Customs	569,842	579,352	—	9,510	560,832	459,489
Excise	540,800	496,900	43,900	—	482,200	425,300
Stamps and Estate etc. Duties	145,400	127,898	17,502	—	117,206	108,616
Other Inland Revenue Duties	712	779	—	67	968	950
Post Office	115,500	116,600	—	1,100	111,000	114,900
Wireless Licences	5,180	4,820	360	—	4,820	4,560
National Defence Contributions ...	35,485	33,304	2,181	—	33,304	30,635
Excess Profits Tax	432,130	477,134	—	46,257	466,668	346,887
	1,845,049	836,787	63,943	56,934	1,777,070	1,491,337
Income Tax and Surtax	1,430,415	1,390,374	41,293	—	1,259,625	1,082,186
	3,275,464	3,227,161	105,236	56,934	3,036,695	2,573,523
Motor Vehicles Duties	43,172	28,965	14,207	—	27,294	28,537
Canadian Government Contribution ...	—	—	—	—	—	224,719
Crown Lands	960	980	—	20	1,060	1,000
Interest on Sundry Loans	11,046	7,520	3,527	—	6,455	4,061
Miscellaneous Receipts	70,561	90,037	—	19,476	77,876	90,533
	3,401,203	3,354,663	122,970	76,430	3,149,380	2,922,373
			NET INCR. £46,540			

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OVERSEAS TRADE OF THE UNITED KINGDOM

for the years 1938–45

Values (c.i.f.) of Imports *
(£ million)

Class and Group.	1938	1939	1940	1941	1942	1943	1944	1945
I.—Food, Drink and Tobacco—								
A. Grain and flour	74·4	55·5	93·7	89·6	53·6	66·0	65·5	81·4
B. Feeding-stuffs for animals	11·4	8·9	8·4	2·5	0·8	0·2	1·5	2·9
C. Animals, living, for food	9·2	11·0	10·9	7·1	13·1	11·4	11·6	12·6
D. Meat	90·7	93·3	97·2	118·8	148·8	167·0	167·9	113·5
E. Dairy produce	80·0	75·9	62·4	66·3	84·4	85·4	91·2	82·0
F. Fresh fruit and vegetables	37·7	35·0	27·2	4·4	5·4	2·3	8·0	15·9
G. Beverages and cocoa preparations	46·5	40·5	45·7	43·4	37·6	51·1	51·8	51·8
H. Other food	56·9	64·7	65·9	71·9	71·0	87·7	89·2	75·6
I. Tobacco	23·3	13·6	8·8	18·0	20·6	42·2	32·7	51·4
Total, Class I	430·1	398·4	420·2	422·0	435·3	513·3	519·4	487·1
II.—Raw Materials and Articles mainly Unmanufactured—								
A. Coal	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0
B. Other non-metalliferous mining and quarry products and the like	4·7	5·1	7·2	6·7	7·1	10·0	9·8	7·7
C. Iron ore and scrap	11·2	9·7	16·6	14·2	7·7	7·9	9·3	15·4
D. Non-ferrous metalliferous ores and scrap	16·3	17·8	24·8	22·5	24·2	25·9	21·3	20·7
E. Wood and timber	42·9	37·1	37·9	24·8	20·4	32·6	36·0	45·5
F. Raw cotton and cotton waste	29·6	34·2	50·3	35·8	52·7	53·7	41·9	48·9
G. Wool, raw and waste, and woollen rags	42·6	40·4	67·2	21·9	27·4	19·0	35·7	36·1
H. Silk, raw and waste, and artificial silk waste	2·1	2·5	4·4	1·4	0·7	1·0	1·9	2·1
I. Other textile materials	11·8	12·8	19·9	9·3	11·1	11·8	12·2	13·9
J. Seeds and nuts for oil, oils, fats, resins and gums	30·6	30·9	44·9	39·6	39·8	55·3	53·6	47·3
K. Hides and skins, undressed	18·2	15·5	15·7	8·6	12·6	10·6	11·7	14·6
L. Paper-making materials	16·1	15·7	13·1	7·9	10·5	9·4	8·8	15·7
M. Rubber	11·5	9·5	23·8	22·2	9·1	13·3	25·7	11·7
N. Miscellaneous raw materials and articles mainly unmanufactured	10·4	9·5	12·4	12·1	12·5	12·1	12·1	14·5
Total, Class II	248·0	240·7	338·2	227·0	235·8	262·6	280·0	294·1
III.—Articles Wholly or Mainly Manufactured—								
A. Coke and manufactured fuel	0·0	0·0	0·0	0·0	—	0·0	—	0·0
B. Pottery, glass, abrasives, etc.	7·4	5·2	1·7	1·6	3·0	2·4	1·0	0·9
C. Iron and steel and manufactures thereof	14·8	18·6	48·2	69·1	47·0	59·0	32·9	6·4
D. Non-ferrous metals and manufactures thereof	40·8	38·7	59·5	57·7	60·9	84·0	66·6	22·1
E. Cutlery, hardware, implements and instruments	7·1	5·8	4·0	5·3	9·2	14·0	10·4	5·9
F. Electrical goods and apparatus	3·1	2·8	4·2	2·7	2·7	8·2	23·9	18·7
G. Machinery	21·8	24·5	41·2	51·2	34·8	42·7	41·5	20·0
H. Manufactures of wood and timber	6·3	5·0	3·9	2·5	2·4	6·5	8·5	13·6
I. Cotton yarns and manufactures	3·1	2·3	2·9	1·7	1·3	2·4	2·5	1·8
J. Woollen and worsted yarns and manufactures	3·8	3·4	1·9	0·1	0·2	0·3	0·7	1·6
K. Silk and artificial silk yarns and manufactures	4·9	3·6	0·9	0·2	0·2	1·8	2·5	2·5
L. Manufactures of other textile materials	4·6	7·1	16·3	4·2	5·7	4·4	10·2	8·2
M. Apparel	8·0	5·7	1·2	0·5	0·6	0·7	5·6	4·8
N. Footwear	2·8	2·8	1·5	1·0	0·5	0·7	0·1	0·7
O. Chemicals, drugs, dyes and colours	13·6	16·0	17·6	16·0	21·9	27·8	25·1	18·6
P. Oils, fats and resins, manufactured	44·1	45·7	72·0	93·4	101·0	153·0	221·0	145·5
Q. Leather and manufactures thereof	6·5	7·7	8·8	3·6	4·1	6·3	4·5	4·8
R. Paper, cardboard, etc.	14·9	15·7	16·3	5·0	4·6	4·6	6·4	11·5
S. Vehicles (including locomotives, ships and aircraft)	4·5	7·9	44·2	106·3	3·3	14·8	6·1	4·4
T. Rubber manufactures	0·8	1·1	1·1	1·3	0·5	0·5	0·4	0·2
U. Miscellaneous articles wholly or mainly manufactured	20·9	19·7	40·1	64·8	7·7	7·2	7·3	8·5
Total, Class III	233·8	239·3	387·5	488·2	311·6	441·3	477·2	300·7
IV.—Animals, not for Food	3·3	3·1	2·4	0·8	2·9	2·3	2·7	4·1
V.—Parcel Post	4·3	4·0	3·8	7·1	10·7	13·1	27·6	15·1
Total	919·5	885·5	1,152·1	1,145·1	996·3	1,232·6	1,306·9	1,101·1
Munitions †	—	—	—	—	209·4	652·2	1,052·9	431·1

* The value of the Imports represents the cost, insurance and freight: or, when goods are consigned for sale, the latest sale value of such goods.

† *i.e.*, aircraft and other vehicles and parts, and arms, ammunition and military and naval stores imported by Government Departments. Not recorded separately prior to 1942.

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Values (f.o.b.*) of Exports of Produce and Manufactures
(£ million)

Class and Group	1938	1939	1940	1941	1942	1943	1944	1945
I.—Food, Drink and Tobacco—								
A. Grain and flour	1·7	1·2	0·6	0·3	0·1	0·2	1·3	2·0
B. Feeding-stuffs for animals	0·7	0·6	0·2	0·0	0·0	0·0	0·0	0·0
C. Animals, living, for food	0·0	0·0	0·0	0·0	0·0	0·0	—	—
D. Meat	1·2	1·2	0·8	0·3	0·1	0·2	0·2	0·4
E. Dairy produce	1·1	1·1	0·9	0·2	0·1	0·4	0·6	2·9
F. Fresh fruit and vegetables	0·4	0·3	0·2	0·1	0·2	0·5	0·8	1·0
G. Beverages and cocoa preparations	13·8	16·0	19·6	17·1	11·5	11·1	10·7	15·2
H. Other food	12·1	10·3	6·3	4·2	2·2	1·4	4·1	22·2
I. Tobacco	4·9	5·0	4·8	5·6	4·3	5·2	5·1	12·1
Total, Class I	35·9	35·7	33·4	27·8	18·5	19·0	22·8	55·8
II.—Raw Materials and Articles mainly Unmanufactured—								
A. Coal	37·4	38·2	25·3	8·0	6·0	6·2	5·0	6·6
B. Other non-metalliferous mining and quarry products and the like	1·0	1·2	0·8	0·6	0·5	0·3	0·3	0·6
C. Iron ore and scrap	0·6	0·3	0·0	0·0	0·0	0·0	0·0	0·0
D. Non-ferrous metalliferous ores and scrap	2·3	1·6	0·2	0·1	0·0	0·0	0·0	0·0
E. Wood and timber	0·1	0·1	0·1	0·0	0·1	0·1	0·1	0·1
F. Raw cotton and cotton waste	0·4	0·4	0·3	0·0	0·0	0·0	0·0	0·1
G. Wool, raw and waste, and woollen rags	6·3	4·7	3·3	2·8	0·9	0·2	0·2	1·7
H. Silk, raw and waste, and artificial silk waste	0·7	1·4	0·6	1·0	0·5	0·6	0·4	1·1
I. Other textile materials	0·3	0·2	0·1	0·0	0·0	0·0	0·0	0·0
J. Seeds and nuts for oil, oils, fats, resins and gums	2·9	2·0	2·0	0·6	0·2	0·1	0·1	1·4
K. Hides and skins, undressed	1·1	1·0	1·2	1·2	1·1	1·0	1·1	1·0
L. Paper-making materials	1·2	0·9	0·3	0·2	0·0	0·0	0·0	0·2
M. Rubber	0·2	0·2	0·2	0·1	0·1	0·0	0·1	0·3
N. Miscellaneous raw materials and articles mainly unmanufactured	2·4	2·2	1·8	1·1	0·8	0·8	0·8	1·9
Total, Class II	56·9	54·4	36·2	15·7	10·2	9·3	8·1	15·0
III.—Articles Wholly or Mainly Manufactured—								
A. Coke and manufactured fuel	3·3	4·0	2·2	1·2	0·8	1·1	0·4	0·5
B. Pottery, glass, abrasives, etc.	9·6	9·5	11·5	13·5	11·4	9·0	9·5	13·0
C. Iron and steel and manufactures thereof	41·6	32·8	31·0	18·9	9·8	6·0	8·5	20·6
D. Non-ferrous metals and manufactures thereof	12·3	12·7	12·4	7·6	6·9	6·5	4·7	11·7
E. Cutlery, hardware, implements and instruments	9·0	8·7	9·4	9·1	7·9	6·7	7·7	10·0
F. Electrical goods and apparatus	13·4	11·2	13·2	11·4	11·2	11·1	12·6	13·6
G. Machinery	57·9	47·4	36·5	30·9	29·9	28·0	41·2	44·6
H. Manufactures of wood and timber	1·2	1·1	0·7	0·5	0·4	0·2	0·2	0·5
I. Cotton yarns and manufactures	49·7	49·1	49·3	44·7	40·1	34·2	37·1	42·7
J. Woollen and worsted yarns and manufactures	26·8	26·7	28·7	29·8	25·0	18·5	15·3	21·5
K. Silk and artificial silk yarns and manufactures	5·5	5·9	8·7	11·8	16·4	12·7	16·4	17·3
L. Manufactures of other textile materials	10·7	11·1	12·4	9·6	8·9	6·3	5·2	7·1
M. Apparel	8·5	8·2	7·5	8·1	7·7	3·4	4·7	11·1
N. Footwear	2·0	1·9	1·8	5·4	1·4	0·7	0·7	1·2
O. Chemicals, drugs, dyes and colours	22·3	22·8	27·7	25·0	23·8	27·5	28·1	37·4
P. Oils, fats and resins, manufactured	5·4	4·7	3·0	2·2	0·7	0·7	1·3	3·0
Q. Leather and manufactures thereof	3·9	4·1	3·7	2·8	3·0	1·7	1·2	1·8
R. Paper, cardboard, etc.	6·9	6·7	9·8	7·2	5·0	4·1	4·1	5·2
S. Vehicles (including locomotives, ships and aircraft)	44·6	39·6	31·2	35·7	9·2	8·7	13·1	18·2
T. Rubber manufactures	1·7	1·4	1·9	1·3	0·8	0·3	0·3	0·7
U. Miscellaneous articles wholly or mainly manufactured	29·0	28·5	31·5	39·4	15·9	13·4	16·3	19·7
Total, Class III	365·3	338·1	334·1	316·1	236·2	200·8	228·6	301·4
IV.—Animals, not for Food	0·7	0·7	0·8	0·3	0·2	0·1	0·3	0·8
V.—Parcel Post	12·0	10·6	6·7	5·5	5·8	3·6	5·2	20·4
Total	470·8	439·5	411·2	365·4	270·9	232·8	265·0	393·4
Munitions †	—	—	—	—	120·1	103·9	62·0	34·5

* The value of the Exports represents the cost and the charges of delivering the goods on board the ship, and is known as the "free on board" value.

† *i.e.*, aircraft and other vehicles and parts, and arms, ammunition and military and naval stores exported by Government Departments. Not recorded separately prior to 1942.

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Values (f.o.b.*) of Exports of Imported Merchandise
(£ million)

Class and Group	1938	1939	1940	1941	1942	1943	1944	1945
I.—Food, Drink and Tobacco—								
A. Grain and flour	1·5	0·9	0·3	0·1	0·1	0·1	1·2	4·6
B. Feeding-stuffs for animals	0·1	0·1	0·1	0·0	—	—	0·0	0·0
C. Animals, living, for food	—	—	—	—	—	—	—	—
D. Meat	0·6	0·5	0·4	0·0	0·1	0·2	6·2	6·0
E. Dairy produce	0·7	0·6	0·1	0·0	0·0	0·1	0·1	7·3
F. Fresh fruit and vegetables	1·3	1·2	0·5	0·0	0·0	0·0	0·0	0·0
G. Beverages and cocoa preparations	6·1	5·6	5·4	2·3	1·3	1·2	2·5	3·0
H. Other food	1·2	1·1	1·0	0·1	0·0	0·1	0·7	3·7
I. Tobacco	0·8	0·9	0·1	0·1	0·1	0·7	0·1	1·6
Total, Class I	12·3	10·9	7·9	2·6	1·6	2·4	10·8	26·2
II.—Raw Materials and Articles mainly Unmanufactured—								
A. Coal	—	—	—	—	0·0	—	—	—
B. Other non-metalliferous mining and quarry products and the like	0·3	0·3	0·2	0·5	0·5	0·0	0·1	0·2
C. Iron ore and scrap	0·0	0·0	—	0·0	0·0	0·0	0·0	—
D. Non-ferrous metalliferous ores and scrap	0·9	0·5	0·3	0·1	0·0	0·0	0·0	0·3
E. Wood and timber	0·3	0·3	0·2	0·1	0·0	0·0	0·1	0·1
F. Raw cotton and cotton waste	1·3	1·2	0·5	0·0	0·0	0·0	—	2·3
G. Wool, raw and waste, and woollen rags	12·6	8·6	4·8	0·1	—	—	—	7·0
H. Silk, raw and waste, and artificial silk waste	0·0	0·0	0·0	—	—	—	—	—
I. Other textile materials	0·8	0·8	0·4	0·0	0·1	0·2	0·2	1·5
J. Seeds and nuts for oil, oils, fats, resins and gums	0·6	0·5	0·5	0·1	0·1	0·1	0·2	1·1
K. Hides and skins, undressed	9·6	7·8	3·6	0·3	0·1	0·2	0·2	0·3
L. Paper-making materials	0·1	0·0	0·0	0·1	—	—	—	0·0
M. Rubber	2·7	3·6	1·3	1·2	0·0	0·2	0·8	0·9
N. Miscellaneous raw materials and articles mainly unmanufactured	1·1	0·9	0·7	0·3	0·2	0·5	0·1	0·4
Total, Class II	30·3	24·5	12·5	2·8	1·0	1·2	1·7	14·1
III.—Articles Wholly or Mainly Manufactured—								
A. Coke and manufactured fuel	—	0·0	0·0	0·0	—	—	—	—
B. Pottery, glass, abrasives, etc.	0·1	0·0	0·0	0·0	0·0	0·0	0·0	0·0
C. Iron and steel and manufactures thereof	0·2	0·1	0·1	0·1	0·1	0·1	0·0	0·2
D. Non-ferrous metals and manufactures thereof	9·1	2·7	0·7	1·1	0·7	0·3	0·9	4·9
E. Cutlery, hardware, implements and instruments	0·7	0·6	0·3	0·1	0·0	0·1	0·1	0·3
F. Electrical goods and apparatus	0·1	0·1	0·0	0·0	0·0	0·0	0·0	0·1
G. Machinery	0·9	0·9	0·5	0·4	0·1	0·4	0·2	0·6
H. Manufactures of wood and timber	0·2	0·2	0·0	0·0	0·0	0·0	0·0	0·0
I. Cotton yarns and manufactures	0·2	0·1	0·1	0·0	0·0	0·0	0·0	0·0
J. Woollen and worsted yarns and manufactures	0·5	0·4	0·2	0·1	0·0	0·0	0·1	0·1
K. Silk and artificial silk yarns and manufactures	0·5	0·3	0·1	0·0	0·0	0·0	0·0	0·0
L. Manufactures of other textile materials	0·1	0·2	0·2	0·1	0·0	0·0	0·0	0·8
M. Apparel	0·4	0·4	0·1	0·0	0·0	0·0	0·0	0·0
N. Footwear	0·0	0·2	0·0	0·0	0·0	0·0	0·0	0·0
O. Chemicals, drugs, dyes and colours	0·5	0·5	0·6	0·4	0·2	0·8	0·1	0·4
P. Oils, fats and resibs, manufactured	1·1	1·0	1·3	0·4	0·7	0·5	0·6	1·6
Q. Leather and manufactures thereof	0·9	0·6	0·4	0·0	0·1	0·0	0·0	0·4
R. Paper, cardboard, etc.	0·1	0·0	0·0	0·0	0·0	0·0	0·0	0·0
S. Vehicles (including locomotives, ships and aircraft)	0·4	0·3	0·1	4·2	0·0	0·2	0·1	0·1
T. Rubber manufactures	0·0	0·0	0·0	0·0	0·0	—	0·0	0·0
U. Miscellaneous articles wholly or mainly manufactured	2·2	1·4	0·7	0·4	0·1	0·1	1·0	0·4
Total, Class III	18·2	10·0	5·4	7·3	2·0	2·5	3·1	9·9
IV.—Animals, not for Food	0·7	0·6	0·2	0·0	0·0	0·0	0·1	0·2
Total	61·5	46·0	26·0	12·7	4·6	6·1	15·7	50·4
• Munitions †	—	—	—	—	6·0	7·3	2·6	0·2

* The value of the Exports represents the cost and the charges of delivering the goods on board the ship, and is known as the "free on board" value.

† *i.e.*, aircraft and other vehicles and parts, and arms, ammunition and military and naval stores re-exported by Government Departments. Not recorded separately prior to 1942.

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BANK OF ENGLAND

Pursuant to the Act 7th and 8th Victoria, cap. 32 (1844),

(000's omitted)

1	2	3	4	5	6	7	8
ISSUE DEPARTMENT						COLLATERAL COLUMNS	
Liabilities		Assets					
Notes Issued	DATES (Wednesdays)	Govt. Debt (£11,015) and Govt. Securities	Other Securities	Gold Coin and Bullion*	Silver Coin	Notes in Hands of Public	Minimum Discount Rate
£		£	£	£	£	£	Per cent.
1,250,242	Jan. 3	1,249,077	916	242	7	1,238,436	2
1,250,242	,, 10	1,249,213	780	242	7	1,231,638	(26.10.39)
1,250,242	,, 17	1,249,280	708	242	12	1,220,368	
1,250,242	,, 24	1,249,330	658	242	12	1,217,700	
1,250,242	,, 31	1,249,405	583	242	12	1,219,635	
1,250,242	Feb. 7	1,249,319	669	242	12	1,221,378	
1,250,242	,, 14	1,249,368	620	242	12	1,218,907	
1,250,242	,, 21	1,249,350	641	242	9	1,215,671	
1,250,242	,, 28	1,249,331	662	242	7	1,217,141	
1,250,242	Mar. 7	1,249,341	649	242	10	1,220,564	
1,250,242	,, 14	1,249,372	618	242	10	1,221,761	
1,250,242	,, 21	1,249,337	654	242	9	1,222,836	
1,250,242	,, 28	1,249,380	614	242	6	1,235,788	
1,250,242	April 4	1,249,284	709	242	7	1,240,265	
1,250,242	,, 11	1,249,285	703	242	12	1,238,686	
1,250,242	,, 18	1,249,342	649	242	9	1,236,290	
1,250,242	,, 25	1,249,272	720	242	8	1,235,220	
1,250,242	May 2	1,249,350	641	242	9	1,240,679	
1,300,242	,, 9	1,299,247	742	242	11	1,250,055	
1,300,242	,, 16	1,299,269	717	242	14	1,261,571	
1,300,242	,, 23	1,299,176	811	242	13	1,266,722	
1,300,242	,, 30	1,299,245	746	242	9	1,269,640	
1,300,242	June 6	1,299,278	715	242	7	1,270,839	
1,300,248	,, 13	1,299,303	688	248 *	9	1,276,016	
1,300,248	,, 20	1,299,240	750	248	10	1,277,475	
1,300,248	,, 27	1,299,283	711	248	6	1,285,156	
1,350,248	July 4	1,349,316	676	248	8	1,294,071	
1,350,248	,, 11	1,349,330	661	248	9	1,297,985	
1,350,248	,, 18	1,349,352	638	248	10	1,301,078	
1,350,248	,, 25	1,349,272	722	248	6	1,305,719	
1,350,248	Aug. 1	1,349,307	687	248	6	1,319,770	
1,350,248	,, 8	1,349,315	673	248	12	1,323,842	
1,350,248	,, 15	1,349,303	687	248	10	1,327,198	
1,350,248	,, 22	1,349,381	605	248	14	1,325,636	
1,350,248	,, 29	1,349,347	641	248	12	1,325,914	
1,350,248	Sept. 5	1,349,376	612	248	12	1,330,936	
1,350,248	,, 12	1,349 422	566	248	12	1,330,179	
1,350,248	,, 19	1,349,467	523	248	10	1,329,476	
1,350,248	,, 26	1,349,363	625	248	12	1,329,916	
1,350,248	Oct. 3	1,349,363	627	248	10	1,333,378	
1,350,248	,, 10	1,349,395	591	248	14	1,330,204	
1,350,248	,, 17	1,349,397	592	248	11	1,325,132	
1,350,248	,, 24	1,349,399	587	248	14	1,319,812	
1,350,248	,, 31	1,349,413	577	248	10	1,322,191	
1,350,248	Nov. 7	1,349,356	634	248	10	1,326,352	
1,350,248	,, 14	1,349,436	556	248	8	1,326,399	
1,350,248	,, 21	1,349,338	652	248	10	1,324,592	
1,350,248	,, 28	1,349,349	642	248	9	1,327,619	
1,350,248	Dec. 5	1,349,288	601	248	11	1,336,549	
1,400,248	,, 12	1,399,426	763	248	11	1,359,527	
1,400,248	,, 19	1,399,297	696	248	7	1,376,022	
1,400,248	,, 26	1,399,331	663	248	6	1,379,942	

* At 168*s.* per fine oz. (at 172*s.* 3*d.* from June 9, 1945).

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WEEKLY RETURN

for Wednesday in each Week, during the Year 1945

(000's omitted)

9	10	11	12	13	14	15	16	17	18
BANKING DEPARTMENT									
Liabilities				DATES	Assets				Totals of Liabilities and Assets
Capital (£14,553) and Rest	Public Deposits	Bankers' Deposits	Other Deposits	(Wednesdays)	Govt. Securities	Discounts and Advances	Other Securities	Reserve (Notes and Coin)	
£	£	£	£		£	£	£	£	£
17,855	16,382	251,359	56,443	Jan. 3	300,838	15,210	12,478	13,513	342,039
17,884	9,449	222,172	54,645	,, 10	258,988	11,173	13,874	20,175	304,150
17,899	7,812	214,102	56,993	,, 17	242,593	8,613	14,271	31,329	296,806
17,916	6,850	208,065	58,207	,, 24	229,983	12,528	14,504	34,023	291,038
17,938	11,562	215,066	57,848	,, 31	240,668	6,632	22,971	32,143	302,414
17,961	22,586	176,541	56,567	Feb. 7	217,248	7,003	19,287	30,387	273,925
17,978	13,247	183,589	59,273	,, 14	216,893	10,207	14,125	32,862	274,087
17,993	10,327	196,674	59,233	,, 21	223,953	9,203	14,907	35,164	284,227
18,015	18,142	207,775	60,473	,, 28	238,003	8,508	23,130	34,764	304,405
18,027	6,257	204,113	57,379	Mar. 7	231,978	7,824	14,772	31,202	285,776
18,040	12,474	190,760	55,973	,, 14	222,704	9,718	14,822	30,003	277,247
18,061	9,664	218,370	55,354	,, 21	247,173	10,212	15,124	28,940	301,449
18,077	8,912	218,935	57,003	,, 28	245,808	18,595	22,600	15,924	302,927
17,653	10,877	187,930	55,980	Apr. 4	225,553	20,660	14,834	11,393	272,440
17,669	10,196	184,613	51,737	,, 11	217,693	19,984	13,642	12,896	264,215
17,687	18,912	191,293	52,545	,, 18	230,203	21,028	13,935	15,271	283,437
17,703	8,474	229,647	50,498	,, 25	256,288	20,051	13,616	16,367	306,322
17,735	13,432	192,290	49,050	May 2	222,968	24,178	14,501	10,860	272,507
17,749	19,547	176,126	51,553	,, 9	181,443	17,619	14,471	60,442	264,975
17,758	17,062	184,870	50,294	,, 16	203,913	13,024	13,091	39,956	269,984
17,775	9,786	219,337	49,951	,, 23	237,738	10,234	14,127	34,750	296,849
17,791	14,791	212,391	50,692	,, 30	230,563	9,584	23,708	31,810	295,665
17,816	18,232	181,171	51,325	June 6	219,428	5,514	12,961	30,641	268,544
17,825	8,445	201,364	51,710	,, 13	235,478	4,489	13,931	25,446	279,344
17,851	10,126	210,572	51,683	,, 20	249,008	3,869	13,331	24,024	290,232
17,866	12,676	262,272	51,590	,, 27	300,818	3,841	23,396	16,349	344,404
17,881	9,068	251,072	58,313	July 4	261,973	3,732	13,266	57,463	336,334
17,889	8,772	218,996	52,719	,, 11	227,938	3,893	13,151	53,394	298,376
17,902	11,012	210,926	52,722	,, 18	227,263	2,148	12,987	50,164	292,562
17,935	10,343	229,050	53,582	,, 25	250,663	1,823	12,975	45,449	310,910
17,950	11,088	220,985	56,053	Aug. 1	256,898	5,814	12,190	31,174	306,076
17,966	15,004	203,092	53,501	,, 8	246,588	2,313	13,652	27,010	289,563
17,983	11,376	210,741	55,979	,, 15	258,618	2,196	11,630	23,635	296,079
17,997	9,761	227,067	55,484	,, 22	269,163	5,179	10,844	25,123	310,309
18,009	16,044	238,165	54,994	,, 29	272,108	7,178	23,160	24,766	327,212
18,016	16,237	214,848	52,678	Sept. 5	263,938	6,205	11,979	19,657	301,779
18,036	18,243	218,561	55,027	,, 12	275,423	2,216	11,836	20,392	309,867
18,053	9,697	248,336	54,583	,, 19	295,225	2,744	20,773	21,047	330,669
18,079	5,524	279,135	53,135	,, 26	322,155	3,614	9,548	20,556	355,873
18,090	9,554	232,934	57,037	Oct. 3	282,540	6,682	11,301	17,092	317,615
17,669	25,060	211,756	56,818	,, 10	275,205	4,262	11,492	20,344	311,303
17,686	21,363	232,451	55,649	,, 17	288,555	747	12,420	25,427	327,149
17,699	14,583	241,564	55,993	,, 24	276,160	10,362	12,523	30,794	329,839
17,709	9,161	244,235	57,439	,, 31	266,875	11,180	22,029	28,460	328,544
17,715	18,223	221,248	53,397	Nov. 7	258,255	10,933	17,046	24,349	310,583
17,747	6,548	225,367	57,669	,, 14	259,260	10,283	13,477	24,311	307,331
17,758	23,128	217,900	56,127	,, 21	262,985	11,983	13,784	26,161	314,913
17,780	10,451	250,195	57,652	,, 28	277,215	11,349	24,402	23,112	336,078
17,788	14,774	219,023	57,168	Dec. 5	274,105	6,903	13,682	14,063	308,753
17,810	20,605	217,139	57,200	,, 12	239,240	18,354	14,077	41,083	312,754
17,844	7,218	228,893	59,874	,, 19	266,005	9,125	14,070	24,629	313,829
17,848	5,276	274,497	58,488	,, 26	313,470	8,414	13,554	20,671	356,109

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Vol. CIX.] [Part II.

Journal of the Royal Statistical Society

PART II, 1946

The Statistical Study of Infectious Diseases

By Major Greenwood, F.R.S.

[Read before the Royal Statistical Society, February 27th, 1946, the President, the Rt. Hon. Lord Woolton, P.C., C.H., in the Chair]

"'Yes,' said the doctor. 'Yes, I think so. It has a strong resemblance at any rate to some sort of epidemic. Probably Epidemic Hypertrophy will meet the case.' 'Epidemic!' said the Vicar. 'You don't mean it's contagious?'

"The doctor smiled gently and rubbed one hand against the other. 'That I couldn't say,' he said."—H. G. Wells, *The Food of the Gods*, Bk. 2, Chap. 1, Sect. 5.

Unfortunately, or perhaps fortunately, growth is neither epidemic nor contagious among adults past middle life, but the question whether a disease, or an opinion, is infectious or contagious is, as the doctor implied, not always easy to answer and has often been answered very confidently and quite wrongly. The object of this paper is to discuss what help the statistician can give in answering the question. It has interested me for years. I have not made much progress, but the account of my partial failures may stimulate quicker minds, and at least I can explain to statisticians who have not read many medical books why the problem is difficult.

It is easy to forget that words, technical words particularly, change their meaning. Thus words translated by contagion or infection were used thousands of years ago by Greek and Roman writers, but the significance they attached to them was different from that we attach to them. The first writer to use the word, or words, in our sense was Fracastorius, who was born in 1483, and published in 1543 a treatise on contagion and contagious diseases. Fracastorius was the first writer to conceive of a *living* principle of contagion and to think of the spread of a contagious (or, infectious *) disease as a biological phenomenon, due to the passing on of a living creature capable of growth, reproduction and death. Before Fracastorius, indeed long after his time, physicians and philosophers found the origin of most, perhaps all, infectious diseases in the "atmosphere." The qualities of the "atmosphere" were corrupted, the corrupted atmosphere discharged "seeds" which set up corruption in the "humours" of a living body; the corruption of these generated (or perhaps simply passed on) seeds or atoms in effluvia, mostly in the expired air or perspiration, and infected others. What these seeds or particles might be nobody enquired closely, certainly nobody thought of them as *we* think of "germs."

This is not a merely archæological digression. The point is that, unlike what we now hold to be the true doctrine, this old belief is spiritual—the seeds, like the atoms of Lucretius, or indeed the atoms of our schooldays, could not be shown on a plate. Also some, indeed a great many, victims were not (by hypothesis) infected at all from another victim, but directly by the "atmosphere."

So there was plenty of scope for disputing whether such or such a disease were infectious or not. The medical historian, Hirsch, was writing not in the seventeenth century, but a little more than sixty years ago, at the beginning of the bacteriological age. He agreed with mothers of large families that measles was a highly infectious disease, and he spoke with some asperity of those who doubted whether bubonic plague were an infectious disease. But he found no reason at all to think that influenza was infectious, and (in spite of Koch's early papers) was very dubious whether infection were of any importance in the spread of tuberculosis. It would seem strange to any reader of this story to doubt the infectious nature of either influenza or tuberculosis, and

* Strictly speaking, a contagious disease is spread by contact only, not, for instance, by the discharge of droplets. The venereal diseases are contagious, tuberculosis contagious and infectious. But the quotations in *N.E.D.* show that the terms were, from quite early times, confused.

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he also knows that, in human beings, bubonic plague is not an infectious disease at all—viz., cannot be conveyed directly from one human being to another.

The explanation is not that Hirsch was a stupid man, but that his criteria were really statistical and that his statistical data—like his statistical methods—were crude.

Mothers of large families noted how measles ran through the families (their own and their neighbours'), and needed no refined statistical method to satisfy them this was an infectious disease. No doubt citizens in plague-time noted the spread of disease through a household and into adjacent households and drew a conclusion which was natural but not correct.

Even if there had been more than one John Graunt in the seventeenth century, there were no arithmetical data of the distribution of cases of measles or of plague in families of different sizes, so it would not have been possible to pass beyond general impressions.

The data are not extensive now; but, with the help of Dr. Percy Stocks, I was able to make a small statistical experiment fifteen years ago, the results of which pleased me; as they were published in a journal which is not read by general statisticians and had too algebraical a flavour to attract medical officers of health, it is fairly safe to say very few people have read them. So I permit myself an exhumation.*

The data of Table I (p. 88) (excepting the figures in brackets) relate to children under the age of 10 years who had not had measles but were exposed within the house to the risk of infection. In most instances these would be families of children, and the data are of what happened to the brothers and sisters of the first patient.

If measles were not an infectious disease, one might expect that the frequencies could be fitted by a binomial or a Poisson series. They cannot; there are far too many multiple sets. For instance, in the series for three children exposed to risk, there are 60 examples of a single infection and 27 of three infections, the binomial (taking p as the ratio of total infections to total exposed) postulates 100·9 and 6·5.

Now, however, consider what pathologists think happens. They think the disease is conveyed by droplets coughed or sneezed out by the first patient, who, however, is only infective for a very short time. Each victim is in turn infective also for only a very short time. This gives one a suggestion—why not try instead of a single binomial a chain of binomials? To make matters easy, we will suppose that all the binomials have the same p. On this hypothesis we might reach three victims in a family of 4 (that is, three exposed to risk) in four different ways.

(1) All three exposed are infected by the first victim—chance p^3.
(2) Two infected by the first exposure and then one—combined probability $3p^3q$.
(3) One from the first and two from the second exposure—combined probability $3p^3q^2$.
(4) One from each exposure—combined probability $6p^3q^3$.

Some rather lengthy but straightforward algebra enables us to equate the mean of the distribution to a sum involving integral powers of q. For instance, for 3 exposed, the mean number of cases is equal to $3 - 3q^2 + 3q^3 - 15q^4 + 18q^5 - 6q^6$. Using this device one obtains the expected frequencies given in brackets in the table. The P's range from 0·46 to 0·75 (two degrees of freedom are absorbed), which is satisfactory enough. One might do better by assuming that the p's vary along the links of the chain, but—apart from the labour—it would mean absorbing more degrees of freedom, and (owing to the decline of the birth rate) one would not have many families of more than 6. For theoretical reasons,† a chain of Poissons might be preferred, but I do not think the arithmetical results would have been much changed, and the binomial method is neat.

I have a paternal regret that nobody has used the method, but as a human being I am not much surprised, because nobody wants to verify the infectiousness of measles, and when the question is of a dubiously infectious disease we shall not have enough data to use a device of this kind. Under dubiously infectious I include not only "new" diseases (for which data will certainly be few), but diseases believed to be infectious but respecting which the vehicle of infection is uncertain.

Before embarking upon another little voyage into algebra, I propose to examine the ways in

* I cannot ask space to reproduce the rather lengthy algebra, see Greenwood, *Journal of Hygiene* vol. 31, 1931, pp. 336–51.

† Greenwood and Yule, *Journ. Roy. Stat. Soc.*, vol. 83, 1920, pp. 256–7.

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which field epidemiologists, caring not at all for algebra and with no passion for arithmetic, do actually work. As an exemplar I shall choose not some classical piece—like Budd's work on Typhoid Fever or Snow's on Cholera—but the investigations of a contemporary, Dr. W. N. Pickles.*

A common—to judge from war-time experience, increasingly prevalent—disease is epidemic catarrhal jaundice. It is characterized by jaundice and symptoms occasionally alarming, but (at any rate in pre-war experience) was rarely mortal. General observations suggested that it was an infectious disease, but the biological principle, the "germ," was, and still is, unknown; there was difference of opinion on the length of the incubation period (*i.e.*, the average interval between infection and falling sick) and the vehicles of infection. It was believed that infection from person to person, as in measles, by droplets was the usual process, but transmission by other means—for instance, contaminated food or drink—was also considered.

Dr. Pickles had practised for many years in Wensleydale, knew the district intimately and kept admirable records of his experience. He had the advantage over town physicians—mentioned by Budd—that the wood was not so thick that he could not know intimately the individual trees. He had another advantage which he was too modest to mention—an intellectual affinity to John Graunt.

In 1928 this disease became epidemic in Wensleydale; in a population of some 5,700 there were 250 cases, in 118 of these the patient was attended by Dr. Pickles or his partner, and he came to the conclusion that the disease had an incubation period of from 27 to 35 days. Instead of abstracting the whole of the evidence, I shall quote in full the author's account of a connected series of 15 cases.

"Next follows what I consider my best series, especially as it includes an instance of the short and only possible exposure:—

A (the first victim)

Not attended personally, but said to have had jaundice in July 1929. It is impossible to trace the infection in his case, but it is significant that he visited Askrigg, a hotbed of infection, in June.

B (A's sister)

Commenced with the disease on August 23rd, and while still obviously jaundiced attended a village fête on August 23rd. All those on the line below were present also and developed the disease on the given dates, all 1929.

C	D	E	F	G
(Sept. 24th) B was known to have been in the house during the day.	(Sept. 24th)	(Oct. 1st) (Who spent the afternoon with B). A maid in a distant village who was proved to have been in contact with the following patients in that village who subsequently infected others.	(Sept. 25th)	(Sept. 28th)

H	J	M	P
(Nov. 5th) Small son of E's employer.	(Nov. 4th) A friend of H and E.	(Nov. 5th) An admirer of E.	(Oct. 27th) Great aunt of E.
	K	N	
	(Dec. 8th) Brother of J.	(Dec. 5th) Brother of M (same house).	
	L	O	
	(Jan. 8th, 1930) A friend of K.	(Jan. 4th, 1930) Sister of N (same house).	

"The five names on the third line are those of the only sufferers from jaundice in the whole district who commenced in the week September 24th to October 1st. Three of these were from the village in which the fête was held, and I found, as I expected, that they had all been present on August 28th. I felt sure also that I was on the track of an interesting discovery

* W. N. Pickles, *Epidemiology in Country Practice*, Bristol (John Wright), 1939, pp. 114.

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when it transpired that the other two from distant villages had this, and only this, experience in common with those three. Someone suffering from jaundice, therefore, had also probably been a visitor at the fête. I tried various sources, and at last, after a long search, I discovered the culprit where I least expected, almost on my own doorstep.

" This was a young girl (B) whom I had actually seen in bed on the very morning of the 28th, and who I never dreamt would be able to get up that day. I have no doubt she exercised considerable skill and elusiveness at the entertainment, for I was also among the throng, and did not know of her presence. This girl had as a friend another girl of 16 (E), who lived in a distant village, and these two girls spent the afternoon together, with the result that E herself commenced with the disease on October 1st, and, as can be seen in the diagram, infected four others in this village. She was employed as a maid, and she infected her employer's small son, this boy's friend, and her own great-aunt, who lived in the village. The infection in all these was easy to explain, but in the fourth (M), a rather pathetic little fellow of middle age, it was not so clear. At last I tackled his sister, who gave him away quite shamelessly. Studies in epidemiology sometimes reveal romances. 'Oh, yes,' she said, 'he's very fond of E. He often goes in at the back door in the evenings, and helps her to wash up.' The brother of M in the same house commenced on December 5th, and the faithless sister, above mentioned, with poetic justice succumbed on January 4th. J, who was a friend of H and E, commenced on November 4th, his small brother on December 8th and this small brother's friend on January 8th. Thus, to my knowledge, thirteen instances of this disease resulted from the determination of one young girl, jaundice or no jaundice, not to be deprived of what she considered her legitimate amusement."

TABLE I*

Frequencies of secondary cases of measles in groups of 2, 3, 4 or 5 children exposed to risk, compared with the expectations of chain-binomials (bracketed figures)

Cases *	(m = 2)		(m = 3)	
0	197 (198·3)	120 (122·4)	84 (89·7)	37 (40·8)
1	104 (101·4)	93 (88·2)	60 (52·7)	34 (27·9)
2	57 (58·3) P = 0·75	86 (88·4) P = 0·58	57 (54·4)	42 (42·7)
3	— —		27 (31·2) P = 0·36	36 (37·6) P = 0·42

	(m = 4)	(m = 5)
0	60 (59·54)	25 (26·86)
1	29 (28·87)	15 (12·42)
2	25 (24·45)	9 (10·63)
3	11 (14·35)	10 (6·90)
4	7 (4·79) P = 0·61	1 (3·30)
5	— —	1 (0·89) P = 0·42

* In the two smaller series only children with *no* previous history of measles were included.

I used to tell my students that field epidemiology was like detecting criminals; but there are not many epidemiologists who can write like Dr. Pickles.

In all Dr. Pickles had 40 instances of patient-to-patient spread; in 6 the interval from case to case was 26 days, in 6, 27 days, in 5, 28 days, in 7, 29 days, in 5, 30 days, in 5, 31 days, in 2, 32 days, in 1, 33 days, in 2, 34 days and in 1, 35 days.

I do not think it necessary to use paper in demonstrating that Dr. Pickles has established a *very* strong case in favour of his contention that the incubation period of the disease as observed in Wensleydale was about a month and, of course, that it was infectious. But, if the layman thinks he will often find in the literature of epidemiology such pretty demonstrations, he will be disappointed. There are many reasons for this. I have mentioned two: ability to do such an investigation is rare, and in a great town it is not easy to simplify human relations; the clues are tangled up. That may happen even in the country when the epidemic is overwhelming (as in influenza) and the incubation period short.

Here again, however, chance will favour a prepared mind, Dr. Pickles' mind.

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"For instance, a few years ago a farmer spent a night in Birmingham, having travelled this distance to bring back a new car. He commenced with influenza a short time afterwards, infected his family, and they in turn a large village. Again, a few years ago, a particularly worthy schoolmistress returned to her home after a Christmas holiday spent with her relatives. On the morning of the school opening, knowing that she was ill, with commendable if mistaken zeal, she attended her school. In the afternoon she was utterly unable to return, but from that brief morning session a crop of 78 cases resulted. I did not realize this myself until I examined my chart afterwards, and with the exception of one case, which might possibly have been derived from a neighbouring town, there was no other conceivable avenue by which the epidemic could have reached the district."

Of course all field epidemiologists do proceed on Dr. Pickles' lines, but, to use the detective analogy, they are like Scotland Yard inspectors sent down when a crime has been committed, suspects numerous and the scent cold. I am not thinking of water-borne or food-borne diseases—in that department *ex post facto* enquiry has often been decisive—but of the kind of disease so far illustrated where transmission from person to person is, *a priori*, the most attractive hypothesis. It is often possible to connect one case with another—or to show, on a spot map, or series of spot maps, what looks like radiation from foci, such as schools. But it is not easy, in fact it is sometimes impossible, to establish connections. One is particularly conscious of this in studying the literature of Acute Poliomyelitis (epidemics of what used to be called infantile paralysis); the most diligent enquiry often fails to establish any human connection (or any common consumption of food) linking together foci widely separated.

But let us suppose a connection is established, and use Dr. Pickles' data as an illustration. Take two of the chains in his diagram, the EMNO series and EJKL series. We have the case dates, October 1st, November 5th, December 5th, January 4th, in one set and October 1st, November 4th, December 8th, January 8th in the other. Taking October 1st as origin, the succession is 0, 36, 66, 96 in one and 0, 35, 69, 100 in the other. The intervals between successive cases are 36, 30, 30 and 35, 34, 31. The connection was actual contact, and what specially interested Dr. Pickles was the uniformity of the intervals, all six between 30 and 37. If one took at random a couple of points on a line 96 units or a line 100 units long, the average intervals formed would be respectively 32 and 33·33 days, but all lengths are possible from 0 to 96 or 0 to 100—if for instance one point fell at 0 and another at 96, then the three intervals would be 0, 96, 0; there is no reason why they should fall within a narrow range about the average. So one might enquire whether the variance of the intervals were smaller than could be reasonably expected as a result of sampling when the points actually had been taken at random. When only two points are in question, this is a soluble problem (see p. 99), the probabilities of random sampling producing variances so small as in these two trials are 0·09086 and 0·05441. A combined probability of 0·0049.

With the other evidence before us, this is a mere slaying of the slain; but, as already said, one does not find in epidemiological literature many such neat demonstrations, so it is at least worth considering whether by a study of the distribution of intervals the statistician can give the epidemiologist any help. In an appendix I have summarized what I could find in the literature or work out for myself on the algebra and arithmetic of interval-distributions; it interested me, but whether it *is* of any use needs careful consideration.

Let us begin with the simplest case. Over a period—a month, a year or what period we please—n cases of disease or n deaths have occurred in a house, family or institution, and we know the dates of occurrence. By one or other of the methods described in the appendix we can determine the probability that the frequency distribution of intervals (or some parameter such as the variance) might have arisen in sampling a random succession. Let us suppose that calculation establishes that the hypothesis is improbable and should be rejected; is this finding of epidemiological value? Very often it will be of no value whatever, it will be mere arithmetical trifling. We do not need to do any sums to satisfy us that the distribution of intervals between successive deaths from enemy action in vulnerable areas is not a random distribution. Unless we can go farther than this we shall not be helpful to the field worker.

One of the most interesting examples in epidemiological literature of only partly solved problems is that of epidemic poliomyelitis or, to give it the popular name, infantile paralysis. As *a clearly*

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recognized epidemic disease, this has not a very long history; indeed, in our country it has only attracted serious attention since 1910, although both Sweden and the United States of America had serious epidemics in the previous decade.

The fundamental biology of the disease is well known, viz. that the *materies morbi* is a virus, which may be conveyed from person to person by droplet infection—discharging of infective particles in coughing or sneezing—or by contamination of food. Study of the literature shows that the earlier writers gave droplet infection the first place, but recently opinion has moved in the direction of emphasizing the alimentary route. For instance, McAlpine * has examined the incidence of poliomyelitis on the troops in the Indian Command during the war. He points out that British officers suffered a much greater incidence than other ranks—British or Indian. In 1942, the attack rate on British officers was 1·7 per 1,000, in 1943, 0·5, and in 1945, 1·4. Other British ranks had incidences of 0·3, 0·1 and 0·3; Indian troops a rate of 0·01 in each year. He attributes this differentiation to poor sanitary supervision of officers' messes and, of course, of the civilian restaurants, which they use much more generously than other ranks.

Pearson and others † record a thorough investigation of families in Fort Worth, Texas, at the time of an epidemic. Here is a summary of their results:—

> "During the 1943 epidemic of poliomyelitis in Fort Worth, an intensive study of the distribution of virus was made in a selected district of the city. Stools from 524 persons were tested for virus by inoculation into monkeys. Six (75 per cent.) of 8 households, representing 27 familial contacts, were positive for virus, as were 8 (18 per cent.) of 45 households containing 80 non-familial contacts and two (1·6 per cent.) of 127 households representing 374 non-contacts.
>
> "Virus was not recovered from specimens of water, sewage, flies, ants, cockroaches or droppings of domestic animals.
>
> "An agent that produced paralysis in mice and cotton rats was obtained from a pool of brains and intestines from 22 Norwegian rats from the city dump. No virus was recovered from several batches of mice and rats collected in various parts of the city."

The inference is that intra-domestic contamination of food *may* be an important means of spread.

It is certain that, in comparison with such diseases as measles, whooping-cough, scarlet fever or diphtheria, infantile paralysis spreads, at any rate as a recognizable illness, slowly. But sometimes it is a real scourge: there are a large number of cases in a small population. Here is an example of a dramatic incident observed during the epidemic of 1911 in South-West England. The hamlet of Stoke Rivers consisted of 18 houses with a total population of 119, and among these 36 attacks were recorded. Only 4 of the houses had no patients and 1 house with 13 inmates 9.

In the epidemic in Sweden of 1905 in a small parish (Trästena) with 500 inhabitants, 49 persons were affected, 26 of these with the paralytic form of the disease—of these 11 died.

A catalogue could be made of grim incidents like this. There is, as might be expected, a goodly list of papers and reports on the subject, but they contain few, in fact hardly any, data like those Dr. Pickles gave us for epidemic jaundice. Sometimes one can find chains of cases in families, schools or institutions, but I have not been able to collect a sufficient number to provide material for the application of the method described in the appendix to this paper to be of any practical use.

The papers I have found most stimulating are two by Aycock and Eaton.‡ In the first place the authors made frequency distributions of the intervals between the first and subsequent cases in the same family (Table III). Of course, if there were more than two cases in a family, that family would contribute more than one entry into the table, and how often that happened is not stated, but it appears from other evidence that there were few instances of three or more cases in a family, so that a great majority of the entries are records of pairs.

The authors also supplied a table of frequencies for scarlet fever and measles. No calculations were required to convince one that the infantile paralysis series was not *in pari materia*

* *Lancet*, 1945, ii, p. 130.
† *Amer. Journ. of Hygiene*, vol. 41, 1945, p. 188.
‡ *Amer. Journ. Hygiene*, vol. 15, 1925, pp. 725–41

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with those for measles and scarlet fever. One also finds that the combination of cases of the grave form of disease, where the victim has paralysis of some group of muscles, with mild cases leads to an ambiguity.

"The difficulties in the recognition of abortive cases of infantile paralysis are well known. The presence of a paralytic case undoubtedly influences the diagnosis. It is our impression that this influence decreases as time elapses from the appearance of the paralytic case; that is, if an indefinite illness, say, a digestive upset with slight nervous symptoms, occurs within a few days of a frank case of infantile paralysis in the same family, its chances of being called infantile paralysis are much greater than if it had occurred some weeks earlier or later than the paralysed case." *

TABLE II

Incubation Periods for Experimental Inoculations of Infantile Paralysis in Monkeys

(Aycock and Eaton)

Period from inoculation in days	Frequency	Period from inoculation in days	Frequency	Period from inoculation in days	Frequency	Period from inoculation in days	Frequency
3	3	10	22	17	3	24	0
4	17	11	25	18	1	25	0
5	30	12	12	19	0	26	0
6	33	13	11	20	2	27	1
7	42	14	7	21	3		
8	37	15	4	22	0	Total	286
9	30	16	0	23	3		

The authors also provide a table of the interval elapsing between the experimental inoculation of the virus and the appearance of physical signs of disease in monkeys. (Table II.) Aycock and Luther have also collected † data of intervals between exposure to risk of infection and onset of disease in human beings, and this evidence is consistent with the opinion that the incubation period is not less than 7 days and, on the average, more. This work was done some time ago, but I do not think we have any more published statistical evidence of the kind.

TABLE III

Infantile Paralysis. Interval between initial and secondary case in same family.

(Aycock and Eaton)

Locus	Interval in days																	
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Sweden 1905	12	22	15	14	11	7	9	7	5	5	5	1	2	(Over 12 days 12)					
New York City 1906	61	45	47	41	46	45	26	17	22	9	8	7	3	3	2	1	3	3	2
Massachusetts 1916	10	21	18	20	12	14	16	14	13	(Over 8 days 54)									
Aycock and Eaton's series:																			
Total	40	46	34	29	28	19	23	27	15	6	2	4	5	4	4	6	4	1	0
Paralysed	9	17	17	12	14	8	10	15	10	1	1	3	4	2	3	3	3	1	0
Abortive	31	29	17	17	14	11	13	12	5	5	1	1	1	2	1	3	1	0	0

This is the position; none of the secondary cases with intervals not exceeding 6 days can, so it appears, have been derived from the primary cases, the patients must have been infected in some other way. But *some* of the patients whose cases were at intervals of more than 6 days from the first cases in family may have been infected within the family. *What* proportion? Suppose one has n pigeon-holes numbered from 1 to n, there are n^2 ways in which two can be independently chosen; in n of these the choice will fall on pigeon-holes with the same index, in $2(n-1)$ there will be a difference or interval of 1, in $2(n-2)$ a difference of 2, and so on. In the table we have

* *Op. cit.*, p. 727.

† *Journ. of Preventive Medicine*, 1929, vol. 3, p. 103.

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series collected by Aycock and Eaton. A rough test of independence would be to find that zero intervals were about half as numerous as unit intervals; this is so for the Swedish, Massachusetts and Aycock and Eaton's "paralysed" series. Taking the latter, one finds that for n from 12 to 14 the "fit" for independence is quite reasonable—a P of about 0·25—but the tails will not fit. We can "explain" in this way about 88 per cent. of the secondaries. But the data are too sketchy and heterogeneous. One wants not aggregated figures, but separate family records—hundreds of short series to which the methods I have suggested in an appendix (or improvements of those methods) can be applied. These data exist, in the files of public health departments, but have never been published.

It is only human that an investigator should try to solve his particular problem—viz., to explain how infantile paralysis really was disseminated through Great Puddleton—whether the Council School, the milkman or, perhaps, little XYZ, or even a fly did the deed. But usually, perhaps always, one can do no more than show how the disease *might* have been spread.

Sometimes an investigator is perplexed. Dr. Jönsson, for instance, a Swedish investigator * having made a table of contacts between children who took the disease and children affected earlier, reached the following results. Of 272 cases, in 59 there had been contact in school, in 20 in the household, in 12 repeated contact outside school or home and in 10 chance contacts, leaving 171 where no contact was discovered. He then, on the advice of Professor Dahlberg, made a control experiment. For each child in the series of cases he took, from a card index of the district, a "control" child and worked out its contacts with patients. This gave him 44 school, 3 household, 18 repeated, 16 casual and 191 no-contacts. Jönsson was rather disappointed; he concluded there was not much difference. Actually if one combines the figures into a 2 by 4 contingency table, the value of P for 3 degrees of freedom is very small. But if we make a 2 by 2 table—compare contacts of some kind with no-contacts—there is no significance, and the author, who knew his material, attached very little importance to the great disparity between household contacts. I do not feel sufficiently sure about the way the experiment was done to be able to reach a conclusion, but it does suggest the advisability of not over-emphasizing the truism that "oft the means to do ill deeds makes ill deeds done."

The way infantile paralysis spreads through a community still seems obscure. What *does* seem certain is that infantile paralysis, like—on a large scale—cerebro-spinal fever and influenza, has increased in importance during our generation. I hope we statisticians may be able to do something about it; we need more data.

So far, I have examined the statistical problem from a point of view which, if not novel—there are few novelties in points of view, for statistics is an old science—is not often taken. I have thought of the statistical universe not as an aggregation of cases, but an aggregation of sets of cases. I have not made much of it, but hope to have suggested to younger statisticians, both field workers and mathematicians, trains of thought. I pass to a review of more familiar work, based on aggregations of cases or deaths. One may study the vicissitudes of infectious disease over many years in a great city or even a nation and try to discover the "law" of change. Again, one may select particular outbursts or epidemics, statistically analyse each of these and again seek uniformities and contrasts.

In our own country the pioneers were William Farr, John Brownlee and Ronald Ross. Nobody will doubt that two of these names are immortal and, in my opinion, Brownlee, of whom older fellows of our Society have kindly memories, had a spark of genius which the smoke of a too philosophical exposition did not permit to give as much light as it should have given. The three were, psychologically and through education, very different people. Farr was primarily a statistician with, as Brownlee once said of him, a genius or intuition for discerning the essential truth behind a statistical table. Brownlee, the only one of the three with the advantage of a university education in mathematics, had had wide clinical experience of infectious diseases and was as erudite as Creighton. Ross, a self-taught mathematician, had, as we all know, a wholly unrivalled first-hand knowledge of malariology.

Farr's contributions to the subject were not numerous, and are concerned with particular problems of the rise, decline and fall of an epidemic as an isolated event. He did not, so far as

* B. Jönsson, *Zur Epidemiologie der Kinderlähmung*—Supplement 98 to *Acta Medica Scand.*, 1938, pp. 125 *et seq.*

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I remember, do any arithmetical work on secular changes. We do not *know* what led him to formulate his two "laws," but one can make a reasonable guess.

He no doubt started with the belief—uncontradicted by experience, otherwise this paper would not be written—that epidemics, however menacing, do come to an end, and the hope which we still have, that it might be possible early in the course to predict when the tide *would* turn. Farr, who was always interested in actuarial science, was surely well acquainted with the simpler methods of graduating rough series used a century ago. Perhaps he began by differencing the weekly scores of deaths, or cases, found that the higher differences did *not* tend to constancy, and then tried the method on the logarithms of the frequencies. There he found that there was *some* hint of convergence. His first epidemic "law" was that the second difference of the logarithms was a negative constant; his second "law" that the third difference was a negative constant. Farr did not perceive that a solution of his first difference equation was the normal function; that was first pointed out by Brownlee, and it was Brownlee who first called attention to Farr's work just 40 years ago.*

Brownlee's ultimate views were formulated in a paper, which appeared a few years before his death, in a medical journal; † a precis of these may be useful because the transactions of a medical society are not familiar ground to statisticians (Appendix 2). Of his work on secular changes, on periodicity, I need say nothing, because much of it appeared in the *Philosophical Transactions*.

Ross's work, most of which was published in the *Proceedings of the Royal Society*, is readily accessible to statisticians.‡

The differences in method of approach of the two investigators are these. Brownlee began empirically—viz., by graduating many recorded epidemics with Pearsonian curves; few of them were perfectly symmetrical, although some did approximate to the Gaussian form. He himself was much impressed by the fact that *if* the infectivity of the disease decreased in geometrical progression from the start of the epidemic, then Farr's first "law" held. He thought that deviations from this norm, which he regarded as, in Plato's sense, the pure idea, were imposed by a great variety of circumstances.

Ross started with the conception of a group the strength and constitution of which were subject to variation in many ways; at the beginning of observation there would be so and so many persons healthy, so and so many suffering from a disease, so and so many recovered from it, of whom some would be immune, some again susceptible. The group would be subject to immigration, natality, mortality, relapse, etc., etc.

In his first studies Ross used the method of differences. So long as one took the various rates as constants, the task was simple. Some readers may remember an illustration in a student's textbook, Runge's *Theorie und Praxis der Reihen*,§ one reaches a system of homogeneous difference equations of the first degree with constant coefficients. If, however, the rates are functions of the time, algebraical difficulties may become formidable. The difference or differential equations set up are not always easily solved and, when solved, not easily tested on available data.

Ross and his successor, the late Col. A. G. McKendrick, a mathematical statistician whose excellent papers are well known to fellows of the Society, were tackling a more general problem than that Brownlee had in mind, so inevitably their results were more complex. To the empiricist, the working statistician, it may be that these investigators have given food for thought rather than working tools. But the steady increase of mechanical aids to computation is making it possible to try out methods before the labour of which an older generation would have quailed.

To those who, like me, take more pleasure in arithmetic than in algebra, the late H. E. Soper's paper in our *Journal* is a source of inspiration.‖ Soper's object was to test whether the apparent periodicity of epidemics of measles could be effectively described in terms of the hypothesis that the flux and reflux of measles were due to the replenishment of the susceptible population after an epidemic had killed or immunized those then present and susceptible. On the elegance and simplicity of Soper's technique I need not dwell; it is enough to say that it was worthy of Soper.

* *Proc. Roy. Soc.*, Ed. 1906, vol. 26, pt. vi, p. 454. *Brit. Med. Journ.*, August 15th, 1915.

† *Proc. Roy. Soc. of Med.* (*Sect. Epidem. and State Med.*), February 1918, pp. 85–132.

‡ Ross reprinted these papers, with some additional notes in a booklet, *A Priori Pathometry*, by Sir Ronald Ross and Hilda P. Hudson. London, 1931 (Harrison and Sons).

§ Leipzig, 1904, pp. 51–6.

‖ *Journ. Roy. Stat. Soc.*, vol. 92, 1929, pp. 34–73.

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I propose now to speak briefly of the results yielded by the application of some of these methods to the data of observation, and to end with an even curter reference to the experimental work in which I collaborated for so many years with Topley and other friends.

The easiest kind of epidemic to have clear in one's mind's eye is that generated by the contamination of a source of food or drink common to a large number of persons. A beautiful example is "The Case of the Methodist Ladies." * On the occasion of a fair in a small Californian town (Hanford), some ladies became amateur caterers and provided, among other good things, an excellent dish of spaghetti which was much appreciated. Unfortunately one of the ladies was a "typhoid carrier"; the dish was succulent and a culture of typhoid germs; of 150 guests, 93 went down with typhoid fever.

Here the "curve" or "law" of the epidemic *ought* to be a reproduction of the frequency distribution of the incubation period of typhoid fever and analogous to that of an experimentally inoculated disease—viz., to the data of experimental poliomyelitis quoted earlier. There will, of course, be blurring, because the guests, unlike the monkeys, will not all receive the same dose. Miner † extracted from the literature the above-mentioned and some similar examples—viz., other examples where there was one opportunity of massive infection—and confined his statistical study to the persons who had, so far as was known, only this opportunity of becoming infected. The result was that, although the means varied, the curves were similar—viz. (like the poliomyelitis data) positively skew with a long tail to the right. In his best set, a Pearsonian Type 1 curve with $\beta_1 = 0{\cdot}6538$, $\beta_2 = 3{\cdot}393$ and skewness $= 0{\cdot}65$ gave a good graduation.

This is well enough, and perhaps we might go so far as to say that if an epidemic of uncertain ætiology is positively skew with β's not *wildly* different from 0 and 3·0, its form is consistent with that of the frequency distribution of the incubation period; so massive infection within a short interval is a hypothesis seriously to be entertained. But any statistician with experience of curve fitting, who therefore knows that wholly disparate equations may produce graphically indistinguishable forms, would decline to go farther.

The next class of epidemics respecting which one has statistics on a large scale is concerned with diseases in which mass infection in the sense of the last paragraph does not occur. The most closely studied example is plague. Brownlee studied the forms of no less than 64 epidemics in great cities ranging in time from 1563 to 1913 and in place from London to Hong-Kong. Although only in rather more than half (36 out of 64) was β_1 less than 0·1 and in 4 greater than 0·5, he was, I think, justified in holding that the form was symmetrical and could be expressed by what he emphatically called the "epidemic curve." This would arise (see Appendix 2) if the following conditions were fulfilled: (1) The number of persons infected during an epidemic is so small in relation to the total population that the supply of susceptible units never becomes appreciably less. (2) Infectivity is very high when the epidemic begins and from the beginning declines in a particular way, which is expressed by the product of a term in a geometrical progression and a power of the number of new cases in a time interval.

Plague in human beings is usually an epiphenomenon of plague in rats, and Brownlee passed to the study of plague in rats, using the results of trappings in twelve districts of Bombay (1905–6). Here one has data not of deaths, but infections. In both species of rat—the house rat (rattus) and the sewer rat (decumanus)—the β's fluctuated a good deal, but the averages for the whole city were 0·070 and 3·159 for rattus; 0·038 and 3·499 for decumanus. The "obvious" explanation of a decline in first the epizootic then the epidemic is that not enough susceptible rats are left alive to keep them going. Brownlee's rejoinder is as follows:—

> "On the theory that the epizootic dies out from want of a sufficient number of susceptible rats, it must be shown that statistically this is compatible with the facts. The data are simple. On the one hand, after the epidemic of plague is over, about a third to a half of the rats at least are found to be still susceptible to plague; secondly, the proportion of black rats affected is only about one-half that of the brown rats. As the evidence is in favour of equal susceptibility to plague in both species under like conditions of infection, this difference must be due to the facilities for infection being smaller in the case of the black rat than of the brown rat, and has been explained on the ground of the observed fact that the number of

* See *Plague on Us* by Geddes Smith (New York, 1941), pp. 209 *et seq.*
† J. R. Miner, *Journ. Infect. Dis.*, vol. 31, 1922, pp. 296–301.

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fleas on the brown rat is twice as great as that on the black rat. Either of the sets of facts regarding the black or brown rats might easily be explained on the theory that the epizootic ends because the population of susceptible rats is not sufficient to maintain it. As far as I can see, however, at the present moment, the data regarding the two epizootics are not easily adjusted so as to be mutually reconcilable. In the appendix I give a diagram with a table of constants of the forms of epidemics which may occur on this hypothesis. The constants include the infectivity assumed, the values of β_1 and β_2, the number of susceptible units out of each initial thousand units remaining uninfected at the end of the epizootic and the standard deviation. The form of the epidemic among the brown rats closely corresponds to that shown in the diagram as being due to an infectivity per infected unit of 1·6, the value of β_1 being in this case small, the value of β_2 3·58 approximately that given by the observations, namely, 3·50, and the proportion of susceptible units left uninfected, 30 per cent. of the original number, a number sufficiently close to the facts for all practical purposes. If it is now assumed that owing to inferior power of transmission of the disease a lower degree of infectivity exists among the black rats, and that the proportion of black rats affected is half that of the brown rats, the infectivity among the black rats must be in the neighbourhood of 1·2, which would satisfy the conditions—namely, $\beta_1 = 0$, $\beta_2 = 3{\cdot}16$. The epidemic in the black rat would, however, require to have a standard deviation in proportion to that of the brown rat of 8·4 to 3·2, whereas the actual values are respectively 2·53 and 2·45. Further, if the theory held, the epidemic in the black rat would culminate very much later than that in the brown unless some other circumstance interfered to suddenly terminate the epizootic. But such a sudden termination of the epizootic implies great asymmetry, and the data are in direct opposition to the possibility of this great asymmetry. Closer approximations to the maxima of the two epidemics in time might be found by taking the infectivity among the brown rats as initially higher, but in this case $\beta_1 + \beta_2$ assume higher values, values quite out of the range of those found from the statistics.

" On the other hand, on the theory of the loss of infectivity on the part of the infecting organism the whole facts are easily explicable. It furnishes an explanation of the symmetry and also of the near synchronism of the two epizootics, while the difference in the number of fleas provides a basis for expecting that the concentration factor already alluded to should be different in the epizootics of the brown and the black rat." *

The statistical argument is, I think, cogent, but if, encouraged by its lucidity, we entertain hopes that the " epidemic curve " is a statistical master-key, a much earlier paper by Brownlee † chills us. In that paper he has fitted Pearsonian curves to 40 epidemics, including the epidemics of influenza (deaths) in 1891 and in 1891–2. For the former the β's are 0·6342 and 4·5863, for the latter 1·2217 and 5·7329. I have taken out the figures for the first wave of the great epidemic (London) of 1918 (the weekly deaths used were 5, 10, 67, 218, 287, 192, 86, 38, 21, 20, 12). The β's are 0·93317 and 4·55778. Neither " the epidemic curve " nor constant infectivity with exhaustion of susceptibles will do at all.

Perhaps a young mathematical statistician will conclude that a systematic algebraical formulation of the consequences of different biological ways in which infection *may* wax and wane in a community, more rigorous and, arithmetically, more efficient than those I have indicated, is within his power. But he will certainly want to know, before undertaking a heavy piece of work, how much real evidence the " doctors " have in favour of (or against) their pet theories. It will have been clear that Brownlee, for instance, believed that changes in the biological properties of the living, infective material were the prime cause of epidemics. So long as we remain in the pleasant land of analogies, a plausible argument can be sustained.

Most educated people know that the properties of " germs " can be altered by passage through living bodies, that a strain may have its virulence—its killing power—increased, or decreased a hundred-fold in this way. Why should not this happen in " nature "? Suppose, for argument's sake, that the virus of influenza is a normal guest of or parasite upon human mucous membranes. This living creature's reproductive cycle is shorter than that of a metazoon. Well, just as at intervals of a century or so a Bonaparte or a Hitler emerges, so may a virus emerge which causes a holocaust and, like a Bonaparte or a Hitler, decays.

* Brownlee, *op. cit.*, pp. 98–9.

† *Proc. Roy. Soc. Edin.*, 1905–6, pp. 485–521.

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This is well enough, but have we *proof* that such things do happen? In collaboration with the late Professor Topley, with Professor Bradford Hill, with Dr. J. Wilson and other colleagues, I have studied the biostatistics of epidemics in herds of mice over a period of more than 10 years. Our principal results were summarized in 1936.* I can resist the temptation of vanity to write at length on this work. If I go to the opposite extreme and seem to dogmatize, all the evidence is accessible.

Virtually all reasoning, from mice to men, is analogical; a herd of mice under the environmental conditions of our experiments is a slum inhabited by cannibals, having just two advantages over Belsen: (1) the inhabitants are abundantly fed; (2) none are deliberately tortured. In our terminology virulence means power to kill, if a measured dose of the germ inoculated into, say, 25 animals kills 20 of them, we should say it was highly virulent. By an infective strain we mean one which spreads a disease under the following conditions. A number of mice, 25, are inoculated with the strain and housed with 100 normal mice for 60 days; if a large proportion of the latter take the disease, whether they die of it or not, we should call the strain infective. The following account of a particular set of experiments shows that the infectivity of a germ may change during the epidemic spread of the disease.

TABLE IV

	Strains				
	P.M.$_3$	P.M.$_3$(2)	P.A.39	P.A.39(2)	P.62
No. inoculated	25	25	25	25	25
No. inoculated that died	20	20	25	21	25
No. contacts	100	100	100	100	100
No. contacts that died	12	3	13	13	62
No. contacts that died showing *Past.* P.M. ...	0	0	6	3	50
No. surviving contacts	88	97	87	87	38
No. survivors showing *Past.* P.M.	0	0	0	1	7

On 6.4.1932, 25 mice were inoculated each with 100,000 *Pasteurella* of a strain that killed 20 out of 25 when inoculated intra-peritoneally with this dose. The 25 were housed with 100 normal mice, and 9 days later one of these died of pasteurellosis; in the following week there was no further spread and 25 more infected mice were added; by 4.6.1932, 11 added mice had died, although the organism was only recovered from the tissues of 8. From this time a small wave of disease began and between 4.6.1932 and 13.6.1932, 30 added mice were dead. At this point we began to add 3 *normal* mice daily, but the mortality subsided, and in the two months ending 15.9.1932 only 18 died in the cage, most of which were devoured, so we had no *post mortem* evidence of infection.

The population was now increasing rapidly, so immigration was stopped. The day after this edict a mouse died of pasteurellosis. The strain recovered from him was injected into 20 mice on 23.9.32 in the usual dose of 100,000 organisms and these 20 mice were added to the herd. On 1.10.1932 the epidemic began to spread, and continued to do so; on 23.10.1932 immigration of 3 normal mice was resumed and the mortality maintained, and until the experiment was discontinued, many months later, the rate of mortality only showed minor fluctuations. So there were three phases: a slow beginning, a flare up of disease which soon burned itself out and then a second and really effective dissemination.

Five strains of the organism were isolated and used in the subsequent experiments:

P.M.$_3$. The original strain.
P.M.$_3$(2). The same strain after repeated subculture.
P.A.39. A strain isolated from a contact mouse dying in herd during the period of abortive spread.

* *Experimental Epidemiology* by Greenwood *et al.*, S.R.S. No. 209 of Medical Research Council, pp. 204, London 1936, S.O.).

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P.A.39(2). The same strain after repeated subculture.
P.62. A strain isolated from the herd during the final period of effective and continuous spread.

With each strain 25 mice were inoculated and herded with 100 normal mice, and each community was observed for 60 days. On the sixtieth all survivors were killed and cultures taken from heart and spleen. All mice dying were examined *post mortem* unless cannibalism prevented this. The results are shown in table IV.

P.62 is differentiated from the others; it was both virulent and infective. A fair inference is that the phase of effective and continuous spread in herd was due to the appearance of a variant which had retained virulence and acquired infectivity.

There is, then, an observational basis for Brownlee's doctrine. When one looks back at a piece of research, one always wishes something had been done differently. We might have repeated these studies, have made them a part of the routine. But this was only an item in our programme, there was much else to do. Experimental epidemiology is time-consuming and mice-consuming. It has *not* been proved that Brownlee's theory always holds.

In one experiment too long to be described * we had no proof that a devastating epidemic of a virus disease was associated with any change of virulence or infectivity. All I conclude is that Brownlee's hypothesis is not merely whimsical. If it is true, why, or rather how, is infectivity enhanced? The riddle of the epidemiological Sphinx is still unread. At the end of the war of 1914–18 the world suffered its greatest epidemiological catastrophe since the fourteenth century, and it was human to think that the miseries of war had something to do with it. We have no proof. At present human misery almost as intense as during the Thirty Years' War is more widespread. That there should be killing infectious diseases due to overcrowding and dirty feeding is a rational expectation. Will there be something much more deadly and widespread—in fact a pandemic of influenza like, but greater than, that of the autumn of 1918? The researches I have described do not give an answer. They do suggest to me that if the rising generation of statisticians, whose mathematical training has been more thorough than that of the pioneers, will give their attention to the problems of statistical epidemiology, a veteran reading a paper here in 1965 will have a more cheerful tale to tell than mine. Greatly as I reverence the noble science of mathematical statistics, I do not think even a Newton or an Einstein could solve these problems by force of sheer genius; the biological conditions must be understood; there must be more experimental epidemiology. But he who can conquer the mathematical difficulties will acquire the other necessary knowledge easily enough.

APPENDIX I

The Problem of Intervals

In 1886, W. A. Whitworth published the result which appears in Proposition LI in the fifth edition of his little classic, *Choice and Chance*, viz. that if an event happens at random once in time T the chance that it will not happen in time t is: $e^{-\frac{t/T}{x}}$. Whitworth did not call attention to the fact that Poisson gave this result as the chance that an event would not happen in m trials when its chance of happening was q, if $q \longrightarrow o$, $m \longrightarrow \infty$ and mq was finite, and did not refer to the use of his proposition in studying the frequency distribution of random intervals. He did, however (in the preface to the fifth edition of his book), attach a good deal of importance to another theorem, which he published in a pamphlet in 1898 and is Prop. LVII on p. 208 of his fifth edition. It is this:—

> "If n random magnitudes, α, β, γ . . . be subject only to the condition $a\alpha + b\beta + c\gamma + \ldots = S$ then every term in the expansion of $(a\alpha + b\beta + c\gamma + \ldots)^m$ is of equal expectation, m being a positive integer. And since the number of terms in the expansion is R^n the expectation of each term is S^n/R^n_m

$$\left[\text{where } R^n_m = \binom{m+n-1}{m}\right].$$

* *Op. cit.*, pp. 64–129.

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This, as I shall try to show, *is* a more important theorem. So far as I know, no statistician took up the subject until 1915, when L. Von Bortkiewicz contributed a long memoir * to the *Proceedings of the International Statistical Institute* on the distribution of random events in time. This paper is not mentioned in Morant's memoir † (the next contribution to the subject which appeared). Morant's investigation was prompted by some quite crude work of my own which I had mentioned to Professor Karl Pearson, to whom I showed von Bortkiewicz's paper. Professor Pearson did not think von Bortkiewicz had added anything to what Whitworth had done. I did not agree then, and am still of the same opinion; others must judge for themselves; the paper is too long to abstract, I content myself with indicating von Bortkiewicz's line of approach. He began by deducing Whitworth's result (he does not cite Whitworth, whose book he is not very likely to have read—after all, the algebra is very simple, and it would not be surprising if the result were independently discovered over and over again). Von Bortkiewicz wrote the frequency distribution of intervals of length z between successive events, $f(z) = ke^{-kz}$, and obtained the moments of the frequency function. He then took the case of n events occurring in a finite period T. He first completes the number of intervals by supposing that the distances from the origin to the first event and from the last event to T together form an interval. As a first approximation he equates $1/k$ to T/n, subsequently showing that $T/(n+1)$ is a better approximation. Then he takes an arithmetical example, the intervals (in days) between successive deaths of members of the International Statistical Institute in the 13 years 1890–1902. There were 65 deaths, $T = 4{,}747$ days and $T/n = 73{\cdot}03$ giving $k = 0{\cdot}01369$. Here are the results:

Interval	Observed	Expected	Interval	Observed	Expected
0– 35 days ...	25	24·7	140–175 days ...	4·5	3·7
35– 70 „ ...	15	15·4	175–210 „ ...	2·5	2·3
70–105 „ ...	9·5	9·5	Over 210 days ...	3·0/65	3·6/65
105–140 „ ...	5·5	5·8			

Applying the x^2 test to this table (five degrees of freedom) the agreement is rather *too* good ($P = 0{\cdot}98$).

Von Bortkiewicz then shows explicitly that the probability of a events occurring in t is $\frac{(kt)^a e^{-kt}}{x\,!}$ and compares the numbers of deaths in each of the 13 years with the Poisson expectation. As he had only 13 years and the expected number never exceeded 2·28, the comparison is not of value.

There is a good deal more than this in the memoir, but perhaps I have sufficiently illustrated the line of approach.

Morant wrote the function in the form $y_t = \frac{N}{m} e^{-(t-\beta)/m}$, where N is the number of intervals and m and β to be determined. The idea of introducing β was to deal with the case of events which could not be simultaneous (he tested his hypothesis by tapping the key of a chronograph when a random event happened, viz. a 5 appeared in the unit place of columns of four figures read from a census report). This paper is readily accessible, so it need not be abstracted. It contains many interesting results, but those relevant to my problems can, I think, be reached more easily.

I returned to the problem as a relaxation during the "Black-outs" of 1940–2, and started from Whitworth's problem of n points taken at random in a line of length L. Using the proposition quoted above, the mean length of an interval is $\frac{L}{(n+1)}$ and the following results are easily deduced:

$$\left.\begin{aligned} &\mu_2 = L^2 n/(n+1)^2(n+2)\ddagger \qquad \mu_3 = L^3 2n(n-1)/(n+1)^3(n+2)(n+3) \\ &\mu_4 = L^4 3n(3n^2-n+2)/(n+1)^4(n+2)(n+3)(n+4) \\ &\beta_1 = 4(n-1)^2(n+2)/n(n+3)^2 \qquad \beta_2 = 3(n+2)(3n^2-n+2)/n(n+3)(n+4) \end{aligned}\right\} \quad . \quad . \quad (1)$$

* *Bulletin de l'Institut intern. de Statistique*, 1915, T. xx, 2nd Livr., pp. 30–111.
† *Biometrika*, 1920–1, vol. 13, pp. 309–337.
‡ This, of course, agrees with Morant's *op. cit.* equation (XXVIII) with change of notation.

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The Pearsonian type curve with these momental values is (unit area, origin at the start)

$$y = \frac{n}{L}\left(1 - \frac{x}{L}\right)^{n-1}$$

and the area from $x = 0$ to $x = x$ is simply $1 - \left(1 - \frac{x}{L}\right)^n$.

The curve passes into the exponential form for n and L tending to infinity, but $\frac{n}{L}$ finite. Indeed, for $L > 100$ and $n > 10$ there is little difference. This is, as Dr. Irwin pointed out to me, not a mere approximation, but an exact solution. The chance that an interval shall exceed x is simply the chance that all n points fall on aggregates of segments adding up to $L - x$, which is $\left(1 - \frac{x}{L}\right)^n$, so $dy = -\frac{n}{L}\left(1 - \frac{x}{L}\right)^{n-1} dx$.

So far no practical advance has been made beyond the position already reached by von Bortkewicz. For n small the form is more exact than the exponential, but, as we see, could have been reached in two lines, and if n is small, unless we have a large number of sets of n—say a good many records for families or houses—no comparison of observed and expected intervals is in the least likely to be of use to the practical statistician. But could one not, using Whitworth's theorem, go a stage farther?

It occurred to me that the variance might be a better object of study. The population variance is, as we have seen, $nL^2/(n+1)^2(n+2)$, and its range from 0 to $nL^2/(n+1)^2$.

The variance of the variance can be obtained by the application of Whitworth's theorem, and proves to be $4nL^4/(n+1)^2(n+2)^2(n+3)(n+4)$. As the range is known, we now have enough information to fit a Pearsonian curve of Type I. Following the usual method,* it appears that $m_1 = (n^2 + n - 6)/4(n+1)$ and $m_2 = (n-1)(n+6)/4$. This is exact in the trivial case $n = 1$, because then $m_2 = 0$ and $m_1 = -0{\cdot}5$, and we shall have equal frequencies for ranges of x from $0 - d^2$, $d^2 - 4d^2$, etc., which must be true since for $n = 1$ the variance is a quadratic function of x. But for n greater than 1 there is no *a priori* reason why the true frequency function should be a Pearsonian type curve at all. After some fruitless attempts to solve the general problem, I consulted Dr. Isserlis who provided me with an elegant solution for $n = 2$.

Taking 0 instead of the mean as origin, the range is from $\frac{L^2}{(n+1)^2}$ to $\frac{L^2}{n+1}$, or, taking out the factor $\frac{L^2}{n+1}$ from $\frac{1}{n+1}$ to 1. So $Sx^2 = 1$ is the upper limit, and we seek the probability that $Sx^2 = \lambda^2$ for λ^2 between $\frac{1}{n+1}$ and 1. If $n = 2$, all the representative points must lie in that part of the plane $x_1 + x_2 + x_3 = 1$ bounded by the intercepts on the axes, viz. must be within that portion bounded by the sides of the equilateral traingle formed by joining the points (1,0,0), 0,1,0) and (0,0,1).

In the triangle ABC if D is $(\frac{1}{2},\frac{1}{2},0)$ the equation of CD is

$$\frac{x_1}{\frac{1}{2} - 0} = \frac{x_2}{\frac{1}{2} - 0} = \frac{x_3 - 1}{0 - 1} = k.$$

So at any point C' in CD,

$$x_1 = x_2 = \frac{k}{2} \text{ and } x_3 = 1 - k.$$

If

$$OC' = \lambda,\ \lambda^2 = \tfrac{1}{4}k^2 + \frac{k^2}{4} + (1-k)^2,$$

or

$$\frac{3k^2}{2} - 2k + 1 = \lambda^2$$

$$(CC')^2 = (\tfrac{1}{2}k)^2 + (\tfrac{1}{2}k)^2 + k^2 = \tfrac{3}{2}k^2$$

$$CC' = k(\tfrac{3}{2})^{\frac{1}{2}}$$

* See, for instance, W. Palin Elderton's *Frequency Curves and Correlation*, 3rd Edition, p. 124.

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If G is the centroid of the triangle,

$$CG = \frac{2CD}{3} = (\tfrac{2}{3})^{\frac{1}{2}} \text{ and } GC' = (\tfrac{2}{3})^{\frac{1}{2}} - k(\tfrac{3}{2})^{\frac{1}{2}}.$$

Hence the probability that any representative point fulfils the condition:

$$x_1^2 + x_2^2 + x_3^2 \not> \lambda^2$$

is:

$$\frac{\pi(GC')^2}{\sqrt{\frac{3}{2}}} = \frac{4\pi}{3\sqrt{3}}\left(1 - \frac{3k}{2}\right)^2$$

or — as $\left(1 - \frac{3k}{2}\right) = \frac{3\lambda^2 - 1}{2}$ it follows that $\frac{\pi(GC')^2}{\frac{\sqrt{3}}{2}} = \frac{2\pi}{\sqrt{3}}(\lambda^2 - \frac{1}{3})$

provided the whole of the circle falls within the triangle

$$ABC\text{—}i.e.,\ GC' \leq GD,\ k \geq \tfrac{1}{3},\ \tfrac{1}{3} \leq \lambda^2 \leq \tfrac{1}{2}.$$

This can be generalized at once;

for

$$\frac{1}{n+1} \leq \lambda^2 \leq \frac{1}{n}$$

the required probability is:

$$\frac{\left(\lambda^2 - \frac{1}{n+1}\right)^{\frac{n}{2}} . \pi^{\frac{n}{2}} . n!}{\Gamma(n+2)/2 . (n+1)^{\frac{1}{2}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

One has merely to write down the volumes of included sphere and including pyramid in space of n dimensions. This means we can write down the probability of values of λ^2 up to $\frac{1}{n}$, which, reverting to the variances, means up to a value of $\frac{n+2}{n^2}$ times the population value. This, for n small, is very useful. If λ^2 is greater than $\frac{1}{2}$ the circle cuts the sides of the triangle and one must subtract from the area of the circle the areas of the three segments so formed, and we now have

$$\frac{2}{\sqrt{3}}(\lambda^2 - \tfrac{1}{3})\left\{\pi - 3\cos^{-1}\left(\frac{1}{\sqrt{6\lambda^2 - 2}}\right) + \frac{3 . \sqrt{3} . \sqrt{2\lambda^2 - 1}}{6\lambda^2 - 2}\right\}$$

The generalization of this has not been obtained. A comparison of the results for the exact solution and the Type 1 graduation with $m_1 = 0$ and $m_2 = 2$ for ranges of λ^2 gave the following results:

Range of λ^2	Correct	Type 1	Range of λ^2	Correct	Type 1
0·333 –0·4	0·243	0·271	0·600–0·667	0·074	0·091
0·4 –0·4667	0·243	0·217	0·667–0·733	0·050	0·061
0·4667–0·533	0·206	0·169	0·733–0·800	0·044	0·037
0·533 –0·600	0·113	0·127	0·800–1·00	0·036	0·029

Clearly the Type 1 curve is a mere graduation, although, as often happens with the Pearsonian family, not a bad graduation.

Before Dr. Isserlis gave me his solution, I had made some experiments on random numbers. The obvious suggestion is to take for L some integer, 50, 100 or whatever is the upper limit of one's patience, and take at random n numbers from 0 to L inclusive, and so form compositions of L into $n + 1$ parts, zero admissible as a part; for instance, suppose $L = 50$ and $n = 3$, and one has taken 3, 7, 25; these give 3, 4, 18, 25 as intervals starting from 0 and ending at 50—viz., a composition. The intervals will be 0, 1, . . . L in length and, writing $n + 1 = r$, there will be r times as many intervals as there are r compositions of L, viz. $r . (L + r - 1)! / L! . (r - 1)!$, if zero is admissible as a part. The compositions containing 0 at least once will be r times the $(r - 1)$ compositions of L, the compositions containing 1 at least once will be r times the $(r - 1)$ compositions of $(L - 1)$, etc. This is a polynomial of degree $(r - 2)$ and each term enumerates the frequency of occurrence of an interval of 0, 1, 2, etc. For the compositions of L when zero is not admissible as a part are:

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$(L-1)!/(L-r)!(r-1)!$ and the frequency of a single zero is $r(L-1)!/(L-r+1)!(r-2)!$, of two zeros $r(r-1)(L-1)!/(L-r+2)!(r-3)!2!$, etc. Multiplying the successive terms by 1, 2, etc., and summing we reach $r(L+r-2)!/L!(r-2)!$ and so on.

In the particular case of $r=4$, the polynomial is $2x^2-2(2L+3)x+2(L+2)(L+1)$. Putting $L=50$ and working out the variance, we have 101·25; the variance for the continuum as given above is 93·75. When $L=100$, the two values are 390 and 375. So L must be very large (and the labour of an experiment very great) to reach a near approximation. It follows that an experiment with L so small as 50 could hardly be expected to give very close results, using the momental values for the continuum, even if the Type I solution of the problem were exact. I did, however, for the sport of the thing, take out from the tables a thousand sets of three numbers,* ranging from 0 to 50 and fitted a Type I curve from the formulæ above deduced. I also fitted a Type I curve from the actual moments of the sample (the mean was actually 98·68, appreciably closer to the value of the polynomial distribution than to the theoretical value). The results are shown in the table.

Range of Variance	Observed	Calculated	Empirical
0– 50	256·5	308·0	282·0
50–100	362·5	298·0	323·0
100–150	196·0	198·0	197·0
150–200	95·0	112·0	106·0
200–250	48·0	54·0	53·0
250–300	29·0	21·0	24·0
300–350	6·0	6·0	10·0
350–	7·0	3·0	5·0
	1,000	1,000	1,000

The fit of the theoretical Type I curve is miserable, owing to wide discrepancies in the first two groups. Isserlis's theorem enables us to check the first subfrequency; it should be 284. It would be special pleading to remark that the mean of the observations being inevitably forced to the right, that would lead to some shift to the right of the subfrequencies, and to note that the subfrequencies above the mean agree quite well with the Type I estimates. To know the truth one must wait until the general problem has been solved, as no doubt it will be, by a better mathematician. Until then, my suggestion is that the Type I approximation for small values of n is probably good enough—using the Isserlis control for the beginning of the range.

In practice, for the kind of problem I had in mind, a serious difficulty arises—viz., that the points cannot be precisely located. We shall be informed that, say, on April 1st m persons fell sick, on April 4th n persons fell sick and can come no nearer to the precise times. If one assumes a random distribution over the 24 hours, one is, *pro tanto*, begging the very question we wish to answer and forcing the data towards randomness; but there is no alternative. Of course, if one has only a single case within a day, and many days intervene between successive cases, the difficulty is trivial. Otherwise it is serious. I have dealt with it in the following way. If m points are taken at random in a line of unit length, the squares of the $(m+1)$ intervals so formed are of equal expectation, and each is equal to $2/(m+1)(m+2)$; we take m times this as the contribution of the first m intervals to the whole sum of squares. The next interval will be between the last point in the first and the first point in the next day with cases. If t blank days intervene, we need $E(x+y+t)^2$, where x is the distance of the mth point from the end of the first day and y the distance of the first of the n points in the next day with points from its start. As the distributions are independent, $E(xy)=Ex\,.\,Ey$. So we have simply: $2(1/(m+1)(m+2)+1/(m+1)(n+1))+2t(1/(m+1)+1/(n+1))+t^2$. Proceeding in this way, we obtain a mean value of the total sum of squares of intervals.

I can see many objections to this plan, but cannot think of a better.

At this point the reader may ask, a little impatiently, whether there is much point in spending

* The expected frequency for any number is 58·82, the range found in the experiments was from 41 to 75 and the χ^2 test (49 degrees of freedom) gave $\chi^2=42{\cdot}4154$ with $P=0{\cdot}52$.

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time on tests of a property which is a consequence of an assumption which can itself be tested directly—viz., the assumption that the distribution of cases themselves is rectangular.

We already possess a variety of methods for deciding whether a sample can reasonably be thought to have come from a rectangular parent population (see particularly, Neyman and E. S. Pearson, *Biometrika*, vol. 20A, 1928, pp. 208–21; K. Pearson, *ibid.*, vol. 25, 1933, pp. 379–410; E. S. Pearson, *ibid.*, 30, 1938, pp. 134–48).

I do not think there *is* any logical advantage in using the interval test, but to a medical statistician it has a psychological attraction; he cannot altogether abandon a hope that somehow one might be able to do what Dr. Pickles did—unveil the incubation period of a disease—and it is therefore natural for him to concentrate on the interval between successive events. It may, indeed, be a hopeless quest for two reasons. The first is that the incubation period is not constant. As the table quoted in the text (p. 91) shows, in the experimental inoculations there is a wide scatter around the mean interval. In the next place, the time during which a patient is capable of passing on infective material to somebody else may be long. The purely arithmetical problem is complex. To solve it will need both more analytical skill and more data than I command.

I am very grateful to Dr. L. Isserlis, Dr. J. O. Irwin, Dr. W. J. Martin and Mr. M. G. Kendall for their help and advice in this work.

APPENDIX 2

Brownlee's Epidemic Frequencies

The earliest of Brownlee's mathematical papers on this subject appeared in *Proc. Roy. Soc. Edin.* June 18th, 1906, pp. 484–521, the latest in *Proc. Roy. Soc. Med. Sect. Epidem. and State Med.* 1918, pp. 85–127. This note is based on the latter. I have retained the author's notation and method of deduction, even when they seemed capable of improvement.

Brownlee, like Karl Pearson, based his system on the limiting ratio of successive ordinates; his y_t is the number of new cases (or deaths)—viz., the number of newly infected persons (or of deaths) at time t; his x_t the integral of y from the origin to time t. In his terminology infectivity is the ratio of successive ordinates. He proceeds by expanding the right hand side of

$$\frac{y + \Delta y}{y} = [\phi(t,y)]$$

giving

$$\frac{\Delta y}{y} = \log [\phi(t,y)]\Delta t + [\log \phi(t,y)]^2 \frac{(\Delta)y^2}{21} + \ldots$$

and so has for his fundamental form:

$$1/y \,.\, dy/dt = \log \phi(t,y) \quad \ldots\ldots\ldots \quad (1)$$

If the infectivity decreases geometrically—viz., is equal to e^{p-qt} then

$$\frac{1}{y} \cdot \frac{dy}{dt} = p - qt \quad \ldots\ldots\ldots \quad (2)$$

and

$$y = c \,.\, e^{pt - \frac{1}{2}qt^2} \quad \ldots\ldots\ldots \quad (3)$$

the Normal Curve. Perhaps some readers may feel a difficulty which Brownlee endeavours to remove in the following words:

> "The method as developed apparently requires that the incubation period of the disease should be infinitesimal, which, of course, never occurs in practice. This, however, is not necessary in all cases."

Brownlee remarks that this form implies that the rate of increase is directly proportional to the number of new cases occurring at that moment, and introduces a power of y as a factor putting

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the infectivity equal to $e^{y^n(p-qt)}$. He always attached great importance to Farr's view that the death rate was proportional to a power of the density of population.

If
$$\frac{1}{y}\cdot\frac{dy}{dt} = y^n(p - qt)$$
$$y = n(c - pt + \tfrac{1}{2}qt^2)^{-1/n} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Which by change of origin reduces to the symmetrical case of the Pearsonian Type IV, sometimes numbered Type VII.

$$y = \frac{M}{(1 + t^2/a^2)^m} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Should n be negative (*e.g.*, partial immunization through non-clinical, unrecognizable attacks of the disease, might reduce liability) we should reach a Pearsonian Type II:

$$y = M(1 - t^2/a^2)^m \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The tacit assumption has been that the number of persons attacked in the whole epidemic is so small a fraction of the exposed to risk that the supply of victims is virtually constant. If the infectivity is constant, the population P and x the number of victims from $t = 0$ to $t = t$, then:

$$\frac{1}{y}\cdot\frac{dy}{dt} = \log k(1 - x/P) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

This cannot be integrated to give y explicitly as a function of t, but a solution can be obtained indirectly (see *op. cit.*, pp. 121–2), and Brownlee tabulated results for infectivities from 1·2 to 2·6 per unit of time and gave the β's, σ's and percentages still unaffected at the end of the epidemic. He also suggested a modification to allow of population increase or decrease—viz.:

$$\frac{1}{y}\cdot\frac{dy}{dt} = kf(x)(P + pt - x) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Except in the particular case of $f(x) = x$ (which had been considered by Ross), Brownlee goes no farther with this. Returning to (3), he considered the effect of allowing for an increase or decrease of infectivity, which could be effected by, *inter alia*, multiplying e^{p-qt} with $(a + bt)$ giving

$$\frac{1}{y}\cdot\frac{dy}{yt} = \log(a + bt) + (p - qt)$$
$$y = C(a + bt)^{a/b + t}e^{(p-1)t - \frac{1}{2}qt^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Brownlee comments:

> "This curve is asymmetrical and the descent of the epidemic is more rapid than the rise. This means that the maximum of the epidemic occurs later and as the infectivity is largely lost by this time the descent is rapid. Similarly it may be found by applying a factor $1/(a + bt)$ that when some progressively inhibiting influence is at work the form of the epidemic is so changed that the decline of the epidemic is less rapid than the rise. Such factors must be considered where the influence of such a condition as varying temperature on the progress of an epidemic is examined, a subject as yet hardly worked at. It may be noted that in the case of the normal curve an exponential increasing or inhibiting factor alters the position of the maximum without changing the form of the epidemic."

Discussion on Professor Greenwood's Paper

Mr. M. G. Kendall: I move the vote of thanks to Professor Greenwood with great pleasure. He has done us a service in reviewing the relationship between the medical and statistical approach to epidemiology, but I cannot help feeling somewhat on the defensive. My difficulties begin with the remark that "Hirsch was not a stupid man, but his criteria were really statistical," the operative word being "but." This is the sort of epitaph which many people would like to write on our graves, and Professor Greenwood has raised in the epidemiological field a question which deserves to be debated at some length. Should research on epidemiology be done by a medical man who has learnt statistics or by a statistician who has learnt medicine? To take an even more con-

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tentious case, who does the best research in economic statistics, the economist who has learnt statistics or the statistician who has learnt economics?

What the answer is I do not know, but I suspect that there is no answer, and that ultimately it makes no difference if the man concerned is sensible. I fully agree with Professor Greenwood in his view that the mathematical statistician is likely to go badly astray unless he understands the subject with which he is dealing. I hope he agrees with me that it is not necessary to have a detailed medical knowledge in order that a statistician may make useful contributions to epidemiology. If so, we are in complete accord and can join forces against the protectionist fallacy that only an economist should be allowed to handle economic statistics.

What, then, are the main directions in which a statistician can contribute to epidemiology? I should have said that there were three:—

(*a*) In the first place the statistician can help to ensure that the data are representative. There are peculiar difficulties in sampling for medical studies, particularly among human populations, and in no branch of science is it more necessary to call in expert statistical advice at the outset of an enquiry.

(*b*) Secondly, the statistician can indicate what sort of information is required for the solution of a problem. Professor Greenwood has given an illustration in the very interesting tables exhibiting the periods of incubation of infantile paralysis. Distributions such as this are, I think, essential for an adequate discussion of epidemics, and I hope that many others will be produced, if only for the benefit of parents like myself, who never quite know how long it is likely to be before children can be regarded as clear of infection from measles or mumps, and spend most of their earlier married lives in a state of suppressed incubation.

(*c*) Thirdly, the mathematical statistician can often throw light on a medical problem from his experience in other fields. This brings us to the problem dealt with by Professor Greenwood in his Appendix A.

As he says, it was Whitworth who seems to have been the first to consider this problem in any sort of generality, but it may be of interest to point out (what is not generally realized) that the essential mathematics of Whitworth's theorem were carried out by Dirichlet earlier in the nineteenth century. Whitworth himself proves his result by a long inductive chain of argument, whereas it follows almost immediately from Dirichlet's integrals.

More recently the problem has been studied from the point of view of finding the distribution of the greatest part into which the magnitude is divided. Whitworth himself solved this problem, but it has been re-discovered and applied, for example, by Fisher to harmonic analysis, by several writers to the test of significance of the variance-ratio in the analysis of variance, and by Garwood and others in problems arising out of the control of traffic and telephone switchboards. More recently still Stevens solved what is essentially the same problem in connection with the distribution of points on a circle, a form which I think was suggested by biological considerations.

Nearly all this work has concerned the distribution of the *greatest* of the fragments into which the random interval is divided. Professor Greenwood has, however, put forward rather a different form of the problem, since he is more interested in the closeness of equality of the parts, and he has accordingly suggested as a test of departure from randomness the distribution of the variance of the parts into which the interval is divided. I do not remember seeing this version before.

The general distribution required by Professor Greenwood as a generalization of Dr. Isserlis's result is equivalent to finding the volumes cut off by hyperspheres in u dimensions on a hyper-tetrahedron. The problem proposed is not insoluble, but it makes a very considerable call on one's powers of geometrical insight. In the time available I have not been able to make any progress with it. There are, however, several other ways of tackling the problem, and I will briefly mention six by way of illustration of the resources at our disposal.

(*a*) Professor Greenwood has fitted a Pearson curve to the distribution by using the first two moments and the range. I think a better fit would be obtained if one used the first four moments and ignored the range. In many of these cases it is better to identify lower moments and to leave the range to look after itself, and I should expect by so doing to get a better fit, at least as regards areas under the curve, which are the main objects of enquiry in this connection.

(*b*) It might be worth while examining the possibility of finding a fit to the distribution function rather than to the frequency function. It appears that for small values of u the distribution is unsymmetrical, and close fits near the start of the curve are likely to be rather difficult to obtain. The cumulative frequency function or distribution function is likely to be a good deal smoother, and therefore more amenable to fitting.

(*c*) It is possible that a transformation of the variate might bring the distribution closer to symmetry or normality, and hence provide a better fit.

(*d*) Although in general the variance is a more convenient measure of dispersion, this particular problem might be mathematically more tractable if one worked with the mean

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deviation. The geometry of the sampling distribution would be complicated by changes of sign in the various quadrants, but at any rate the volumes cut off would be bounded by planes and not by spheres.

(*e*) Another possible measure to test the departure of the parts of the interval from equality would be the difference of the greatest and the least parts, or the ratio of the greatest to the least. I should expect the latter to be quite a sensitive index for the purpose, and the distribution theory might be a good deal simpler than for the variance or the mean deviation.

(*f*) Finally, some sort of test might be obtained by examining the correlation between successive intervals. If the division of the magnitude is random, one would expect zero correlations within sampling limits, but if the division is regular, positive correlations might appear, although in practice no doubt the situation would be complicated by the variation in incubation periods.

DR. FRASER ROBERTS: I am sure that Professor Greenwood must be accustomed to being thanked, not only for very valuable papers, but for his great gift of making everything he describes so interesting. In the course of his paper Professor Greenwood mentioned the difference in the incidence of infantile paralysis amongst officers and amongst other ranks in the Army in India during the war. I wonder whether he would apply the same explanation to the similar difference found in the Navy in the incidence of infective jaundice. Throughout the entire Navy the incidence of jaundice was considerably higher among the officers than among the ratings. The same thing was true in the Army and Air Force—particularly, I believe, in the Army in the Middle East.

I am afraid that I am not qualified to take part in the mathematical discussion, although I shall listen to it with great interest. I feel guilty in many ways because I should have taken much more interest in epidemiology at an earlier age. My own native town contains a remarkable, a unique, monument, not, indeed, to an epidemiologist, but to a man who struggled with a particular epidemic of cholera about 100 years ago. He was a doctor who was renowned for his eccentricity as well as for his courage. He attended his patients during that epidemic, he refused to leave them, and when they died they were buried by another local eccentric who also refused to leave the place.

The epidemic passed, and the grateful townsfolk desired to raise a memorial fund. They suggested scholarships for local boys, but the doctor said he would much prefer to have opposite his house a very tall column with a statue of himself on the top. In spite, I regret to say, of a very handsome donation from the doctor himself, the fund did not quite run to it, and so what was clearly regarded as the least essential detail had to be left out. However, there now arises in this town in a small public garden an enormous column 40 or 50 feet high, and on the top of it is a statue, not of the worthy doctor himself but a second-hand one of Lord Palmerston—whom I believe the doctor did somewhat resemble!

The vote of thanks was then put to the meeting and carried unanimously.

DR. J. A. SCOTT said that it was a great pleasure for him, as a medical officer of health who was not a statistician, to pay his tribute of admiration to Professor Greenwood. In the early part of his paper Professor Greenwood had mentioned that continued growth was not "epidemic" beyond middle life, but in so far as mental growth was concerned he thought Professor Greenwood would have to modify his observation in relation to people who had the benefit of contact with his mind and thought. A medical officer of health had some sympathy with the observations made by the proposer of the vote of thanks on the question of which came first, the hen or the egg—in other words, the doctor or the statistician. In a field like medicine, in which qualification took a very long time, a combination of those qualities was rarely found. It was of some interest that Professor Greenwood was probably only the fourth of the very few medical statisticians concerned with epidemiology in this country. He had mentioned three of them—namely, Farr, Brownlee and Ross—and the speaker thought that no one had intervened between these three and Professor Greenwood himself.

Probably from the point of view of knowledge of the statistical side of epidemiology the only possible workaday arrangement would be some liaison between the epidemiologist-qua-medical man and the statistician-qua-statistician. He was particularly interested in Dr. Pickle's observations on the incubation period of epidemic catarrhal jaundice. This took his mind back to 1937, when he was medical officer of health for Fulham and had an interesting experience of the same disease. In a day nursery an epidemic originated from a probationer nurse of 18 who had scarlet fever. She was in hospital for four months, and while in hospital had an attack of jaundice. She was discharged on April 3rd, and had a fortnight's sick leave, coming back to the nursery on April 13th. She shared a bedroom with four other nurses. On May 6th one of these nurses fell ill, and on May 10th showed signs of jaundice. On May 9th two other probationer nurses felt unwell and subsequently got jaundice. A fourth nurse a few hours later started the same cycle, jaundice being marked on May 14th. From the first development of the illness to the actual exhibition of jaundice two or three days elapsed. But within a month of this probationer nurse coming back to the nursery four nurses sharing the same bedroom went down with the complaint.

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There was a great deal of this kind of statistical information available to the medical officer of health on which the statistician could work. How the medical officer and the statistician were to work together he was not quite clear. He himself had had forty children under the age of 5 in this day nursery. The nurses were with them all day long. Two or three of those children, during the time this probationer nurse was affected, had mild attacks of vomiting with bile salts in their urine, but not a single one had jaundice. That seemed to underline the need for a liaison between the medical officer and the non-medical statistician. The only criterion of epidemic catarrhal jaundice could not be jaundice alone, any more than in infantile paralysis could it be paralysis alone, for with every case of paralysis there were four or five others which had a related illness, but not infantile paralysis.

Between the statistician and the medical man, the epidemiologist was essential if much progress was to be made in working out all the biological laws underlying epidemiology. From one's own routine experience one could bring forward similar instances in connection with measles. In his borough measles was a notifiable disease. In London and in some other places measles occurred in biennial epidemic waves, and it was of some interest to him to examine the incidence of measles in children under 5 who were attending school and children under 5 who were not. He started with the theory that there was a bigger risk of infection among the children attending school and mixing with other children, and he had the opportunity of working out a few figures which did satisfy him that there was a greater liability during an epidemic period for a school child under the age of 5 to have measles than a child of similar age who did not go to school.

Material of that kind was available, but was not utilized as it should be by the statistician. It would certainly throw a little more light on biological phenomena, and he personally would plead for some further co-operation and team work between the statistician and the medical officer. How that team work should be done was not for him to say. The medical officer was usually frightened out of his wits by the controls and requirements of the statistician, but it should be possible in large towns to have a statistician in the department of the medical officer of health. Certainly such statisticians would be very useful in view of the future development of the hospital services which was looming ahead. In the smaller towns the work might be done through the regional university. In that way more use could be made of the material which was available.

One final observation on Professor Greenwood's mathematics. As a field worker it all passed over his admiring and non-comprehending head, but it was very pleasant to see it on paper, and he hoped that Professor Greenwood and his successors would have an opportunity of applying it more fully to the material which medical officers of health possessed.

Dr. Melville Mackenzie expressed his deep appreciation of Professor Greenwood's paper, not only from the point of view of its own merits, but because of the offer which it made of co-operation. He was quite convinced that in many fields of epidemiological enquiry there should be a much closer contact with statisticians and at an earlier stage. Many medical officers investigating epidemics had very little idea of what material was likely to be of use to statisticians, and it would be a great step forward if they could be informed in some detail as to the type of material which could be utilized. In much of the work, of course, they were concerned primarily with stopping the disease, but in a certain number of enquiries they did undertake work entirely from the research point of view. One of these was anterior poliomyelitis, in which little could be done for prevention, and such enquiries as were carried out were by way of research. At the same time, one had to admit that it was impossible to get down to an enquiry with a cut-and-dried scheme. Factors and conditions varied enormously, but if a general guide could be laid down it would be of great help to the Ministry of Health, which would be most happy to collaborate in any way which Professor Greenwood might suggest.

He had been very much pleased also to hear Professor Greenwood's plea for further research generally in epidemiology. He believed that during the war the public learned a very great deal about the value of research in the progress of the war, and that at present the public were ready to spend money on research, as they had not been before the war. The Ministry of Health and the Public Health Laboratory Service looked forward to carrying out a considerably increased amount of work during the next few years. Concerning the controversial question of statisticians *versus* doctors, it seemed to him that the only way in such a field as anterior poliomyelitis was to have the clinician, the statistician and the field epidemiologist all working together.

He desired to say a word about influenza. He had just come back from Austria and Germany, where the conditions from the epidemiological point of view were perfect for the spread of influenza and for a high mortality rate. But in actual practice in the British zone there had been relatively few cases of influenza and absolutely few deaths. That raised another point on which possibly Professor Greenwood might be able to help them statistically—namely, the question whether under-nourishment did make for the spread of epidemics or not; in other words, whether the under-nourished were more easily infected in the first instance, and whether, having been infected, they died more frequently. Most of them would say that epidemics spread more easily in a badly nourished population, but the question was constantly recurring as to whether there was a great

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danger of epidemics in an ill-nourished community. He had had occasion to search for any authoritative statement in support of the idea that bigger rations would prevent epidemics, but there was not much scientific support for such a statement.. The question was of great importance. Actually the great pandemic of influenza in 1918 and 1919 occurred amongst the well-nourished and younger portion of the community. He believed also that under-nourished monkeys could not easily be infected with anterior poliomyelitis, and that hedgehogs under-nourished could not be infected with foot-and-mouth disease. This made it all very difficult.

Concerning poliomyelitis from a statistical angle, this was an unfortunate theme to discuss. From the epidemiological point of view they were completely lost in the disease. They knew that there were precocious carriers—people who had the virus in their stools before they developed the disease—but there were also ordinary carriers—convalescents and others—and there were certainly many persons who had mild attacks which were not regarded as poliomyelitis at all. The number of cases of anterior poliomyelitis which occurred in the north midland region during the six years of war was 231. These cases were spread amongst sixty-eight local authorities; seventy of them occurred in the area of five local authorities. Therefore they were scattered in ones and twos throughout the entire area. This made investigation exceedingly difficult. One other point about anterior poliomyelitis from a statistical angle. In a recent outbreak in a boys' camp there were nine cases, eight out of the nine had had while in camp some form of transient illness, one did not develop drowsiness until ten days after a second attack. Of the thirty boys who did not get the illness six had a transient attack. One boy had a similar second attack with stiffness in the back and neck which would never have been spotted had not the practitioner attending the case been aware of the occurrence of the epidemic. This boy ten days afterwards had a similar clinical attack with extensive paralysis.

The other point he desired to mention was in connection with a recent visit to the United States, where he had been very much struck by the extensive use made not only of statistics in epidemiology, but also with the interest of the community in disease, both in terms of death-rate and incapacity to work. At every stage the capacity to work was determined by statistical examination. There was quite a considerable need for the further use of statistics in this country in these investigations.

In thanking Professor Greenwood, he again promised the willingness of the Ministry of Health to help him in any way possible.

Dr. Irwin said there were two reasons why he could speak on this paper with a certain propriety, apart from the fact that it gave him great pleasure to do so. First the subject was one in which his interest went back about twenty years.

When he was a young man on Professor Karl Pearson's staff, he tried to tackle the problem of predicting the course of an epidemic from a knowledge of some weeks of initial cases. He assumed the form would be adequately described by one of Pearson's Types, and used the method of moments. He failed, as Professor Pearson had done before him. The reason might have been because the hypothesis of a Pearsonian form of distribution was wrong, or because in fitting the tail of a distribution which, considered in itself, was naturally a very skew curve, the method of moments, which he tried, was a very inefficient method of fitting.

He supposed he was one of the few people, if indeed they were few, who read Professor Greenwood's 1931 paper on chain binomials. Indeed, he well remembered feeling honoured at being asked to give advice on it, shortly after joining Professor Greenwood's staff, one of the first duties which he had to perform. Later he became interested in Kermack and McKendrick's theoretical studies, and in particular their application to some of Greenwood and Topley's data on mouse ectromelia and mouse typhoid. He still thought, if it was not presumptuous to say so, that more might be done on those lines.

However, he supposed that he would be expected to say something on the problem considered in Appendix 1.

He had to thank Professor Greenwood for giving him some happy hours in trying to solve it, and it might be worth while stating what he did. He obtained the solution—that is, the distribution of the variance of the intervals, for 2 and 3 intervals, the latter by a method which involved calculating the area common to an ellipse and a right-angled triangle, and verified that the variances of the distributions so obtained agreed with those calculated directly. Later he saw that Isserlis's method, which involves calculating the area common to a circle and a symmetrically placed equilateral triangle, would be the easier to generalize, and obtained, as Dr. Isserlis did, the distribution of the variance for any number of intervals and sufficiently small values of the variance. The problem of four intervals involves the volume common to a regular tetrahedron and a symmetrically placed sphere, and there is some difficulty in the case when the sphere cuts the edges as well as the faces of the tetrahedron. In the case of three intervals the distribution of the variance is shaped rather like a Wellington boot. It is rectangular for $0 \leq v \leq \frac{L^2}{18}$, and the frequency then declines to zero at $v = \frac{2L^2}{9}$. The tangent has one point of discontinuity. For four intervals the curve rises

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to a mode, and there are two points at which the analytical form changes. For a number of intervals $(n+1) \geq 4$ there is of course a mode, and there are $(n-1)$ points at which there is an analytical change of form. He (the speaker) obtained the distribution of the variance for two and three intervals in a form which was rather more explicit than that given by Professor Greenwood, and this seems worth putting on record. The period of exposure to risk being unity, the interval length being x and defining the variance of the $(n+1)$ intervals determined by n cases by

$$v = \frac{1}{(n+1)} \Sigma \left(x - \frac{1}{n+1}\right)^2$$

the distribution of v is:

for $n+1=2$ $\quad \frac{dv}{v^{\frac{1}{2}}} \qquad 0 \leq v \leq \frac{1}{4}$

for $n+1=3$ $\quad 2\sqrt{(3)}\pi dv \qquad 0 \leq v \leq \frac{1}{18}$

$$\left[2\sqrt{3}\pi - 6\sqrt{3}\cos^{-1}\left\{\frac{1}{\sqrt{(18v)}}\right\}\right] dv \;.\; \tfrac{1}{18} \leq v \leq \tfrac{2}{9}$$

Both Mr. Eliezer, a pupil of his at Cambridge, and he spent some time in trying to get the complete generalization for $(n+1)$ intervals, but were not successful. Someone with Professor Fisher's geometrical intuition seemed required.

If one confined one's attention to the total period between the first and last cases one could, on the hypothesis tested, regard the resulting $(n-1)$ intervals as a random sample of $(n-1)$ values from an exponential distribution and the variance $\frac{1}{(n-1)} \Sigma \left(x - \frac{1}{n-1}\right)^2$ would have a distribution of the same form as before. Dr. M. S. Bartlett had pointed out to him that the variance was not necessarily the best measure of heterogeneity in the sense of departure from the exponential form, and had suggested another whose distributional properties might be easier to determine.

Finally Dr. Irwin would like to say that during the war he did not see as much of his old chief's writings as he had done previously, and it was a real pleasure to read this paper through and to find that the war, which was wearing for them all, had changed neither the profound learning nor the literary fascination of his style. As an illustration of the latter he would like to read a quotation from each of two authors.

Dr. Irwin then read Macaulay's description of the pestilence among Schomberg's troops in Ireland (*History of England*, Vol. II, Chap. XIV, p. 539, in the Everyman edition) and Greenwood's description of plague in India (*Epidemics and Crowd Diseases*, p. 297). He concluded by saying that Professor Greenwood's style combined with the accuracy of the statistician all the picturesqueness of Macaulay.

Dr. M. S. Bartlett desired to amplify by a word or two the suggestion he had made to Dr. Irwin on the problem set in Appendix 1 concerning random intervals following an exponential distribution. This distribution was itself a distribution of a variance with two degrees of freedom from a normal sample, and therefore the problem was identical with testing homogeneity or heterogeneity of a set of such variances. A test was immediately available if one regarded it from that point of view.

The χ^2 method which he had proposed * for making this test was not too good an approximation if, as here, the degrees of freedom per variance were only two, but other writers † had investigated this particular case in more detail with a view to getting more precise significance levels.

Mr. P. A. Moran said that at the suggestion of Mr. M. G. Kendall, he had been studying the distribution of the sum of the squares of the intervals formed by the random division of a line. This was equivalent to considering the volume common to an n-dimensional sphere and an n-dimensional regular simplex of the same centre. The calculation of the exact distribution would be extremely awkward. It was, however, possible to express the result for n-dimensions in terms of that for $n-1$ dimensions. It was also possible to calculate the lower moments by using Dirichlet's integral. Moreover, by using a new type of non-linear Central Limit Theorem it was possible to show that the distribution tended to normality for large n. He hoped to publish these results shortly.

Mr. Mason said it was not clear from the discussion of Aycock and Eaton's infantile paralysis results whether it was their view or Professor Greenwood's that "none of the secondary cases with intervals not exceeding six days could have been derived from the primary cases, and that the patients must have been infected in some other way." This assumption was not logically acceptable

* *Proc. Roy. Soc.*, A, **160** (1937), 268.

† See, for example, Bishop, D. G., and Nair, U. S., *J.R.S.S. Suppl.*, **6** (1939), 89.

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unless the period of incubation could be shown to be the same as that before infection was transmissible. He felt infection *could* take place before incubation was complete or diagnosed, so that it was quite possible for a case in the "interval 5" or even "interval 1" column of the table to be related to one in the "0" column. Thus the interval in days for this purpose should be reckoned from some earlier time x from which the infection might start. These possibilities should, therefore, he felt, be exhausted first.

PROFESSOR GREENWOOD, in reply: I am very grateful for the generosity with which my friends and colleagues have treated this paper. On the mathematical side, I shall profit from the suggestions and comments of Mr. Kendall, Dr. Irwin, Mr. Moran and Dr. Bartlett. I mean I shall profit personally; to the amateur it is delightful to wander in the mathematical paradise; in doing algebra—often quite ineffectively—I almost forgot the *Blitz* which was in full swing when most of this paper was written. Of course the problem will be solved, if it is worth solving, not by me but by one of the speakers. My concentration on the variance was partly mere tradition, partly a notion that as, in practice, the samples would be very small, the variance was likely to be the most useful measure. On the epidemiological side I am equally grateful. I think Mr. Mason is right that the tacit assumption I made and, I think but am not quite sure, Haycock and Eaton made, that an infected person cannot convey the disease during the incubation period is arbitrary. I shall try to investigate the point. I wish to emphasize what Dr. Scott and Dr. Mackenzie said of the importance of data which are not used, really because those in possession of them have other, more urgent, tasks to perform. I have reserved to the end of my reply the first point raised in the discussion—viz., should statistical research in epidemiology be done by a medical man who has learnt statistics or by a statistician who has learnt medicine. I have deliberately restricted the generality of Mr. Kendall's question by introducing the adjective "statistical," because I think other speakers have made it clear that study in the field can be done only by medical men, and by medical men who have had experience wider than that an undergraduate training in a medical school provides. Even so limited, the question is hard to answer.

I think this paper and the comment upon it bring out the bad and the good points of the kind of work done by a person whose upbringing has been fundamentally medical, whose mathematical knowledge, such as it is, has been largely self-taught.

The bad points are obvious enough. Having posed my mathematical-statistical problem, my handling of it has been unskilful because I had neither the knowledge nor the facility (which comes only from long training) to switch over to an alternative approach, but stolidly proceeded with a frontal attack on old-fashioned lines. That is the incurable weakness of the amateur. On the good side, an interest in the problem for its own sake, an appreciation of its biological importance and, perhaps, even memories of visiting sick people in the East End of London when I was my father's assistant more than forty years ago, may, perhaps, have enabled me to bring the data more vividly before a casual reader, than a mathematical statistician could have done.

Perhaps there is some analogy between the situation I am considering and that of the legal profession. The primarily medical statistician is the solicitor, the primarily mathematical statistician the Chancery counsel. The solicitor *must* know enough law not to lead his client to disaster, the more law he knows the better, but, above all, he should know human beings.

I cannot answer Mr. Kendall's question decisively. I am sure that some men whose primary training has been statistical can acquire a great deal of medical knowledge: I can think of conspicuous instances. I know how difficult it is to acquire fundamental mathematical knowledge after undergraduate age. On the whole, speaking statistically, I think the medical experience should come first, but I should be sorry to see a barrier erected.

As a result of the ballot taken during the meeting, the candidates named below were elected Fellows of the Society:—

Walter George Bailey, F.I.A.
Reginald Francis Hugh Banister, B.Sc.
Marcus Bridger.
Sydney Cyril Boxer, B.Sc. (Econ.)
John Peter Burman.
Dudley Albert Clark, B.A. (Econ.)
James Alfred Cornell.
Rudolph Dalberg, Dr. phil., Dr. jur.
Edward Thomas George Emery.
Vernon Eric Gough.
John Baur Greenwood.
Phyllis Ann George.
Stanley Richard Harvey.
Alec Richard Holmes, B.Sc. (Econ.)
Arthur Howarth, M.Sc.
Lieut-Col. R. D. H. Jones.
Thomas Herbert Kelly.
Per Kirstein.
Elsie Anna Grace Knowles.
Derrick Norman Lawley.
Anthony Herbert Leppard.
Walter Levy, B.Sc. (Econ.)
George Osborne McLean.
John Hylton Madge.
Bernhard John Albert Martin.
John Mars, M.A., M.Com., B.Litt.
Irena Mathison.
Gomer Hanworth Morris.
Alfred Stephen Musk, F.I.A.
Philip Edward Naylor.

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Percival Albert Outridge, B.Sc., F.I.A.
Harry Pratt.
Andrew Bandi Reisz.
Campbell Secord.
Edmund Fleetwood Sheppard.
Reza Shimy, B.Sc.
Edward Hugh Simpson, B.Sc.
Edwin Frederick Winchester Sumner, F.I.A.
Thomas Ronald Suttie, F.I.A.
Agnes Teleky, Ph.D.
Major Maurice Arthur Warneford Vale.
Julia Rule Weatherburn.
Leslie Thomas Wilkins.
Thomas Wilson.

Corporate representatives.

Malcolm Alexander Campbell Macneill, *representing* the Anglo-Iranian Oil Company, Ltd.
Walter Taplin, *representing* The Economist Newspaper.

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Improvements in Methods of Census and Survey Analysis

By John P. Mandeville, A.M.I.Mech.E.

[Read before the Royal Statistical Society, March 28th, 1946,
Professor M. Greenwood, F.R.S., in the Chair.]

This paper aims primarily at outlining certain opportunities that appear to exist for the improvement in the methods of analysis of the population census. It is presented at this time because the setting up of a Royal Commission on Population affords opportunity to make changes in the character of the census—both in the nature of the census itself and in the methods and equipment with which the results are produced. The Treasury are bound to reconsider the vote for the census in the light of the findings of the Royal Commission and at that instant there is presented a chance of introducing a fresh approach to the problem of machine analysis of the census. It is therefore important that desirable possibilities of technical improvement be considered in time. Before embarking on such consideration let us ask ourselves a straight question: "Are decennial censuses going to be as important in the future as they have been in the past?"

The answer is not so obvious as it might appear. The fact that the demand for information is so urgent at present may well force a change in outlook which will affect the importance of the population census fundamentally. The 1931 census is fifteen years old and information formerly obtained by hook or even by crook from census data is now beyond reach. The departments in need of information on which to base vital decisions must therefore follow the example of the Royal Commission and have recourse to sample surveys. The increasing use of such enquiries and improvements in the technique of choosing samples both indicate that the importance of the 1951 census may consist chiefly in checking conclusions already reached by sample enquiries and in providing more up-to-date data on which to base future surveys by sampling.

If this situation is faced boldly by the Royal Commission there would seem but a short step to the introduction of a new form of census-taking which would very much enhance the value of any improvements in methods introduced. I refer to the method of continuous enumeration. Suppose, for instance, that the organization built up to provide for the enumeration of the sample of married women were to be made the starting-point of a permanent team of enumerators. This team could work steadily at the continuous enumeration of the population, not dealing with a scattered random sample, but making a total enumeration in a given geographical slice of the country within a fixed time. The complete enumeration of the country might then take place in a series of, say, twenty slices, each of which was designed to form a representative sample of the total population. If each slice were completed in three months the entire enumeration would be finished in five years—say in the five years from 1946 to 1951. A decennial-type census should still be taken in 1951 but clearly its importance would have changed.

It is not within my competence to discuss such a proposal in detail. I would, however, value the opinion of experts as to the practicability of continuous census, for such a method of census-taking would open the way to the full advantages of the improvements in method that will be outlined later. It would appear likely that the flow of the population, released from war strain, stimulated by service training and diverted into new industries, will for some years to come require to be measured by a series of sections made at comparatively small time intervals. Continuous census would provide such sections, each of which, treated as a sample, would supply fresh evidences of population trends, internal migration and so forth. Besides fixed questions required for the consistent enumeration of the population, such regular enumeration would allow additional questions to be asked, varied in each slice according to the urgent requirements of departments. Properly trained enumerators in regular employment working under the expert control of regional field statisticians would provide more accurate information than is to be expected with enumerators in temporary employment.

There would seem at any rate to be an excellent case for the adoption of this system temporarily, to provide information before the 1951 census results can be published. It is my personal opinion that continuous census over the next five years would, in the hands of our sampling experts, prove such a valuable tool that it would not thereafter be abandoned.

The importance of the decennial census would be further reduced if it were possible to estimate

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the Industries and Occupation tables from data obtained elsewhere—within the Ministry of National Insurance, for example. It is no longer outside the bounds of practical machine analysis to match census cards with cards recording much more complete data. All that is required is for the information to be on punched cards containing a common reference number on all cards relating to the same person. This would require that the National Insurance reference number be recorded on the census schedule. If the 1951 census was matched with the records of the Ministry of National Insurance, already prepared with this aim, the relationship between data collected by that department and the Occupation and Industry tables of the census could be established; thereafter these tables could be estimated from the departmental records. Incidentally such a linking of information could enable individual income totals to be transferred to census cards and built up into family incomes.

Whilst it is in regard to this preface and to the conclusion that I expect discussion to prove most valuable, I shall now make a survey of past improvements in order to place these matters in their proper perspective and point the way to what I believe to be the line of future improvements.

Past improvements in methods of census analysis

In the *Journal* of this Society of 1894 there is a paper by Hooker on Modes of Census-taking in the British Dominions.[1] This paper contrasted various methods of hand tabulating and pointed out the advantages of the individual card method, in which the data relating to each person are written or marked on a card that could then be conveniently sorted by hand in various ways; the cards were afterwards counted by hand. Later in the same volume we find the first description of an electrical tabulating machine and it is here that we begin the review of improvements.

"This system of electric tabulating may perhaps most readily be described as the mechanical equivalent of the well-known method of compiling statistics by means of individual cards upon which the characteristics are indicated by writing. As it would be difficult to construct a machine to read such written cards, I prepare cards by punching holes in them."[2] It was in this manner that Dr. Herman Hollerith described to this Society, just over fifty years ago, the basis upon which his new system of electrical tabulating was founded—namely, the punched card. His address on December 4th, 1894, followed the reading of a paper on the eleventh census of the United States of America, by the Superintendent of that census, the Honourable Robert Porter. Porter claimed that by the use of this system of tabulation it had become possible "for the first time in the history of statistical work to aggregate from the schedules all the information that appears in any way desirable."[3] In support of this claim he submitted, in an appendix, a comparison between the detail obtained by hand methods in 1880 and that obtained by the use of electric equipment in 1890.

In surveying the evolution of the punched-card system (first suggested in this country in 1848 by Charles Babbage, Professor of Mathematics in Cambridge University), it is well to spend a little time considering the first machine described by Hollerith. The background of his invention was as follows. The 10th census of the United States had taken more than seven years to complete. Towards the close of the census work Hollerith, an engineer employed as a special agent of this census, was stimulated by Dr. Billings,[2] one of the leading authorities on vital statistics, to devise some mechanical arrangement for facilitating the compilation of population and similar statistics. It was a natural suggestion. American inventive genius was at that time extremely active, led as it was by Edison, and it must have seemed high time indeed that some inventor applied himself to the problems of statistical analysis.

The first object in such an invention must have been the provision of a device for automatic tallying, to avoid the labour and errors of hand tabulation. Hollerith's solution was to provide a number of electrically-operated counters capable of advancing one unit every time the circuit was made, with a recording capacity up to 9999. To make the circuit he had recourse to using cards with holes punched in them. For punching these holes the patent suggested the use of "any convenient form of punch," adding in brackets "an ordinary conductor's punch cutting a round hole about $\frac{3}{16}$-inch diameter is very satisfactory,"[4] but the punch generally used in the first thirty years of the system was the pantograph punch, in which the operator located a position with a pointer moving over an enlarged card layout and, on depressing the key, punched a hole in the corresponding position in the actual card.

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The census card had 288 spaces, $\frac{1}{4}$ inch square; " to each of these squares some particular value or meaning was assigned—a hole in one place meaning a white person and in another place a black. Here it meant a certain age group, there it gave the exact year in that group. . . ." [2]

The card after checking against the original document was ready for tabulating in the electric machine. " This machine consists," said Hollerith, " primarily of a press or circuit-closing device, the upper and movable part of which is provided with projecting spring-actuated needles or points, corresponding in number and relative position to the holes that may be punched in the record card; the lower or fixed plate consists of a piece of hard rubber provided with a corresponding number of cups partly filled with mercury, through which suitable wires are connected with the binding posts of a switchboard. If a punched card is placed in the bed and the handle depressed, wherever there is a hole in the card the needle will dip down into the mercury while at all other points the needles will be pressed back. . . . If we now imagine a counter connected to each cup it is evident that if all the cards are successively placed in the press the counters will ultimately give the total number of times any given hole occurs in a card, or, in other words, a total showing the frequency of the different holes or items." [2]

From this extremely simple design in its " beer-engine " stage, the system has evolved by dint of meeting successive necessities which were the successive mothers of a series of successive inventions; Hollerith's genius is more clearly seen in slow, painstaking progress than in brilliant flashes of inspiration.

Here was one of his problems. " It would require 800 counters to compile a table of 800 columns." Here is his solution. " I have recourse to the use of a sorting box. . . . A box divided into compartments (usually 24) is placed by the side of the operator. The lids of these compartments are controlled by electric magnets. If these magnets are connected to the mercury cups corresponding to age groups, for example, and the cards successively placed in the press, for each card a lid is automatically opened." [2] "As the card is taken out of the press it is dropped into the open division of the sorting box, and the lid then closed. This is done with the right hand while another card is placed in position in the press with the left hand." [4] Hollerith was evidently conscious of the principles of the economy of motion, although motion study had yet to be born. " Each card having been deposited in the compartment opened by it, we now have all our cards sorted according to 20 age groups. If now each of these age groups is passed through the machine provided with fifty counters we obtain a result equivalent to a table of 800 columns." [2]

A description of the American 1890 census machines describes a visit to the machines in these words: "As one enters, the ear catches the sound of crisp bell-ringing, for all the world like that of sleighing. This music comes from the Hollerith machines on each of which . . . a bell rings every time a card is counted, while its failure to ring indicates that there is something wrong with the card or that it has not been slipped in properly." [21]

Cards could be counted at the speed of 6,900 per day of $6\frac{1}{2}$ hours—a very creditable speed considering the hand operations and the transcribing. An account of the Austrian census of 1891 given at the 5th session of the International Statistical Institute [22] reported that in tables where the transcribing of totals was not very great the speed of counting rose from 823, the average overall speed per hour, to as much as 1,250 an hour. Incidentally the punch operators were paid by the hundred cards and averaged about 600 a day, the best operators doing 1,400 a day. When next you look at a Hollerith sorter sorting cards at 400 a minute in what appears to be an unbroken stream of cards flowing like magic into the right pigeon-holes, cast your mind back to the days when 18 cards a minute were put into the sorting box " with the right hand." You will appreciate the patient persistence of the inventor.

Another such problem is described in the original patent of 1889: " In compiling statistics of manufacture, agriculture, or other similar statistics, it is necessary to add, instead of the single units as hereinbefore described, aggregate totals." [4] The solution of this problem was quite ingenious; certain columns of spaces were numbered from 1 to 9, one of these columns was allotted to each digit to be added and the mercury cups for the nine spaces in each column were connected together to one counter. A special press was used in which each of the needles was fed by a separate circuit which came from a device Hollerith called the integrator. The card was inserted and the press lowered on to it. The integrator, which was a cylinder with a varying number of ribs on different parts of its circumference, was then given one complete turn. According to the number of ribs through which electrical contact was made to the needle passing through a hole a number of

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impulses would pass, each adding one unit to the counter; thus a hole in the ninth space would allow the nine impulses coming to it from the integrator to add nine successive 1's on to the counter. This integrator, or " barrel-organ," in which the total of each digit was obtained separately, was the father of the modern tabulating machine, which adds six separate amounts, each of up to ten digits, at 9,000 card passages an hour.

The original patent of 1889 was followed by two in 1894, one of which concerned the arrangement of circuits so as to detect common errors such as categories missed out, double punching, cards put into the press crooked, etc.;[5] the other was a scheme for carrying out the operation by pneumatic power.[6] In 1901 a semi-automatic tabulator, or improved integrator, was patented in which wheels were turned through different arcs according to the digits punched on the card.[7] The press was still hand-operated, but later in the same year there came a device for providing automatic card feed,[8] and in 1904 an automatic horizontal sorter.[9] All the above machines retained the feature of the card being sensed in a stationary condition. It was not until 1907 that mechanism for sensing moving cards was patented in conjunction with a fully automatic tabulator or sorter.[10]

The 1907 tabulator is worth considering as it is the predecessor of the modern one; it allows a simple guillotine motion to feed the cards downwards, between a row of metal strand brushes and a revolving metal drum connected to a source of electric current. When a hole reaches the brush sweeping its particular column a circuit is made and the armature of the magnet, instead of ticking up 1 on a counter, causes a clutch to engage which makes a positive connection between a driving-shaft, that makes one revolution for each card passage, and a numeral wheel bearing the numbers 0 to 9. The card is fed down with the 9 position first; by the time the 9 position is opposite the brush, the driving-shaft will have turned $\frac{1}{10}$th of a revolution. If there is a hole in this position the clutch will be thrown in at this point in the cycle; the wheel will then be turned through the remaining $\frac{9}{10}$ths of the revolution. When the clutch disengages at the end of the card passage the wheel will have turned from 0 to 9. If the next card is also a 9, the wheel will be turned through another $\frac{9}{10}$ths of a revolution from 9 through 0 on to 8 by the end of the card passage, but as it passes through the zero position it will cause a carry-over to take place so that the next wheel on the left reads 1; thus 18 will have been registered. This principle remains practically unaltered to this day.

Hollerith in 1907 had put in roughly twenty years' work developing the punched-card system; he had improved the machines out of all recognition and had succeeded in applying them most successfully to a number of very hopeful fields such as railway accounting. However, in this year he was destined to take a step that had very considerable effect on the future of the system, for owing to a disagreement with the Director of the United States Census Bureau he withdrew his machines. As the 1910 census approached, the census authorities were forced to produce tabulating equipment for themselves. This they succeeded in doing, and they then cast about for a punching machine. James Powers of Brooklyn, N.Y., was engaged to develop card-punching machines. Powers worked in the mechanical laboratory of the Census Bureau and developed a punch employing the principles of the full keyboard—*i.e.*, a key for each possible hole on the card.[11] This punch had the additional feature of being able to register an initial count by sex.[12] Meanwhile, a patent was granted to Hollerith in 1909 covering the use of a mechanical system of tabulating. This probably reflected his efforts to restrain the Census Bureau from using this possibility. He was unsuccessful, however, in the case brought against the U.S. Government for infringement.

On the termination of his work with the Census Bureau, Powers developed a system of mechanical tabulation with the idea of utilizing his punch. Thus it was that a rival system sprang up just at a time when Hollerith might have expected to be able to relax his efforts. From the users' point of view it was probably very fortunate that this natural competition sprang up, because otherwise certain advantages of the mechanical system might never have been recognized, and also the whole tempo of development might have flattened out like the top of a normal growth curve. In order to have a chance against the known Hollerith system, Powers had to produce machines with features that were ahead of Hollerith's design. This pressure to produce new features was all the greater as in mechanical tabulation it was necessary to stop the cards in order to sense their contents—a limitation that carried with it a certain handicap in the running speed of the machine. So the Powers tabulator was designed to print totals.[13] At that time Hollerith tabulators did not print totals, although they could print information from cards. As the equipment was chiefly employed up to 1910 by very large undertakings such as railways, the number of times the machine had to be

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stopped in order to write down totals was small compared with the great volume of cards. But the fact that the Powers tabulator could print was undoubtedly a step towards adapting the punched-card system to smaller commercial work. Hollerith was not long in producing a printing tabulator.[14] The Powers Company still maintained their lead in printing by patenting an alphabetical printing device.[15]

Printing having been achieved, group control became the next feature to be improved. In 1916 the Powers Company patented a device for automatic control.[16] Until then, cards were either put into the tabulator in the groups for which totals were required, or the groups were separated by "stop" cards, which stopped the tabulator at the end of each group. As long as the operator had to write down the totals, these stop cards were no disadvantage, but as soon as the machine was capable of printing totals there was a clear call for a device that would enable the machine to sense the end of a group of cards, print and clear the totals, and restart adding. Powers got there in 1916 and Hollerith in 1917.[17] *

From this time onwards each item of the original Hollerith equipment was improved upon, both in its electrical form and in its mechanical rival. Thus the conductor's punch gave birth to a host of special punches, including of course that which launched Powers as a manufacturer, and which was later improved into the automatic key-punch, which still remains a most successful present-day product of the Powers-Samas Company. There are now high-speed gang punches running at 400 cards a minute, copying information on to cards. A so-called reproducer punch provides the means of copying information in one pack of cards into another pack, as well as the facility for gang-punching cards; the speed of this punch is 100 cards a minute. The multiplying punch—a truly marvellous piece of equipment—will sense a multiplier and multiplicand punched in the same card, produce the product (in £ *s. d.* if required) and punch it in the card; its speed is 10 to 25 cards a minute. This punch can also be used for giving the sum or difference of two (and more) numbers on the card.

The original patent showed the sorting boxes as an afterthought to the counting gear, but in the course of years the sorter has become a general purpose machine included in all installations, and it is only in certain special cases, such as machines for census work, that it has counting equipment fitted, so that counting of the holes in one, two or three columns can take place simultaneously. This equipment has been further improved by adding printing facilities; its speed is that of the sorter—400 cards a minute. A recent development derived from the original sorting box is the collator or interpolator, as it is variously called. This is a machine that may be described as a mechanical filing clerk, capable of merging two packs of cards at an overall speed of 24,000 cards an hour. It is this machine that could be used to file the census card relating to a person behind the National Insurance card relating to the same person, as was suggested earlier.

It is the "barrel-organ" integrator, however, which has the greatest variety of offspring, for the adding of aggregates is far more widely needed than the counting of frequencies. It has already been related how automatic listing, adding and printing tabulators came to be produced. Since 1917 certain main trends may be observed. Firstly there is the growth in versatility of the tabulator, enabling it to add or subtract totals in one counter to or from those in another. Secondly there is the development of facilities for general book-keeping operations; among these are statement devices, ledger-card selection and posting mechanisms. Thirdly there is the production of miniature equipment in the endeavour to secure a more general market.

The analysis of the population census

In the general report of the 1911 census of the United Kingdom it was noted that this census was distinguished by the fact that for the first time tabulation had been carried out using the Hollerith system. Since then the two rival systems have competed for the contract. The 1921 and 1931 censuses were both analysed on Powers-Samas equipment.

The analysis of the population census is at present a simple and straightforward problem of organization. The success depends more on the time-table than on the type of machines used and the time-table depends on the extent of the vote. The speediest analysis could probably be

* Note:—It has been pointed out that as both Hollerith and Powers were American inventors the dates of the British patents do not necessarily give a true picture of the progress made by these two inventors. Nor do any patent dates give a clue to the dates at which devices became reliable enough to be included in standard equipment. These are both fair criticisms of my references to British patents. J. P. M.

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effected by using both types of punched-card system and allowing the natural competition between them to be demonstrated in the speeds achieved by the rival teams. The Austrian census of 1934 was analysed in this manner, and it was claimed that the 2,000 pages of tables were produced in record time.[18] In this country the rivalry between the two companies has been used by the Treasury to obtain a cut price for the census contract, which is given to the company that quotes the most advantageous rates. This has had the effect of making any expenditure on special machines or special research for the census unattractive to the companies.

An alternative that might well be considered is the straightforward duplication of the census cards after punching. This would allow two sets of machines to run on two sets of tables simultaneously. Such a duplication would cost about £20,000, and if the other machines are hired there should be no further increase in the cost of the analysis.

Before considering other alternatives it is necessary to outline the main processes of census analysis. After checking, the schedules have to be coded with numerical codes for all the details that are not already in figures, with the exception of those such as sex and marital status, the codes for which can be remembered by the punch operators.

The punching then takes place. A team of operators is created from girls who have just left school. The work is checked—that is to say, the punching operation is repeated in some form of verifying punch, until such a time as the individual operator shows a percentage of error below a certain limit. The speed obtained naturally depends on the number of figures to be punched, but the average for a census is probably between 300 and 400 cards an hour. The speed of production of the census tables could be raised by an increase in the size of the punching team, but this in turn would call for an increase in the coding team. As both these operations are performed by temporary operators or clerks they deteriorate in quality if the teams are made too big and if the jobs have too short a duration.

The cards are counted on three-bank counting and sorting machines capable of counting the number of holes in three columns at the same time, each bank having a counter for each hole in a column. The results are printed at the end of each group of cards; the printed sheets then pass to the controlling section whose duty it is to check their totals against the control totals that have been established. Final table-making can then take place.

This outline of operations conveys nothing of the considerable administrative detail necessary if such a large volume of documents and cards is to flow harmoniously to programme. It is the very excellence of this planning by the Registrar-General's staff that gives rise to a feeling of injustice whenever the progress of the census is criticized; they cannot reply "It's all the fault of Treasury limitations." Yet this is actually the case, for, as asserted at the beginning of this section, the time-table depends upon the vote—taking this in the wider sense to include the machinery supplied by H.M.S.O. on Treasury authority.

Moving back along the line of authority and putting ourselves into the shoes of the Treasury officers concerned, we have to consider the financial aspect and the provision of equipment as two separate problems. The officer recommending the scale of the vote will no doubt reflect that however rapidly the census is analysed the departments using the figures will have to make increasing adjustments as the years go by; he will assume that the technique of making such adjustments is well understood by those concerned. He will be unlikely therefore to find grounds for increasing the 1931 vote any more than is absolutely necessary. The officer concerned with the sanctioning of equipment will see no advantage in going outside the properly serviced well-proven machines of the ordinary commercial type that can easily be disposed of at the conclusion of the machining programme. As long as the census remains a solitary and enormous peak in the wide ten-year plain, both these restraining influences must, in fairness to the taxpayer, be considered reasonable.

This paper, however, is based upon the conviction that surveys by sampling will fill up the plain and abolish the peak. If this is accepted, the Treasury officers concerned would be freed from precedent and a complete reassessment of vote and equipment would be admissible. To prepare for such a contingency the engineer of to-day would be well advised to free himself from the precedents set by Hollerith and make a fresh approach to the technical problems. We shall find it profitable to consider the broad direction of such an approach.

The first effect of any permanent census organization is to encourage the consideration of special equipment. The first direction in which the aid of special equipment is likely to be applied is to the

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counting of more columns than the commercial counter-sorter permits. The machine designed and installed by the staff of the Federal Bureau of Census of Canada illustrates this.[19] This machine has 520 counters as against the 39 of a three-bank counter-sorter. It can therefore count on 40 columns at a time instead of 3. The card used has only 45 columns, possibly because the machine is pneumatically operated. Special equipment is also used in the U.S.A. and Italy.[20]

There is however another direction not yet generally explored which offers even greater promise—the possibility of eliminating hand punching. Hand punching is the weakest link in the chain of punched-card procedure; it depends for its economic success on an adequate supply of cheap juvenile labour of school-leaving age, and is a tedious job that must be hand checked. A first step in its elimination has been taken in a new Hollerith development—the mark-sensing device, in which cards marked with pencil lines are sensed in a reproducing punch which copies into fresh cards, not holes found in an old pack, but holes corresponding to the pencil marks. A current is made to pass along the pencil mark and is amplified by thermionic valve circuits to set up the magnets of the punch.

With mark-sensing, punching has been fully mechanized, takes place at 6,000 cards an hour instead of 300, and includes automatic verification. It is true that the system suffers from the defect that at least two columns are required for each digit to be indicated by a pencil line, thus the capacity of the card falls to 40 digits. Again, the marking of the card, although in part to be set off against the normal work of coding, would entail more clerical work if applied to the census. Mark-sensing may well prove to be the answer to certain immediate problems in census and survey analysis. However, in my opinion, the full advantage of such a system is unlikely to be pressed by the patentors, because its logical consequence is none other than the complete elimination of the punched card. The impulses used to punch the holes in a mark-sensing reproducer could be used to operate counters. You will remember that Hollerith said that as it would be difficult to construct a machine to read written cards, he had recourse to punching holes; the situation he faced in the old days no longer exists. We now have a machine capable of reading a marked card—we do not really need the punched card in census work any longer!

But if the patentors will not allow the mark-sensing device to be used apart from the punching of cards, we must think again. In this further search we shall do well to seek a form of writing rather than a form of marking—namely, a method of visual indication depending on outline rather than on position. I have planned such an alternative; it has no general application as yet to commercial work but is readily applicable to the census and to surveys. It consists in replacing the coding in arabic numerals by coding in a new script that I shall call the electronic script. This script is functionally designed to interrupt the passage of light falling on a photo-electric cell and call into play the instantaneous electrical effect that variation in the electrons flowing in such a cell can cause. It has been designed in a simple of manner and its use would at once obviate the very real difficulty of constructing a machine to read arabic figures written by temporary clerks (which no doubt was what Hollerith had in mind).

Before illustrating the script I must explain the basic requirements of such a system of writing. Imagine that we wish to compare the Roman numerals I, II and III by passing them under a beam of light the size of a pin-head; they would clearly be distinguished by making one, two or three interruptions of that beam. It would hardly matter where the beam traversed them—it would be interrupted by the opacity of the writing medium the same number of times. However badly the clerks wrote the Roman numerals, and at whatever angle they sloped, the pin-head of light would be interrupted once, twice or thrice, provided it traversed these numerals at approximately half their height.

Suppose instead of one pin-head of light we submit our script, still consisting of three Roman numerals, to two such beams, write two " decks " of the numerals, and traverse the top deck under one beam and the lower deck under the other, we can then obtain combinations of these six possible interruptions; these can be used to represent the numbers 0 to 9 (also 11 and 12 if wanted). By playing around with this scheme it is easy to design flowing strokes that will give us the required combinations of interruptions in the upper and lower decks. In Figure 1 I show an electronic script made from the letters *i*, *n*, *m*, *y*, *h*, *U*, and the sign /, which rather resembles copybook writing. Such a script would be written in rectangles already printed on the census schedule or survey form in some faintly-coloured ink. Its outline could be varied by the individual clerks without destroying

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the actual interruptions caused to pin-head beams of light arranged to traverse the rectangles at half their height.

The first three numbers are made by writing the letters *i*, *n* and *m* in the upper deck with nothing in the lower deck; the numbers 4 and 5 can be regarded as *n* with a hanger and a pothook or else as *y* and *h*, the numbers 6, 7 and 8 are the letters *m*, *n*, and *i* in the lower deck and nothing in the upper deck, 9 is *U* extending to both decks and 0 is / also extending to both decks, 11 and 12 are *iy* and *hi*. The upper and lower beams would be pin-head lights normally penetrating the paper and falling on photo-electric cells. The document would be fed with mechanical accuracy in such a

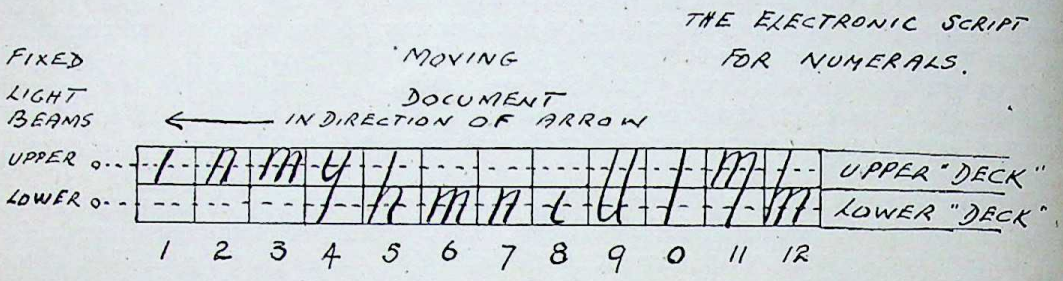

FIGURE 1.

way as to cause the paper to move across the beams and interrupt them with the successive strokes of the script. The interruptions arising from the strokes within any unit traverse (*i.e.*, the passage of the two rectangles formed by the upper and lower deck in which a unit sign is written in the script) would together set up a combination of relays. A current passed through these relays could then actuate punch magnets or counters.

Figure 2 shows a specimen run of census data coded in the script. From this it will be seen that local Registrars should have little difficulty in training their staff to use it.

It may be noted that if the interruptions of the upper beam are read as if they were of the lower beam, and vice versa, 1 will read as 8, 2 as 7, 3 as 6 and 4 as 5; 9 and 0 will remain the same. This scheme was adopted at the suggestion of Mr. J. R. Womersley as providing a method of obtaining complements to 9 by simple switching over of circuits.

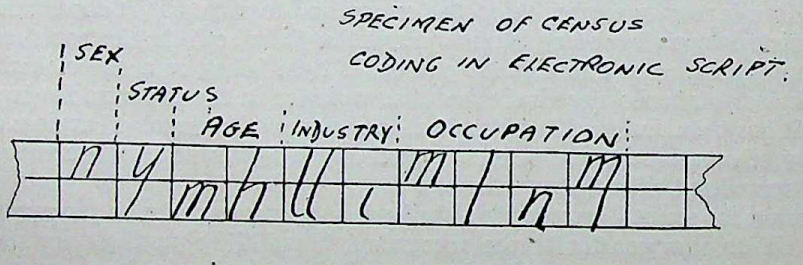

FIGURE 2

As far as I know, the approach to photo-electric sensing has hitherto demanded the typing of arabic numbers of uniform outline quite outside the possibility of use for the census, and this special electronic script has not been suggested before. I believe the reason to be that it would be impracticable at present to introduce it in ordinary business and it therefore possesses little attraction for the machine-making companies. But where surveys are coded by clerks they can be coded in anything that clerks can be taught to use; I claim that such a script is as simple as it could well be.

The machine necessary for the sensing of such documents could be made up of well-proven features. For instance, the sheet-feeding device that enables the printer to feed sheets of paper time and time again for different colour printings at up to 8,000 sheets an hour, and yet obtain hairline register of each colour, could be used for feeding schedules. Photo-electric detection of the strokes would be largely an optical problem; the setting up of relays by electronic circuits and the counting or punching of cards through such relays are established techniques. It might well

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be found desirable to equip the station having the photo-electric gear with air conditioning, as paper-stretch due to humidity is likely to be a more serious cause of distortion than handling the document.

Such a system has two advantages over the commercial system of mark-sensing. Firstly, photo-electric scanning gives us back the full 80-column card if it is carried out in conjunction with the Hollerith system; this increase in capacity is most important if the station is to be used for surveys other than the census. The second is that as the system is not patented (to the best of my knowledge) the punched card can be eliminated altogether if desired.

If we combine the use of a larger number of counters with photo-electric sensing we have the means of leading the world in the central machining of large-scale statistics. We have the great advantage of not being committed to any existing system.

Now let us take a step inside an imaginary survey analysis station of the future. Here is a room with banks and banks of counters arranged on curving stands which allow rotating cameras to photograph their contents. The counters themselves are recording, not arabic figures, but electronic script. As they are being photographed so they are being scanned, and elsewhere summary cards are being punched recording the final tables of the survey. The scanning of the counters not only causes the cards to be punched, but also checks the counter totals against their subtotals and against control totals. In fact it does so before it allows the cards to be punched.

The punched-card equipment is in another room, in which the summary cards containing the census table totals are listed on blanks pre-printed with headings, etc. As soon as the carbon copy has passed the control section, the results are photographed, offset printing plates are prepared from the negatives and the tables are provided. All the tedious process of typesetting and proof-reading is thus eliminated. This is no novelty; the Civil Service Commission adopted it in the 1930's for publishing examination results.

In another room collating or interpolating machines are engaged in matching cards punched from returns (coded in electronic script) made by panel doctors, with the standing file of patients' history cards. The matched cards are being completed by the copying of certain data from the history cards by high-speed gang-punches; the two sets of cards are afterwards sorted apart and finally tabulated.

In through an airlock we pass to the photo-electric sensing station where half a dozen machines are quietly feeding themselves with schedules of the continuous census at 6,000 an hour. It is from these machines that the counters we saw in the first room are operated.

The Royal Commission on Population has the opportunity of taking the first step towards the realization of such a photo-electric dream. It can ask for an investigation to be made into the possibility of improving the machinery of census analysis and can point to the body competent to undertake such an investigation. Whereas in 1931 there was no branch of the Civil Service competent to undertake research into the design of equipment for statistical purposes, we now have within the Department of Scientific and Industrial Research the Mathematics Division of the National Physical Laboratory whose terms of reference include the undertaking of research into new computing methods and machines. The Superintendent of this Division is a member of the Council of this Society. Could there be a more favourable moment to ask for such an investigation? Further, should it be found that special equipment on the lines indicated is thoroughly practical, the chances of it being made available are exceptionally favourable at this moment, since the necessary components could be secured before the huge war surplus of radio components has been dispersed.

Although there are clear advantages to be derived from such equipment even if it were stored from one decennial census to the next, its full benefit cannot be felt unless the character of the census is changed. Thus the main burden of future development is not something that can be turned over to the N.P.L.; it falls on the statistician—not on the engineer. It is the task of grappling with the problems of continuous census, such as overlap and duplication, the task of planning samples on a large scale and of providing ever-increasing refinement in treating the results of such samples. Continuous census could be started this year. It might well be that the first few slices would raise new problems of great complexity, but I cannot conceive that the Fellows of this Society would allow these to go long unsolved, and I remain firmly convinced that 1951 would find continuous census established as a permanent feature of national self-survey.

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Conclusion

Having traced the genesis of the punched-card system as a solution to census analysis, I have suggested that the story may be completed by the exodus of the punched-card from this field. One solution of the difficulty of designing a machine to read written cards is to write differently—in functional " electronic " script. A more important problem is to change the character of the census itself. Both developments can be tested. It is upon the Royal Commission on Population that the responsibility for future progress now rests. If, in the words of William Morris, they have " imagination enough to conceive; courage enough to will; power enough to compel . . . then, I say, the thing will be done."

References

1. Hooker (1894, " Modes of Census-taking in the British Dominions," *J.R.S.S.*, **57**, 331.
2. Hollerith (1894), " The Electric Tabulating Machine," *J.R.S.S.*, **57**, 678 f.
3. Porter (1894), " The Eleventh Census of U.S.A.," *J.R.S.S.*, **57**, 657 f.
4. Hollerith, British Pat. 327 of 1889.
5. *Idem*, British Pat. 8,101 of 1894.
6. *Idem*, British Pat, 17,738 of 1894.
7. *Idem*, British Pat. 10,502 of 1901.
8. *Idem*, British Pat. 19,372 of 1901.
9. *Idem*, British Pat. 27,072 of 1904.
10. *Idem*, British Pat. 27,067 of 1907.
11. Durand (1909), *J. Amer. Stat. Soc.*, **11**, 608.
12. Powers, British Pat. 11,720 of 1911.
13. *Idem*, British Pat. 26,896 of 1913.
14. Hollerith, British Pat. 19,461 of 1914.
15. Foster, British Pat. 2,384 of 1915.
16. Powers, British Pat. 104,703 of 1916.
17. Hollerith, British Pat. 117,985 of 1917.
18. *Revue de l'Institut International de Statistique*, 1936, 75.
19. *Ibid*, 1936, 545.
20. *Ibid.*, 1935, 308.
21. *Electrical Engineer* (*American*), November 11th, 1891.
22. Rauchberg (1895), *Bulletin de l'Institut International de Statistique*, **9**, 249.

DISCUSSION ON MR. MANDEVILLE'S PAPER

MR. J. WOMERSLEY: I find myself a little bewildered. Mr. Mandeville has, I think deliberately, given me a very great deal more to think about than I can very well digest in time to be relevant about it this afternoon, but I would like on your behalf to pay the warmest possible tribute to him for even raising this question, apart from the very fine suggestions he has put forward in the paper.

I was particularly charmed by the early history of the Hollerith equipment, which was entirely new to me, and I could not help noticing that there is a kind of historical parallel between the situation in, say, 1890 and to-day. Then there was only one job worth making new machines for—the census. Engineering developments that had taken place in the nineteenth century were first applied to the census, and the commercial use came very much later. It seems to me that now we are in an analogous position. Once again there is one big job for which it will be worth while to try out new ideas—the census : the commercial use may again come later. The electronic developments that have taken place since Dr. Hollerith began making his " beer-engines and barrel-organs " have now reached a stage when we can think quite seriously about their practical application.

On the question of the reform of the census and of taking it continuously rather than in lumps at intervals of ten years I do not feel competent to speak ; there are others here to-night who can say a good deal more about that than I can. It is certainly much more worth while making new machines if you have a steady load. However, it was worth while in 1890 making special machines for a load that was discontinuous, and it may well be equally worth while now. I shall want to think a little about the electronic script and its practicability, but I can say now that I foresee some very remarkable increases in speed if this thing does work, and there have been developments during the War on the radio side which make it far nearer practical application than might at first appear.

Of course, at the National Physical Laboratory our normal terms of reference would only include the possibility of using this in scientific work, and I do not know how far the policy of the Laboratory would be to encourage us to take part in the technical development for the census. In conclusion may I, on behalf of you all, thank Mr. Mandeville for a very interesting and entertaining discourse.

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DR. L. J. COMRIE: I remember, twenty-five years ago, when I sent in my first paper to a learned society, the president of that society sent a letter to the secretary, which I still have, saying, "Who is this Comrie?" I expect some people to-night have been asking, "Who is this Mandeville?" I will, therefore, say just a little about him.

He is not to be confused with the author of the same name who wrote *The Voyage and Travels of Sir John Mandeville*. Judging by the language, this book was written about 600 years ago, and in fact Sir John Mandeville was buried in 1372 at St. Alban's Cathedral. The chief thing about this book is the imagination it displays. One cannot believe that Sir John Mandeville was actually able to sail his ship under one of the loops of the sea serpent; while in describing a foreign country, he says, "And there grows a manner of fruit as though they be gourds, and when they be ripe men cut them in two and men find within a little beast in flesh and bone and blood as though it was a little lamb, and men eat both the fruit and the beast and this is a great marvel."

Although this John Mandeville cannot claim to be descended from Sir John Mandeville, he has the same asset—imagination—and he has put before us a very good scheme. I hope we shall be able to develop this scheme in less than seven years and, after all, why should we not? In his paper we read how there has been development in the United States on machines for the census, and also in Canada and Austria, but very little in this country. In one part of his paper he put his finger on what I regard as our weakness—our Treasury. The Treasury procedure in the past has been to take the two commercial machines, get the firms to compete against each other, and then take the lower price, and such a price that it does not pay either firm to take on the job: therefore they have supplied the machines, taken them back after the census, and had nothing over for development. If the Treasury would take notice of what Mr. Mandeville has said, they might now be able to encourage a little development.

Returning to Mr. Mandeville's career, in his early days he entered Dartmouth College as a Naval Cadet. He served during the 1914–18 War, left the Navy after reaching the rank of Lieutenant (E) and joined the Royal Navy Emergency List. In 1929 he went to Russia with the Anglo-Russian Trade Delegation—he was representing machines, particularly linotype and printing machines. He learned to speak Russian, and he has used that to good effect recently, translating from the original Russian into English *The Extraordinary Adventures of Karik and Valya*. If only we could be seven again, as A. A. Milne would have us, we would enjoy very much reading that book. I found it quite fascinating, even though I am more than seven. Then came the slump, and he joined Calculating Statistical Service, which later amalgamated with the British Tabulating Machine Company. There he got his first experience of punched-card machines, and became one of their experts in the service department and an authority on the machine analysis of market research: he was always putting forward bright ideas, as he has so clearly done in this paper.

Later he left them and went down, down, down!—first to the Ministry of Supply, then to the Treasury, then to the Ministry of Aircraft Production. Now I think he has risen again, because he is an independent consultant on the application of the punched-card system. I want to see him continue with his ideas, and I want to see the Treasury take up his ideas and give him a chance to exploit them. On your behalf I should like to join the proposer in thanking him for a very interesting paper.

The vote of thanks was put to the meeting and carried unanimously.

PROFESSOR HOGBEN wished to join the two previous speakers in expressing what he thought to be the sentiment of every one present—one of cordial appreciation for a timely, lucid, highly instructive and, what was not necessarily the same thing at a learned Society's meeting, an enjoyable exposition. When he had first come into the august presence of the Hollerith machines he had felt, like many others, that the ground upon which he trod was holy, but as daily familiarity blunted the fine edge of reverence he had become uncomfortably aware that the technical design of the machine for a particular purpose might dictate disadvantageously the conduct of an enquiry with an entirely different end in view. For that reason it was of the utmost importance that progress in the speaker's branch of engineering should proceed with the closest possible collaboration between inventor and user. Also, for that reason, officers of the Society were to be highly congratulated on providing a common meeting-ground for those interested either in the design or in the use of modern methods of tabulation.

It would be presumptuous for him to express any view concerning the value of the speaker's proposals with reference to the improvement of tabulating machines for use on the census; but one point he wished to submit for Mr. Mandeville's further consideration emerged naturally from a remark made in the course of his paper, when he said that the bottleneck of mechanical sorting is the process of punching the card. That was probably true of the Hollerith in so far as it was concerned with tasks set it by the census and by the day-to-day work of commercial firms who used it, but an organization such as the Directorate of Army Medical Statistics had to take an alternative view. The punching of cards is, of course, a bottleneck, but where documents themselves contain

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highly technical information, experience of Army Medical Statistics disclosed another and more serious bottleneck—namely, the task of extracting that information from the documents in the form of a code for use by the team to carry out the mechanical work. In so far as they had discussed among themselves, as users, the sort of amenities which they would like the engineering skill of inventors to provide, the thoughts of those engaged in Army Medical Statistics had run less on the lines Mr. Mandeville had discussed in connection with his proposed electronic script, than on the process of eliminating the job of coding altogether.

With great diffidence in the presence of an expert on the mechanics of the Hollerith machine, he therefore wished to make a suggestion. In so far as such machinery was going to be used increasingly, not merely to deal with problems of the census and accountancy, but also to a greater extent in scientific research—more especially in medicine—of a statistical kind, coding was the key issue. Clearly, the problem of eliminating the bottleneck of the code was the problem of transferring the labour of interpreting a document to the level at which recording the information itself involved interpreting the data. This was an entirely soluble problem when one was clear about the object of an enquiry. One could then design a record card on which all class items were explicit, so that the recorder had merely to signify which item was applicable by a mark of one kind or another.

A record card so designed to make for economical hand-sorting was, of course, equally usable as a code slip if suitable numbers were attached to the items so treated ; and it was not a far cry from a self-coding, self-recording document so designed for a field enquiry to provision of a marginal interpretation specifying how to black-in appropriately numbered squares. Was it not possible that a development along such lines would prove to be as generally serviceable as the electronic script? If it were indeed possible to adopt a system of blacking-in such numbered squares as a method of recording quicker than writing a description and eliminating the necessity for a coding slip, would it not be possible to rig up a photo-electric circuit to make the process of punching an entirely unnecessary stage in the process of sorting and tabulating?

Mr. V. P. A. Derrick had read Mr. Mandeville's paper with great interest, but thought that there would be difficulty in offering constructive comment because the conditions envisaged by the author were so far beyond those experienced hitherto in actual practice. Regarding the census, Mr. Mandeville's suggestions were interesting, but they seemed to be based on premises of doubtful tenability. For example, in preparing the ground for his proposed developments in tabulating machinery, the author had come to the conclusion that they would not function unless the census was altered to meet them. That, it seemed, was putting the cart before the horse : the statistical objective of the census was surely above and independent of any mechanical appliances that might be there to help it. For the earlier censuses no machines at all were available, and even under modern conditions quite a lot of tabulation was still undertaken without recourse to machines. Part of the housing tabulation of the 1931 census and the whole of the National Register statistics, which comprised a considerable analysis of the population of all areas of the country, were undertaken without any machines at all.

Although the machine had undoubtedly greatly enlarged its possibilities and scope, it had no material influence, so far, either on the nature of the facts to be collected or on the design of the statistics : in every case it was the machines which had to be adapted to the statistics rather than the census adapted to the machines. That was an important principle, and he would personally be very reluctant to depart from it; we should, of course, make the fullest use of the available machine capacity in its widest sense, but always subject to the overriding claims of the census itself.

Another suggestion put forward by the author to support his proposals was that of dividing the country into geographical regions and spreading the total census over a period of years by taking independent censuses, region by region, at successive intervals of time. That proposition was understandable as far as it went, but the author had then gone on to make the somewhat startling claim that the total enumeration of the country would thus be completed in a series of slices, each of which would form a representative sample of the total. That was difficult to understand: the differences between regions of the country were known to be great, both in kind and scale, and one of the first analyses of any population study was to classify the results by regions, with the express objective of disclosing and measuring the regional differences. Moreover, by taking partial censuses at intervals it would be impossible to ensure their mutual exclusiveness, because of the tremendous population changes going on continuously: at the present time removals from town to town or from town to country were taking place at the rate of about 100,000 a week. With a scale of movement of that kind there was going to be a very great deal of change from region to region over a period of five years, and it would also happen that many persons would be repeatedly picked up in successive regions and would get counted twice or three times over, while others would escape enumeration altogether. The regional enumeration would be of limited application—it would merely be a local census without much relation to the wider National field of the whole population.

The author's concern with the census, however, was clearly a secondary one; as an engineer,

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the main and immediate objective of his paper was obviously the advocacy of eliminating the manual punching of machine cards in favour of some other sensing device. It would be generally agreed that card-punching was a laborious and tiresome operation, but having read the paper very carefully, he could not see that either of the suggested alternatives was likely to be any less boring, mechanical and monotonous. The author had explained the mark-sensing device and referred to the fact that it would enable cards to be punched mechanically at a rate of 6,000 an hour, but the marking of the cards was a correspondingly laborious operation. It was just as easy to punch a card as to put a specific pencil-mark upon it: easier, in fact, because the punch automatically found the place on the card as well as punched it, and it would be a much slower business to put pencil-marks on the card and to ensure their being correctly placed.

The same type of criticism applied to the electronic script-sensing which the author had envisaged. It could be assumed that, even if only for record purposes, the primary census data would continue to originate in script form provided by a private individual or an enumerator. In that case before it could be fed to a machine it would have to be subjected to a process converting the script into a form in which the machine could handle it, and it was the conversion process as a whole which had to be judged in deciding which method was the better. Hand-punching of cards was very simple and was within the capacity of anybody; it was usually performed by a low-grade staff, and, having been done, the punched holes were readable by everybody who had anything to do with the machines.

When the whole conversion operation was brought under review, it seemed very doubtful whether the alternatives suggested had any overriding advantage; in any event, a great deal more experimental work had yet to be done before they would be justified in giving up well-tried and established procedures in favour of such methods.

MR. A. THOMAS was much interested in the matter under discussion, having been personally responsible for the design of the counting machines used in the 1921 and 1931 census.

He thanked Mr. Mandeville for his paper and for drawing attention to the census. This subject of Census Machinery was apt to be neglected until the census became imminent, and then somebody would find, say in 1938, that they wanted a nice new machine to do everything, and wanted a number of them in 1940. This did not give a fair chance for proper development.

Mr. Mandeville had rather assumed that the five hundred counters said to be used by the Canadian Census Office must be better than the thirty-six counters used by the English census authorities, but this did not follow at all. The Canadian Census Department had given publicity to their methods, but an official from another place had said that Canada was only using obsolete United States equipment.

He would rather disagree with the inference made by a previous speaker that this country had been behindhand in the provision of special machinery for the census, and that the Registrar-General's Department had been limited to the use of the ordinary commercial machines as designed for general use. Speaking for the Powers Company, and also for their good friends of the Hollerith Company, he could say that a large amount of development had been carried out on special census machines in collaboration with and to the needs of the Registrar-General's Department, and he felt that the machinery which had been developed in this country, inefficient though it might be from some aspects, was the most advanced in the world, except perhaps that of the United States Census Office, which had a large engineering department making their own machines. When he last visited Washington in 1937 the Census Department were just changing over from twenty-four-column cards to forty-five-column cards.

With regard again to the number of counters used, Mr. Mandeville had described how Dr. Hollerith had found that by using a sorting machine he could obtain on fifty counters a result which would otherwise need eight hundred counters.

Census work was an analytical job demanding the use of sorting and counting in their proper places.

The photo-electric cell was becoming rather popular these days, but was actually old when Dr. Hollerith made his first machine. There was no doubt that great advances had been made by science in all directions, and improvements in technique should be expected as a result, but it should be pointed out that in order to use science properly it was above all necessary to set out the proper problems.

What was the reason for these efforts to do away with the holes in the cards? There seemed to be an idea that it was making the holes that was the nuisance, and that the holes were only necessary to enable the machines to read the information. This was not the fact. Machines could be made to read Braille embossed records or marks of various kinds.

A card was punched for two reasons: first to get the information on to a standard-sized card, and secondly to place it in correct position on the card.

His reaction to the proposed electronic script was "Why write these things free-hand when a machine could be used?"

It would be a real solution to the problem if we could find a machine which would read all sorts of original documents and put the information in proper order on standard cards or forms.

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At present the easiest, and fastest, means of recording was to punch a card. Punching was the only recording means that did not need maintenance or replacement. Punching-machines had been in use for twenty years without any attention having to be given to the cutting punches themselves, whereas a pencil had to be sharpened at intervals, a pen had to be filled with ink, and no printing or typewriting process would go on without periodic replenishment of the recording medium.

Mark-sensing had been mentioned. He would say that this method of recording, although quite efficient in sensing the marks and translating them into holes, was not successful owing to the difficulty of getting people to make proper marks and to put the marks in the right places.

The mark-sensing was invented seventeen years ago, and the inventor had met with this difficulty then. When the mark was made with the required thickness of line and in the right place, the device worked, but it was so difficult to get the operators to do this that he continued his development by making a little key-operated machine, just like a punching machine, to make the pencil-mark!

MR. D. M. SEN mentioned a similar mechanical development which was now being commercially operated in America for facilitating marketing research. The engineer was helping the statistician all the way from the beginning of the history of marketing research. Until recently there was no other way of radio-programme rating than by the telephone research method. A special electronic device had now been developed in the Nielsen Audimeter, which was well known in America in technical circles, and which recorded every movement of the knob of a radio set by means of a recording tape inside the set. In other words, whenever the radio set was tuned in to whatever wave-length, the audimeter would record it. The audimeter recorded the wave-lengths listened to and the times of listening. The collection of these recording tapes, however, involved some delay in the processing of the data. It had now been possible to reduce materially the field cycle and to cut down the time for the receipt of listening records by an engineering development which allowed an instantaneous transmission from the audimeter radio sets to an urban collecting centre, followed by overnight mailing of data to the national collecting centre. These centres could collect information in this way from about fifty to one hundred homes, thus automatically collecting data all the time of the programmes being listened to and the length of time the set was switched on. All this was made possible because a similar electronic device was perfected and put into operation in 1942, after seven years of continuous research. It had now proved a commercial success, and was now earning a dividend after two million dollars had been spent on the project.

From the Atlantic to the Pacific coasts they were now recording habits of radio listening in America, and fifty different organizations—advertising agencies, radio-programme planners, net works, stations and all sorts of people connected with advertising—were actually paying for it.

So it could be said that however difficult Mr. Mandeville's electronic script might sound, it might quite well inaugurate a similar new era in general marketing research in this country.

More than $350 million were spent in 1944 on advertising through the medium of the radio in America. Obviously it was ultimately of great value to the advertising agencies, as well as, generally speaking, to the economy of the country, to have a device by which one could check which was the most economical way of advertising.

Similarly, if Mr. Mandeville's script proved a commercial success, it might lead towards economy from the national point of view, whether in marketing research or in census or survey work, by saving time and labour. As those who were engaged in continuous market research would know, the bottleneck was usually in the punching of holes. The firm for whom he was working at the present time had to deal every month with 7,000 different Hollerith punched cards; actually the number of holes came to about 4,000,000. Everything else went quite smoothly, but the sequence of operations was often held up by the punching-room in the Hollerith section. But, none the less, one could see a difficulty with mark-sensing and the electronic script, in that learning a new script and being able to use it automatically without having to stop to think was no easy matter. Where data had to be adjusted, checked or processed before they could be tabulated, obviously the staff for such checking would be employed for quite other reasons than their ability to read electronic script. To train them to read electronic script would take some time, and the commercial firm using this script would then be investing an annual amount in training the electronic-script readers and writers. Similarly, supervisors who were perhaps chosen for their knowledge of the kind of data the firm was handling would have to pass through an initial period of a very special kind of training. In these days of comparative full employment, ordinary checkers and comptometer operators did not stop at any job for long. Commercial firms would therefore be loth to invest money in training their staffs any more than was necessary, and would be eager to put them to productive work as soon as possible. It was therefore just possible that the time gained by the elimination of hand-punching would not be offset fully by man-hours lost in training staff, comptometer operators, supervisors, etc., in the electronic script. Especially when staff-turnover was likely to be very high, as it was to-day with juvenile labour, much of the money invested in specialized training would yield no dividend to the firm.

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Mr. Sen, however, wholly agreed with Mr. Mandeville that a way out must be found, and probably, with certain modifications and additional mechanical devices, the electronic script would be the answer.

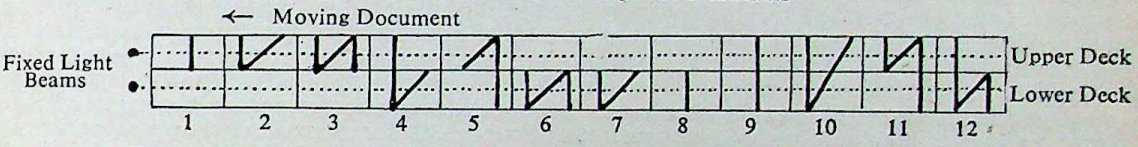
The following comments were received in writing:—

DR. DAVID HERON: I read Mr. Mandeville's paper with great interest and listened to his admirable exposition of his proposals with great pleasure, and therefore regret all the more that his suggestions are not based on surer foundations.

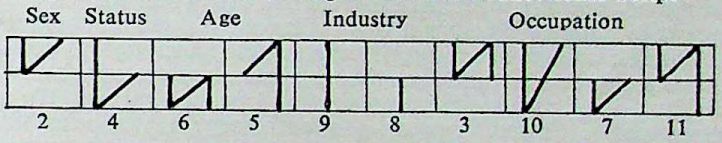
In advocating continuous sampling as an alternative to the decennial census, Mr. Mandeville suggests that each sample should cover "a total enumeration in a given geographical slice of the country within a fixed time." It would thus fail to measure accurately migration from one part of the country to another—one of the most important aspects of population changes and one which is satisfactorily dealt with by the decennial census.

His suggested electronic script is also unsatisfactory. Whatever instructions are given, *m* will often degenerate into *ʍ* and give five light-beam interruptions instead of three, while *y* will often degenerate into *ɥ* with three instead of two light-beam interruptions in the upper deck, as is indeed apparent in the second symbol of his Figure 2. If there is to be any future for an electronic script, it must be less likely to cause such errors. I suggest as an alternative the script given below:

Alternative Electronic Script for Numerals



Specimen of Census Coding in Alternative Electronic Script



I feel, however, that coding in any electronic script would be likely to cause an even greater bottleneck than the punching of cards, which have the advantage of being easily read by any one.

DR. E. R. BRANSBY: Mr. Mandeville has done a service in drawing attention to the need for further thought about the census. It is now generally accepted that there should be periodical stock-takings to assess the state of our economic, social and physical health. Further, it is generally recognized by those dealing with social and economic data that in the past there was a sad dearth of reliable data for such an assessment. Before the war there were a number of official enquiries on a national scale, such as the census, and the housing and cost of living surveys. That there were important gaps in our knowledge is illustrated by the fact that when Professor Bowley was asked by the Ministry of Health's Advisory Committee on Nutrition to study the distribution of population according to income per head, he had to base his report on incomes of 1937 and population and conditions of employment in 1931. For the population part of the enquiry he used a sample of the census, as did Sir John Orr, in conjunction with the Market Supply Committee, for certain calculations in Food, Health and Income. The present is surely the time to review the whole question of the collection of national data, including the census, and to plan for the integration of the various needs into one system of surveys. A number of matters need to be settled. What further questions should be asked in the census itself? What information should be obtained from existing records in Government Departments and what further routine information should the Departments collect to help complete the national picture? What special enquiries, perhaps of a more detailed and precise character, should be made on a sample of the census population? and how often should the various surveys be made? This review would include the desirability of modifying the census, and in this connection I would like to know whether, in view of present knowledge on the technique of sampling, it is really necessary to collect information from the total population, or whether a sample of the population would be sufficient. The whole problem could usefully be examined by a specially appointed committee composed of representatives in interested Government Departments and learned societies.

There is a further matter of a different kind to which I wish to refer. When I heard that Mr. Mandeville was to give a paper dealing with punched-card machines, I felt that the Society was doing a useful service to those who, like myself, use such machines for analysing social and other data, and who wish to be kept up to date in the latest developments. Mr. Mandeville's paper does

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not deal with this perhaps somewhat elementary aspect, but during the war there have been very considerable developments of machining techniques which remain hidden away in the departments or other centres in which they originated. I am sure that a meeting of the Society or of the Research Section devoted to this aspect would be welcomed by many.

MR. E. T. G. EMERY: An electronic device of the type proposed could be as sensitive to punched holes in a card as to written or typed script, and therefore I suggest the first question to be answered is "Has electronic sensing any advantages over existing methods?" If the answer is "yes," then we may consider which method of producing a record is most suitable, having regard to speed, likelihood of errors, need for special training, etc. It appears to me that the author's script could be produced more easily on a special twelve-key typewriter than by handwriting.

The limiting feature in the use of all "seeing" machines is the production of the kind of record necessary to their special form of vision, and I submit that a more profitable development might be the production of a "hearing" machine. If we had a machine sensitive to, say, twelve single-syllable words, we could build up codes from these and *read* each document to the machine.

DR. J. WISHART: Mr. Mandeville has made it clear that a census of the continuous recording order is amenable, through coding and feeding of the forms to the machines, to the system of photo-electric sensing and continuous recording of results by photographic methods. There is, however, an important alternative to the continuous census in certain fields where periodical statistical tabulation is also required. This may be illustrated from service personnel data. The Royal Navy may be said to have done pioneer work in this respect, for it was as early as 1918 that personnel particulars began to be recorded by means of punched-card machines. The system is that a continuous census takes place of *new entrants*, in that a form containing the required particulars is supplied, the particulars are coded and a punched card is prepared. But such particulars require to be continually amended, and the problem is to keep the central index correct and up to date in regard to each individual, and at the same time to provide for periodical standard statistical tabulation of the contents of the file, as well as for *ad hoc* counts, either of the whole or of selected samples, for special purposes. The administrative machine provides for particulars to be supplied every time a change takes place. These are of two kinds: (*a*) an amendment to a card which remains live—for example a change in rating; (*b*) a report that a rating has been discharged, which results in the card becoming dead. Sufficient census particulars are supplied with the "change" to identify the individual—*e.g.*, name, official number and existing rating. To supply more would be a work of supererogation. Two things are now necessary: (1) to bring the index up to date and (2) to prepare a fresh statistical return. The collator is a very useful machine for the first of these tasks. After, say, a month's "trading," provided we prepare, in addition to the batch of new entrant cards, new cards to take the place of old to index changes, and extraction cards to match those to be withdrawn as discharged, one run through the machine will do the job. The second task can be tackled along the lines of full analysis of the month's "trading," before collation with the index, using the results to correct the last return, but, unless we are very careful, it is possible to go adrift. To do a large-scale analysis of the full index, whether for standard or special purposes, conflicts with the maintenance task unless two files are kept, which in itself adds to the maintenance problem.

Now I have no doubt that there is a solution to this problem along Mr. Mandeville's lines, which would make the index almost instantaneously up to date, and enable statistical returns to be prepared at great speed and be photographically reproduced from the apparatus, thereby economizing very greatly in man-power and money. But just because it is not the same problem with which he dealt, but implies the matching of partial new information with that already recorded, the problem is commended to the attention of those who are engaged in working out the practical details.

DR. H. O. HARTLEY: Mr. Mandeville has invited a critical discussion from the expert statisticians, of his idea of a "continuous census" and I feel that it is a great pity that this has received hardly any attention.

The only critical discussion is that by Mr. Derrick, who has largely misunderstood Mr. Mandeville, possibly because of the rather sketchy outline of the idea of continuous census given in his paper. Far from being an expert in these matters, I have nevertheless endeavoured by personal contact to ascertain more of what was at the back of Mr. Mandeville's mind, and feel that another attempt to invite discussion by the experts (possibly within the framework of the revived Study Section) is worth while.

If we were to take the census for all localities in England and Wales in (say) ten "slices" $S_1, \ldots S_{10}$, to be surveyed consecutively (instead of simultaneously) each slice would constitute a 10 per cent. sample of all localities. The first slice could be built up as a stratified sample of localities using the well-known technique of dividing all localities into strata of geographically, socially, industrially, etc., "similar" localities, and selecting from each stratum the appropriate quota of $\frac{1}{10}$ as a random sample to contribute to the first slice.

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The census for this first slice would then provide information for the whole population of precisely the same accuracy and within precisely the same limitation as a properly stratified 10 per cent. sample survey. (This answers Mr. Derrick's doubts about regional bias. Apparently Mr. Derrick wrongly concluded that Mr. Mandeville's slices corresponded to actual compact regional units.) To form the second to tenth slice, further samples of localities had to be selected from each stratum *not* completely at random, but the selection would not be unlike the well-tractable case of sampling without replacement. If for the ten slices thus obtained the census is taken at times $t_1 \ldots t_{10}$, respectively, then the ordinary simultaneous census merely becomes a special case of the "continuous census," in that we choose $t_1 = t_2 = \ldots = t_{10}$ and repeat the census for all ten items, say, 10 years later. The claim, then, that the simultaneous census is the best is thereby equivalent to claiming that of the great variety of possible choices of $t_1 \ldots t_{10}$, the above particular choice is the most efficient in every respect, or at least in most respects.

The onus of the proof of this lies mainly with those who say that no change should be made. There are undoubtedly certain statistics for which the simultaneous census is the most efficient. For instance, if the *total* population of England and Wales is required (possibly broken down by age groups) at discrete time intervals a simultaneous census in those particular years appears to be best, since, as Mr. Derrick indicates, the estimation of these statistics from a continuous census would involve an allowance for migration within England and Wales. However, this is hardly the only information that we expect the census to provide, and one can think of a variety of statistics in which a continuous census would yield more information, particularly through providing current estimates based on up-to-date information.

With a continuous census, national or regional figures *at any time* can only be obtained by methods of statistical estimation similar to that practised in sample survey technique. It is not by any means obvious that such methods should be inferior in accuracy to the statistical estimation consisting of interpolating, or indeed extrapolating the information of the census years.

MR. MANDEVILLE said he welcomed the controversy which had started, and looked forward to some of the experts taking his side with regard to the continuous census. The whole point of the paper was that the matter should be thoroughly investigated, and if he had done anything towards bringing this question to the attention of the Royal Commission so that they might set in motion such an investigation, he thought the rest could be left to more competent hands to deal with.

MR. MANDEVILLE subsequently wrote as follows: I should like to express my gratitude for the encouragement and stimulation afforded me by the discussion and written comments. These have enabled me to get an improved perspective of the problems raised in the paper.

The paper, perhaps, obscured the logical sequence of the problems. Some of these are in the foreground, and are therefore sharply defined, some are in the middle-distance, whilst some are background problems on the horizon of the picture. Right in front of us is the question whether the whole matter should not be officially investigated. No one contested the desirability of this forward move; but Dr. Bransby has contributed a valuable suggestion in advising the widening of the interests of those sponsoring the enquiry so that a committee representing all the users of the Census data, together with those responsible for various social surveys, should undertake the control of the complete investigation. It may well be that the Lord President of the Council will supply the executive action necessary to bring into being such a committee. This would save the Royal Commission on Population the necessity of stretching their terms of reference to include such an enquiry.

The question then arises as to who should carry out the technical investigation. My solution is that the appropriate body is the National Physical Laboratory, and in particular the Mathematics division of this branch of the Department of Scientific and Industrial Research. Mr. Womersley, the superintendant of this division, showed his personal approval of the direction in which my paper pointed, in proposing the vote of thanks, but was cautious as to the policy of the Laboratory. As his supreme commander is the Lord President of the Council it is likely that the difficulties regarding policy may be cleared up.

Behind this series of foreground problems there arise the middle-distance problems, which are less sharp in definition and for which there is more time for consideration. These will in fact become the foreground of the new picture that will face any investigators. The first of these problems is to decide the order of importance of improvements in the machinery of census analysis. Mr. Sen has told us how the Nielsen Company of America spent two million dollars on perfecting the audimeter for measuring the radio listening habits of samples of listeners. How fortunate is private enterprise when headed by such an enlightened man as "Art" Nielsen! By turning the sampling technique used in power-house efficiency tests to measure the flow, not of gases up a flue, but of goods over a counter, this engineer-statistician has so revolutionised market research that his company can well afford two million dollars on this further application of sampling technique. It would seem reasonable to expect that our present Government, in the advance we hope they may make towards the more subtle techniques of control without "controls", will be

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forced to pay serious attention to the effective provision of up-to-date machinery for national self-survey. One quarter of the amount the Nielsen Company spent on their " audimeter " would provide a machine analysis station far ahead of anything we have had before.

It would be unfair to Mr. Derrick, as one of the officers in the Registrar-General's department most familiar with the orderly observance of well-tried, traditional routines so necessary to the analysis of such magnitude, to expect him to adopt at first any other attitude than one of caution. Equally it would be difficult for Mr. Thomas, who has spent years of his life designing mechanical as opposed to electrical devices for machine analysis, to develop rapidly any enthusiasm for electrical devices. It is disappointing however to find these two recognised census experts making statements that may mislead those less well-informed. Mr. Derrick, for instance, says, " the author had come to the conclusion that his proposed developments would not function unless the census was altered to meet them. That, it seemed, was putting the cart before the horse. . . ." Whereas in fact the paper runs thus: " Although there is a clear advantage to be derived from such equipment even if it were to be stored from one decennial census to the next, its full benefit cannot be felt unless the character of the census is changed." Evidently I must have confused Mr. Derrick by the manner in which I associated the continuous census with the electronic script.

Actually, continuous census is a background problem, but the adoption of electronic script is a middle-distance problem independent of continuous census. It depends on certain factors that can be fairly sharply defined straight away and others that must await the results of experiments. The cost of the machine, for instance, is most difficult to guess, and must be left till it arises as a practical issue. The uses of a device for rendering punching a machine process can however be considered straight away. With the original census schedules coded in electronic script and automatically fed into the machine at a speed resulting in cards being produced at 6000 an hour, twenty girls punching at 300 an hour and, say, five girls verifying (as only a proportion of the census is verified) could be displaced by each machine, and the gross saving would therefore be of the order of £50 a week. Against this would have to be set-off the cost of the machine, which is unknown, and the extra staff required for coding.

From Mr. Derrick's remarks and those of Mr. Thomas, the reader unfamiliar with census analysis would gain the impression that census schedules were just filled in by the private individual or enumerator and thereafter punched. Mr. Derrick says, " Before it (the schedule) could be fed to a machine it would have to be subjected to a process converting the (handwritten) script into a form in which the machine could handle it, and it was the conversion process as a whole which had to be judged in deciding which method was better. Hand-punching was very simple and within the capacity of anybody. . . ." Mr. Derrick thus completely skipped one of the most tedious processes of ordinary census routine—namely, the necessary process of hand coding all the schedules. Mr. Thomas's reaction to the proposed electronic script was " why write these things free-hand when a machine could be used? " Surely the various census codes have to be written freehand as it is. Even if a punch can be worked for twenty years without needing oiling or the attention of the specialist, punch mechanics employed by the companies can still only punch figures, and before Occupation or Industry, etc. can be recorded, someone has to write code numbers on the schedule. My reasonable suggestion is that this coding should be carried out in electronic script instead of in arabic script. The schedule is numbered by means of a numbering stamp. This stamp can just as well be in electronic script. The schedule is coded for district by one coding for a whole batch of documents. My automatic punch would punch the first card with the district code taken from a single coding in electronic script; the remaining cards in the batch would be automatically gang-punched from this first card. Sex and marital status would be picked up by using Professor Hogben's idea of indicating by a tick placed against one or other set of pre-printed electronic codes. Age must be rendered in electronic script, whereas in ordinary practice the punch-operator can punch the numbers written on the schedule straight away. Had the two experts on census analysis asked " What about age coding? " I should have felt that they had really followed what I was proposing. Again they might have said that an electronic script representation of a digit would require more hand movement than would the arabic script representation, and the time required for coding would, therefore, be longer. I should agree to that. The coding operation will cost more, something of the order of 10 per cent., as far as I can estimate at present. Against this it is possible to insert an age-group code when coding for age, which will render the gang-punching of this code, usually included in the machining operations, unnecessary.

The fact that the photocell was old when Dr. Hollerith made his first machine (which Mr. Thomas seems to think has some bearing on the subject under discussion) reminds me that Wyatt's invention of " mechanical fingers " for rendering spinning an automatic process was forgotten for thirty years before Arkwright turned the principle into one of the key inventions of our industrial revolution. Mark-sensing is also, according to Mr. Thomas, an old device, but that does not mean that it may not be very effective. It was probably awaiting developments in the means of amplification of the current passing along the pencil line, just as Whittle's turbine had to wait for certain alloys before it could be the practical success that it now is.

Dr. Heron raises a middle-distance issue, the exact design of the script. I am grateful to him

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for his alternative script, which certainly avoids any danger of double lines. It must be subject to careful test when the machine comes into being.

Mr. Emery goes right beyond my horizon with his hearing machine, but far be it from me to say that it is impossible.

Dr. Wishart raises the issue of the " on " and " off " method. This would be a possible solution if we had a permanent system of registration, as is the case in Holland and other countries. As long as any such system has as its main object keeping an orderly card index of the whole population it is unlikely to be popular in this country. If, on the other hand it is the by-product of some scheme in which the majority stands to gain, such as national insurance, it may well be that we shall find quite important totals thrown up by just such a method of recording changes.

Finally, in the background of our picture is sketched the possibility of introducing the continuous census. Here Dr. Hartley has helped me considerably by arguing that the matter is worthy of closer study. He has suggested that the Study Section of the Society should consider it. My feeling is that the Study Section might well consider the manner in which the research into the possibility of continuous census should be undertaken. The research itself, I would suggest, is likely to require some years of work. It would appear for instance that the 1931 Census results could be used to demonstrate the errors arising from applying trends observed between the 1921 and 1931 figures of localities selected to form a stratified sample, to the remaining localities held to be similar in type. The Study Section might consider the validity of such a procedure and offer alternative lines of research for budding doctors of philosophy. At least it might offer new lines of research not open to the criticism, sometimes made of Ph.D. theses, that they too often consist of the ceremonial " removal of bones from one graveyard to another."

If a careful reconsideration of the actual uses of the census figures were to be made it might well be found that most practical present-day requirements could best be satisfied by approximations to be derived from continuous census results, which are current, rather than from exactitudes that are already out of date. To make this point stronger Mr. Derrick himself said, " at the present time removals from town to town or from town to country are taking place at the rate of about 100,000 a week. . . ." This surely means that upwards of 5,000,000 such movements may be expected to take place in a year. It is just this very condition that gives me a feeling that a static decennial census for 1951, published at an interval which allows millions of such movements to have taken place, is in danger of being a museum piece in, say, 1954. I cannot believe that if we have an Economic General Staff this body will be content to navigate the ship of State through the present shoal waters using the old-fashioned method of " heaving the lead ", particularly when it takes three years thereafter to hand it up and to work out the " depth." They are bound to hear at least a rumour of electric sounding devices giving the continuous profile of the bottom of the sea.

In conclusion I should like to thank both Mr. Womersley and Dr. Comrie for their help in pruning the verbiage of my original paper to the point at which it could decently be included in the dignified records of this Society.

As a result of the ballot taken during the meeting, the candidates named below were elected Fellows of the Society:—

Major Gordon Vernon Bayley.
Ronald William Bevan.
Leonard Richard Billingham.
John Howard Black.
Bertram C. Brookes.
Archibald Reid Gunn Burt, B.Com.
Diego de Castro.
Kenneth David Coley.
Wilfred John Corlett.
Henry Dowbekin.
Major Noël de Guingand.
Frederick Hertz.
Major John Michael Hammersley.
Leslie Frederick Houchin.
Margaret Jones.
David Joel Liston.
Ronald Lionel Michaelson, F.I.A.
A. F. J. Mullins.
George Reuben Bertram Owen.
Major George Romney Rawes.
Amode Ranjan Sen.
Clive Colquhoun Spicer.

Corporate representative.

Arthur Graeme Cranch, *representing* Mather and Crowther, Ltd.

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MISCELLANEA

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The Number of Nations on Each Side of a War

By Lewis F. Richardson

Summary

The historical facts to which this investigation relates are contained in two lists of wars. Each list is comprehensive, world-wide, and covers more than a century. The first step was to notice how many nations fought on the two sides of each war, and to classify the wars accordingly as 1 versus 1, or 2 versus 1, or in general as r versus s. The number of wars of each of these types was then counted, and denoted by $n(r, s)$. It is a bivariate distribution of a peculiar shape, unlike the ordinary correlation table. Attention was thus diverted from the peculiarities of named nations at particular times to some habits of belligerents in general. The observed frequency, when allotted half to the point (r, s) and half to the point (s, r), is found to vary as $(rs)^{-2\cdot5}$. That is to say, wars became rapidly rarer as the number, rs, of pairs of opposed belligerents increased. Various explanations, numbered I, II, . . . , XIII, have been attempted. One is that international relations might be likened to the molecular chaos in a gas, in so far as ternary encounters are rarer than binary encounters, for reasons which can be expressed by probabilities, after the manner of Bernoulli. This idea is developed in Theory VIII, but it fails to fit the historical $n(r, s)$ when 60 nations are regarded as all involved. Fewer nations would somehow fit better. Accordingly, in Theory X disputes are regarded as localized; and it is found that a tolerable agreement with the historical $n(r, s)$ is obtained by supposing that each dispute interested only eight nations, and that the probability that any two of the eight came to war with each other about it was 0·35. Such a theory, however, fails to explain the rare wars in which $r + s > 8$. To remedy this defect, the restriction of the number of interested nations is expressed in Theory XI by classifying them into local powers and world-powers. Any two local neighbours are assumed to be at war with one another during a total fraction z of the time. It is found that z increased with the number, rs, of pairs of opposed belligerents in the following manner:—

$rs =$	1	2	3	4
$z =$	0·008	0·020	0·046	0·119?

This increase of z can be explained by the infectiousness of fighting.

Any guidance to the practical statesman which this enquiry can offer is thus the indication of a chaos, restricted by geography, and further modified by the infectiousness of fighting. There are also some suggestions in Theory XIII as to a possible future effect of aviation.

PART I. HISTORICAL FACTS AND SMOOTHING

Introduction

The very practical questions whether alliances make for world-peace, and whether the great victor Powers of the Second World War are peculiarly well qualified by their past history for maintaining world-peace, can be answered only by comparison with the average behaviour of States in general during centuries. It is this general background of human nature which is here examined, not the peculiarities of named States at particular times. The discussion involves collections of facts lettered *A*, *B*, . . . , *H*, *J*, and theories numbered I, II, . . . , XII, XIII.

Wars can be classified by the number of groups of people on the two sides: for example, 1 nation versus 3 nations, 2 states versus 2 other states, 1 government versus 1 set of insurgents; or, in general, r belligerent groups versus s belligerent groups. The decision as to what groups should

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be counted in r or s was made in accordance with the ordinary common-sense practice of historians—that is to say, token declarations of war were ignored, so also was any support that was only diplomatic or financial or by provision of arms; but, on the other hand, groups who did a notable amount of fighting were counted, even if they joined late or left early.

The type, r versus s, of each war having been thus recognized, the next step was to count how many wars there were of each of the types 1 versus 1, 2 versus 1, 3 versus 2, and so on.

The author has abstracted, from numerous works on history, a card-index and list of fatal quarrels that ended since A.D. 1820 in all parts of the world. This collection is subdivided according to the number of war-dead on all sides jointly. A fatal quarrel that caused 10^{μ} deaths is said to be "of magnitude μ." For most purposes fatal quarrels of magnitude less than 3·5 have been omitted, because, in popular language, they would seldom be called wars. This collection is fundamental for the present purpose, and is so far not fully published; but extracts from it appear in the present paper as Collections A, D, F, D, J.

There have been a very few wars which cannot be regarded as conflicts between two sides. For example, there was a three-sided fight in Galicia in 1846, but it was only of magnitude 3. A larger exception to the proverb, "Your enemy's enemy is your friend," was the Mexican revolution of 1910–20, which was of magnitude 5. F. A. Kirkpatrick (1938, p. 349) describes it as chaotic. It has been omitted from the present study of two-sided wars.

The number of wars of each type

Obviously the counted numbers should be arranged in a table of rows and columns in the (r, s) plane. That process has been applied to Richardson's list with the following result.

Collection A. Wars of all magnitudes greater than 3·5 *which ended from* 1820 *to* 1939 *A.D. inclusive*

s	1	2	3	4	5	6
6						0
5					0	0
4				0	0	0
3			0	1	0	0
2		3	2	1	1	0
1	42	24	5	5	2	1
	1	2	3	4	5	6

(r)

There were also beyond bounds of this table:
2 wars of 7 versus 1
1 war of 9 versus 1
1 war of 15 versus 5
thus making a total of 91 wars.

For many wars there is no doubt as to how the individual persons should be grouped; so that the assignment of r and s is quite simple and obvious. For example, in 1904–05 there was a war between Russia and Japan, plainly of the type 1 versus 1. On the contrary, for some other wars there is room for considerable doubt: for example, how many German groups took part in the war of 1870–71 against France? It is therefore particularly desirable to have independent opinions. Prof. Quincy Wright has provided one by publishing (1942, Appendix XX) the number of participants on each side of each war in his impressively long list. By comparison of particular wars it is found that Wright's word "participant" usually means a belligerent State selected from a restricted set of sovereign States. But he lists 34 wars as each involving only a single participant. He explains his principle thus (p. 637):

> ". . . there was the difficulty of deciding what entities were to be regarded as participants, and in this respect actual independence before or after the war rather than legal status under international law was the criterion used."

For example, Wright assigns only one participant to the American Civil War of 1861–66, because the Southern Government did not exist either before or after the war. That may be satisfactory from a special legal point of view—Prof. Wright is a professor of international law—but it is of no use for common-sense purposes, nor for those of psychology. Some amendment had therefore to be made. It would not be sufficient merely to alter Wright's wars from type 0 versus 1 to type 1 versus 1; for his principle of "actual independence before or after the war" may have had other effects. He explains his practice further thus:

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" Unrecognized princes like the Aztec and Inca rulers, semi-dependencies like the Barbary states, feudal principalities like the German and Italian states nominally subject to the Empire, and successful insurgents like the United States in 1775 have been listed as independent participants. Unsuccessful revolutionists, rebels, or insurgents which lacked even *de facto* status, except during the war itself, have not been so listed, and many of the small feudal principalities of the Holy Roman Empire have been ignored on the ground that, while enjoying some legal status though short of full independence under international law, their actual political importance was very slight."

It thus becomes necessary to correct the excessive legalism whereby Wright ignored unsuccessful rebels; and yet it is desirable to maintain a due respect for Wright's opinion on other matters. A fairly satisfactory device for attaining both these ends is to omit from Wright's list all those wars which he marked *C* for civil.* They are distributed thus when expressed in the notation *r* versus *s*.

Collection B. Civil wars in Wright's list

Type	0 versus 1	0 versus 2	1 versus 1	1 versus 2	1 versus 3	1 versus 4	1 versus 6	1 versus 18	Total
Number of wars	34	1	27	10	3	1	1	1	78

Wright does not actually use the absurd symbols 0 versus 1 and 0 versus 2, but he implies them. After the rejection of these 78 civil wars, there remain in Wright's list 200 other wars which are distributed as follows. They include four large wars which Wright states both as wholes and in parts. Here the parts are omitted and the wholes are included.

Collection C. Wars not marked civil in Wright's list from A.D. 1480 *to* 1941 *inclusive*

s \ *r*	1	2	3	4	5	6
6						0
5					0	0
4				0	0	0
3			1	1	1	0
2		4	6	0	1	1
1	117	28	12	12	3	2

There were also, beyond the bounds of this table:
1 war of 7 versus 1
1 war of 8 versus 1
1 war of 11 versus 1
1 war of 16 versus 1
1 war of 20 versus 1
1 war of 7 versus 3
1 war of 8 versus 5
1 war of 20 versus 5
1 war of 33 versus 5
1 war of 35 versus 7
1 war of 9 versus 8
thus making a total of 200 wars.

The quadrantal arrangement

In the foregoing tables the observations have been spread over the octant for which $r \geqslant s$, leaving a blank space where $r < s$. When $r = s$, there is only one place in the table for the number of wars, and it is accordingly shown there; but when $r \neq s$ there are two places, such for example as (3, 1) and (1, 3); and there is no clear reason for choosing one or the other. If we could, always and infallibly, name the first aggressor, that decision would enable us to distinguish 1 versus 2 from 2 versus 1; but in fact the question "Who began it?" usually leads to controversy. Accordingly, the number of wars might be concentrated in either place, or distributed between the two places in any manner. In the following two tables the number of wars is distributed equally, half of them in each place, when $r \neq s$. The only justification that can be given for this choice at the present quite empirical stage of enquiry is that equal distribution makes the observed facts look neater, smoother, more consistent, and more intelligible. Another justification will appear when we come to theories. The two types of table will be called octantal and quadrantal; and these adjectives will be applied also to the formulæ descriptive of the two tables. It is, of course, essential to avoid confusions between the octantal and quadrantal arrangements.

To facilitate comparisons between the two sets of statistics the frequencies in each have been divided by their respective totals.

* For the suggestion of this amendment I am indebted to another lawyer, Sheriff A. M. Hamilton, K.C.

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Collection D, obtained from Collection A

Outliers beyond the bounds of the table, 0·0440. Total 1·0004.

s \ r	1	2	3	4	5	6
6	0·0055	0	0	0	0	0
5	0·0110	0·0055	0	0	0	0
4	0·0275	0·0055	0·0055	0	0	0
3	0·0275	0·0110	0	0·0055	0	0
2	0·1319	0·0330	0·0110	0·0055	0·0055	0
1	0·4615	0·1319	0·0275	0·0275	0·0110	0·0055

Collection E, obtained from Collection C

Outliers beyond the bounds of the table, 0·0550. Total 1·0000.

s \ r	1	2	3	4	5	6
6	0·0050	0·0025	0	0	0	0
5	0·0075	0·0025	0·0025	0	0	0
4	0·0300	0	0·0025	0	0	0
3	0·0300	0·0150	0·0050	0·0025	0·0025	0
2	0·0700	0·0200	0·0150	0	0·0025	0·0025
1	0·5850	0·0700	0·0300	0·0300	0·0075	0·0050

Separation of essential from accidental features

Some features of the historical facts are presumably accidental. We shall avoid useless meticulosities by referring back to the counted numbers in the previous tables, and by remembering that any counted number n of wars is likely to have a standard uncertainty $\sqrt{n}$, because the distribution of wars in time is Poissonian (Richardson, 1945).

There are strong resemblances between the two sets of statistics. In both the type 1 versus 1 is the most frequent. In both the cell-frequency in general decreases with increase of r and s. In both the cell-frequency decreases more rapidly along the diagonal where $r = s$ than it does along the lines $r = 1$ or $s = 1$. In both the quadrantal type 3 versus 2 is rarer than the quadrantal type 4 versus 1, although each involves 5 belligerents. The chief disagreements between the two sets of statistics are now seen to be that in Wright's set the wars of type 1 versus 1 are commoner, and those of types 2 versus 1 and 1 versus 2 are rarer, than in the other set.

It is desirable to reject accidental features and to retain only the meaningful essence by fitting empirical formulæ to the distributions, or to their mean.

Let $n(r, s)$ be the counted number of wars in the r, s cell of the quadrant, the table having been made symmetrical in the manner described above . . . . . (1)

Let $\phi(r, s)$ be the smoothed function which we seek as a substitute for $n(r, s)$ . . . (2)

Let N be the total number of wars, so that

$$N = \sum_{r=1}^{\infty} \sum_{s=1}^{\infty} n(r, s) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The infinite upper termini here merely indicate that we must include all the field as far as it is occupied; for beyond the most complicated wars $n(r, s)$ is everywhere zero.

The first step towards the choice of a formula is to notice the shape of the isopleths of n—that is to say, the loci on which $n(r, s)$ is constant. They are hard to discern on the octantal tables, but fairly obvious on the quadrantal. So let us confine attention to the latter. On the diagonal lines that slope upwards to the left, so that along them $r + s$ is constant, n is occasionally constant, but usually is less in the middle than at the extremes. Along the rectangular hyperbolæ on which rs is constant, n is on the average moderately constant. The conclusion so far is that a function of rs would fit better than a function of $r + s$, but that a function of some variable intermediate between rs and $r + s$ would be best. That is to say: *the number*, rs, *of pairs of opposed belligerent groups is somewhat more relevant than the total number*, r + s, *of belligerent groups.* It is also desirable

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to notice, as forms that must be definitely rejected because their isopleths are hollow on the wrong side, $\phi = f(r^2 + s^2)$ and $\phi = f(r!s!)$. The former, and its elliptic generalizations, are associated with correlation tables. The latter often occurs in probability-theories, and tends to prevent them from fitting these facts.

The largest counted numbers have the least fractional uncertainty. The largest is N, the next is $n(1, 1)$. With these ϕ should especially agree. When we are selecting a formula, it is convenient to take $\phi(1, 1)$ as the standard, because $\phi(1, 1)$ is obvious; whereas the total

$$\sum_{r=1}^{r+s=m} \sum_{s=1} \phi(r, s) = \Phi, \text{ say} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

may be troublesome to calculate or may be ill-defined. For the upper termini of the sums are connected by $r + s = m$, where m is the number of possible belligerents in the world. The difficulties of calculation have in many cases been overcome, but there remains the uncertainty arising from our ignorance of m. Certainly the number of "sovereign and independent" States was of the order of 60 in the year 1930; but some belligerents were insurgents; so that m was an ill-defined number of the order of 100. We shall be fortunate if the form of $\phi(r, s)$ is such that no appreciable increase in the sum (4) occurs when m is increased from 60 to ∞. Here follows a selected list of expressions, regarded as candidates for the position of empirical formula, together with their chief qualifications for that appointment.

Select list of candidate-formulæ

Region of the r, s quadrant	$rs = 1$	The whole field	$rs = 2$	$rs =$ 3 to 6	$rs =$ 7 to ∞	$r + s =$ 61 to ∞
Observed Ratios { Collection A	1·000	$2{\cdot}1_{67}$	$0{\cdot}5_{71}$	$0{\cdot}42_{9}$	$0{\cdot}16_{7}$	0·000
Ratios { Collection C	1·000	$1{\cdot}70_{9}$	$0{\cdot}23_{9}$	$0{\cdot}33_{3}$	$0{\cdot}13_{7}$	0·000
$(rs)^{-2\cdot5}$	1·000	1·800	0·354	0·303	0·143	<0·009
$\frac{1}{2}(r + s)(rs)^{-3}$	1·000	1·977	0·375	0·361	0·241	0·034
$2r^{-2}s^{-2}(r + s)^{-1}$	1·000	1·688	0·333	0·257	0·097	<0·004
4^{1-rs}	1·000	1·684	0·500	0·184	0·001	0·000
$(rs)^{-2\cdot4}$	1·000	$1{\cdot}91_{3}$	0·379	0·347	$0{\cdot}18_{8}$	—

The calculation of the double sums to infinity, for columns 3 *and* 7 *of the above table.* The easiest of these sums is

$$\sum_{r=1}^{\infty} \sum_{s=1}^{\infty} (rs)^{-\kappa} = \sum_{r=1}^{\infty} r^{-\kappa} \sum_{s=1}^{\infty} s^{-\kappa} = [\zeta(\kappa)]^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where $\zeta(\ )$ is the Zeta Function of Riemann, given in the tables of Jahnke and Ende.

It was also necessary to approximate to sums by way of integrals. The following modification of the Euler–Maclaurin approximation was found to be especially appropriate for estimating remainders after the larger terms had been summed by arithmetic. Let the r, s plane be divided into square cells by lines at $r = \frac{1}{2} + j$, $s = \frac{1}{2} + j$, where j is 1, 2, 3, 4, . . . Let $\phi(r, s)$ be regarded as the ordinate at (r, s) of a surface over which r and s vary continuously. In any cell the volume under the surface ϕ numerically exceeds the ordinate at the centre of the cell, because these surfaces are all concave. Therefore

$$\sum_{r=r_1}^{\infty} \sum_{s=s_1}^{\infty} \phi(r, s) < \int_{r_1-\frac{1}{2}}^{\infty} \int_{s_1-\frac{1}{2}}^{\infty} \phi(r, s) dr ds \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Most books neglect to mention this shift of a half in the termini; but it brings the above inequality much nearer to equality. In any cell the volume numerically exceeds the ordinate at the centre by $\frac{1}{24}(\partial^2\phi/\partial r^2 + \partial^2\phi/\partial s^2)$ + terms in fourth derivatives; and this correction transforms by Green's theorem, but it was not required. One of the integrals used for this purpose was

$$\int_{\alpha}^{\infty} \int_{\beta}^{\infty} \frac{2dr ds}{r^2 s^2 (r + s)} = \frac{2}{3}\left\{\frac{\alpha + \beta}{\alpha^2\beta^2} - \frac{1}{\beta^3} \log\left(\frac{\alpha + \beta}{\alpha}\right) - \frac{1}{\alpha^3} \log\left(\frac{\alpha + \beta}{\beta}\right)\right\} \quad . \quad . \quad (7)$$

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The fringe beyond the sloping line $r + s = 60$ is, for some ϕ, troublesome to calculate. However, all that we need to know about it is whether it is negligible. So I have been content with rough bounds. Of the upper bounds the easiest to obtain is the sum beyond the square whose four sides are $r = 1$, $s = 1$, $r = 30$, $s = 30$. A closer upper bound is the sum beyond the hyperbola $rs = 59$. For those ϕ which are functions of the single variable rs, an approximation to the sum beyond $rs = 59$ is

$$\int_{rs=59\cdot5}^{\infty} \phi(r, s)\bar{\tau}(rs)d(rs) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where $\tau(rs)$ is the number of divisors of rs, including 1 and rs; and $\bar{\tau}$ is a smoothed value of the wobbly function * τ. I took

$$\bar{\tau}(rs) = 1{\cdot}1544 + \log_e(rs) - \frac{1}{24r^2s^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Selection, and final test of an empirical formula

It would, of course, be possible to fit two formulæ, one to Collection A, another to Collection C. But a more useful question is whether a single formula can be found to specify a hypothetical large collection, such that both Collections A and C can be regarded as random samples from it. The preceding table suggested that $(rs)^{-2\cdot5}$ might be a suitable proportionality. It was therefore selected for a χ^2 test. For this purpose the double sum to infinity must first be made to agree with N, the observed total number of wars, by altering the formula to

$$\phi(r, s) = \tfrac{5}{9}N(rs)^{-2\cdot5} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Each set was then given its appropriate N—namely, 91 for Collection A, 200 for Collection C. The χ^2 test was applied to the four cells for which rs is severally 1, 2, 3 to 6, 7 to ∞. The index, $-2\cdot5$, had been adjusted, but had certainly not been chosen to suit each set separately. It seemed proper therefore to regard N as the only adjustable constant, and to take the degrees of freedom as three. The results are:

Collection	χ^2	Probability of a greater χ^2
A	4·0	0·26
C	4·4	0·22

Thus the formula (10) is satisfactory, in so far as it credibly describes both sets of observations, and is smoother, and easier to remember, than they are. Much closer agreement could be obtained if the index, instead of being fixed at 2·5, were suited to the sets separately; but such an elaboration might not be an improvement. For, where samples can differ, a perfect fit is a misleading ideal.

The total duration of wars of these types

For testing Theory VIII which follows in Part II it was necessary to make a special collection of historical facts in the form of total durations. These facts, being of wider interest than the theory which led to their collection, are here stated separately. They are derived from two lists of wars, Wright's and my own, based on different definitions and by rather different processes. The impressive fact is that these two contrasted systems lead on the average to similar results.

In Richardson's list the intention has been to record the duration of hostilities. Wright, in some cases, has placed an extraordinary emphasis on the final legal conclusion of the quarrel. Thus he makes the Napoleonic wars end with the Final Act of the Congress of Vienna, a few days before the battle of Waterloo. Also he makes the First Carlist war continue until an amnesty dated September 2nd, 1847, seven years after the end of hostilities as they are stated by Hume or by the Cambridge Modern History.

No great accuracy can be expected of statements of the duration of hostilities. Although for most fatal quarrels the duration is fairly obvious; yet for others judgment is made difficult by such features as the following;

(i) Provocative incidents prior to the main outbreak. For example, a war between the Burmese and British is here taken to have begun in September 1823, although the British declaration of war did not follow until March 1824. (E 4, 432.)

* For advice about τ I am indebted to Prof. G. H. Hardy, F.R.S.

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(ii) The outbreak of hostilities without any declaration of war, as in Armenia 1892–94.

(iii) Armistices. For example, the First World War was taken to have ended in November 1918, but, because the blockade of Germany continued with fatal consequences, June 1919 might be a more reasonable date.

(iv) Pauses, not agreed between the belligerents as armistices. The usual pause in winter has been included as part of the hostilities. On the contrary the 12-year pause from 1864 to 1876 in the war between the Chinese and the Mohammedans of Sinkiang has been omitted from the duration, because the Chinese army was then far away.

(v) Guerilla warfare continuing after the main defeat, as in South Africa from 1900 to 1902, or in Madagascar 1897.

(vi) The cessation of hostilities without any declaration of peace. This occurred between the Chinese and the Mohammedans in Sinkiang in 1864.

As examples of wars that had extremely opposite characteristics one may pick out Bolivia versus Paraguay 1930–36 as having a duration of hostilities so indefinite that it might be stated as 1·4 or as 5 years; whereas in the American Civil War of 1861–65 the hostilities lasted 4·0 years, definite to a month. Formulæ will here be fitted to the total duration of many wars; so that the uncertainty of the particulars will to a considerable extent be averaged out.

These total durations have one advantage over the numbers of wars—namely, that they enable us to give a clearer description of those wars in which some belligerents joined after the beginning, or quitted before the end. In such a case the war was divided into non-overlapping intervals which were allocated to different types. On the contrary, it can be argued that the beginnings of wars are more relevant than their durations.

As a preliminary to an analysis by the numbers of belligerent groups, *r* versus *s*, some overall summaries are of interest. According to Wright (1942, p. 651), the average duration of the wars that occurred from A.D. 1480 to A.D. 1941 was 4·3 years each. Outstanding deviations from the mean are recorded in the names of the Thirty Years and Seven Weeks Wars. The fraction of a political State's time which was spent at war has been tabulated by Woods and Baltzly (Wright, 1942, p. 653). Over the interval A.D. 1450 to A.D. 1900 this fraction averaged 0·115 for Denmark, 0·175 for Sweden, 0·235 for France, 0·250 for Great Britain, 0·275 for Austria, 0·300 for Russia, 0·305 for Turkey, and 0·330 for Spain; the mean for those 8 States being 0·248. The facts for Prussia are there given only over the shorter interval A.D. 1600 to A.D. 1900; and for it the Prussian fraction was 0·17. This should be compared with Lord Vansittart's thesis that Germany has always been the chief aggressor.

After these preliminaries let us turn to the analysis by types *r* versus *s*, from the two aforesaid lists. Richardson's list makes possible a cross-classification by magnitudes, with the following result.

Collection F. Fatal quarrels that ended from A.D. 1820 *to* A.D. 1939 *inclusive*

Magnitude	Octantal types							
	1 versus 1	2 versus 1	3 versus 1	2 versus 2	4 versus 1	3 versus 2	More complicated	All types
	Total durations in years							
7·5 to 6·5	0	0	0	0	0	0	4·3	4·3
6·5 to 5·5	28·8	0	5	0	0	0	·0	33·8
5·5 to 4·5	71·0	10·2	0·8	0	2·4	0·6	1·9	86·9
4·5 to 3·5	69·2	40·6	6·6	14·5	1·3	6·6	28·4	167·2
7·5 to 3·5	169·0	50·8	12·4	14·5	3·7	7·2	34·6	292·2

The irregularity of the distribution in the central part of the above table shows that the classification has been rather too detailed. So the further discussion has been confined to the totals shown in the bottom row. An objection to the use of these totals is that they go as far as a lower bound at magnitude 3·5, which is an arbitrary choice. It is quite conceivable that if (by an enormous labour that no one has yet performed) the statistics were extended to magnitude 2·5, it would be

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found that the total distribution among types r versus s was notably different. The best that can be done is to *remember the arbitrariness of the lower bound.* It corresponds, very roughly, to choosing fatal quarrels that are large enough to be called wars in the popular sense of the word.

Prof. Wright's collection does not permit an analysis by magnitudes. The separate wars into which he subdivided the Thirty Years War, the French Revolution, the Napoleonic Wars and World War I are here fundamentally rearranged as non-overlapping intervals of type r versus s. The wars which he marks civil are again omitted. A few obvious slips are amended. To make the two sets of statistics comparable, each has been divided in the following table by the intervals of years to which it relates.

	Octantal types						
	1 versus 1	2 versus 1	3 versus 1	2 versus 2	4 versus 1	3 versus 2	$r+s \geqslant 6$
	Total duration divided by interval						
Collection G from Collection F. Magnitudes 7·5 to 3·5 ...	$1{\cdot}40_8$	$0{\cdot}42_3$	$0{\cdot}10_3$	$0{\cdot}12_1$	$0{\cdot}03_1$	$0{\cdot}06_0$	$0{\cdot}28_3$
Collection H, from Wright, omitting civil wars	1·107	0·229	0·133	0·034	0·116	0·049	0·244

It is seen that there is a moderate degree of concord between the two Collections G and H. The larger duration of type 2 versus 2 in Collection G arises mainly from the struggle against Rosas of Argentina 1835–52. Wright lists this war as of type 2 versus 1 from 1839 to1852. The statistics of duration agree with those of numbers of wars in Collections A and C at least in so far as in both of them the type 1 versus 1 predominates, and the type 2 versus 1 comes next.

A gloomy feeling may be engendered by the fact that the total duration of wars of the type 1 versus 1 exceeded the historical interval in the ratio $1{\cdot}_3$. It is therefore desirable to point out * that the total duration of such wars might conceivably have been much larger than it actually was. For we can imagine that the 60 States in the world were arranged in 30 wars of type 1 versus 1, wars which went on all the time. If so, the ratio would be, not $1{\cdot}_3$, but 30.

PART II. THE SIMPLER ATTEMPTED EXPLANATIONS—NAMELY, THEORIES I TO X

General preface to particular theories

An accurate and detailed record of historical events is the necessary preliminary to any forecast of the future. Yet such a record is not, in its raw state, the best guide, for two reasons:—

(i) Every set of historical events is to be regarded as a sample, and subject to the variation of samples. This doctrine has already been admitted in so far as the observed distribution $n(r, s)$ has been replaced by a smooth empirical formula $\phi(r, s)$.

(ii) Circumstances change. Unless the events can be analysed back to recognizable causes, we cannot allow for changed circumstances, such for example as the death of leading persons, or the decrease in a population, or the increase of aviation, or the innovation of atomic energy.

Let us therefore now attend to explanations which are at least intelligible. They have various commendations: some have been popular, some are plausible at first glance, and some would simplify the mathematics. But one of the tests to be applied to them is whether they give a *quantitative* agreement with the observed $n(r, s)$ distribution, and this test is found to be remarkably critical.

The contrast between an empirical formula and a theory resembles the contrast in a factory between an old reliable foreman and an imaginative graduate fresh from the university. The graduate might at first make frightful blunders, but in the course of a decade he is likely to improve the technique more than the foreman ever could.

* I am indebted to my wife for doing so.

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A general characteristic of the facts is that wars involving more belligerent groups were rarer than wars involving fewer belligerent groups. We may seek a hint leading towards explanation by noticing other phenomena which have the same characteristic. Amidst the unorganized activities in the playground of a Scottish elementary school, fights of one versus one were observed to be more frequent than other fights.*

In a gas, the collision of two molecules occurs much more frequently than the collision of three (Hinshelwood, 1940, p. 142). What have these diverse fields—the world, the playground, and the gas—in common? The answer seems to be: lack of organization. Those who supported the League of Nations often complained about the international chaos. The fights in the playground began in the absence of the teachers. In his standard treatise on *The Dynamical Theory of Gases* Jeans (1916) frequently mentioned the molecular chaos. One of our interpretive ideas should therefore be chaos, with its characteristic property that complicated events are rarer than simpler events. In statistical theory chaos is analysed somewhat in the following manner: the complicated events are regarded as built up from simpler elements, some of which are statistically independent, so that the probability of their happening together is the product of their probabilities of happening separately. The more such elements, the less resultant probability. This is certainly a leading idea; but it leaves us to seek historical elements that are statistically independent.

As a model of theoretical procedure let us bear in mind Bernoulli's classic expression for the binomial distribution, namely

$$\left\{\frac{k!}{x!(k-x)!}\right\}[p^x(1-p)^{k-x}]$$

for although it is not appropriate in detail to wars, it gives nevertheless the following useful hints: we should expect a factor, analogous to p^x, expressing the probability that x elements are in the war, and another factor, analogous to $(1-p)^{k-x}$, expressing the probability that many other elements keep out of the war; we may also expect a factor, analogous to that in $\{\ \}$, expressing the number of mutually exclusive wars for each of which the probability is the factor in $[\ ]$.

As the application of these general ideas of Bernoulli to the encounters between nations is impeded by the complications and the passions of politics, anyone who is conversant with physics might well prefer to begin by considering their application to the encounters between molecules in a gas. I worked that as a preparatory exercise (Richardson, 1946). There x is the number 2, 3, 4, 5, . . . , of molecules in the bunch, and $k-x$ is the enormous number of molecules that might be in, but are not. Yet at low densities p is so near to zero that $(1-p)^{k-x}$ is almost unity, and may be omitted, as in the classical theory of gases. An almost similar situation for nations occurs in the following Theory XI. On the contrary, for dense gases $(1-p)^{k-x}$ is an important factor. The corresponding factor for nations is important in Theory X.

The rejection of certain large probabilities

It often happens that the total duration of all the wars of a particular specification can be explained by two different values of p, one emphasizing factors like p^x, the other those like $(1-p)^{k-x}$. That is to say, the equation for p has more than one root. Here the overall summary of Woods and Baltzy is a guide. They found that eight large Powers were each at war on the average for $\frac{1}{4}$ of the time. It seems likely therefore that the average fraction of the time spent at war between any two *named* Powers was much less than $\frac{1}{4}$, and more likely to be of the order of 0·01. For this reason various much larger alternatives should be rejected.

Equations to be solved

The Bernoullian logic of probability, when brought into connection with historical fact, leads at simplest to many equations each of the type

$$p^b(1-p)^c = a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which a, b, c, p and $1-p$ are each positive . . . . . (2)

The unknown is p. As equation (1) is not a common feature in the textbooks, a few remarks on its solution may be in place here. The function

$$p^b(1-p)^c = f(p), \quad \text{say,} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

has a maximum at $p = b/(b+c) = p_m$, say; . . . . . (4)

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and the value at the maximum is

$$f(p_m) = b^b c^c/(b + c)^{b+c} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

If $a > f(p_m)$, equation (1) has no real roots, and the proposed theory receives a knock-out blow . . . . . (6)

If $a < f(p_m)$, equation (1) has two real roots p_1, p_2, which for definiteness we may dispose thus

$$0 < p_1 < p_m < p_2 < 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Two methods are available for finding these roots.

By nomogram

Let $$\log_{10} p = \xi, \quad \log_{10} (1 - p) = \eta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where ξ and η are Cartesian co-ordinates. The identity

$$p + (1 - p) = 1 \quad \text{is then equivalent to} \quad 10^{\xi} + 10^{\eta} = 1 \quad . \quad . \quad . \quad . \quad (9)$$

This represents a fixed curve, independent of a, b, c. This curve is plotted once for all. The equation (1) is equivalent to

$$b\xi + c\eta = \log_{10} a, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

which represents the straight line joining the point $\xi = 0$, $\eta = c^{-1} \log_{10} a$ to the point $\xi = b^{-1} \log_{10} a$, $\eta = 0$. Where this straight line (10) cuts the curve (9), there the equation (1) is satisfied. Let the abscissæ of the points of intersection be ξ_1 and ξ_2, the former being not the greater. Then $p_1 = 10^{\xi_1}$, $p_2 = 10^{\xi_2}$. Unequal scales for ξ and η were taken, so as to suit the prevalent values of b and c in the set of equations.

Successive approximations. We can nip each root between numbers approaching it from opposite sides. For it can be proved that any sequence $l_0, l_1, \ldots, l_n, \ldots$, which obeys the recurrence formula

$$l_{n+1}^b (1 - l_n)^c = a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

converges to the lesser root p_1 from below provided that

$$0 \leqslant l_0 < p_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

and converges to the same root from above provided that

$$p_1 < l_0 < p_m \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Also any sequence $u_0, u_1, \ldots, u_n, \ldots$, which obeys the recurrence formula

$$u_n^b (1 - u_{n+1})^c = a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

converges to the greater root p_2 from below or above according as

$$p_m < u_0 < p_2 \quad \text{or} \quad p_2 < u_0 \leqslant 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)\ (16)$$

A table of subtraction-logarithms shortens the computation.

The proofs in the four cases (12), (13), (15), (16) are much alike. For example, take the case (13). In that range of p—namely, $p_1 < p < p_m$—the function $p^b(1 - p)^c$ increases with p; so that we have from (1), (4), (11),

$$p_1^b (1 - p_1)^c = a, \quad l_0^b (1 - l_0)^c > a, \quad l_1^b (1 - l_0)^c = a \quad . \quad . \quad (17)\ (18)\ (19)$$

From (17) and (19) $$(l_1/p_1)^b \{(1 - l_0)/(1 - p_1)\}^c = 1$$

but by (13), $$(1 - l_0)/(1 - p_1) < 1.$$

Therefore $$p_1 < l_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

Again from (18) and (19) $$l_1 < l_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

Similarly at the next stage $$p_1 < l_2 < l_1 < l_0.$$

By induction the sequence (l_n) is monotonic decreasing, is bounded below by p_1, and has no greater lower bound . . . . . (22)

Therefore p_1 is its limit.

Example: to find the greater root of $p(1 - p)^{86} = 0{\cdot}0007$. The nomogram gave $p_2 = 0{\cdot}048$ roughly. A sequence controlled by (14) began

$$u_0 = 0{\cdot}048, \quad u_1 = 0{\cdot}047973, \quad u_2 = 0{\cdot}047966.$$

Another sequence, also controlled by (14), began

$$u_0' = 0{\cdot}04790, \quad u_1' = 0{\cdot}04795, \quad u_2' = 0{\cdot}047961.$$

Therefore $$0{\cdot}047961 < p_2 < 0{\cdot}047966.$$

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Symmetry. Confusions might easily occur between the octanta and quadrantal arrangements, and would ruin any theory. To prevent them, all quantities peculiar to the octantal arrangement have a star affixed. This new convention does not change the meanings of n and ϕ, for they were defined over the quadrant, but we now have an octantal n^* connected with n thus:—

$$n^*(r, s) = n(r, s) + n(s, r) \quad \text{if} \quad r \neq s$$
$$n^*(r, r) = n(r, r)$$

and similarly for ϕ^* and ϕ. It is n^* which is taken directly from history.

Various general remarks.

In the collections of historical facts the number of belligerent groups has been counted without any attention to their heroism or wickedness, and with attention to their populations only in so far as groups of insignificant population were ignored altogether. It would be of no avail to look in the statistics for distinctions of ethics or of power which had been sieved out by the method; and therefore it will be best to refer to particular states incognito as A, B, C, D . . . instead of by their usual names. All that we can hope thus to make is an analysis of the average motives of humanity in general, but the probability attached to "any particular nation," such as A or B or C or D, is usually quite different from the probability attached to "some unspecified nation," because the latter designation leaves open more choice.

The foregoing general principles can be applied in company with various special assumptions. Strongly held opinions as to the causes of wars may be heard or read on all sides. Yet I had difficulty in finding any hypotheses which would give a *quantitative* agreement with the historical facts in Collections A, C, E, F, G, H. It seemed best therefore to begin with some theories which can easily be refuted, then to introduce improvements, and so gradually to approach realism.

Theory I. The decisions of so-called "free, sovereign and independent" States are statistically independent.

States have been in the habit of claiming more freedom, sovereignty, and independence than they ever in fact possessed. For a belligerent can hardly be at war by itself alone; and States which have been threatened have mostly felt compelled to defend themselves. The assumption "let p be the probability that any state is at war" is self-contradictory; for if one State is at war, another must be.

Theory II. The wars in the world were mainly due to aggression by Germany, Italy and Japan

This theory should perhaps be mentioned because it was widely believed in Britain during the years 1936–44. It will not, however, describe the facts over the longer interval 1820–1939, as may be seen from any reasonably full list of the names of belligerents—for example, Wright's list.

Theory III. The facts can be explained by the well-known tendency to join the winning side

That tendency can indeed explain the fact that, in the octantal plan, $n^*(3, 1) > n^*(2, 2)$. For if a war of 2 versus 1 had begun, a fourth belligerent would on this theory tend to join the larger group making 3 versus 1, not 2 versus 2. Similarly the fact that $n^*(4, 1) > n^*(3, 2)$ can be explained, but this theory fails utterly to explain why wars of 1 versus 1 are the commonest type. Any general tendency to join in fights must evidently be restrained by other motives. Humanity in general is not like the Irishman who is said to have asked: "Is this a private fight, or may anybody join?"

Theory IV. The sense of justice is universal

It can be argued that wars tend to be of the type many-versus-one because justice is a widely venerated ideal, that shines above national prejudices, and so leads many nations to combine against the transgressor, who is thus left isolated. To complete the theory we have also to suppose that transgression breaks out sporadically in different places at different times, as Ranyard West has emphasized (1942). Some at least of the kaleidoscopic patterns of nations-at-war can be so explained; for example, the coalition against Napoleon in 1815. Then in 1854 the Turks, French, British and Piedmontese combined, with the approval of Austria and Prussia, against the Russians, not from any permanent affinity with one another, but because they were all indignant at the Czar's unreasonable aggressiveness. In the next war between the Russians and the Turks, 1877–78, it

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was the turn of the Turks to be isolated, and to be opposed by many; this time by Russians, Rumanians, Serbs, Montenegrins, Bosnians and Herzegovinians, who were all indignant about the extortions by Turkish officials and the atrocities committed by Turkish soldiers.

On this view we should not lament the fickleness of alliances, saying that it is sad evidence of the untrustworthiness of friends; instead we should rejoice over the changing scene, because it reveals the control by the higher ideal of justice. This theory undoubtedly contains some truth, but it is nevertheless an exaggeration. For if the sense of justice had been strictly universal, there would not have been any transgressors; and if only rare lapses from justice had occurred, we may reasonably suppose that nearly all wars would have been of the police-action type: society versus a criminal, many-versus-one. The United Nations Organization designed at San Francisco in 1945 is suited to a future world of that character. In the past, however, most wars were of the type one-versus-one, which suggests ordinary quarrelling, rather than police-action. We must therefore admit that the sense of justice abroad in the world was flickering, confused, local, and fallible; so that its quantitative results must be represented by probabilities.

Theories V, VI, VII, VIII

We now come to a group of theories all of which ignore geographical restrictions. They are therefore more appropriate to a future age of aviation than to the past age of land and sea forces, but, because they are simpler mathematically than those theories which express geographic restrictions, it is convenient to discuss them first.

Theory V. Mutually exclusive wars are equally probable

The number of combinations that can be formed from m nations or other possibly belligerent groups, r at a time, is $m!/\{r!(m-r)!\}$. That having been done, there remain $m-r$ possibly belligerent groups, and from them the number of combinations s at a time is $(m-r)!/\{s!(m-r-s)!\}$. The number of arrangements in wars jointly of the types r versus s and s versus r is the product of these two expressions—namely, $m!/\{r!s!(m-r-s)!\}$ provided $r \neq s$, but if $r = s$ we have, in the above argument, counted each arrangement twice. Let A, with suitable attached symbols, denote the number of distinct arrangements. Then in the octantal plan

$$A^*(r, s) = m!/\{r!s!(m-r-s)!\} \text{ if } r \neq s,$$

but

$$A^*(r, r) = \tfrac{1}{2}m!/\{r!r!(m-2r)!\}.$$

It would be troublesome thus to have different formulæ according as $r = s$ or $r \neq s$. The quadrantal plan relieves us of this difficulty because

$$A(r, s) = A(s, r) = \tfrac{1}{2}A^*(r, s) \text{ if } r \neq s$$

and so

$$A(r, s) = \tfrac{1}{2}\frac{m!}{r!s!(m-r-s)!} \text{ for both } r = s \text{ and } r \neq s \quad . \quad . \quad (1)$$

Example, $m = 4$. States named A, B, C, D. The possible wars of the type 2 versus 2 are A and B versus C and D, A and C versus B and D, A and D versus B and C; three distinct wars and no more, agreeing with $A^*(2, 2) = 4!/(2!2!0!2)$. Again, the possible wars of the types 3 versus 1 and 1 versus 3 are jointly A and B and C versus D, B and C and D versus A, C and D and A versus B, D and A and B versus C; four distinct wars and no more. This is the octantal method of counting and it agrees with $A^*(3, 1) = 4!/(3!1!0!)$.

If r and s are given, $A(r, s)$ is the number of *mutually exclusive* wars. The probability of the whole set of them is the *sum* of their separate probabilities. Thus the expression (1) for $A(r, s)$ is likely to occur in many theories. It is, however, an embarrassment to the theorist, because its isopleths have the wrong curvature. To see this, let us attend to the sloping straight lines on which $r + s$ is constant. On such a line $A(r, s)$ varies as $(r!s!)^{-1}$. The cases which occur most frequently are:—

$r+s$	Successive values of $1/(r!s!)$ along the line $r + s$ = constant								
4			1/6		1/4		1/6		
5		1/24		1/12		1/12		1/24	
6	1/120		1/48		1/36		1/48		1/120

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These fractions are greatest in the middle and least at the extremes, and so are distributed in a manner opposite to the historical facts.

Moreover, $A(s, s)$ is greatest near $s = m/3$. That is to say, if $A(s, s)$ were the only consideration, and if $m = 60$, then wars of 20 versus 20 would be the mode; whereas in fact such a war has never occurred in the interval studied.

It is thus abundantly evident that there must be a restraining factor. Guided by the Bernoullian model, we may expect it to be the probability of a war of the type r versus s between *particular* States. This factor will need to overcome the tendency of $A(r, s)$ to give isopleths hollow on the wrong side.

Theory VI. The behaviour of any pair of States is statistically independent of the behaviour of every other pair of States

We have not here the crude absurdity of Theory I; for there can be war inside the pair of States A and B without necessarily involving any other pair. The historical statistics in Collections D and E, moreover, emphasize the importance of the number, rs, of pairs of opposed belligerents, because $n(r, s)$ is almost a function of the single variable rs. The present theory asserts that the probabilities, that the different pairs are pairs of enemies, combine by multiplication. This theory accordingly explains the fact that $n(r, s)$ decreases as rs increases. Whether the theory will give the observed functional form of the decrease is a further question, which will be examined.

In particular, the tendency to join the winning side, which was the basis of Theory III, can equally well be expressed as a reluctance to form pairs of opposed belligerents. For example, if a war of 2 versus 1 is going on, a fourth belligerent would make one more such pair by converting the war to 3 versus 1, but would make two more such pairs by converting the war to 2 versus 2.

Although Theory VI has these merits, there is abundant historical evidence against it in the form of relations between different pairs. The most obvious of these is an indirect result of *alliance* between States. For example, in 1914, when Britain went to war with Germany, the ancient British alliance with Portugal began to lead Portugal into the war in which otherwise she had no great interest (E **18**, 283). For our present purpose the point is that war inside the pair Britain–Germany caused war inside the pair Portugal–Germany. Another relation between pairs was that of *distraction*. For example, in 1857, conflict inside the British–Indian pair delayed hostilities inside the British–Chinese pair (E. **5**, 536) for some British troops on their way to China were diverted to India. Again for example, Bismarck, after organizing the defeat of France in 1871, encouraged her colonial enterprises in Africa and Indo-China in order to distract her from revenge (E **3**, 665). A third relation between pairs has arisen when a State having armed forces ready for one purpose has taken the opportunity to use them for another. This may be called *opportunism*. For example, in 1858, a French fleet, ready for war with China, bombarded a port in Annam (Priestley, 1938, p. 114). That is to say war inside the French–Chinese pair facilitated fighting inside the French–Annamese pair.

We must therefore amend Theory VI.

Theory VII. Because of the fickleness of alliances and the flitting of distraction and opportunities, different pairs of States behaved in a manner statically independent of one another when the facts are collected from an interval of at least a century

Quincy Wright (1942, p. 774) remarks that:

> "Alliances and confederations intended to be permanent have seldom proved reliable unless carried to the point of federation, transferring much of legal sovereignty and the conduct of external affairs to the central organs. Such a development has seldom been possible unless geographic and cultural factors have conspired to unite the group. Alliances purely for defence have broken up if the State against which they are directed ceases to be menacing."

The objections already stated against Theory VI are for the most part not valid against Theory VII. The latter therefore is worthy to be compared with fact in more detail. This, however, requires additional hypotheses, which will now be stated.

Theory VIII. A simple chaos uniform among sixty nations

This is a special case of Theory VII and is based on precise assumptions. The probability of an event is regarded as the sum of the fractions of a long historical time-interval during which the

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event occurred. There are assumed to be m possible belligerents in the world. Some are States, some insurgents. For brevity they are all called nations . . . . . (1)

Every schoolboy knew that his own nation was remarkably different from, and better than, the rest. Yet as our present study is of mankind in general let us, in Theory VIII, try-out the simplest assumption—namely, that the probability of war between any two nations is:—

the same at all times, ... (2)
the same for all pairs of nations, ... (3)
independent of any other wars that are going on; . . . . . (4)
and let us denote this constant probability by p . . . . . (5)

These assumptions specify a hypothetical world which is simpler and more chaotic than the actual world. So they should do; for the art of beginning theories is to leave out of consideration all but the most essential features.

Consider a war between r nations named $A, B \ldots$ on one side, versus s nations named $Z, Y, \ldots$ on the other. This war involves rs pairs of opposed nations. The probability of war inside each pair is p by (2), (3), (5); and by (4) the pairs are statistically independent. The probability of war simultaneously inside rs pairs is therefore p^{rs} . . . . . (6)

It is essential to the specification that no other nations should be in this war, but the specification ought not to exclude quite disconnected, though simultaneous, fightings. Thus the specification does not require all the remaining $\frac{1}{2}m(m-1) - rs$ pairs to have peace inside them. The notion of disconnection may, in practical politics, involve the causes of the quarrels, which have no representative in the present abstract theory. Nevertheless we can represent the usual kind of isolation of the conflict by assuming that: The nations $A, B, \ldots$ are not at war with any other nations except the nations $Z, Y, \ldots$; and vice versa . . . . . (7)

That is to say, there are $(m - r - s)$ nations not at war with the s, nor with the r nations. Altogether they form $(r + s)(m - r - s)$ pairs which have peace inside them. The probability of peace inside any pair is $(1 - p)$. So the probability of peace between the belligerents and the neutrals is $(1 - p)^{(r+s)(m-r-s)}$ . . . . . (8)

in the specified war, whatever wars the neutrals may have between themselves. The probability of the war of the r specified nations versus the s specified nations is, from (6) and (8) equal to

$$p^{rs}(1-p)^{(r+s)(m-r-s)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

They could at most be at war during the whole time-interval. So expression (9), being the ratio of their time at war to the whole time-interval, lies between 0 and 1 in the manner characteristic of a probability.

The number of mutually exclusive sets of specified nations of the type r versus s, was shown in connection with Theory V to be, in the quadrantal plan,

$$A(r, s) = \tfrac{1}{2}m!/\{r!\,s!\,(m-r-s)!\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Each of these sets is at war during the fraction (9) of the time. So the total duration of the wars of this type between unspecified nations, when divided by the historical interval, is expected to be

$$\tfrac{1}{2}\frac{m!}{r!\,s!\,(m-r-s)!}p^{rs}(1-p)^{(r+s)(m-r-s)} = Q(r, s), \quad \text{say} \quad . \quad . \quad . \quad (11)$$

This $Q(r, s)$ can exceed unity, and so is not a probability.

The number m of nations in the world averaged about 60 . . . . . (12)

For the types of war which occurred most frequently in history we obtain from (11), on inserting $m = 60$,

$$Q(1, 1) = 1770p(1-p)^{116} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$
$$Q(2, 1) = 51330p^2(1-p)^{171} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

The first members of these equations are given by the historical facts in Collections G and H, according to their respective definitions of "war." For the type 2 versus 1 it is necessary to divide the octantal observed numbers by two, so as to make them comparable with the theory, which is

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quadrantal. The equations are then soluble for p. The two roots, p_1 and p_2, were found by the processes already described, and are set out in the following table.

Type of war	Collection of facts	p_1	p_2
1 versus 1	G	0·0009	0·031
	H	0·0007	0·034
2 versus 1 } 1 versus 2 }	G	0·0025	0·032
	H	0·0017	0·037

. . . . . (15)

Theory VIII will be valid if the various historical data can all be explained by the same number for p, within the uncertainty of sampling. Only one of p_1 and p_2 need be considered; but we do not know which. So far, p_2 looks the more consistent, but before we can come to any conclusion, we need to consider wars of other types. The most reliable of the historical facts is the total duration of wars of all types. This total, however, is not convenient for comparison with theory, because the theoretical $\sum_{r=1}^{r+s=60} \sum_{s=1} Q(r, s)$ is troublesome to compute, even when p is given numerically.

On the contrary, the two simplest of the theoretical relations are

$$Q(2, 2)/Q(3, 1) = \tfrac{3}{2}p, \qquad Q(3, 2)/Q(4, 1) = 2p^2 \quad . \quad . \quad . \quad (16)\ (17)$$

Their great advantage is that they do not involve m. They depend on observations which are rare and correspondingly erratic. Let us therefore take the longer historical interval—namely, Wright's, as represented in Collection H. After converting the data to the quadrantal plan, we obtain:—

$Q(2, 2)/Q(3, 1) = 2 \times 0{\cdot}03/0{\cdot}13$ whence, by (16), $p = 0{\cdot}31$
$Q(3, 2)/Q(4, 1) = 0{\cdot}05/0{\cdot}12$ whence, by (17), $p = 0{\cdot}46$.

The discrepancy between 0·31 and 0·46 is not interesting, for it can be attributed to the random occurrence of wars of these rare types, but we must take seriously the mean result that p is of the order of 0·38 . . . . . (18)

This is quite inconsistent with either p_1 or p_2 as found above in (15) by putting $m = 60$ together with the observed values of $Q(1, 1)$ and $Q(2, 1)$. Moreover, $p = 0{\cdot}38$ is quite incredible; for it asserts that in a very long historical interval each of the m nations was at war with each of the other $(m - 1)$ nations for 0·38 of the time. The world was, in fact, certainly not so war-ridden, if $m = 60$.

Thus Theory VIII has crashed. No one will be surprised that a theory, which began by assuming all nations to be statistically alike, should have come to a bad end; but the precise manner of its failure could hardly have been foreseen.

Theory IX, being the Theory VIII modified by the assumption that only a few of the nations were involved in wars

We might argue thus: That incredibly large value $p = 0{\cdot}38$ may have come from an aberrant sample of history. Without violence to the facts we could replace it by $p = 0{\cdot}2$. That, moreover, would be less incredible if only a few bellicose nations were concerned. Also by reducing m from 60 to say 10 or 6 we could raise the p obtained from the wars of types 1 versus 1 and 2 versus 2 up towards 0·2. As thus amended, the theory would be much more self-consistent.

We meet, however, difficulties fatal for Theory IX when we try to *name* the bellicose minority and the non-belligerent majority. In fact it is impossible to name them, because belligerency was too widespread. Suppose, for example, that we choose as notable belligerents the ten nations: Britain, France, Russia, Turkey, China, Piedmont and its successor Italy, Prussia and its successor Germany, Spain, U.S.A., Japan.

Of the 91 wars that ended from A.D. 1820 to A.D. 1939 according to Richardson's list, 13 wars did not involve any one of those 10 notable belligerents. Also the total number of belligerents' names in those 91 wars, including names of insurgents, but not counting any name twice, was 108 —far in excess of ten.

Thus Theory IX fails, but it leaves us with the hint that although many nations were involved, yet the probability p was related to only a few of them at a time. That suggested the next theory.

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Theory X. Localized quarrels

Most wars have been localized. Litvinov's doctrine that "Peace is indivisible" was announced to the League of Nations. It is commonly believed in 1946 that, in view of the new United Nations Organization, future threats of war will usually concern the whole world; but that was certainly not so prior to 1918.

Let us therefore try the theory that disputes occurred at random over the world; and that each dispute concerned only l possible belligerents, where l is much less than 60, and may be more like the average observed number, 5 or 6, of geographical neighbours.

In Theory VIII the elementary probability denoted by p was the fraction of a very long historical time during which the two specified nations were at war with one another. In Theory X we have again an elementary probability, but it has a different meaning. It is now the probability that the pair of nations went to war with one another about a particular dispute, and it will be denoted by θ. The conclusions of Theory X will therefore be comparable, not with the total durations of wars, but with the numbers of wars. With those modifications the argument of Theory VIII is so far applicable that we obtain a formula like that for $Q(r, s)$, but with l replacing m and θ replacing p, thus

$$\tfrac{1}{2}\frac{l!}{r!\,s!\,(l-r-s)!}\theta^{r+s}(1-\theta)^{(r+s)(l-r-s)} = U(r, s), \quad \text{say} \quad . \; . \; . \; . \; . \quad (1)$$

Here $U(r, s)$ is not a total duration, but is proportional, for variations of r and s, to the probability of a war about a particular dispute. To obtain a number equal, instead of merely proportional, to the probability we should have to divide $U(r, s)$ by $\sum_{r=1}^{r+s=l}\sum_{s=1} U(r, s)$. But, as the double sum is troublesome to calculate, it is preferable to use proportionalities.

Let us now, if possible, choose l and θ so as to make $U(r, s)$ agree with the historical facts. As in Theory VIII, the two simplest of the theoretical results obtainable from (1) are that

$$U(2, 2)/U(3, 1) = \tfrac{3}{2}\theta, \quad U(3, 2)/U(4, 1) = 2\theta^2 \quad . \; . \; . \; . \; . \; . \quad (2)\ (3)$$

for they do not involve l. On referring to the octantal collections A and C of numbers of wars, and making the corrections to the quadrantal plan, we obtain the following:—

	From Collection A	From Collection C
$U(2, 2)/U(3, 1)$ =	6/5	8/12
therefore, by (2), θ =	0·8	0·44
$U(3, 2)/U(4, 1)$ =	2/5	6/12
therefore, by (3), θ =	0·45	0·50

. . . . . (4)

The mean of these four values of θ is 0·55. The objection, previously noted, to large values of the probability p, does not hold against large values of the probability θ, for the unspecified probability of a dispute intervenes. But as the counted numbers of these wars are small, there is much uncertainty due to sampling. As a deviation from the mean to $\theta = 0{\cdot}80$ occurs in the sample, we cannot reject the possibility of an equal but opposite deviation to $\theta = 0{\cdot}30$.

An upper bound to θ is provided by the observed fact that wars of the type many-versus-one have been rare in comparison with those of the type one versus one. It follows from (1) that

$$\frac{U(l-1, 1)}{U(1, 1)} = \frac{1}{l-1}\left\{\frac{\theta}{(1-\theta)^2}\right\}^{l-2} \quad . \; . \; . \; . \; . \; . \; . \quad (5)$$

To agree with history, the first member must be near to zero when l is 7 or more. This can occur if $\theta < (1-\theta)^2$, but not if $\theta > (1-\theta)^2$. The equality $\theta = (1-\theta)^2$, which separates these two cases, is satisfied by $\theta = 0{\cdot}382$, or by $\theta = 2{\cdot}618$, of which the former only can be a probability. Accordingly θ *must be less than* 0·382 . . . . . (6)

The conditions (4) and (6) pull θ opposite ways, so as to restrict it to a range between about 0·30 and 0·38. The value $\theta = 0{\cdot}35$ will be taken as typical of this range.

The simplest method for estimating l is to deduce from (1) that

$$2U(2, 1)/U(1, 1) = (l-2)\theta(1-\theta)^{l-5} \quad . \; . \; . \; . \; . \; . \; . \quad (7)$$

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The first member was taken from the counted numbers of wars and the equation was then solved for l to the nearest integer, rejecting irrelevant roots between $l = 2$ and $l = 3$.

Collection of wars	A			C		
$2U(2, 1)/U(1, 1)$		24/42			28/117	
Assumed θ	0·3	0·35	0·4	0·3	0·35	0·4
Therefore l	8	8	8	12	11	10

(8)

Remembering that l is the number of nations, or other possibly belligerent groups, which are concerned in each quarrel, we see at once that a theory which restricts l to be 12 or less cannot explain world-wars. But world-wars have been very exceptional. Thus $l = 11$ would exclude only 8 wars out of the 200 in Collection C; and $l = 8$ would exclude only 2 wars out of the 91 in Collection A. It is worth while, therefore, to make a further comparison between Theory X and the distribution of ordinary wars.

The most weighty of the present set of historical facts remains to be considered. It is the ratio of the total number of wars to the number of wars of type 1 versus 1. The theoretical ratio was obtained from formula (1) by laborious arithmetic after inserting for θ and l numbers that had been indicated as approximately suitable by the foregoing simpler methods. For comparison with these results of Theory X it seemed proper to truncate the Collections A and C by omitting from them the small minority of wars for which the theory offers no explanation—namely, wars involving more than l belligerents. The comparison is to be seen in the following table. The numbers in the block common to the last three columns and the lower four rows are ratios of total numbers of wars to the number of wars of type 1 versus 1.

		8	9	10
$\frac{1}{U(1,1)} \sum_{r=1}^{l} \sum_{s=1}^{r+s=l} U(r, s)$	$\theta = 0{\cdot}35$	2·2910	1·8695	1·5635
	$\theta = 1/3$	2·2004	1·8346	1·5600
Truncated collections of facts A		$2{\cdot}1_2$	$2{\cdot}1_2$	$2{\cdot}1_4$
Truncated collections of facts C		1·62	1·63	1·64

The resemblances between theory and fact in the above table are sufficient to make worth while further comparisons by way of the χ^2 test due to K. Pearson and R. A. Fisher. For this purpose the data were grouped in cells according to the number, rs, of pairs of opposed belliegerents, thus:—

rs	1	2	3, 4, 5	>5	Total
Collection A, truncated	42	24	15	8	89
Theory X with $l = 8$, $\theta = 0{\cdot}35$ ...	38·85	22·40	18·80	8·95	89·00

It follows that $\chi^2 = 1{\cdot}24$. Three parameters have been adjusted—namely, the total and, less accurately, θ and l. So, as there are four cells, only one degree of freedom remains. Yule's table (Kendall, 1943, p. 445) gives the probability that χ^2 would exceed 1·24 to be 0·27; which indicates that *Theory X passes the χ^2 test for agreement with the truncated Collection A.* There may be some readers who mistrust the hypotheses on which Theory X is founded; but even they must admit that it has provided respectable formulæ, which, however, they will have to call empirical.

On the contrary, a satisfactory fit between Theory X and Collection C appears to be impossible for any values of θ and l. Here is the best of my attempts:—

rs	1	2	3, 4, 5	>5	Total
Collection C truncated	117	28	31	15	191
Theory X with $\theta = 0{\cdot}35$, $l = 10$ ...	122·16	39·69	15·91	13·24	191·00

From this table χ^2 comes to 18·2; and the probability that a larger value should occur by chance in the historical sample is less than 0·0001. I cannot offer any precise explanation of why Theory X should fit Collection A but not Collection C. The empirical formula $\phi(r, s) = \frac{5}{8}N(rs)^{-2\cdot5}$ fitted both of them. But of course I prefer the definition of war on which Collection A is based.

Summary on Theory X. With the exception of a small minority of very complicated wars, Theory X succeeds in explaining, not particular wars nor the behaviour of named States, but the average behaviour of the world over an interval of 120 years or more. Its hypotheses are of the simplest. It is supposed that disputes occur at random over the globe; that each dispute interests eight nations (or other belligerent groups) and no more than eight; that the probability of war about a dispute between every pair formed from those eight nations is 0·35, the same for every pair, the same for every dispute, and the same in every year. It is remarkable that such simple

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hypotheses lead to a quantitative agreement between the theory and the observed facts. But it is well to remind ourselves that a theorem does not prove its converse. Here, in particular, the observed facts do not prove that the aforesaid probability was always the same.

PART III. THE INFLUENCE OF GEOGRAPHY AND OF INFECTIOUSNESS

Multitude of contacts. Sea-powers counted separately

In order to amend Theory X so as to include world-wars, let us attend to the fact that some States had much more numerous contacts than others, and therefore more opportunities either for friendship or for quarrelling. We need both a collection of historical facts, appropriately classified, and an abstract theory of the probabilities. They will be stated in turn.

Collection J of facts

Aviation is now tending to put every nation into contact with every other, but for the interval A.D. 1820 to A.D. 1929, the opportunities for quarrelling were greatest for the sea-powers. Contact across land-frontiers had a similar effect, but not to the same extent. For whereas considerable power at sea put its possessor into contact with about forty sovereign States, the greatest number of land frontiers possessed by any other State was eleven for Brazil. It is thus proper to attend to the world-wide sea-Powers as a class apart; but coastal defence should here be classed along with the defence of land frontiers.

No sharp distinction between ubiquitous and local Powers existed in fact, for Portugal, Spain, Holland and Belgium had distant colonies. Yet, for the purpose of comparing fact with theory, a meaningful dichotomy can be made by decision; provided that the same classification is made in the facts, as in the theory. After consulting the comparative tables published by Q. Wright (1942, pp. 670, 671), together with articles on some national navies in the *Encyclopædia Britannica*, I took the many-contact States to be as follows: for the whole interval A.D. 1820 to A.D. 1929, Britain, France, Russia; from 1870, U.S.A.; from 1880, Italy; from 1900, Japan; from 1895 to 1918, Germany; from 1910 to 1918, Austria-Hungary. The number of such powers was thus three from 1820 to 1869, then rose gradually to a maximum of eight from 1910 to 1918, and afterwards settled at six for the rest of our interval, which ends with the year 1929. The mean number was 4·6.

The notation "r versus s" now becomes inadequate, for both r and s must be partitioned. Let the new notation be "r_1, r_2 versus s_1, s_2". Let the connection with the former notation be that

$$r_1 + r_2 = r, \qquad s_1 + s_2 = s.$$

Let r_1 and s_1 be the numbers of the many-contact powers defined by the above list; so that r_2 and s_2 are the numbers of any other belligerents.

The classification of wars has thus become four-dimensional. Distinctions of symmetry remain essential, for otherwise factors of 2 or ½ will enter by mistake. The two-dimensional ideas "quadrantal" and "octantal" require to be generalized; yet we lack familiar words applicable to four dimensions. The distinctions can however be made plain in the special notation. Thus, for example, some historical facts are set beside the symbol (0, 1 versus 0, 1) because they belong to that subtype alone, but other facts are set beside the two symbols (1, 0 versus 0, 1) and (0, 1 versus 1, 0) because they belong to those subtypes, not severally, but jointly.

The following table was abstracted from Richardson's unpublished list of the wars that ended during the 110 years from A.D. 1820 to A.D. 1929, for all magnitudes greater than 3·5. The interval is here cut off ten years earlier than the final date for Collection A in order to avoid the larger effects of air-power. The statements in the last two columns are only loosely connected with one another. For the total durations refer strictly to one subtype, and so may include portions of more complicated wars; whereas in the third column each war is counted as a whole after being classified by the subtype of its most complicated phase.

In comparison with Collections A, C, G and H the present more detailed classification has many more empty compartments. They occur even among the subtypes of 2 versus 1, 3 versus 1, 2 versus 2. Any probability-theory of occurrences must take account also of non-occurrences. We

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Collection J

Type r versus s	Subtypes r_1, r_2 versus s_1, s_2	Number of wars attaining that subtype	Total duration of subtype in years
1 versus 1	0, 1 versus 0, 1	22	110·4
	1, 0 versus 0, 1 0, 1 versus 1, 0	13	37·7
	1, 0 versus 1, 0	1	1·5
2 versus 1 and 1 versus 2	1, 1 versus 0, 1 0, 1 versus 1, 1	12	17·3
	1, 0 versus 0, 2 0, 2 versus 1, 0	5	15·8
	0, 2 versus 0, 1 0, 1 versus 0, 2	5	$14{\cdot}5_5$
	2, 0 versus 0, 1 0, 1 versus 2, 0	1	2·5
	1, 1 versus 1, 0 1, 0 versus 1, 1	1	$0{\cdot}2_5$
	2, 0 versus 1, 0 1, 0 versus 2, 0	0	0·0
3 versus 1 and 1 versus 3	0, 3 versus 0, 1 0, 1 versus 0, 3	1	8·5
	0, 1 versus 1, 2 1, 2 versus 0, 1	3	2·8
	2, 1 versus 1, 0 1, 0 versus 2, 1	0 *	$0{\cdot}7_5$
	2, 1 versus 0, 1 0, 1 versus 2, 1	1	0·4
	The other subtypes	0	0·0
2 versus 2	0, 2 versus 0, 2	2	14·2
	1, 1 versus 1, 1	1	0·3
	The other subtypes	0	0·0

* There was no war of sub-types 2, 1 versus 1, 0 and 1, 0 versus 2, 1 at its most complicated phase; and yet 0·75 year of the Crimean war of 1853–56 belonged to those subtypes.

could not, for example, discuss the probability of the First World War unless we distributed its occurrence over some of the many empty surrounding compartments, by a process of smoothing in four dimensions; and the choice of a suitable process would be a question of considerable difficulty. As an alternative, the following discussion is restricted to the parts of the distribution which show a tolerable regularity. There is evidently a considerable correlation between the total durations and the numbers of wars in the above table.

Inspection of the above Collection J shows at once that wars involving *only* the long range sea-Powers have been remarkably rare. There is only one in the table—namely, the Russo-Japanese war of 1904–1905. That may be because such powers were few in number, or because they were cautious of attacking anyone of like strength, or because they were peace-loving. We cannot say which, unless we have a probability-theory. One will now be offered.

Theory XI. Chaos restricted by a formalized geography

If we were to attempt to take account of all the features of the political map, we should get no further. Nevertheless a search on atlases brought to notice a few facts which suggest the basis for a theory:—

There is not any point where four independent States meet one another, nor any point where three such States meet the sea. This is a remarkable fact . . . . . (1)

The number of different governments with which a State was in contact by land has ranged from 0 for Japan to 11 for Brazil, with a mean 5·4 when taken over 33 persistent States . . (2)

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There is some, rather loose, connection between political geography and Euler's theorem that $f - e + v = 2$, where f, e, v are the numbers of faces, edges and vertices of a polyhedron. To make the polyhedron resemble the political map, each vertex must be shared between three faces, so that $3v = 2e$. It follows that the average number of edges surrounding a face is $6 - 12/f$. If $f = 60$ this comes to 5·8 edges, not far from the mean of 5·4 mentioned in (2). But the political map differs from the polyhedra as to curvature, lack of closure, single frontiers, encirclement and detached portions. Euler's theorem moreover represents a delicate compensation which is easily upset. It is therefore preferable to lay aside Euler's theorem and to base Theory XI on robust assumptions among which the fact (1), being simple, will be included as it stands; but the facts (2), being complicated, will be replaced by the following assumption (3).

Let our formalized world consist of:—

γ land-locked countries each touching five different States.
β local coastal countries each touching the sea, also touching five different States by land.
α world-wide sea-Powers which are in contact with so many other Powers by sea that their contacts by land alone can be ignored in comparison. . . . . (3)

At the stage of numerical computation the following values will be assumed.

$$\alpha = 5, \quad \beta = 44, \quad \gamma = 11 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

because they are a fair representation of the average during the interval A.D. 1820 to A.D. 1929, insurgents being included, for they are so in Collection J.

As in Theory VIII, let the probability p of war between the two members of any pair of States be defined to be the fraction, of any very long historical interval, during which such war occurred. We now have to consider three kinds of States, which can form six kinds of pairs, to each of which we must assign a probability. Fortunately, however, it is possible to make theoretical progress with only four distinct probabilities, one of them being zero. They are defined in the following table:—

Members of the pair	Symbol for the probability p
(i) Two world wide sea-Powers	x
(ii) A world wide sea-Power and a local coastal Power	y
(iii) Two local coastal Powers	z if in contact, or if a local belligerent touched both; otherwise zero.
(iv) A local coastal Power and a land-locked Power	
(v) Two land-locked Powers	
(vi) A world wide sea-Power and a land-locked Power	Zero, unless a local coastal belligerent put them in contact, when p becomes y.

These definitions will be referred to as . . . . . (5)

It is assumed here for simplicity, as in Theory VIII, that the probabilities defined in the above table are constant in time and independent of all circumstances other than those specified in the definitions (5). . . . . (6)

In the definitions (5) of the probabilities it has been assumed that two States which have no contact can yet be at war with one another, provided that either is allied to an intermediary belligerent whose land extends from one to the other so that their armies can march across it. This principle may be briefly called "*conduction by land.*" . . . . . (7)

It has only a slight effect on the probabilities x, y, z deduced from the wars of type 1 versus 1, but a large effect for more complicated wars.

There is a further question about geography: Are widely separated local Powers ever regarded as allies? In the First World War the formal list of "Allied and Associated Powers" (Q. Wright, 1942, Table 42) contains the names of Peru and Rumania. But if the fightings of Britain with Argentine and with Afghanistan had happened to coincide in time, as they nearly did in 1847 and 1848, they would presumably have been called two separate wars of type 1 versus 1, not a single war of type 2 versus 1. The answer thus seems to be that: *widely separated local belligerents are regarded as allies only if two or more world-powers are fighting each other and one at least of them is allied to both those local belligerents.* . . . . . (8)

In this matter the mathematics must conform to the mental habits of historians.

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The existence of States which could not possibly be at war with one another makes vast changes in the distribution of mutually exclusive wars. According to Theory V there could be war inside any pair, and the number of mutually exclusive wars of the type r versus s was therefore

$$A(r, s) = \tfrac{1}{2}\frac{m!}{r!\,s!\,(m-r-s)!}.$$

In particular, for $m = 60$ States in the world,

$$A(30, 30) = \tfrac{1}{2}60!/(30!)^2 = 5{\cdot}9 \times 10^{16}.$$

It is conceivable that, in the future, aviation may make these $5{\cdot}9 \times 10^{16}$ varieties of all-in war each possible. In Theory VIII the total duration of all such wars was made negligibly small by a factor p^{900}. In history no such war has ever occurred. On the present Theory XI not a single one of them was possible, because of zero values of p. So, in one way or another, all-in wars are satisfactorily dismissed. The great advantage of Theory XI is that it, unlike Theory X, admits the occurrence of wars such as those of 1914–18 and 1939–45, which were world-wide, although not all-in.

Let us first consider the *number of mutually exclusive possible wars*, alias "varieties," belonging to each of the subtypes of 1 versus 1.

Subtype 1, 0 *versus* 1, 0, that is a war between two world-wide sea-Powers. There are $\frac{1}{2}\alpha(\alpha - 1)$ varieties.

Subtypes 1, 0 *versus* 0, 1 *and* 0, 1 *versus* 1, 0 *jointly*, that is any war between a world-wide sea-Power and a local coastal Power. Of these there are $\alpha\beta$ varieties.

Subtype 0, 1 *versus* 0, 1, a purely local war. The first belligerent can be selected in $\beta + \gamma$ ways. This having been done, the second belligerent can, according to the assumption (3), be selected in five ways, not in $\beta + \gamma - 1$ ways. This manner of selection takes every pair twice. Thus there are $\frac{5}{2}(\beta + \gamma)$ varieties. We have further assumed that $\beta + \gamma = 55$, so making $137\frac{1}{2}$ varieties, which can be rounded off to 138, as the half is absurd. The reason for the half is that the assumption (3), that each State has five neighbours, though a reasonable approximation to the actual geography, is not exact; and that moreover there are some necessary logical relations, connected with Euler's theorem and sea-coasts, which would be too complicated to introduce into this theory.

Each of these mutually exclusive varieties involves one pair of belligerents with probability x or y or z. With reference to any war, let us use the phrase "outward probability" to denote the probability that the belligerents remain at peace with all the neutrals. These outward probabilities were found to need close attention, thus:

Subtype 1, 0 *versus* 1, 0. The two belligerents form $2(\alpha - 2)$ pairs in peace with the remaining $\alpha - 2$ long range sea-Powers. The probability of peace is $1 - x$ in each such pair, and so is $(1 - x)^{2\alpha-4}$ for the $2\alpha - 4$ pairs jointly. Moreover, according to principle (8), any or all of the β local coastal Powers could join in this war; they form 2β pairs with the belligerents; and the probability that there is peace inside all these pairs is $(1 - y)^{2\beta}$. The outward probability is thus $(1 - y)^{2\alpha-4}(1 - y)^{2\beta}$.

Subtypes 1, 0 *versus* 0, 1 *and* 1, 1 *versus* 1, 0. The long-range sea belligerent forms $\alpha - 1$ pairs with other Powers of its own kind, thus giving a factor $(1 - x)^{\alpha-1}$. The local Powers which might, but have not, joined the war are the five neighbours of the local coastal Power, and no others. It is assumed that so long as they are not in the war, the possibility of land conduction of more distant powers does not arise . . . . . (9)

There is accordingly a factor $(1 - z)^5$. The index of $(1 - y)$ is the sum of two parts. One part is the $\alpha - 1$ pairs formed by the local belligerent and the $\alpha - 1$ long-range neutrals. The other part is the five pairs formed between the long-range belligerent and the five neighbours of the local belligerent. If land-conduction were rejected this part of the index would be less than five because probably some of those neighbours have no sea-coast. On collecting factors we obtain $(1 - x)^{\alpha-1}(1 - y)^{\alpha+4}(1 - z)^5$ for the outward probability.

Subtype 0, 1 *versus* 0, 1. The countries of the two local belligerents may be imagined as irregular pentagons having a side in common. A remarkable fact of political geography is that there is practically never a point where more than three States meet. So the eight peaceful frontiers of the belligerents touch six, not eight, neighbouring States. According to the principle (7) of conduction by land, any one of those six neighbours might be, but is not, at war with either bel-

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ligerent, thus giving a factor $(1 - z)^{12}$. The number of pairs formed between the two local belligerents and the long-range neutrals is 2α; but for a few of these pairs the probability of war is zero. To admit the possibility that a distant Power might attack either of the belligerents by sea, it suffices to suppose that *one* of the belligerents has a sea coast; for the other could then be attacked by land-conduction. With accuracy sufficient for a minor correction we may take the probability that neither belligerent has a sea-coast to be $\gamma^2/(\beta + \gamma)^2$. The index of $1 - y$ is accordingly $2\alpha\{1 - \gamma^2/(\beta + \gamma)^2\}$. The outward probability for this subtype is therefore

$$(1 - y)^{2\alpha\{1-\gamma^2/(\beta+\gamma)^2\}}(1 - z)^{12}.$$

All these results are collected in the following table.

Subtypes of 1 versus 1 (10)

Subtype	Mutually exclusive varieties	Probability of each variety	Expectation * of subtype when $\alpha = 5,\ \beta = 44,\ \gamma = 11$
1, 0 versus 1, 0	$\frac{\alpha(\alpha-1)}{2}$	$x(1-x)^{2\alpha-4}(1-y)^{2\beta}$	$10x(1-x)^6(1-y)^{88}$
1, 0 versus 0, 1	$\alpha\beta$	$y(1-x)^{\alpha-1}(1-y)^{\alpha+4}(1-z)^5$	$220y(1-x)^4(1-y)^9(1-z)^5$
0, 1 versus 1, 0			
0, 1 versus 0, 1	$\frac{5}{2}(\beta+\gamma)$	$z(1-y)^{\frac{2\alpha\beta(\beta+2\gamma)}{(\beta+\gamma)^2}}(1-z)^{12}$	$2\frac{7}{2}z(1-y)^{9\cdot6}(1-z)^{12}$

* By "expectation" is here meant the total theoretical duration divided by the historical interval, when the interval is very long. This expectation can exceed unity, and so is not a probability.

The theoretical expectations in the last column of the above table were equated to the observed fractions of 110 years taken from Collection J for the respective subtypes. One of these fractions is greater than unity. Thus were obtained the following set of three equations, simultaneous in x, y, z.

$$x(1-x)^6(1-y)^{88} = 0{\cdot}00136 \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$
$$y(1-x)^4(1-y)^9(1-z)^5 = 0{\cdot}00156 \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$
$$z(1-y)^{9\cdot6}(1-z)^{12} = 0{\cdot}00730 \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

One root of these equations is

$$x = 0{\cdot}00159, \quad y = 0{\cdot}00166, \quad z = 0{\cdot}00819 \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

It was found by two sequences of approximations which approached it from opposite sides. The method resembled that already explained for the single equation $p^b(1 - p)^c = a$.

Considerable changes in the outward probabilities make only slight changes in the position of this root. For example, if the hypothesis (7) of land-conduction be rejected, the brackets in (13) become $(1 - y)^8(1 - z)^9$, and instead of (14) the root becomes $x = 0{\cdot}00159$, $y = 0{\cdot}00165$, $z = 0{\cdot}00795$. This change is within the uncertainty of the historical facts. We shall meet a quite different relation for more complicated wars.

There remains the question whether the present set of three equations has any other root of historical significance. The search for such a root was conducted graphically, by regarding x, y, z as rectangular co-ordinates, so that each equation represents a surface, and any point common to all three surfaces represents a root. Because x, y, z are probabilities, we are not concerned with anything outside the cube bounded by their values 0 and 1. The first equation represents a cylinder which lies in the range $0 < y < 0{\cdot}041_5$. The third equation represents another cylinder having its generators at right angles to those of the first. These cylinders cut one another in two separate arcs, which lie in the following ranges: $0{\cdot}0081 < z < 0{\cdot}014$ and $0{\cdot}21 < z < 0{\cdot}26$. Any z in the latter range is incredible, for it would assert that every local Power was fighting each of its five neighbours for more than 0·21 of the historical interval. In the former range of z one credible root has already been found. The search for another was pursued by plotting the section, by the plane $z = 0{\cdot}014$, of the surface represented by the middle equation. It thus appeared that there may be a root near $z = 0{\cdot}4_8$; but if so it is of no interest.

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The only root of historical significance is that stated in (14).

A searching test of Theory XI will be to notice whether the same numerical values of the probabilities x, y, z, explain the occurrences of wars of various types. We shall come to that later.

In the meantime, and for the type 1 versus 1, z is nearly five times x. That is to say, *the world wide sea-Powers made much less pugnacious use of their opportunities for contact than did the local Powers.* The statistics here gathered do not afford any explanation; but on other grounds alternative explanations may be tentatively suggested, thus: (i) Local contact existed all the time, whereas distant contact by sea could be made or not, as desired by the sea-Power. The more persistent contact, the more opportunity for quarrels. (ii) A nation's capacity for fighting is bounded. Nations having very many contacts are likely to have been satiated with fighting; and therefore to have been, to that extent, peace-loving.

Leaving aside surmises, such as (i) and (ii), let us return to more definite investigations. The statistical analysis which was applied above to wars of the type 1 versus 1, has been extended to the types 2 versus 1 and 1 versus 2, jointly. As no new geographical principles are involved, it may suffice to state the results in tabular form. The broad second column shows the theoretical formula for the expectation. A few of the indices of $1 - y$ and of $1 - z$ are averages over slightly different cases. The third column contains the historically observed fraction of 110 years taken from Collection J. The fourth column headed "numerical expectation," will be explained presently.

Subtypes of (2 versus 1) and (1 versus 2) (16)

Subtype	Expectation of subtype	Historical fact	Numerical expectation
2, 0 versus 1, 0 1, 0 versus 2, 0	$\frac{\alpha!}{2!(\alpha-3)!}x^2(1-x)^{3\alpha-9}(1-y)^{3\beta}$	0·0000	0·0000
2, 0 versus 0, 1 0, 1 versus 2, 0	$\frac{\alpha(\alpha-1)\beta}{2}y^2(1-x)^{2\alpha-4}(1-y)^{\alpha+8}(1-z)^5$	0·0227	0·0734
1, 1 versus 1, 0 1, 0 versus 1, 1	$\alpha(\alpha-1)\beta xy(1-x)^{2\alpha-4}(1-y)^{\alpha+2\beta-4+\frac{10\gamma}{\beta+\gamma}}(1-z)^5$	0·0023	0·0012
1, 1 versus 0, 1 0, 1 versus 1, 1	$5\alpha\beta yz(1-x)^{\alpha-1}(1-y)^{2\alpha+4}(1-z)^{12}$	0·1573	0·2093
0, 2 versus 1, 0 1, 0 versus 0, 2	$\frac{5\alpha\beta}{2}y^2(1-x)^{\alpha-1}(1-y)^{2\alpha+\frac{13}{3}}(1-z)^{\frac{38}{3}}$	0·1436	0·0770
0, 2 versus 0, 1 0, 1 versus 0, 2	$10(\beta+\gamma)z^2(1-y)^{3\alpha-3\alpha\gamma^2/(\beta+\gamma)^3}(1-z)^{21}$	0·1323	0·1190
	Totals	0·4582	0·4799

If we equate each expression in the second column to the historical number in the third column and put $\alpha = 5$, $\beta = 44$, $\gamma = 11$ as before, we obtain six equations simultaneous in x, y, z; and, of course, somewhat inconsistent.

As a preliminary let us consider the first equation by itself. The previously noted objection to large probabilities is here illustrated in an extreme form. For we could explain why the subtypes (2, 0 versus 1, 0) and (1, 0 versus 2, 0) never occurred in the history, by inserting $x = 1$ in the first row of the table. But $x = 1$ would also entail the utterly false assertion that each of the five sea-Powers was fighting each of the others all the time. Instead, therefore, we must suppose that x is zero or near to it.

The last equation, when taken alone, provides an under-estimate of z. For $0 < y < 1$, and therefore $z^2(1-z)^{21} > 10^{-4} \times 2{\cdot}405$; the solution of which is $z > 0{\cdot}019$.

The column headed "numerical expectation" was obtained by inserting

$$x = 0{\cdot}0004,\ y = 0{\cdot}0150,\ z = 0{\cdot}0200 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

into the formulæ for the theoretical expectation. These numbers are a compromise, intended to fit all six of the simultaneous equations tolerably. They were obtained by trial and adjustment after the manner of the successive approximations explained on page 139, except that at each stage inconsistent results had to be averaged. The product moment correlation

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coefficient between the last two columns is 0·78. The chief obstacle to making the correlation nearer to unity is the disagreement between the two equations in y^2. In the history, two local Powers were fighting one sea-Power for about six times longer than one local Power was fighting two sea-Powers. In the theory the expectations of these subtypes are roughly equal. It is too soon to say whether this misfit is of political interest, or is merely an extreme effect of sampling. The distribution of wars in time has been shown to be described by the Poisson law (Richardson, 1935), a law which also describes the emission of alpha particles from radioactive substances (Rutherford, Chadwick and Ellis, 1930). That is to say, the historical facts are to be regarded as random samples, when wars are counted as wholes. The present detailed classification is likely to have increased the uncertainty of sampling. In this manner the misfits between the last two columns of the above table may be tentatively excused. Whoever says that these results are rough, should compare them with our previous blank ignorance.

The excess of z over x, which we noted for type 1 versus 1, is increased for the types 2 versus 1 and 1 versus 2. The political meaning of this difference in probability has already been discussed.

Both the probabilities y and z are greater for wars involving two pairs of opposed belligerents than for wars involving only one pair. This new phenomenon makes it desirable to study wars involving more than two pairs, in order to notice whether in them the probabilities are further increased.

Although the geographical principles are the same as before, one crucial piece of argument is worth stating. It relates to the subtypes (0, 3 versus 0 1) and (0, 1 versus 0, 3), jointly. The isolated belligerent can be chosen in $\beta + \gamma$ ways. The mutually exclusive varieties can then be classified according to the number of direct contacts which the isolated belligerent A has with its three enemies B, C, D. The statements which follow were obtained by sketching pentagons, with attention to the fact (1) that not more than three States meet at any point.

	Varieties $(\beta + \gamma)x$	Neutral neighbours	
A touches B, C, and D. There are two subclasses:			
One of B, C, D touches A, but not the other two ...	5	7	
The remaining subclass	5	6	
A touches two only of B, C, and D:			
These two touch one another	10	7	(18)
	5	6	
These two do not touch one another	20	8	
A touches one only of B, C, and D:			
This one touches the other two	5	7	
The remaining subclass	20	8	
Total	70		

The mean number of neutral neighbours, weighted by the number of varieties, comes to 52/7. Any one of the four belligerents might have been at war with 52/7 neighbours on the average. Therefore the outward probability contains a factor $(1 - z)^{208/7}$, or practically $(1 - z)^{30}$. When a similar argument was applied to the subtypes (1, 2 versus 1, 0) and (0, 3 versus 1, 0), some slight ambiguities appeared. It seems that the geographical assumptions (1), (3), (5), (7), (8) do not quite fix the expectation for these subtypes, although they fix it satisfactorily for all the others so far considered. The ambiguity is not important; no easy device for avoiding it appeared; and so it was passed over by a factor $\beta + \varepsilon\gamma$ where ε is undetermined except that it lies in the range $0 < \varepsilon < 1$. The results are set out concisely in the table on the following page.

As before, let us assume that $\alpha = 5$, $\beta = 44$, $\gamma = 11$, and let us try to equate the theoretical expectations in the second column to the historical facts in the third column. The latter are the fractions of 110 years during which the subtypes occurred, according to Collection J. Although the eight equations so obtained are strictly inconsistent, it is not difficult to obtain a useful compromise solution, starting from the presumption that x, y, z, have values resembling those found from the wars of 2 versus 1 and 1 versus 2. Then the last equation almost fixes z; and there are two pairs of equations which almost fix y/z consistently. The column which is headed "numerical expectation" in the above table contains the results of inserting

$$x = 0{\cdot}001,\ y = 0{\cdot}009,\ z = 0{\cdot}046 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

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into the formulæ for the expectations. The product moment correlation coefficient between history and expectation over the eight subtypes of octantal 3 versus 1 then exceeds 0·99, which is satisfactory.

The successive increases of z from 0·008 to 0·020, and thence to 0·046, have so far accompanied increases of both $r + s$ and rs. There is accordingly a special interest in extending the investigation to the subtypes of 2 versus 2; for thereby $r + s$ will remain at 4, while rs will increase from 3 to 4. The most frequent of these subtypes in Collection J is the purely local 0, 2 versus 0, 2. The number of its mutually exclusive varieties can be deduced from the corresponding number for the subtypes (0, 3 versus 0, 1) and (0, 1 versus 0, 3), jointly, a number which we have seen to be $70(\beta + \gamma)$.

For consider any bunch of four countries touching one another, so that the conduction of troops by land would be possible. A solitary belligerent, opposed to the remaining three, can be selected from them in four ways; but a pair of allies, opposed to the remaining two, can be selected in $4 \times 3/2! =$ six ways. Therefore the number of mutually exclusive varieties of wars of the

Subtypes of (3 versus 1) and (1 versus 3) (19)

Subtype	Expectation of subtype	Historical fact	Numerical expectation
3, 0 versus 1, 0 1, 0 versus 3, 0	$\frac{\alpha!}{3!(\alpha-4)!}x^3(1-x)^{4\alpha-16}(1-y)^{4\beta}$	0·0000	0·0000
3, 0 versus 0, 1 0, 1 versus 3, 0	$\frac{\alpha!\beta}{3!(\alpha-3)!}y^3(1-x)^{3\alpha-9}(1-y)^{\alpha+12}(1-z)^5$	0·0000	0·0002
2, 1 versus 1, 0 1, 0 versus 2, 1	$\frac{\alpha(\alpha-1)(\alpha-2)\beta}{2}x^2y(1-x)^{3\alpha-9}(1-y)^{\alpha+3\beta-6}(1-z)^{\beta-1}$	0·0068	0·0000
2, 1 versus 0, 1 0, 1 versus 2, 1	$\frac{5\alpha(\alpha-1)\beta}{2}y^2z(1-x)^{2\alpha-4}(1-y)^{2\alpha+9}(1-z)^{13}$	0·0036	0·0037
1, 2 versus 1, 0 1, 0 versus 1, 2	$\frac{5\alpha(\alpha-1)\beta}{2}xy^2(1-x)^{2\alpha-4}(1-y)^{2\alpha+2\beta-2}(1-z)^{2\beta+2}$	0·0000	0·0000
1, 2 versus 0, 1 0, 1 versus 1, 2	$20\alpha(\beta+\epsilon\gamma)yz^2(1-x)^{\alpha-1}(1-y)^{3\alpha+4}(1-z)^{21}$	0·0255	{ 0·0261 to 0·0327
0, 3 versus 1, 0 1, 0 versus 0, 3	$5\alpha(\beta+\epsilon\gamma)y^3(1-x)^{\alpha-1}(1-y)^{3\alpha+\frac{11}{3}}(1-z)^{20}$	0·0000	0·0003
0, 3 versus 0, 1 0, 1 versus 0, 3	$70(\beta+\gamma)z^3(1-y)^{4\alpha}(1-z)^{29\cdot71}$	0·0773	0·0772
	where $0 < \epsilon < 1$		

subtype 0, 2 versus 0, 2 is $\frac{6}{4} \times 70(\beta + \gamma) = 105(\beta + \gamma) = 5775$. The expectation of subtype 0, 2 versus 0, 2 is accordingly $5775z^4(1 - y)^{20}(1 - z)^{29\cdot71}$. In a previous case we obtained a lower bound to z by omitting the factor $1 - y$ and equating the rest of the expectation to the observed fraction of 110 years, which here is 14·2/110. But, just in this particular, Collection J may be an aberrant sample, because 11 of those 14·2 years come from a single war (Rosas) which Q. Wright classifies differently. The same contrast occurs between Collections G and H. Accordingly, to make sure of obtaining an underestimate of z, let us omit the eleven years and put

$$5775z^4(1 - z)^{29\cdot71} = 3\cdot2/110 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

This equation just misses having any real root in the range $0 < z < 1$. But if the observed duration had been 2·96 years instead of 3·2 years, as it well might, there would have been two coincident roots at $z = 0\cdot119$. We cannot escape from the conclusion that z is here larger than it was for wars of lesser rs.

Summary on Theory XI. In a general way Theory XI explains the rare occurrence of very complicated wars such as that of 1914–18; but in such a case detailed calculations would be too troublesome. The application has been made to wars involving not more than four nations. The probability, x, of fighting between two world-wide sea-Powers is found to be of the order of $x = 0\cdot001$; but is not sharply determined, because such wars have been rare. The probability, y, of fighting between a world-wide sea-Power and a local Power is found to have values $y = 0\cdot002$, 0·015, 0·009, according to circumstances. These look like sampling variations around a mean of $y = 0\cdot009$, which is distinctly greater than x. The probability, z, of fighting between two local

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Powers is more closely determined than either x or y, because there have been many local wars. It is plain that z considerably exceeds x. That is to say, the world-wide sea-Powers have made much less pugnacious use of their opportunities for contact than have the local Powers. Theory XI assumes that z is the same in all circumstances, but when Theory XI is combined with geographical and historical facts, a different result emerges, thus:—

Number, *rs*, of pairs of opposed belligerents	1	2	3	4
Probability, z, of war between neighbours	0·008	0·020	0·046	0·119
Compare the empirical formula $z = 0{\cdot}008\ (2{\cdot}549)^{rs-1}$ ...	0·008	0·020	0·048	0·119

Thus Theory XI fails, but in a manner so regular as immediately to suggest suitable improvements

Theory XII. Chaos restricted by geography and modified by infectiousness

Fighting is infectious. When it is known that a war has begun, neutrals begin to ask: How will this affect us? Are our interests endangered? What is the right and wrong of the dispute? Ought we to join in? On the contrary, in Theories VIII and XI it was assumed for simplicity that the probability of war between any two nations was not affected by the peacefulness or belligerency of other nations. I do not apologize for this assumption. A condition for discovery in applied mathematics is that the assumptions must at first be simple. But now the comparison between Theory XI and fact has shown definitely that the assumption is wrong, and has indicated the numerical amount of the required amendment. Theory XII is simply an acceptance of that amendment; by assuming that the probability, z, of war between a pair of local powers is to increase with the number of pairs of opposed belligerents in the manner indicated by the last table. It is unnecessary to deduce the consequences, for that, in effect, has already been done; and they have been found to agree with historical fact.

Looking back at Theory III we notice that some rather similar gregarious tendencies were there mentioned and rejected as an explanation. Certainly infectiousness alone will not explain the facts; chaos alone is nearer the mark; it is only as a modifier of a geographical restricted chaos that infectiousness is admitted into Theory XII.

Theory XIII. A uniform chaos modified only by infectiousness

Although Theory XII has attained agreement with history, we may for logical satisfaction wish to see whether we could do as well by simpler hypotheses, retaining chaos and infectiousness, but ignoring geography, as aviation may make it ignorable in the future.

Let us suppose that the trouble begins with the existence of a world-wide controversy, for example as to the relative merits of communism and individual enterprise. Let our elementary probability be denoted by θ, not p, because here, as in Theory X, it is the probability of war about a dispute and not a fraction of the time. In contrast with Theory X, let us now suppose that θ increases with the number κ of pairs of opposed belligerents who are already fighting about the dispute, so that we write θ_κ. Also in contrast with Theory X, let us consider a whole world of m nations, instead of a local group of l. Let the expectation of a war of the type r versus s be denoted by $V(r, s)$ in the quadrantal plan. As in Theory X, we are concerned only with the ratios of V for different r, s. By modifying Theory VIII it is easily seen that

$$V(1, 1) = \frac{m(m-1)}{2}\theta_0(1-\theta_1)^{2(m-2)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

which differs from Theory VIII by involving θ_1 and θ_0. If another nation joins the war it does so with probability θ_1 and then, for the neutrals, the probability rises to θ_2. Thus the expectation of a war of the quadrantal type 2 versus 1 is

$$\tfrac{1}{2}\frac{m(m-1)(m-2)}{2}\theta_0\theta_1(1-\theta_2)^{3(m-3)} = V(2, 1) \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

A fourth nation can join the war in either of two ways. It may join the pair of allies making a war of 3 versus 1 with expectation, quadrantally

$$\tfrac{1}{2}\frac{m!}{3!1!(m-4)!}\theta_0\theta_1\theta_2(1-\theta_3)^{4(m-4)} = V(3, 1) \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Or the fourth nation may join the solitary belligerent, thus forming *two* new pairs of opposed belligerents; so that the expectation of a war of 2 versus 2 is

$$\tfrac{1}{2}\frac{m!}{2!2!(m-4)!}\theta_0\theta_1\theta_2^2(1-\theta_4)^{4(m-4)} = V(2, 2) \quad . \quad . \quad . \quad . \quad . \quad (4)$$

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Similarly

$$V(4, 1) = \tfrac{1}{2}\frac{m!}{4!\,1!\,(m-5)!}\theta_0\theta_1\theta_2\theta_3(1-\theta_4)^{5(m-5)} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

From (4) and (5) we obtain

$$\frac{V(4, 1)}{V(4, 1)} = \frac{1}{6(m-5)}\frac{\theta_3}{\theta_2}(1-\theta_4)^{m-9} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

whence, if $m = 60$,

$$\theta_3/\theta_2 > 330V(4, 1)/V(2, 2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

but the observed values of $V(4, 1)/V(2, 2)$ are 5/6 from Collection A, 6/4 from Collection C. Whence

$$\theta_3/\theta_2 > 275 \quad \text{or} \quad > 495 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

These ratios are incredibly great. *Thus the introduction of infectiousness alone will not serve. We are also constrained to reduce the number of nations by attention to geography.* Theory XIII is an anticlimax. We are directed back to Theory XII.

Acknowledgments

Mr. Oliver M. Ashford, B.Sc., started all this enquiry by sorting the author's card-index into a form which has now become Collection J.

The author wishes to thank the Government Grant Committee of the Royal Society for the loan of a calculating machine.

References

E refers to the *Encyclopædia Britannica*, 14th edition.
Hinshelwood, C. N. 1940. *The Kinetics of Chemical Change*. Clarendon Press, Oxford.
Jeans, J. H. 1916. *The Dynamical Theory of Gases*. Cambridge University Press.
Kendall, M. G. 1943. *The Advanced Theory of Statistics*. Griffin, London.
Kirkpatrick, F. A. 1938. *Latin America*. Cambridge University Press.
Priestley, D. I. 1938. *France Overseas*. Appleton–Century Co., New York.
Richardson, L. F. 1941. Letter to *Nature* of November 15th.
——. 1945. *J. Roy. Stat. Soc.*, **107**, 242–250.
——. 1946. *Proc. Roy. Soc.*, A, **186**, 422–31.
Rutherford, Chadwick and Ellis. 1930. *Radiations from Radioactive Substances*. Cambridge University Press.
West, R. 1942. *Conscience and Society*. Methuen, London.
Wright, Q. 1942. *A Study of War*. Chicago University Press.

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On the Interpretation of Within and Between Class Analysis of Variance When the Intra Class Correlation is Negative

By J. O. Irwin

The classical interpretation of within and between class analysis of variance (1) assumes the variate to be the sum of two portions $(x + y)$. x remains constant within a class and has variance A from class to class. One observation of x and k observations of y are assumed to be made in each class. The k values of y are independent and have variance B. The intra class correlation is then given by $\rho = A/(A + B)$.

It is easily shown that if s_1^2 and s_2^2 are respectively the between and within class mean squares in the analysis of variance (with d.f. $(n-1)$ and $n(k-1)$ for n classes) then s_1^2 and s_2^2 are unbiased estimates of $(kA + B)$ and B respectively. The components of variance A, B are then estimated by

	Value	Estimate
Between classes	A	$(s_1^2 - s_2^2)/k$
Within classes	B	s_2^2
Total	$A + B$	$(s_1^2/k) + s_2^2(1 - 1/k)$

This interpretation is satisfactory if A is positive and s_1^2 significantly greater than s_2^2, but if ρ is negative and s_2^2 is significantly greater than s_1^2 it breaks down. For if A is a variance it cannot be negative and ρ must be positive.

The following interpretation meets both the cases when ρ is positive and when ρ is negative.

The variate is assumed to be the sum of two portions $(x + y)$. x has variance V and the k values of y in each class are samples from populations with variance σ^2. They are however correlated with correlation coefficient ρ_1. The variance of $x + y$ is then $V + \sigma^2$ and

$$\text{cov}(x + y_1)(x + y_2) = V + \rho_1\sigma^2 \quad \text{so that} \quad \rho = (V + \rho_1\sigma^2)/(V + \sigma^2).$$

Write $A = (V + \rho_1\sigma^2)$, $B = \sigma^2(1 - \rho_1)$ then $\rho = A/(A + B)$

as before and s_1^2, s_2^2 are still estimates of $(kA + B)$ and B, for the expectations of s_1^2 and s_2^2 are given by

$$E(s_1^2) = kV + \sigma^2(1 + (k-1)\rho_1) = kA + B$$
$$E(s_2^2) = \sigma^2(1 - \rho_1) = B.$$

Since we are interested in the case when ρ_1 is negative we write

$$\rho_1 = -\rho_2 \qquad \rho_2 > 0$$

and find

$$\rho = -(\rho_2\sigma^2 - V)/(V + \sigma^2)$$

while the components of variance are

	Value	Estimate
Between classes	$-(\rho_2\sigma^2 - V)$	$-(s_2^2 - s_1^2)k$
Within classes	$\sigma^2(1 + \rho_2)$	s_2^2
Total	$V + \sigma^2$	$(s_1^2/k) + s_2^2(1 - 1/k)$

We note that it is impossible to estimate ρ_2, σ^2 and V separately, but since

$$1/(k-1) > \rho_2$$

we have

$$\sigma^2/(k-1) > \rho_2\sigma^2 > V.$$

If k is at all large V must be quite small compared with σ^2 and ρ_2 quite small.

An actual example of this occurred in spectrophotometric tests of vitamin B_1 (2). The extinction coefficients are measured by means of exposure of a photographic plate and duplicate determinations are made on each plate by a matching process. If matchings are made by eye it appears there is a tendency for the second determination to be high if the first one is low and vice versa. Four

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workers using different instruments carried out duplicate determinations on a number of sets of two plates.

There were differences between sets due to differences of wave length, solution used and thickness of cell employed but we are not concerned with these here.

One worker used a photoelectric method of matching and the other three matched by eye. For the first worker s_1^2 was significantly greater than s_2^2, for the second the contrary was the case, for the other two there was no significant difference.

The results for the first two workers are worth quoting. There were 28 sets of two plates covering seven different wave lengths, 1 cm. and ½ cm. cells and aqueous and alcoholic solutions.

The analyses of variance are as follows :—

	Worker (1)			Worker (2)		
	S. of S.	D.F.	Mean Sq.	S. of S.	D.F.	Mean Sq.
Between plates	9·7072	28	*0·3467*	5·8325	28	0·2083
Within plates	2·2624	56	0·0404	33·3854	56	*0·5962*
	11·9696	84		39·2179	84	

For worker (1), who used a photoelectric method of matching, s_1^2 is significantly greater than s_2^2; for worker (2), who matched by eye, the reverse is the case.

The components of variance are as follows :—

	Worker (1)	Worker (2)
A	0·1531	— 0·1939
B	0·0404	0·5962
Total plate error	0·1935	0·4023
S.D.	0·4398	0·6343
Mean	34·78	33·29
C.V.	1·27%	1·91%

For worker (1) we may assume ρ_2 to be zero; *A* then measures the plate error excluding matching error and *B* measures the matching error and is about 20% of the whole. For worker (2), total plate error is less than the matching error. This is because the matching error of a single observation is swollen by the negative correlation between observations on the same plate. But this correlation does not affect the variance of observations one on each plate. It is also worth noting that while the matching error of a single observation is $\sigma^2(1 + \rho_2)$ that of the mean of two observations is $\frac{\sigma^2}{2}(1 - \rho_2)$. The former is increased, the latter reduced, by the negative correlation.

References

(1) Fisher R. A. *Statistical Methods for Research Workers.* Section 40.
(2) Holiday, E. R., and Irwin, J. O. *Quart. J. Pharm. Pharmacol.* 1946. **19**.

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REVIEWS OF STATISTICAL AND ECONOMIC BOOKS

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3.—*Kuczynski* (*Jürgen*). A Short History of Labour Conditions under Industrial Capitalism. Vol. 3. Part I. 160

1.—*Industrial Experimentation.* By K. A. Brownlee, M.A., Directorate of Royal Ordnance Factories (Explosives), Ministry of Supply, London, H.M.S.O. 1946. 6″ × 9¾″. 116 pp. 2*s*.

This monograph was based on a memorandum produced by the Directorate of Ordnance Factories (Explosives), for the use of those concerned with pilot-plant and plant-scale experiments on chemical manufacturing processes. Practically all the standard text-books on practical statistics have been written for research workers in agriculture and biology, and there is certainly a need for a corresponding book dealing with the application of statistical methods to industrial experiments. Without such a book, the considerable scope for this application will not be developed.

The present monograph contains a description of most of the statistical tests and processes which industrial experimenters will require, but it cannot be recommended as an adequate treatment of the subject. The elementary statistical concepts receive scant treatment, and the book consists almost entirely of a description of the arithmetical aspect of statistical techniques. Incorrect application of statistical methods will often result from knowledge of this one aspect only. The author's treatment of the analysis of variance is particularly unsatisfactory. He has devoted approximately half of the book to particular cases, none of which presents any difficulties once the general procedure is understood. Few, if any, of the difficulties which may arise in the application of this technique are described. The emphasis throughout is on the estimation of components of variance, which is more relevant to sampling studies than to experimentation. Finally, it is surprising that a book on industrial experimentation should contain so little discussion of the design of experiments: as far as the reviewer could see, there was no mention of the necessity of randomization to ensure unbiased estimation of treatment effects and experimental error. O. K.

2.—*Intergovernmental Commodity Control Agreements.* Montreal, International Labour Office. 1943. 10″ × 6½″ lviii + 221 pp. 8*s*. *Commodity Control: a Study of Primary Products.* By P. Lamartine Yates. London, Jonathan Cape, 1943. 8½″ × 6″. 248 pp. 15*s*.

The first of these volumes is a collection of the texts of the existing intergovernmental commodity control agreements, and of extracts from the principal international pronouncements on commodity control policy. These are introduced by a preliminary discussion of some of the social issues which the control schemes present. In the footnotes to this introductory chapter there is a very useful select bibliography of the literature of control schemes in the English-speaking countries. This introductory chapter is the most significant part of the book, since it affords an indication of the state of thinking on this important subject among the officials of the I.L.O. The fact that they recognize that there is a lack of an authoritative statement of the principles which should guide such agreements, and that they point out that without an unambiguous statement of the objectives of control there will be no sufficiently clear principles to guide the policy of the control authorities and no adequate criterion for the evaluation of the results achieved (see pp. xxxv–xxxvi) makes one scan these pages the more eagerly. But one is disappointed. The objective is admirably stated: Control schemes must be fitted into a system designed " to assure that the fullest possible use is made of the resources of production, power and material, of the skill and enterprise of the individual, of available scientific discoveries and inventions so as to attain and maintain in all countries a stable economy and a rising standard of living " (see p. xxvi). But there is no hint that the authors of this report have grasped what transformations in economic institutions are required if this aim is to be pursued effectively. The purpose of commodity control schemes is, in their view, to assist the world economy in regaining " equilibrium " whenever the emergence of surpluses of particular commodities indicates the absence of it. The guiding principles which they propose are merely designed to ensure that in the process of adjustment no harm is done to established consumer-interests and to the ideal of non-discrimination in commerce. They also advocate that the control-measures adopted should be co-ordinated with anti-slump policies, and that they should attempt, indirectly, to increase labour-mobility and diminish exploitation of unorganized labour.

The underlying tacit assumption which hampers the I.L.O. economists in coming to grips with the real problems of commodity control is that they still persist in the belief that full resource-utilization can be equated with the antiquated classical text-book notion of "equilibrium." But

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this notion has no counterpart in the real world, and a set of guiding principles based on the assumption that they have, is a guide to a dangerous futility.

The same error and the difficulties occasioned by it stand out very clearly in Mr. P. L. Yates's book, which contains most lucid and in many ways instructive chapters on the problems of wheat, sugar, coffee, cotton, rubber, tin, copper and mineral oil. The material for these chapters was originally collected by a group of Fabians—Dr. V. Anstey, Messrs. A. Clegg, O. Gross, M. K. Joliffe, J. Stone, B. and J. Vernon and Yates—but owing to the difficulties of group deliberation in wartime, Mr. Yates eventually became responsible for the recommendations at the end of each chapter, and for the general conclusions. These are invariably disappointing. One would expect that members of the Fabian Society would be inclined to relate problems of commodity production and control directly to the crying need for satisfying human needs on a more adequate scale throughout the world, and that this would assist them in emancipating themselves from laisser-faire dogmas when they deal with material of the kind which has been brought together in this book. But Mr. Yates—like the authors of the I.L.O. report—regards commodity problems as arising from the "maladjustment" of the supply and demand of certain commodities, which arise from the irresponsiveness of the supply of primary products to price-fluctuations in the face of changing demand. His diagnosis and his remedy are summed up by his dictum: "Our capacity to transfer will largely measure our capacity to progress" (p. 217). His criterion for the need to transfer is based, not on any assessment of the real needs of human consumption and productive development, but on the trend of effective demand under capitalism. Realism, in his view, requires for example that Great Britain should face up to the loss of her coal export markets, and should assist the transfer of labour out of coal mining, and although he notes in passing that in the past many Governments have felt that it would be morally wrong to curtail wheat production as long as there were millions of half-starved coolies in India and China, he is nevertheless for grappling "seriously" with the "problem of wheat surpluses" and praises the U.S. Government for having made an attempt at organizing the restriction of wheat production on a large scale under the New Deal (pp. 37, 215). All this makes curious reading.

Mr. Yates professes to be in favour of "expansionism." But this merely consists in advocating that the unsaleable surpluses produced by the capitalist economy should not be squandered if there are people in the world who are in need of them (p. 218). The real logic of this view is nowhere followed up. In the opening chapter of the book Mr. Yates shows himself conscious of the fact that the problem of primary products is closely bound up with large problems in the field of world trade, investment, migration and politics. But having said that he goes on to say, without proof and in the reviewer's view without justification, that the "main difficulties" in the realm of primary products can be discussed in isolation without detailed regard to these broader problems (see p. 15). It is his reliance on the textbook notion of "equilibrium" which enables Mr. Yates to maintain this unrealistic attitude. Correspondingly his only remedy for our troubles in this wider field is free trade, and he even takes the view that without it, "expansionism" cannot work (p. 219). When will economists in the English speaking world learn to go *behind effective* demand, and *money* cost, and when will they emancipate their thinking from that of the "stupid, bothering stockbrocker," Ricardo? J. K.

3.—*A Short History of Labour Conditions under Industrial Capitalism.* Vol. 3, Part 1. *Germany, 1800 to the Present Day.* By Jürgen Kuczynski. London, Frederick Muller, 1945. $7\frac{1}{4}'' \times 4\frac{3}{4}''$. 268 pp. 9*s.* 6*d.*

Mr. Kuczynski has now published not only this German companion volume to similar studies for Great Britain and the United States, but has also already published a sequel to it in his *Short History of Labour Conditions in Germany under Fascism.* A similar volume on France is stated to be in preparation. In the present book under review he has rendered valuable service by extracting wage data for Germany which were previously tucked away in almost inaccessible pamphlets and monographs. These wage data are presented in special appendices to the first three chapters. They are useful raw material for further analysis, all the more since Mr. Kuczynski's own inferences from these data are not always acceptable. He is quite undaunted by previous criticism of his treatment of wage data, and again arrives at the startling conclusion (so contrary to other evidence and common observation) that a century or more of capitalistic development has left the industrial worker hardly better off than in 1820.

This result is partly reached by the introduction of such an intangible factor as "intensity of work" into the statistical analysis. With this the author endeavours to prove that "intensity of work" has steadily increased so as to wipe out most of the apparent improvements. His main evidence is an increase in accident rates—but surely there is a simpler explanation of that which makes them useless as evidence of greater strain. The more machinery there is the greater the chances of getting hurt. Is that too simple to be true?

Similarly, the increased incidence of ill-health under the Insurance Scheme is very poor evidence —except of a greater willingness for workers to pay attention to illness and prolong convalescence— evidence of rising standards of living, surely, not of the contrary. Moreover, the author seems

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to "have it both ways" by explaining the decline of reported ill-health during the 1929–1933 depression as "not really an improvement but the growing fear among workers of losing their jobs" (p. 251). That may be true, but what light does that throw on the previous increase?

This desire to "have it both ways" is evident throughout the book. If a lengthening of working hours is a symptom of "exploitation," a shortening is also a symptom of a different kind of exploitation ("intensive exploitation"). In fact, the word "exploitation" loses all its meaning, except the colourless one of "use of labour."

In general, the author's attitude is not, in the opinion of the reviewer, truly scientific. In the introduction he writes (p. 8): "It is obvious that if I had written this volume in Germany before Fascism came to power I would have had more material at my disposal, although I do not believe that any of the statistical conclusions would have been affected." How can one tell in advance whether additional material would affect one's conclusions or not? The purpose of the analysis is, in fact, stated with disarming frankness as "to fill some less important gaps" in the Marx–Engels analysis (p. 109).

The book raises other problems of statistical presentation. The term "wages" is used throughout, and the reader is never certain how far the figures refer to wage rates, average hourly earnings, weekly or yearly figures. At first one is under the impression that one deals with earnings, only suddenly to find (p. 75) that at least one wage series used is one of piece rates. Wage statistics are notoriously ambiguous, and lack of definition makes them hopeless. Nor does the text to the tables always agree with the figures; for instance, it is said that "there was probably little change in the period under review (1814–23 to 1844–53) in the frequency and severity of accidents" yet the table on which this statement is based shows a drop in man-days lost by accidents of over 25%.

When the author combines his results of movement of real wages with output statistics the result is utterly unbelievable. Thus it is argued that real wages fell relative to the national output by two-thirds in the fifty years to 1909–14, with a further drop since. We prefer to think that there is something wrong with wage figures leading to such a conclusion.

Mr. Kuczynski mentions the case of the Prussian statistician who had to combine his thesis of steady economic progress with due reverence for the achievements of the ruling monarch's ancestors. One feels that he himself is in a similar predicament—having to square his thesis of lack of improvement and an early "golden age" with a reverence for facts and figures.

It will be clear from this review that this is in many ways an irritating book. Borrowing the famous *dictum* of Clausewitz' one might call the author's use of statistics a "continuation of the class struggle by other means." The main value of the book would seem to be in the data presented for economic analysis, and also in the problems raised for the economic historian, especially about the alleged "golden age" of the 1820's and the extraordinary rapidity of economic progress, in Germany, during the 1850's.

H. W. S.

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STATISTICAL NOTES

In Part I of the *Journal* we gave, on pages 79–81, tables summarizing the oversea trade of the United Kingdom, for each class and group distinguished in the statistics, for the years 1938 to 1945. We now bring the story up to September 1946 in the following summary table; the figures for each of the first three quarters of the year are shown, together with the comparable figures for the second half of 1945 and for 1938.

The adverse balance shown by the official statistics amounted to £82 million in the first quarter of 1946 and £89 and £75 million in the second and third quarters. These figures rather underestimate the real adverse balance. Recorded exports include shipments abroad by N.A.A.F.I. as well as exports of relief and rehabilitation supplies (mostly U.N.R.R.A.), and in neither case does foreign exchange accrue from the export. These special exports were recorded separately in the official statistics as from June 1946, and amounted to £4·4 million (N.A.A.F.I.) and £6·7 million (U.N.R.R.A.) in the third quarter. There is also the rather complicated question of diamonds, though this has the opposite effect on the adverse balance as recorded. Almost all the diamonds (which are essentially for industrial uses) imported this year have been—for the first time—included in the statistics, but exports and re-exports of diamonds are essentially unrecorded; the value of diamonds included in the import figures amounted to under £1 million in the first quarter, and about £4 and £1½ million, respectively, in the second and third quarters.

Total imports in the first nine months of 1946 amounted to £925·7 million, an increase of one-third on the 1938 rate; average values, however, were about double those in 1938, the index-numbers for the individual quarters being 195, 206 and 209. Re-exports were still below the 1938 rate even by value, and were lower by three-fifths in terms of volume. Allowing for the rise in average values, the volume of retained imports in each quarter was 63, 69 and 70 per cent. of 1938. The bulk of the rise in volume was in the first half of the year. Comparing with the average for July–December 1945, the increase in retained imports of food, drink and tobacco in the first quarter (£29·1 million) exceeded that in the total by £1 million, but this class accounted for only about half the increase

£ million

Movements and Classes	1938	July–Dec. 1945	1946		
	Quarterly average		1st. Quarter	2nd. Quarter	3rd. Quarter
Retained Imports, c.i.f.					
Food, drink and tobacco	104·5	106·3	135·4	157·5	157·2
Raw materials and articles mainly unmanufactured	54·5	76·9	74·3	89·3	88·3
Articles wholly or mainly manufactured ...	53·8	49·9	48·4	54·0	63·8
Other articles	1·7	4·8	7·7	6·9	6·4
Total	214·5	237·9	265·8	307·7	315·7
United Kingdom Exports, f.o.b.					
Food, drink and tobacco	9·0	14·3	15·4	15·4	18·6
Raw materials and articles mainly unmanufactured	14·2	4·6	8·1	8·3	7·4
Articles wholly or mainly manufactured—					
Metal goods	43·9	34·8	73·4	98·7	105·0
Textiles	25·3	25·8	39·0	44·4	49·0
Other manufactures	22·1	24·4	40·0	46·5	53·3
Other articles	3·2	5·8	8·3	5·9	6·4
Total	117·7	109·7	184·2	219·2	239·7
Exports of Imported Merchandise, f.o.b. ...	15·4	13·5	12·3	10·9	13·2

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between the first and second quarters. Part of the increase in the first quarter was due to the general commencement of entry at the full c.i.f. valuation (as from last January) of foodstuffs previously imported under Lend-Lease arrangements, a substantial proportion of which had been recorded at their former Lend-Lease valuations in the final months of 1945. Imports were, moreover, low in the last quarter of 1945, partly as a result of the termination of the Lend-Lease and Mutual Aid agreements at the beginning of September. Among the different groups, the outstanding increase in the first quarter was in dairy produce (£18·0 million for July–December, 1945, as against 26·7, 29·3, and 31·5 million £'s in the three succeeding quarters). Retained imports of meat have been rising continuously since the end of the war; the quantity imported in the first nine months of 1946, however, was one-tenth lower than in 1938. Retained imports of miscellaneous foodstuffs, after rising from £20·2 million to £31·7 million between the first two quarters (as a result of abnormally large supplies of sugar from the Caribbean area), fell to £21·2 million in the third. There was a marked increase for fresh fruit and vegetables in the third quarter (£18·2 million, as against £7·2 million and £5·9 million in the first and second quarters). Part of the rise was due to seasonally high supplies of tomatoes from the Channel Islands, but there were also marked increases for bananas, oranges, and peaches and nectarines, while—for the first time since the end of the war—large supplies of Italian and Dutch grapes arrived in the third quarter. There is normally a marked seasonal movement in imports of tobacco, the figures for each quarter this year being 12·8, 16·7 and 9·0 million £'s, the low figure for the third quarter being mainly due to the import in July and August of the small final consignments of the 1945 American crop.

For raw materials, there was an increase of £15·0 million (one-fifth) between the first and second quarters, followed by a small reduction (£1·0 million) in the third quarter. Over half the rise in the second quarter was due to abnormally high imports of raw rubber from Malaya and Ceylon, together with larger supplies of synthetic rubber from the United States; retained imports in the second quarter were the highest on record and more than double those in 1938. The other large increase was in raw cotton (from £7·9 to £12·3 million), shipments from Brazil, the United States and Egypt all being more than double those in the first quarter; this was followed by a decline to £9·3 million in the third quarter, due partly to seasonal factors, and partly to abnormal re-exports which were twice as high as in 1938. Retained imports of timber were low in the first half of 1946, but showed a considerable improvement in the third quarter, these movements being essentially seasonal; imports of pitprops in the third quarter exceeded the 1938 average and were the highest for seven years, but hardwoods and softwoods were three-fifths below pre-war. Apart from rubber, the only other groups having a volume index higher than in 1938 in any quarter this year were hides and skins (index 163, 147 and 67 in each successive quarter), and iron ore and scrap (89, 115 and 116).

Retained imports of manufactured articles rose by £5·6 million (one-tenth) in the second quarter and by a further £9·8 million (one-sixth) in the third. The main group concerned was non-ferrous metals, retained imports of these being 4·1, 7·1 and 14·1 million £'s in the successive quarters (volume index-numbers 32, 53 and 132 per cent. of 1938); re-exports of copper and nickel, which were important before the war, have been small this year, so that the volume index for total imports was only 104 in the third quarter. The large rise in the third quarter was essentially due to the commencement of imports of aluminium under the Government's Canadian contract; the quarterly total—56,000 tons—exceeded the record average for 1943 and was nearly five times the 1938 figure. Imports of iron and steel have been rising each quarter; those of semi-finished steel in the third quarter (91,000 tons) were the highest since the end of 1944 and together with steel ingots (23,000 tons) exceeded the pre-war aggregate by one-fifth. Imports of finished steel (17,000 tons) were nearly three times as large as in the second quarter, and the highest since the end of the war, but were still only 14 per cent. of the 1938 average. Retained imports of refined petroleum were 857, 782 and 839 million gallons in the three quarters, the last figure being nearly one-third greater than pre-war.

The value of United Kingdom exports during January–September 1946 was £643·1 million, almost double the rate of exports in the second half of last year. The rise was continuous between

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January and May. The recorded figures are rather difficult to interpret because of the variation in the length of the working month. Taking 26 days (excluding Sundays) as a normal month, exports in the first five months of 1946 were 55, 65, 67, 75 and 82 million £'s respectively; the average for June and July on this basis was £81 million, and that for August and September £76 million, the reduction in the latter case being essentially due to the general holiday season.

Average values have risen by about 2½ per cent. each quarter this year, the figures being 186, 191 and 196 (1938 = 100); of the three main categories of manufactured goods, average values of textiles were more than two and a half times those in 1938 (index 265 in the third quarter), while the increases for metal goods and miscellaneous manufactures were 73 and 95 per cent. (again for the third quarter). In terms of volume, the third quarter was the first since the outbreak of war for which exports exceeded the 1938 rate (index 104), the index numbers for the first and second quarters being 84 and 98. The recorded figure for the first quarter was, however, inflated by the inclusion of some goods which had accumulated during the strike period at the docks last autumn. As mentioned above, the recorded figures include shipments abroad by N.A.A.F.I. and U.N.R.R.A., and these represented some 5 or 6 per cent. of the total in each of the first three quarters of the year.

Exports of food, drink and tobacco rose by £3·2 million (one-fifth) in the third quarter, after having remained fairly constant since the middle of 1945. Almost the whole of the increase in the third quarter was accounted for by emergency shipments of barley to Germany (51,000 tons in all), and by substantially increased exports of sugar—countries in the Middle East taking two-fifths of the total. Coal exports—still the biggest single item in the raw materials class—remained essentially unchanged each quarter at only one-seventh of the pre-war rate.

Exports of manufactured articles have been rising rapidly, the quarterly figures this year being 152·3, 189·6 and 207·2 million £'s. The greater part of the rise of £37·3 million in the second quarter was caused by metal goods, but the smaller rise (£17·6 million) in the third quarter was fairly evenly spread among the three main categories. The volume of exports of all the metal groups taken together rose from 103 per cent. of 1938 in the first quarter to 133 in the second and 139 in the third. The most rapid increase over the year has been for vehicles (16·6, 26·0 and 30·8 million £'s in the three quarters); the number of new motor cars exported increased from about 6,800 in the first quarter to 15,500 in the second and 20,600 in the third, the last figure being about 85 per cent. higher than the 1938 rate. Exports of machinery rose throughout the period (the quarterly figures being 20·4, 26·1 and 28·7 million £'s), exports of agricultural machinery (including shipments by U.N.R.R.A.) in the third quarter being over three times, and those of cranes, hoists, etc., almost double, the 1938 average. There were also marked increases for electrical goods—exports of radio sets rising from 32,000 in the first quarter to 74,000 and 102,000 in the second and third quarters, the latter being nearly five times the 1938 average—and, in the second quarter only, for non-ferrous metals, the quarterly figures in this case being 7·0, 11·2 and 9·8 million £'s. Exports of iron and steel had shown the strongest upward trend in the early months of the year (£17·7 million in the first quarter as against a quarterly average of only £7·4 million for July–December 1945), but there was only a small increase in the second quarter to £22·2 million, followed by a reduction to £20·4 million in the third, as a result of the acute supply position; even so, the quantity exported in the third quarter (553,000 tons) exceeded the 1938 average by 15 per cent.

Exports of textiles have progressed steadily, the rise in the second and third quarters being £5·5 million and £4·6 million, respectively; the volume index-numbers were 62, 68 and 73 per cent. of 1938. The principal increase over the year was in apparel exports (3·5, 6·9 and 9·6 million £'s); the rise was mainly in commercial exports (these being the highest by value for over 25 years), the value of U.N.R.R.A. shipments in the third quarter being only £0·8 million. Exports of cotton and of woollen and worsted goods remained substantially unchanged from quarter to quarter, the volume index-numbers being below the 1938 average by three-fifths and one-quarter, respectively. Silk and rayon exports were high throughout the war, and have further increased over the past year, the volume index in each quarter of 1946 being about 150 per cent. of 1938.

Exports of chemicals rose to almost £16 million in each of the first two quarters of this year from an average of £10·5 million for the second half of 1945, and there was a further rise of £1·0 million in the third quarter to a volume index of 163 per cent. of 1938. Pottery, glass, etc., showed

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a marked rise in the first and second quarters (the volume index-numbers respectively being 131 and 153), but there was no essential change in the third quarter.

A VALUABLE contribution to our knowledge of agricultural conditions in this country is contained in a Report issued by the Ministry of Agriculture dealing with the Statistical Results obtained by the National Farm Survey of England and Wales during the years 1941–43. (Stationery Office. Price 2*s*.) It supplements the Annual Agricultural Statistics, and provides a mass of information on such points as the classification of occupiers by economic type and of holdings by type of farming and by size; tenure, rent and length of occupation; condition of permanent buildings, and number and condition of farm cottages, managerial efficiency of occupier, farm water supply and use of electricity.

The basic information was obtained through the agency of the County War Agricultural Committees in the course of farm visits and inspections made as part of the Committees' war-time duties, and consisted partly of particulars of matters of fact, such as kind of tenure, rent, number of cottages, source of water supply, etc., which were obtained from the occupier himself, and partly of qualitative information as to condition of land and buildings, management, drainage, etc., which depended on the personal judgment of the field recorder. The enquiry covered all holdings of 5 acres and upwards to the number of nearly 300,000, but some of the holdings, though separate in position, were farmed together, so that the total number of self-contained units dealt with by the Survey was reduced to 290,600.

After the records had been completed, they were as far as possible checked for internal consistency and general accuracy by the Advisory Economists attached to the eleven Provincial Agricultural Advisory Centres located either at Universities or Agricultural Colleges, and were also compared with the relative return made by the occupiers in connection with the Agricultural Statistics. The results given in the Report are based on a random sample drawn by the survey staffs of the Advisory Economists by taking a varying proportion of all the holdings in each of five size groups. The sample comprised 14 per cent. of the holdings included in the Survey—viz., 40,000 out of 290,000. The methods of statistical analysis adopted for the purpose of the Report were largely devised by Dr. F. Yates, Head of the Statistical Department of Rothamsted Experimental Station, and Mr. O. Kempthorne, also of the Department, in consultation with the Ministry and, under the general direction of the Farm Survey Supervisory Committee. An account of certain aspects of the statistical analysis, such as the definition of the population, the adequacy of the sample, and the treatment of rejected records, is given in an Appendix.

One interesting feature of the Survey is that it provides an answer to a difficulty which has always been felt in connection with the Agricultural Statistics—viz., that while as a general rule, and particularly in the case of the larger holdings, it could be assumed that the figures related to "farms" in the ordinary sense of the word there was an undefined number of holdings used for agricultural purposes, but which it was misleading to describe as farms, or even as small holdings. For various reasons the distinction is not very easy to draw; the size and physical equipment which are often useful guides are not final tests, the essential feature being economic rather than physical. For this reason, the basis adopted by the Survey was that a holding was to be treated as a "farm" when it provided the main employment of, and the chief source of livelihood to the occupier—regardless of its size or equipment. On this general line, a number of economic types were distinguished, of which the largest was the "full-time" occupier (including market gardener, horticulturist, and poultry keeper) who was fully engaged in farming as an occupation and wholly or substantially dependent on it for a living. In this group there were 215,900 holdings, or 74·3 per cent. of the total number, covering 21,296,000 acres, or 88 per cent. of the total cultivated area. A second group comprised the "part-time" occupier—that is, persons who had some other occupation, but to whom farming was still the main occupation and source of livelihood. This group accounted for 32,500 holdings and 1,379,000 acres, so that the two groups, which on this definition could be described as "farms", taken together represented 85·5 per cent. of the total holdings and 93·7 per cent of the total area. Outside these there are a number of smaller groups, such as (1) the "spare-time" occupier who had some full-time or nearly full-time employment, but still occupied his holding for profit; (2) the "hobby" occupier—that is, a person not dependent on farming for a

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living, and who, though he may make a profit, is occupying the land for some other purpose, such as pleasure or amenity; (3) the occupier of accommodation land; and (4) others not farming primarily for profit, as in the case of home farms and farms run by educational and other institutions. Of these groups the spare-time occupier was the most important in point of numbers (9·8 per cent.), though less so in point of area (3·2 per cent.), the remainder representing 4·7 and 3·1 per cent. respectively. The classification was not one which permitted absolute precision, particularly in the case of the smaller holdings (5–25 acres), but it is believed to reflect the true position reasonably well. It marks the first attempt at a fairly comprehensive classification of occupiers according to economic type.

Another point dealt with by the Survey on which there has hitherto been some uncertainty is the extent to which land is cultivated by tenants or by owners. It was found that no very precise line can be drawn, as many occupiers own part and rent part of the land they occupy, and do so in very varying proportions. On the whole, it is considered that 65 per cent. of the total occupiers are tenants and 35 per cent. owners, the proportion of land so occupied being very similar, so that it may be said that tenants and owners are in the proportion of two-thirds and one-third, both as regards numbers and area.

CURRENT NOTES

In August 1945 the "Vereeniging voor Statistiek" (The Netherlands Statistical Association) was founded. The purpose of this Association is to stimulate the development and the use of mathematical statistics and to advance the application of statistical theory as it has been developed during the last few decades. It hopes to bring together statistical workers of all interests and to encourage the use of statistical methods in those fields in which statistics have been relatively neglected either in the Netherlands or in general. It will publish a bi-monthly journal, *Statistica*, devoted to articles on statistical applications in any field and the development of new theories. Most articles will be written in Dutch, but each will include an English summary. The first issue will be published in August 1946. The Association particularly welcomes co-operation with foreign statistical institutes and societies. The President of the "Vereeniging voor Statistiek" is Mr. J. van Ettinger, who has been active in the development of industrial statistics since 1929. The Secretary is Mr. J. H. Enters, to whose address, Postbox 27, Rijswijk, Z.H., Netherlands, all inquiries concerning the work of the Society should be directed.

There has recently appeared a Report of the Committee on the Provision for Social and Economic Research, presented by the Lord President of the Council and the Chancellor of the Exchequer (Cmd. 6868). The Committee consisted of:—

The late Sir John Clapham,
Sir Alan Barlow,
Sir Alexander Carr-Saunders,
Sir Henry Clay,
Sir Hector Hetherington,
Sir Walter Moberly,
Professor L. Robbins,
Professor R. H. Tawney,
T. M. Wilson.

The Committee call attention to the necessity for an increased provision of funds for social and economic research, and also for the desirability of putting that provision on a permanent and routine basis. They suggest the formation of a standing interdepartmental Economic and Social Research Committee to be set up to survey and advise upon research work in Government Departments. The following paragraph will be of particular interest to Fellows of the Society :—

> "We cannot leave this part of our report without some comment on the quite inadequate provision which at present prevails in universities in the United Kingdom for posts in statistics. An adequate supply of statistical competence is quite fundamental to the advancement of

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knowledge of social and economic questions. It is no less essential in public administration. Yet the number of chairs in statistics throughout the country could be counted on the fingers of one hand. It is small wonder that, during the War, statisticians were probably the scarcest of scarce commodities and that, now that the war is over, a chronic struggle is going on between the universities, business and the public service for the services of a supply of statisticians which is in the aggregate equal to only a tiny fraction of the demands immediately manifesting themselves, and still less of the demands which are certain to accrue as current policies develop and projected institutions are brought into being. There are few more urgent needs to-day than the increase of the supply of first class statisticians. But this need will not be met if further provision is not forthcoming at the universities."

The Trustees of the Houblon-Norman Fund on the recommendation of the Advisory Committee, have made the following awards for 1946–47:—

Fellowships: R. F. Henderson, M.A., "Financial limitations to investment by private firms and joint-stock companies." C. T. Smith, Ph.D., B.Sc., "The External relations of insurance companies." W. E. Tate, "The enclosure movement of the eighteenth century." P. J. Watkin, B.A., "Financial facilities for industry in South Wales."

Research Grants: N. L. Armstrong, Ph.D., B.Sc., B.Com., "The relative efficiency of English banking in different parts of the country during the nineteenth century." P. S. Sumner, M.Sc., B.A., "The economics of the Stock Exchange and company finance."

An offer of awards for 1947–48 will be made early in 1947. Further information may be obtained from The Secretary, Houblon-Norman Fund, c/o The Bank of England, E.C.2.

The Colonial Office has asked us to make it known that the Secretary of State, on the advice of the Colonial Research Committee, has instituted a number of *Colonial Research Fellowships* to encourage qualified scientists to give special attention to colonial problems and to enable them to pursue research work in the British Colonial Empire. Provision has been made for twenty-five such Fellowships within the period 1944–49, but it is hoped the interest promoted by the scheme will ultimately justify its becoming a permanent institution.

The award of these Fellowships will be made by the Secretary of State on the advice of the Colonial Research Committee. The Secretary of State hopes that the Universities and other institutions will be willing to grant applicants, if already members of their staffs, leave of absence for this period in order to enable them to take up the Fellowships.

The Fellowships will normally be reserved for University graduates in the natural or social sciences under 35 years of age from any part of the British Commonwealth and Empire.

Fellowships, which will be tenable for a period of two years, subject to a satisfactory report at the end of the first year, and may be extended for a third year, carry remuneration at the rate of £400 per annum which may be increased to a sum not exceeding £750 per annum if the Fellow is married or in any other appropriate circumstances. Travelling expenses and the cost of any apparatus or material required for the Fellow's research will also be provided.

Further particulars and application forms may be obtained from The Secretary, Colonial Research Committee, Palace Chambers, Bridge Street, London, S.W.1.

A Temporary Statistical Commission was established by the first session of the Economic and Social Council as the result of suggestions made by the Preparatory Commission of the United Nations and adopted by the General Assembly in January, 1946. This Commission met in New York in May, 1946, with the following Members present at some or all of its meetings: Mr. Stuart Rice (Chairman), (U.S.), Professor A. Sauvy (France), Mr. H. Campion (U.K.), Professor P. C. Mahalanobis (India), Dr. D. K. Lieu (China), Mr. Pavel Fedesimov (temporary representative, U.S.S.R.). Written suggestions were made by Mr. Gunnar Jahn (Norway). The following is a summary of the recommendations included in its Report to the Economic and Social Council:

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1. That a permanent Statistical Commission of not more than twelve members be established by the Economic and Social Council, and that members be appointed to the Commission in their individual capacities on the basis of technical competence and professional eminence.

2. That the Statistical Commission be charged with the task of assisting the Council in:

(*a*) promoting the development of national statistics and the improvement of their comparability;

(*b*) the co-ordination of the statistical work of specialized agencies;

(*c*) the development of the central statistical services of the Secretariat;

(*d*) advising the organs of the United Nations on general questions relating to the collection, interpretation and dissemination of statistical information;

(*e*) promoting the improvement of statistics and statistical methods generally.

3. That suitable provision be made for co-ordinating the statistical activities of the United Nations and specialized agencies which are brought into relationship with the United Nations.

4. That there be established within the Secretariat of the United Nations an adequate, thoroughly competent central statistical unit to serve the statistical needs of all departments and organs of the United Nations.

5. That action be taken promptly to assure maintenance without interruption of the valuable statistical work of the Economic, Financial and Transit Department of the League of Nations and of other statistical activities carried on under the sponsorship of the League.

6. That recognition be given by the United Nations to the important contributions made by the International Statistical Institute and other organizations to the improvement of world statistics. The Commission hopes to give further study to the rôle of such organizations in relation to the United Nations.

7. That recognition be given likewise to the rôle of regional statistical organizations such as the Inter-American Statistical Institute.

8. That a Sub-Commission on Statistical Sampling of not more than nine members be established.

The full Report of this Commission will be issued shortly. Documentation may be consulted at the Library Information Service, Church House, S.W.1.

NUMEROUS requests have been received from employers during the past year for the names of statisticians who wish to be considered for vacancies or new posts of a statistical character. Such requests sometimes come direct to the Assistant Secretary of the Society and sometimes to Fellows.

The Council think it is desirable, under present conditions, to help in putting prospective employers and employees in touch with each other. It has, therefore, been arranged that the Assistant Secretary will keep a list:

(*a*) of Fellows who indicate their interest in obtaining statistical posts; and
(*b*) of posts which are notified to the Council by employers as being vacant.

Fellows who wish to be put on the first list should write to the Assistant Secretary giving particulars of age, education, qualifications, experience and present employment, and should also specify which types of statistical work they are interested in. Employers who have vacant posts to be included on list (*b*) should similarly communicate with the Assistant Secretary specifying their requirements. Particulars of these vacant posts will then be circulated to such Fellows on list (*a*) as would appear to be interested in the type of work so that they can themselves make contact with the prospective employer. Names of Fellows seeking posts will not be given directly to any employer.

It is hoped that this regularization of the position will help both employers and Fellows of the Society in making contact.

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OBITUARY

Lord Keynes

The death of Lord Keynes on the 22nd April, 1946, has been universally felt as a grievous loss both to the country and to the world. Of the fine record of achievement in his many-sided career it would not be in place to attempt a general appraisement here. It must suffice to dwell upon one side of his activities.

The Royal Statistical Society is one of the many circles where he will be sadly missed. He became a Fellow in 1909, being proposed by his father, Dr. J. N. Keynes, who had been elected in 1883 and is still with us as a Fellow.

Keynes served three terms on the Council of the Society, 1915–19, 1933–37 and 1941–45. Amid the many absorbing interests of his active and fruitful life, he did not often find time to participate in the ordinary meetings. The only paper he contributed was not on any specifically statistical subject: it was his study of the life, personality and work of Jevons in April 1936, on the occasion of the centenary of Jevons's birth. It was a notable manifestation of Keynes's pre-eminent power of writing such studies, elsewhere exemplified in his memoir of Marshall, and in his description of the leading actors in the framing of the Treaty of Versailles.

But it would be a mistake to suppose that Keynes took little interest in statistics. His *Treatise on Probability* (the dissertation for which he received his fellowship of King's, Cambridge) was an important and original contribution to philosophy, and included sections on probability in relation to statistics which threw new light on that subject.

And it was not only the philosophy of statistics that appealed to him; no economist was ever more alive to the need for a statistical foundation for economic policy. The book which first gained him world-wide fame, *The Economic Consequences of the Peace*, was in essence a statistical argument. His scathing denunciation of the levity which had ignored the difficulties in the way of extracting reparations from Germany was supported on a solid statistical foundation. The question was one of *magnitude*. His thesis was that a burden, which he calculated at £450 millions a year, far transcended Germany's power of making external payments. His genius transformed what might have been a dry survey of numerical data into a brilliant and vivid narrative of contemporary history, permeated by a generous moral purpose.

That book was the sequel of four years' strenuous experience at the Treasury of the administration of the country's external finances in war-time—a service which he was destined to repeat at the end of his life. And once again his judgment in employing statistical reasoning in the great issues of practical life was strikingly illustrated. His culminating achievement, the negotiation of the American loan in 1945, had to be firmly based on statistical evidence both of the war effort of the United Kingdom and of the economic position to which the country has been reduced.

And the influence of Keynes's appreciation of statistical material as the indispensable instrument of economic policy is traceable in the widened activities of the Government in the statistical field, for example in the White Papers on National Income which have shed light on the annual financial statements of recent years.

R. G. H.

Sedley Anthony Cudmore

The sudden death of Dr. S. A. Cudmore on 17th October, 1945, while attending the meetings of the United Nations Food and Agricultural Organization came as a great shock to those who were privileged to know him. His health had been undermined by the strain of organizing the Canadian statistics to cope with the many demands for new information arising out of the war, and the strain was increased rather than lessened by the coming of peace with its no less difficult problems.

Prior to 1935 he had for many years been principal assistant to Dr. R. H. Coats, Dominion Statistician, being editor of the Canada Year Book, and it is due to that fact and to all the statistics in Canada being organized and mainly collected centrally that he owed his appointment in that year as Statistician for Palestine, with the task of setting up a central bureau of

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statistics there. It was not an easy duty that he undertook, as can well be realized, and the excellence of the statistical information now available regarding all the economic and social aspects of life in Palestine affords an abiding recognition of his outstanding ability both as an all-round statistician and as an organizer. He had only been seconded by the Dominion Bureau of Statistics, and when, after three and a half years, he had the Palestine statistics well organized, he returned to the Bureau and on Dr. Coats's retirement in 1942 became Dominion Statistician.

It was in 1935, immediately before Dr. Cudmore's departure for Palestine, that he took part in the Conference of Empire Statisticians held in Ottawa. All those who were present at that Conference were impressed by his clear thinking and sound judgment as well as by his comprehensive knowledge of all aspects of Government statistics.

He became a Fellow of the Royal Statistical Society in 1920 and was elected as a Member of that select body, the International Institute of Statistics, in 1929. He was also a Fellow of the Royal Economic Society and the Royal Society of Canada and a Member of the American Statistical Association and the Inter-American Statistical Institute. His publications included *A History of the World's Commerce*, *Applied Economics* and *The National Income of Canada*.

H. L.

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STATISTICAL AND ECONOMIC ARTICLES IN RECENT PERIODICALS

UNITED KINGDOM—

The Banker—

March 1946—Will the safety-valves work?: *E. H. Stern.* Inflation in Palestine: *A. R. Prest.*

April 1946—Failure in manpower budgeting: *Walter Hill.* Reconversion in iron and steel: *F. W. Forge.* The motor industry's problems: *E. D. O'Brien.* Engineering and reconversion: *F. H. Happold.* Cotton hamstrung by labour shortage: *W. Gwyn Pilkington.* Reconversion in the rayon industry: *C. H. Ward-Jackson.*

May 1946—Death of a genius: *The Editor.* Lord Keynes: an appreciation: *Paul Einzig.* National finance in the transition. Industrial Wales (5 articles): *Sir Frederick Rees, Sir Lewis Jones, H. T. Norton, John Pares, W. Fennell.*

June 1946—Britain's balance of payments: *T. Barna.* Anglo-American co-operation: *R. F. Harrod.* Pay-as-you-earn reform: *G. A. Bauer.* Canada's post-war economy: *H. Dargan Roberts.*

Bankers' Magazine—

April 1946—The Banks' information service. Bank of England Charter (text).

May 1946—The problem of international trade.

Biometrika—

June 1946—Statistical techniques in applied psychology: *E. G. Chambers.* A useful method for the routine estimation of dispersion from large samples: *A. E. Jones.* Inequalities in terms of mean range: *C. B. Winsten.* Tables for testing the homogeneity of a set of estimated variances: *Catherine M. Thompson* and *Maxine Merrington.* The design of optimum multifactorial experiments: *R. L. Plackett* and *J. P. Burman.* On the solution of some equations in least squares: *D. V. Lindley.* Some generalizations in the multifactorial design: *R. L. Plackett.* The growth, survival, wandering and variation of the long-tailed fieldmouse. II. Survival: *H. P. Hacker.* Table of percentage points of the *t*-distribution: *Elizabeth M. Baldwin.*

Economica—

February 1946—The London School of Economics, 1895–1945. The subjective theory of value and accounting "cost": *G. F. Thirlby.* The original security bank: *S. R. Cope.*

Eugenics Review—

April 1946—The trend of national intelligence: *Godfrey Thomson.*

Institute of Bankers, Journal—

January 1946—The Cohen Report in relation to banking: *H. B. Lawson.* Pronouncements of the Institute of Chartered Accountants *ex cathedra*: *E. E. Spicer.*

April 1946—The future of the city: *O. R. Hobson.* The new budgetary approach to public finance: *G. L. Schwartz.* Thirty years of monetary change, 1914–1945: *W. F. Crick.*

Lloyds Bank Review—

July 1946—1946 and 1919: *Professor A. C. Pigou.* Inflation and its aftermath: *W. Manning Dacey.* Indian cotton: *P. G. Oulton.*

Oxford Institute of Statistics, Bulletin—

April 1946—Grocery sales and rationing: *T. Schulz.* Europe's average standard of living: *K. Forschheimer.* Economic conditions in Norway: *K. Petersen.*

May 1946—The Budget and reconstruction: *T. Steindl.* The national income and expenditure of the U.K. 1938–1945: *D. G. Champernowne.* Earnings, hours and mobility of labour: *J. L. Nicholson.*

June 1946—United Kingdom export target: *F. A. Friday.* Low cost family diets and individual nutrition: *T. Schulz.* Post-war economic prospects in the U.S.S.R.: *M. H. Dobb.*

Quarterly Journal of Mathematics—

March 1946—The five-dimensional geometry of the curvature tensor in a Riemannian V_4: *H. S. Ruse.* Note on the distribution of the intervals between prime numbers: *Lord Cherwell.*

Statistical and Social Inquiry Society of Ireland, Journal—

Vol. XVII—Irish actuarial data: *W. A. Honohan.* Emigration and rural Ireland: *T. W. Freeman.* The development of local government in Ireland: *J. J. Horgan.* The problem of full employment: a discussion. Some thoughts on the making of Irish index numbers: *R. C. Geary.*

INDIA—

Sankhyā—

April 1946—On the linear combination of observations and the general theory of least squares: *C. R. Rao.* Theories of probability: *J. G. Singh.* A statistical approach to the problem of

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chronology of Shakespeare's plays: *M. R. Yardi.* Sample surveys of crop yields in India: *P. C. Mahalanobis.* Sampling for rice yield in Bihar and Orissa: *J. A. Hubback.* Confounded designs in quasi-Latin squares: *C. R. Rao.* Enumeration of non-isomorphic solutions of balanced incomplete block designs: *H. K. Nandi.* A further note on non-isomorphic solutions of incomplete block designs: *H. K. Nandi.* Impossibility of symmetrical incomplete block design with $\lambda = 2$, $k = 7$: *Q. M. Husain.* On the order of approximation involved in Laplace's central limit theorem in probability: *M. N. Ghosh.* A note on concentration of income in Bengal villages: *R. K. Mukerjea* and *M. M. Mukerjea.* A criterion for testing whether two samples have come from the same population without assuming the nature of the population: *K. K. Mathen.* Price of agricultural commodities and land transfer in Bengal. *N. R. Dasgupta.*

UNION OF SOUTH AFRICA—

South African Journal of Economics—

March 1946—Some reflections on full employment: *Sheila T. Van der Horst.* What's the use of economists?: *W. F. Crick.* Fluctuations in income concentration: *J. de V. Graaff.* Survey of the income and expenditure of Indian employees of the Durban Corporation, living at the Magazine Barracks, Durban: *V. Sirkari Naidoo.*

UNITED STATES—

American Academy of Political and Social Science, Annals—

May 1946—The Netherlands during German occupation.

American Economic Review—

March 1946—National budgets and national policy: *Jacob Mosak.* Psychological analysis of business decisions and expectations: *George Katona.* Shortcomings of marginal analysis for wage-employment problems. Family size and residential construction: *Eric Schiff.* The Soviet Union's war budget: *T. A. Sumberg.*

May 1946—Papers and proceedings of the fifty-eighth annual meeting of the American Economic Association.

American Statistical Association, Journal—

March 1946—The advancing statistical front: *W. A. Shewhart.* On the design of a sample for dealers' inventories: *W. Edwards Deming* and *Willard Simmons.* War-time aluminium statistics: *M. D. Goldstein.* Historical note on the purchasing power concept, and index numbers: *W. F. Ferger.* A method of making actuarial estimates for a compulsory health insurance system: *P. A. Dodd.* Approximation of chi-square by "probits" and by "logits": *Joseph Berkson.*

American Statistical Association, Biometrics Bulletin—

April 1946—The covariance analysis of multiple classification tables with unequal subclass numbers: *L. N. Hazel.*

Annals of Mathematical Statistics—

March 1946—Some distributions of sample means: *G. W. Brown* and *J. W. Tukey.* Unbiased estimates for certain binomial sampling problems with applications: *M. A. Girshick, Frederick Mostellar* and *L. J. Savage.* Distribution of sample arrangements for runs up and down: *P. S. Olmstead.* The theory of unbiased estimation: *P. R. Hálmos.* Some significance tests based on order statistics: *J. E. Walsh.* Chains of rare events: *F. Cernuschi* and *L. Castagnetto.* A note on some single sampling plans requiring the inspection of a small number of items: *J. H. Curtiss.* On the use of the sample range in an analogue of Student's *t*-test: *Joseph F. Daly.* An inequality for deviations from medians: *J. W. Tukey.* On the independence of the extremes in a sample: *E. J. Gumbel.* A note on sampling inspection: *Paul Peach* and *S. B. Littauer.* On an equation of Wald: *David Blackwell.* Correction to the paper "on a problem of estimation occurring in public opinion polls": *H. B. Mann.*

Econometrica—

April 1946—Macroeconomics and the theory of rational behaviour: *L. R. Klein.* Theory of the firm and of investment: *Leonid Hurwicz.* Capital expansion, rate of growth, and employment: *Evsey D. Domar.* Multiplier effects of a balanced budget: some monetary implications of Mr. Haavelmo's paper: *G. Haberler.* The implication of a lag for Mr. Haavelmo's analysis: *R. M. Goodwin.* Further analysis: *Everett E. Hagen.* Reply: *Trygve Haavelmo.*

Harvard Business Review—

Summer Number 1946—How good is marketing research?: *F. L. Thomsen.* Job evaluation—a case history: *A. J. Percival* and *G. B. Cross.* Aims of our foreign investment policy: *O. H.*

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Korican. Note on turnover in government personnel: *J. W. Culliton.* Private *v.* public management of airports: *L. L. Bollinger.*

Journal of Political Economy—

April 1946—Neumann's and Morgenstern's new approach to static economics: *J. Marschak.* The first twenty years of the Bank of Spain. II: *E. J. Hamilton.* Professor Knight and the theory of demand: *R. L. Bishop.*

June 1946—Experience under intergovernmental commodity agreements, 1902–45: *J. S. Davis.* Some economic aspects of the margarine industry: *W. H. Nicholls.* Interest and employment: *M. W. Reder.* Some implications of full-employment policy: *A. L. Meyers.*

Milbank Memorial Fund Quarterly—

April 1946—Nutritional status of aircraft workers in Southern California IV, Effects of vitamin supplementation on clinical, instrumental, and laboratory findings, and symptoms: *H. Borsook, J. W. Dubnoff, G. Keighley, D. G. Wiehl, E. Goolden* and *J. G. Williams.* Urbanization in Latin America: *Kingsley Davis* and *Ana Casis.*

Quarterly Journal of Economics—

May 1946—Reparation labor—a preliminary analysis: *Paul Fisher.* Federal reserve policy in transition: *C. R. Whittlesey.* "Ability to pay": *Bernard W. Dempsey.* The supply of labor to the firm: *L. G. Reynolds.* Resource conservation and economic stability: *S. v. Wantrup.*

Review of Economic Statistics—

May 1946—Life, liberty, and the pursuit of happiness: *Sir William Beveridge.* A symposium on fiscal and monetary policy: *Messrs. Mints, Hansen, Ellis, Lerner, Kalecki.* Debt policy and banking policy: *H. C. Simons.* Innovations and then irregularity of economic cycles: *Richard Goodwin.* Gold sales as an anti-inflatory device: *R. F. Mikesell.*

EGYPT—

L'Egypte Contemporaine—

July 1946—The future of world trade: *P. Kantorovitz.* Le commerce extérieur de l'Egypte pendant les deux guerres mondiales: *J. Schatz.*

FRANCE—

Société de Statistique de Paris, Journal—

Novembre–décembre 1945—L'école moderne de statisticiens italiens: *Pierre Thionet.* Chronique des statistiques agricoles: *Michel Augé-Laribé.* Variété: étude statistique des tendances en littérature: *M. L. Dufrénoy.*

Janvier–février 1946—L'école moderne de statisticiens italiens (*suite*): *Pierre Thionet.* Variations des prix et de la circulation monétaire en Syrie et au Liban au cours de la deuxiéme guerre mondiale: *Felix Rosenfeld.*

Mars–avril 1946—La statistique et l'étude des opinions: *Jean Stoetzel.* Quelques aspects de la production industrielle française en 1945: *René-Ch. Marchand.* La construction d'immeubles d'habitation en France entre 1928 et 1939: *Lucien Flaus.* Valeur de la preuve en dactyloscopie: *Lucien Amy.* Montesquieu et la constitution des États-Unis: *M. L. Dufrénoy.*

MEXICO—

Estadística—

March 1946—Metodología en Estadísticas Agrícolas de las Americas: *Hernan Montoya.* Calculation of life tables for population of countries and regions: *Nathan Keyfitz.* Renta Nacional, su significado y medición, parte II: *Loreto M. Dominguez.* El Censo industrial de Colombia de 1945: *Luis Ortiz C.* Machine tabulation: *John J. Mahaney.* Prueba de Registro de Nacimientos en Paraguay: *Jorge Roig Ocampos* and *Charles C. Bennett.*

INTERNATIONAL—

International Labour Review—

March–April 1946—Wage stabilisation in New Zealand: *A. R. Low.* Economic post-war problems in Denmark: employment, wage and financial policies. Land aspects of labour problems in Kenya.

Revue de l'Institut International de Statistique—

1944—Colin Clark's "Economics of 1960." Die Lebensjahre einer Bevölkerung. Bemerkungen zu Liebmann Hersch's "Potentiel-vie": *W. Winkler.* Quelques précisions sur la méthode des potentiels-vie et ses notions fondamentales. En réponse aux critiques du Prof. Wilhelm Winkler: *L. Hersch.* La statistique méthodologique italienne de 1939 à 1942: *G. Pietra.*

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LIST OF ADDITIONS TO THE LIBRARY

Since the issue of Part I, 1946, the Society has received the publications enumerated below:—

I.—OFFICIAL PUBLICATIONS

(a) United Kingdom.

Colonial Office. Nutrition in the British West Indies: report by B. S. Platt. London, H.M.S.O., 1946 (Colonial 195). 38 pp. 9½″. 9*d.*

Food, Ministry of. The world food shortage. London, H.M.S.O., 1946. Cmd. 6785. 23 pp. 9½″. 4*d.*

Fuel and Power, Ministry of. Forest of Dean regional survey report. London, H.M.S.O., 1946. 58 pp. Maps. 9½″. 1*s.* 3*d.*

Health, Ministry of. Reports on Public Health and Medical Subjects No. 93. A report on the re-adjustment in civil life of soldiers discharged from the Army on account of neurosis, by E. Guttman and E. L. Thomas. London, H.M.S.O., 1946. 72 pp. 9½″. 1*s.* 3*d.*

Health, Ministry of, and Department of Health for Scotland. Report of the Inter-Departmental Committee on Remuneration of General Practitioners. London, H.M.S.O., 1946. Cmd. 6810. 31 pp. 9½″. 6*d.*

Industrial Health Research Board. Report No. 89. Artificial sunlight treatment in industry: a report on the results of three trials—in an office, a factory and a coalmine, by Dora Colebrook. London, H.M.S.O., 1946. 64 pp. 9½″. 1*s.*

Medical Research Council.

Special report Series No. 235. The chemical composition of foods, by R. A. McCance and E. M. Widdowson. 2nd ed. London, H.M.S.O., 1946. 156 pp. 9½″. 6*s.*

—— No. 254. An experimental study of rationing, by R. A. McCance and E. M. Widdowson. London, H.M.S.O., 1946. 61 pp. 9½″. 1*s.*

Parliament. Statistics and documents relating to the world grain position. London, H.M.S.O., 1946. Cmd. 6737. 10 pp. 9½″. 2*d.*

Scotland, Department of Health for.

New Towns Bill. Memorandum by the Secretary of State for Scotland. London, H.M.S.O., 1946. 38 pp. 9½″. 9*d.*

Rehabilitation: the report of the Medical Advisory Committee (Scotland). Edinburgh, H.M.S.O., 1946. 35 pp. 9½″. 6*d.*

Scottish hospitals survey. General introduction to the reports. 15 pp. 6*d.* Western region. 128 pp. 2*s.* South-Eastern region. 104 pp. 1*s.* 6*d.* Eastern region. 52 pp. 1*s.* North-Eastern region. 68 pp. 1*s.* Northern region. 48 pp. 9*d.* Edinburgh, H.M.S.O., 1946. 6 parts. 9½″.

Scottish Education Department. Technical education: a report of a special committee of the Advisory Council on Education in Scotland. Edinburgh, H.M.S.O., 1946. Cmd. 6786. 118 pp. 9½″. 2*s.*

Supply, Ministry of. Iron and steel industry: reports by the British Iron and Steel Federation and the Joint Iron Council to the Minister of Supply. London, H.M.S.O., 1946. Cmd. 6811. 107 pp. 9½″. 1*s.* 6*d.*

Supply, Ministry of. Directorate of Royal Ordnance Factories. Explosives. Industrial experimentation, by K. A. Brownlee. London, H.M.S.O., 1946. 116 pp. 9½″. 2*s.* (Presented by the author.)

Town and Country Planning, Ministry of. New Towns Bill. Memorandum by the Minister of Town and Country Planning showing application and modification of provisions of the Town and Country Planning Act 1944. London, H.M.S.O., 1946. Cmd. 6801. 38 pp. 9½″. 9*d.*

Town and Country Planning, Ministry of. & Department of Health for Scotland. Interim report of the New Towns Committee. Cmd. 6759. 21 pp. 4*d.* Second interim report of the New Towns Committee. Cmd. 6794. 25 pp. 6*d.* London, H.M.S.O., 1946. 2 parts. 9½″.

Trade, Board of.

Report of the enquiry on the ball-clay industry. London, H.M.S.O., 1946. 14 pp. 9½″. 3*d.*

Working Party reports. Cotton. 1946. vi, 278 pp. 9½″. 3*s.* 6*d.* Pottery. 1946. 52 pp. 1*s.* 3*d.* London, H.M.S.O.

Transport. Ministry of. Road accidents: report submitted to the Minister of Transport, by J. Harry Jones . . . London, H.M.S.O., 1946. 80 pp. 9½″. 1*s.* 3*d.*

Treasury. National income and expenditure of the United Kingdom 1938–1945. London, H.M.S.O., 1946. Cmd. 6784. 49 pp. 9½″. 9*d.*

(b) International and Others.

Australia—*Commonwealth Rationing Commission.* Rationing in Australia: a review of the work of the Commonwealth Rationing Commission, June 1942–December 1944. Melbourne, 1944. 32 pp. 8¼″.

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France.

Direction de la Statistique Générale.

Études Économiques No. 1. Documents sur le problème du logement à Paris. Paris, 1946. 165 pp. 10½″.

Statistique des institutions d'assistance. années 1937, 1938, 1939. Paris, 1945. lviii, 117 pp. 10½″.

International Institute of Agriculture. An outline of the basic problems of post-war agricultural reconstruction, by George Pavlovsky. (International Problems of Agricultural Reconstruction, New Series No. 1.) Rome, 1944. 68 pp. 9½″. 5 *Lire*. (Presented by the Author.)

League of Nations.

Economic, Financial and Transit Department.

Economic demography of Eastern and Southern Europe, by Wilbert E. Moore of the Office of Population Research, Princeton University. (1945 II.A.) Geneva, 1945. 299 pp. 10*s*.

Industrialization and foreign trade (1945 II.A.10). Geneva, 1945. 171 pp. 9″. 7*s*. 6*d*.

Raw-material problems and policies. Part I: Problems, proposals and policies. Part II: Lessons from the past. (1946 II.A.2.) Geneva, 1946. 116 pp. 9″. (London, Allen & Unwin.)

Norway. *Statistisk Sentralbyrå.* Nasjonalinntekten i Norge 1935–1943. Realkapitalen 1939 og kapitalreduksjonen under krigen. Okkupasjonskostnadene. (Norges Offisielle Statistikk X. 102.) Oslo, 1946. iv, 169 pp. 9½″.

Poland. *Glownego Urzedu Statystycznego.* (Wiadomości Statystyczne. Statistical news of the Chief Bureau of Statistics of the Republic of Poland). **18** (1935) No. 1 August—4–5 November–December. Special vol. I (1946). Provisional results of the general summarized population census on February 14th, 1946. 11 pp. Warsaw, 1945–6. 11¾″. 4 parts.

Southern Rhodesia. *Department of Statistics.* Statistical handbook of Southern Rhodesia, 1945. Salisbury, 1945. 33 pp. 9¾″.

Union of South Africa. *Office of Census and Statistics.* Special report No. 127. New retail price index numbers, 1938. Pretoria, 1939. 35, 33 pp. 13″. 6*d*.

II.—AUTHORS AND MISCELLANEOUS

Carnegie Institute of Technology. Quality control reports. 1. Introduction of statistical quality control at Illinois Testing Laboratories, by J. A. Fizzell. 8 pp. 2. Organizations concerned with statistical quality control, by H. Working and E. G. Olds. 38 pp. 3. Statistical quality control at the St. Louis division of American Stove Company, by L. K. Kauffman. 22 pp. 4. Quality control at the Lukas-Harold Corporation, by C. G. Schmid. 10 pp. 5. Statistical quality control in radio manufacture, by F. Trowbridge and J. T. Clark. 13 pp. 6. An application of statistical quality control at the Hoover Ball and Bearing Company, by R. C. Trombly. 9 pp. 7. An adaptation of statistical quality control at Aldens', by J. M. Ballowe. 13 pp. 8. An application of statistical quality control at the John Deere Tractor Company, by E. L. Fay. 10 pp. 9. Statistical quality control at Lockheed, by J. R. Crawford and P. C. Hammer. 17 pp. 10. Some aspects of statistical quality control in the steel industry, by R. E. Ede and E. W. Mahaney. 13 pp. 11. Bibliography on statistical quality control. 48 pp. 12. Some experiments illustrating principles of quality control, by I. W. Burr. 12 pp. Pittsburgh, Pa., Issued by Quality Control Program, Carnegie Institute of Technology, for the Office of Production Research and Development, War Production Board. 1945. 12 parts. 11″.

Clough (Shepard B). A century of American life insurance: a history of the Mutual Life Insurance Company of New York, 1843–1943, New York, Columbia University Press, 1946. (London, Oxford University Press.) xiv, 402 pp. 9¼″. 30*s*.

Conseil Permanent International pour l'Exploration de la Mer. Bulletin statistique des pêches maritimes des pays du nord et de l'ouest de l'Europe. Vol. II (1905) to Vol. XXVIII (1938) inclusive. Copenhagen, A. F. Høst & Fils, 1908–1939. 27 vols. 10½″. (Presented by the Bureau du Conseil.)

Elsas (M. J.). Umriss einer Geschichte der Preise und Löhne in Deutschland vom ausgehenden Mittelalter bis zum Beginn des neunzehnten Jahrhunderts. Zweiter Band, Teil A. Leiden, A. W. Sijthoff, 1940. iv, 649 pp. 10″. *Fl*. 13, 10 unbound. *Fl*. 14, 70 bound.

Godwin (H. J.) and Hartley (H. O.). Tables of the probability integral and the percentage points of the mean deviation in samples from a normal population. (*Biometrika* **33** (1945). pp. 252–265.) London, Biometrika Office, 1945. 10¾″. 2*s*. 6*d*. (Presented by Prof. E. S. Pearson.)

Hansen (Alvin H.). Fiscal policy and business cycles. New York, Norton & Co., 1941. (London, Allen & Unwin.) 462 pp. 8½″. £4.

Hogben (Lancelot) *editor*. Political arithmetic: a symposium of population studies. London, Allen & Unwin, 1938. 531 pp. 9¼″. 30*s*.

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Digitized by Arya Samaj Foundation Chennai and eGangotri

INSTITUTO BRASILEIRO DE GEOGRAFIA E ESTATISTICA. Tábuas de mortalidade e sobrevivência Brasileiras. Distrito Federal e Municipio de São Paulo. Rio de Janeiro, 1946. 194 pp. 10½″. (Presented by Prof. G. Mortara.)

INTERNATIONAL STATISTICAL INSTITUTE. Statistique international des grandes villes. Statistique du tourisme. . . . 1929–1934, 1939. 89 pp. Territoire et population. . . . . 1928–1934, 1939. 300 pp. Statistique de l'électricité, du gaz et de l'eau 1934, 1939. 97 pp. Statistique du logement . . . 1928–1934, 1940. 170 pp. The Hague, Office Permanent de l'Institut International de Statistique, 1939–40. 4 vols. 10½″. (Presented by the Institute.)

KENDALL (M. G.). Contributions to the study of oscillatory time-series. (National Institute of Economic and Social Research, Occasional Papers, ix.) Cambridge University Press, 1946. [8], 76 pp. 9½″. 7*s.* 6*d.*

LÉVY (PAUL). Calcul des probabilités. Paris, Gauthier-Villars, 1925. viii, 350 pp. 10″. 175*f.*

LIVERPOOL, UNIVERSITY OF. SOCIAL SCIENCE DEPARTMENT. The import trade of the port of Liverpool, by G. C. Allen, F. E. Hyde, D. J. Morgan, W. J. Corlett. Liverpool, University Press, 1946. vi, 109 pp. 8½″. 6*s.*

LYLE (PHILIP). Regression analysis of production costs and factory operations. 2nd ed. Edinburgh and London, Oliver & Boyd, 1946. xii, 204 pp. 8¼″. 15*s.*

MACKRILL (A. H.). Mechanised accounts of a municipal authority: proven schemes. London, Gee & Co., 1946. 135 pp. 8½″. 20*s.*

Mathematical tables and other aids to computation: a quarterly journal edited on behalf of the Committee on Mathematical Tables and other Aids to Computation, by R. C. Archibald, D. H. Lehmer, with the coöperation of L. J. Comrie, S. A. Joffe. Vol. I, 1–12, 1943–1945. Vol. II, 13–14, 1946. Lancaster, Pa., National Research Council. (London, Scientific Computing Service. 9¾″. 14 parts.

MENDERSAUSEN (HORST). Changes in income distribution during the great depression. (Studies in Income and Wealth, 7.) New York, National Bureau of Economic Research, 1946. xviii, 168 pp. 9¼″. $2.50.

MORTARA (GIORGIO). The calculation of life tables for populations lacking reliable birth and death statistics, with application to Brazil. (Paper presented to the Statistical Section of the 8th American Scientific Congress, Washington, D.C. 1940, pp. 321–334.) Washington, 1940. 9″.

—— Los factores demográficos del crecimiento de las poblaciones americanas en los ultimos cien años. Cordoba, Universidad Nacional, 1942. 37 pp. 9″.

—— Contribuição ao estudo das influências da imigração sôbre a taxa da natalidade. (*Rivista Brasileira de Estatistica*, **3** (1942), pp. 575–584.) 10½″.

—— Os erros nas classificações por idade dos vivos e dos falecidos, e sua influência na construção de tábuas de mortalidade. (*Revista Brasileiro de Atuária*, **2** (1942). Rio de Janeiro, 1942. 36 pp. 9″.

—— Nota sôbre as possibilidades de aproveitmento dos resultados do censo demografico para a determinação da fecundidade total da mulher Brasileira. (*Revista Brasileira de Estatistica*, **4** (1943), pp. 79–82.) Rio de Janeiro, 1943. 10½″.

—— Nota acêra do aproveitamenta do censo demográfico para a determinação da freqüência das variações de estado civil em função da idade. (*Rivista Brasileira de Estatistica*, **4** (1943), pp. 435–438.) Rio de Janeiro, 1943. 10½″.

—— A riddle resolved: Brazil's population. (*Estadistica*, I, (1943), pp. 141–151.) Mexico, 1943. 9″.

—— As tábuas de sobrevivência e suas aplicações na demografia. (*Revista Brasileira de Estatistica*, 5 (1944), pp. 67–100, 317–358.) Rio de Janeiro, 1944. 10½″. (Presented by Prof. G. Mortara.)

NATESAN (L. A.). State management and control of railways in India: a study of railway finance rates and policy during 1920–37 . . . with a foreword by Sir Ralph Wedgwood. University of Calcutta, 1946. xxiii, 496 pp. 9½″. *Rs.* 12.

NATIONAL BUREAU OF ECONOMIC RESEARCH. Studies in income and wealth 8. Conference on Research in Income and Wealth. New York, N.B.E.R., 1946. xiv, 297 pp. 9¼″. $3.

NEW YORK CITY, WORK PROJECTS ADMINISTRATION. Tables of the exponential function e^x. Federal Works Agency, Work Projects Administration for the City of New York . . . conducted under the sponsorship of the National Bureau of Standards, 1939. New York, W.P.A., 1939. [18], 535 pp. 10¾″. 20*s.*

'OCCAM.' Hints on the study of elementary statistics, by 'Occam.' London, Gee & Co., 1946. 52 pp. 8½″. 4*s.*

PAVLOVSKY (GEORGE). Agricultural planning and the business cycle. (*Monthly Bulletin of Agricultural Economics, Rome* **32**. (1941), pp. 63–97.) 9½″.

—— The effects of the war upon the agricultural situation in the non-belligerent countries in 1939–40 and 1940–41. (*Monthly Bulletin of Agricultural Economics, Rome*, **32** (1941), pp. 339–392.) 9½″.

—— Some observations concerning the spatial organization of agriculture. (*Monthly Bulletin of Agricultural Economics, Rome* **33** (1942), pp. 321–355.) 9½″.

—— The representative method and some of its applications in agricultural economics. (*Monthly Bulletin of Agricultural Economics, Rome* **33** (1944), pp. 227–265.) 9½″. (Presented by the Author.)

PIGOU (A. C.). Income: an introduction to economics. London, Macmillan, 1946. vii, 118 pp. 5*s.*

PONTIFEX (BRYAN). The City of London livery companies. London, Methuen, 1939. viii, 93 pp. 7½″. 6*s.* (Presented by the Author.)

ROBINSON (E. A. G.). Monopoly . . . with an introduction by C. W. Guillebaud . . . (Cambridge Economic Handbooks xi). London, Nisbet, and Cambridge University Press, 1946. xvi, 298 pp. 7¼″. 7*s.* 6*d.*

SINGH (TARLOK). Poverty and social change: a study in the economic reorganization of Indian rural society. London, Bombay, etc. Longmans Green, 1945. viii, 200 pp. 6*s.*

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Digitized by Arya Samaj Foundation Chennai and eGangotri

SLACK (MARGARET). The probability distributions of sinusoidal oscillations combined in random phase. (*Journal of the Inst. of Electrical Engineers*, **93** (1946), pp. 76–86.) 12″. (Presented by the Author.)

SOMERVELL (HUBERT). Labour shares production: ten years' experience of a novel system of annual wages based on production . . . Reprinted from *The Accountant*, with an appendix. . . . London, Gee & Co., 1946. 44 pp. 8½″. 2*s*.

WILSON (THOMAS). Fluctuations in income and employment with special reference to recent American experience and post-war prospects. 2nd ed. London, Pitman, 1945. xi, 213 pp. 8½″. 18*s*.

WYATT (T. W.). Money and the machine age. London, Staples Press, 1946. 132 pp. 7¼″. 8*s*.

YULE (G. UDNY). A mathematical theory of evolution, based on the conclusions of Dr. J. C. Willis, F.R.S. (*Phil. Trans. of the Royal Society*, B. 403 **213** (1924), pp. 21–87). 12″.

—— On a method of investigating periodicities in disturbed series, with special reference to Wolfer's sunspot numbers. (*Phil. Trans. of the Royal Society*, A. 642, **226** (1927), pp. 267–298). 12″. (Presented by the Author.)

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Wool in the World Economy

By Gerda Blau, Ph.D.

[Read before the Royal Statistical Society, April 25th, 1946, Dr. E. C. Snow, C.B.E., in the Chair.]

Wool is one of the world's main staples. In 1938 the value of world wool exports ranked fourth in the list of world export values of all raw materials entering international trade, and surpassed the value of world exports of all the main categories of food-stuffs, with the sole exception of wheat.*

Some wool is produced in nearly every country in the world; but the bulk of the world's clothing wool entering international trade comes from five countries in the Southern Hemisphere—from the Dominions of Australia, New Zealand and South Africa, and from two South American countries, Argentina and Uruguay. Together, these five countries accounted for 80 to 90 per cent. of total apparel wool exports before the war.†

Some indication of the relative importance of wool in the economies of the five chief wool-exporting countries can be given by the ratios of values of wool exports to values of total exports from each country. With the exception of Argentina, pre-war wool shipments from each of the main exporting countries represented from one-fifth to nearly one-half of the values of their total pre-war exports.‡

The level of world wool values in any one year has thus a strong influence on the state of prosperity in the main wool exporting countries, and the annual wool cheque is one of the factors determining their capacity to import other goods.

Whilst production of wool for export is concentrated in five countries in the Southern Hemisphere, the use of this wool in manufacture is widely diffused over the globe. Wool textile industries of varying sizes are established in 46 different countries. With the exception of the United States and the Soviet Union, all of the world's ten most important wool consuming countries use imported

* League of Nations, *Network of World Trade*, Geneva, 1942, p. 30, Table 12; *World Exports of Twenty-six Products in 1938* (new gold dollars)."

† *Wool Shipments from Chief Exporting Countries Compared with Shipments from Other Countries*

(In millions of lb., actual weight)

Exports from	1931	1932	1933	1934	1935	1936	1937	1938
Australia, Argentina, New Zealand, South Africa and Uruguay ...	1,704	1,834	1,969	1,410	1,798	1,762	1,625	1,811
Other countries	246	181	291	256	290	309	313	248
World total	1,950	2,015	2,260	1,666	2,088	2,071	1,938	2,059
Share of chief exporting countries (per cent.)	87	91	81	85	86	85	85	88

Source: Imperial Economic Committee, *World Production and Trade, 1938/39.*

‡ *Proportion of Wool Exports to Total Exports (Values), excluding Gold, Silver and Specie*

	Australia.	New Zealand.	South Africa.	Argentina.	Uruguay.
	%	%	%	%	%
1936	43	24	37	8	47
1937	34	29	34	6	46
1938	36	21	34	9	44

Source: League of Nations, *International Trade Statistics, 1938.*

Australia's overseas income from wool was larger than from any other commodity. In South Africa wool was the most important exportable agricultural product, and in New Zealand wool exports ranked next in importance to butter and meat.

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wool for 70 to 90 per cent. of their normal requirements, and their manufacturing industries are thus largely dependent on the proper functioning of the world market.

In view of the importance of wool in world trade and in the economies of so many different countries, the economic future of the wool-growing and wool-using industries may be expected not only to depend on, but also to contribute to the future state of general world economic prosperity. Plans for the future of wool marketing are therefore of more than sectional interest.

At present the importance of adequate plans for world wool disposals is enhanced by the large accumulation of stocks resulting from the economic dislocation of war. Stocks of Dominion and South American wool are estimated at about 4,000 million lb.—*i.e.*, an amount which is more than twice as large as the annual average exported from the five main Surplus Countries before the war; about four times as large as the accumulation of stocks left over at the end of the first world war; and more than thirty times as large as average end-of-season stocks during the inter-war period.

Stocks of Dominion wool represent over three-quarters of total world stocks, and are valued at about £170 million. The inter-governmental Wool Conference held in London in April 1945

TABLE I

Estimated Stocks at June 30th, 1945

(In millions of lb., actual weight)

	Country of origin						
	Australia	N.Z.	South Africa	Argentina and Uruguay	U.K.	U.S.A.	Total
U.K. owned in Dominions	1,605	370	505	—	—	—	2,480
U.K. owned in U.S.A.	225	20	110	—	—	—	355
	1,830	390	615	—	—	—	2,835
U.K. owned in U.K.	230	150	30	—	70	—	480
	2,060	540	645	—	70	—	3,315
U.S.A. Government owned in U.S.A.	—	—	—	—	—	200	200
South American owned	—	—	—	500	—	—	500
Total	2,060	540	645	500	70	200	4,015

Source: *Report and Recommendations of the London Wool Conference*, S.M.(B) 1302–1000 (28.8.45), R.M.5, Appendix A.

estimated that "on reasonably optimistic assumption, the period required to dispose of existing Empire stocks alongside new clips would be on average 13 years from June 30th, 1945." *

In the following we shall attempt to give a general survey of major factors influencing the economic position of the world wool industry. Our examination will start with a brief account of the commercially important characteristics of the wool fibre; followed by a survey of trends and short-term changes in total size and distribution of wool production and consumption; a brief analysis of pre-war markets and price movements, and methods and effects of war-time control; and an appraisal of certain new factors likely to influence future trends. On the basis of these data, we shall then proceed to consider plans and prospects for the future of wool marketing.

I. WOOL PRODUCTION

Wool—The Commodity

There are three main qualities of wool—merino, crossbred and carpet. Merino—*i.e.*, the finest and most expensive wools—and crossbred or medium types, are classed together as "apparel" wools, and constitute the bulk of all wools entering the international market.

In the British wool market the distinction between merino, crossbred and carpet types is denoted

* *Report and Recommendations of the London Wool Conference*, S.M.(B) 1302–1000 (28.8.45), R.M.5, Appendix A.

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by quality numbers; 60's and above for merino, 58's to 40's for crossbred, and all numbers below 40's for carpets. This classification is based primarily on fineness and length of the wool fibre.*

In addition to fineness and length, there are, however, many other quality characteristics which must be taken into account in determining the commercial value of the wool fibre—such as uniformity of diameter, dry and wet strength, handle and surface structure (scaliness), colour, lustre, crimp or waviness. These factors are related to some extent—*e.g.*, the finest fibres are likely to be the shortest, and those with the strongest crimp. However, the proportions in which these factors are combined vary widely according to breed, country of origin and climatic conditions during the wool season. In fact, quality characteristics may be different for each fleece of the same breed from the same country, and qualities of fibres from different parts of the same fleece also differ widely. The difficulties of standardizing wool types are illustrated by the fact that under the British Government Purchase Scheme there are 1,500 classified types of Australian wool, 950 of New Zealand wool, and 350 of South African, with relatively few types in any one clip common to the clip of another Dominion.

Additional marketing problems arise from difficulties of standardizing *quantity*. Raw wool is sold in the grease, but the value is based on the estimated clean yield, which varies not only as between main types, but also as between individual lots.

Most of the commercially important characteristics of the wool fibre can best be accurately ascertained by personal inspection. It is not surprising, therefore, that before the war a large share of the world's wool was sold by auction—*i.e.*, under a system which enables traders to examine each unit, and to bid according to the results of this personal examination.

Summary Survey of World Wool Production

Annual world production of wool averages around 4,000 million lb., greasy basis, of which about four-fifths is classified as "apparel" wool (merino and crossbred), and one-fifth as carpet wool.

A summary survey of output by main groups of wool-producing countries is given in Table II. As can be seen from these figures, the five main Surplus Countries of the Southern Hemisphere

TABLE II

World Wool Production

Summary Survey by Main Groups of Wool-producing Countries

Average 1939/40–1943/44.

Groups of wool-producing countries	Millions lb. greasy	Per cent. of world total	Group percentages
I. *Apparel Wool Production* (Merino and Crossbred)			% of apparel wool production
1. Five Main Surplus Countries (Australia, Argentina, New Zealand, South Africa, Uruguay)	2,277	57·1	70·2
2. Five Main Deficit Countries (United States, United Kingdom, France, Germany, Canada)	660	16·6	20·3
3. Other Countries	307	7·7	9·5
I. Apparel Wool Production, Total	3,244	81·4	100·0
II. *Carpet Wool Production*			% of carpet wool production
1. Three Main Carpet Wool Producing Countries (U.S.S.R., China, India)	465	11·7	62·8
2. Other Countries	276	6·9	37·2
II. Carpet Wool Production, Total	741	18·6	100·0
World Wool Production, Total	3,985	100·0	—

* The quality numbers are related to the "counts" of yarns—*i.e.*, to the number of "hanks" (560 yards of yarn) which can be spun from 1 lb. of scoured wool. Thus, a 60's quality wool was originally so called because of its ability to be spun into 60 "hanks," each of 560 yards length, the total weight of which would be equal to 1 lb. At present, quality numbers have come to be based primarily on fineness of diameter measurement, and a 60's wool or top need not necessarily be able to spin a 60's count of yarn (*cf.* S. G. Barker, *Wool Quality*, London, 1931, p. 8).

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account for 70 per cent. of total apparel wool output, and for well over one-half of world wool production. The second main group of "apparel wool producers" comprises the five main Deficit Countries—the United States, United Kingdom, Canada, France and Germany—accounting together for 20 per cent. of total apparel wool supplies. The United States is by far the most important producer in this second group. Indeed, the American domestic wool clip is the third largest in the world, ranking next in importance only to those of Australia and Argentina. At the same time, the United States belongs to the class of Deficit Countries because her own output is not sufficient to meet domestic needs, and considerable quantities of Dominion and South American wool are imported. Together, the ten countries comprising the Surplus and Deficit Groups account for 90 per cent. of the world's apparel wool output, and 73 per cent. of the world clip.

The world's three main carpet wool producing countries are the U.S.S.R., India and China, which account for 63 per cent. of total carpet wool output, and 12 per cent. of world wool output. The Balkans, Turkey, Iran, French Africa and some South American countries produce the bulk of the remaining 37 per cent. of carpet wool production, or 7 per cent. of the total for all wools. Carpet wools are largely home consumed, and represent less than 10 per cent. of world wool exports. These coarse wools are therefore of relatively minor importance for this study, which is primarily concerned with wool as an international commodity.

Apparel Wool Production—Historical Development

The concentration of the world's exportable surplus of apparel wool in five countries in the Southern Hemisphere is a relatively new development, which grew to its present dimensions only during the second half of the nineteenth century. Until that time the world's most important wool-growing countries were England, Spain and Germany. Up to the earlier part of the fourteenth century most of England's raw wool was exported to the Continent, mainly to the Netherlands, for processing and dyeing. Edward III effected the first transfer of Flemish workers to Britain and at the same time he prohibited exports of raw wool and imports of cloths. By these measures Britain became both the chief wool-growing and wool-manufacturing country of Europe, and in 1454 Parliament declared that "the making of cloth within all parts of the realm is the greatest occupation and living of the poor commons of this land." Under Elizabeth, England's dependence on wool increased, and towards the close of the seventeenth century woollen and worsted manufactures represented nearly two-thirds of all exports of the country.*

Up to the middle of the eighteenth century the British wool industry was mainly a domestic one, but the discoveries of Kay,† Hargreaves,‡ Arkwright,§ and Crompton ‖ made mechanical spinning and the development of large-scale industry possible. England's wool production became insufficient for her needs, and wool began to be imported.

The revolutionizing changes in size and distribution of the world's wool production date from the time when the first merino sheep were brought to the countries of the Southern Hemisphere towards the end of the eighteenth century. Climatic conditions in these overseas areas proved favourable, and the labour-extensive methods of sheep farming were well suited to the agricultural development of new and thinly populated countries with vast land resources. Besides, wool is a reasonably non-perishable commodity and, compared with other agricultural products, has a relatively high value per unit of weight and bulk.¶ These were additional important factors influencing overseas farmers, whose produce could not be sold in their relatively quite insignificant internal markets, but had to be shipped to Europe to meet the rapidly growing needs of the expanding industrial population in the older countries. This development was, of course, greatly facilitated by the improvements of communications during the latter part of the nineteenth century.

Exact records of sheep numbers in the nineteenth century are not available; but some indication of the growth of the overseas sheep population is given by the following approximate estimates collated from different sources.

* E. Lipson, *History of Woollen and Worsted Industries*, London, 1921.

† John Kay (1704–64). In 1733 Kay took out patents for the fly-shuttle. By this method the use of one hand only was required, the other being employed in beating up or closing the weft.

‡ James Hargreaves (1710–78), inventor of the spinning-jenny.

§ Sir Richard Arkright (1732–92), constructor of the spinning-frame.

‖ Samuel Crompton (1753–1827), inventor of the mule.

¶ J. E. Nichols, *A Study of Empire Wool Production*, Leeds, 1932.

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Sheep Numbers in Certain Areas, 1840–1909 *

	Main old European sheep-growing areas (Britain, France, Germany, Spain)	Main new overseas sheep-growing area (Australia, New Zealand, South Africa Argentina, Uruguay).
	Million head	Million head
1840	90	5
1860	90	44
1880	87	164
1909	72	239

The sheep industries of New Zealand (where the first merino sheep were introduced in 1844) and of the South American countries were decisively influenced by the development of cold-storage transport from 1880 onward. The merino, though growing the finest wool, is a poor mutton sheep, and with the development of the facilities for sending meat overseas, the South American countries and New Zealand found it more profitable to evolve breeds of a dual-purpose animal for which climatic conditions in many parts of Australia and in South Africa are not suitable. To-day crossbred wool constitutes 98 per cent. of the New Zealand clip, and 86 per cent. of the clips of Argentina and Uruguay.

Apart from the increase in sheep numbers, the expansion of overseas wool production was greatly accelerated by improved breeding, which led to substantial increases in average yields per sheep. In New South Wales, for instance, the weight per fleece was increased from an average of 4·9 lb. (greasy) in 1876–81 to an average of 7·23 lb. in 1909–12; and 8·18 lb. in 1935–39. †

The development of wool production in the three Dominions during the period from 1800 to 1940 is shown as follows:

Wool Production in Australia, New Zealand and South Africa, 1800–1940 ‡

(In lb. greasy)

	Australia	South Africa	New Zealand
1800	20,000	30,000	—
1840	51,000,000	1,000,000	—
1880	307,000,000	42,500,000	69,000,000
1920	625,000,000	182,000,000	208,000,000
1940	1,077,000,000	260,000,000	333,000,000

As can be seen from these figures, the expansion of overseas wool production did not come to a stop in the twentieth century, but was continued during the inter-war period. Mainly owing to the influence of overseas growth, the combined total of apparel-wool output in the ten most important producing countries in 1939–43 was nearly 60 per cent. higher than in 1890–94. This is shown in Table III, which also illustrates changes in the proportions of merino and crossbred output over that period. The figures reveal the drastic shift from merino to crossbred during the 'nineties. From the beginning of the twentieth century onward the merino/crossbred ratio remained, on the whole, fairly stable, output of each section representing about one-half of the total. The reduction of the merino share from 55 per cent. of the total in 1909/13 to 46 per cent. in 1914/18 was partly due to war-time demand for coarser types, but was also much accentuated by a severe drought in 1915/16, which caused an absolute decline in merino output. During 1939/43, on the other hand, merino production showed a considerable absolute increase, and, as a result of this, the effect of war-time conditions on the merino/crossbred ratio by groups was relatively small, despite an increase in the Australian clip's crossbred percentage from 16 per cent. in 1938/39 to 23 per cent. in 1943/44. At the same time it may be expected that the time-lag in the influence of war-time changes in relative demand on shifts in supply, coupled with the severe and protracted effects of the 1944/45 drought on merino production, will lead to a further relative increase in the average of crossbred supply during the next three-year period.

* Sources: Sauerbeck, *Production and Consumption of Wool*, 1878. W. L. Senkel, *Wollproduktion und Wollhandel im 19 Jahrhundert*. H. Behnsen and W. Genzmer, *Weltwirtschaft der Wolle*, Berlin, 1932. Empire Marketing Board, *Wool Survey*, 1932. New Zealand Official Yearbooks. F. Cronje, *The World Production, Consumption and Prices of Clothing Wools, with Special Reference to Merino Wool* (mimeogr.), Cambridge, 1937.

† Cf. *Dalgety's Annual Wool Review*, 1935/36, p. 4, "Average Yield per Sheep"; and C.S.I.R., *Wool and Australia—Some Facts*, June 1945.

‡ Sources: W. L. Senkel, *Wollproduktion und Wollhandel im 19 Jahrhundert*. H. Behnsen and W. Genzmer, *Weltwirtschaft der Wolle*, Berlin, 1932. Empire Marketing Board, *Wool Survey*, 1932. New Zealand Official Yearbooks.

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TABLE

Apparel Wool Production by Main Groups of

Five-year averages,

Five-year averages	PART I						PART	
	Group A: Five main surplus countries						Group B:	
	Australia	New Zealand	South Africa	Argentina	Uruguay	Total, Group A	U.S.A.	U.K.
								Section I (millions
1909/10–1913/14 ...	757	220	154	341	112	1,584	347	114
1919/20–1923/24 ...	700	229	186	326	108	1,549	304	88
1924/25–1928/29 ...	878	256	243	328	122	1,827	338	116
1929/30–1933/34 ...	981	275	305	361	122	2,044	439	119
1934/35–1938/39 ...	996	299	250	378	111	2,034	453	110
1939/40–1943/44 ...	1,112	324	258	457	126	2,277	467	98
								Section II (Index:
1909/10–1913/14 ...	100	100	100	100	100	100	100	100
1919/20–1923/24 ...	93	104	121	95	97	98	88	77
1924/25–1928/29 ...	116	116	158	96	109	115	98	102
1929/30–1933/34 ...	130	125	198	106	109	129	127	105
1934/35–1938/39 ...	132	132	162	111	99	128	131	97
1939/40–1943/44 ...	147	145	168	134	113	144	134	86
								Section III
	% of total (group A)						% of total	
1909/10–1913/14 ...	48	14	10	21	7	100	58	20
1919/20–1923/24 ...	45	15	12	21	7	100	61	18
1924/25–1928/29 ...	48	14	13	18	7	100	61	20
1929/30–1933/34 ...	48	13	15	18	6	100	66	18
1934/35–1938/39 ...	49	15	12	19	5	100	67	17
1939/40–1943/44 ...	49	14	11	20	6	100	70	15

TABLE III

Production of Merino and Crossbred Wool in Ten Countries (a).

Five-year averages, 1889/93–1939/43

Five-year averages (*b*)	Millions of lb. greasy			Index: 1889/90–1893/94 = 100			Per cent.	
	Merino	Cross-bred	Total	Merino	Cross-bred	Total	Merino	Cross-bred
1889/90–1893/94	1,210	631	1,841	100	100	100	67	33
1894/95–1898/99	1,030	933	1,963	85	148	197	53	47
1899/1900–1903/04	831	965	1,796	69	153	98	46	54
1904/05–1908/09	999	923	1,922	83	146	104	52	48
1909/10–1913/14	1,194	986	2,180	99	156	118	55	45
1914/15–1918/19	955	1,114	2,069	79	176	112	46	54
1919/20–1923/24	932	1,117	2,049	77	177	111	45	55
1924/25–1928/29	1,271	1,114	2,385	105	176	130	53	47
1929/30–1933/34	1,435	1,270	2,705	119	200	147	53	47
1934/35–1938/39	1,380	1,329	2,709	114	210	147	51	49
1939/40–1943/44	1,478	1,459	2,937	122	231	160	50	50

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IV

Countries Compared with World Wool Production

1909/13–1939/43.

II				PART III			
Five main deficit countries				Groups A and B compared with world production			
Canada	France	Germany	Total, group B	Group A total	Group B total	Groups A and B total	World total production
of lb. greasy).							
13	78	44	596	1,584	596	2,180	3,063
20	43	45	500	1,549	500	2,049	2,857
17	46	41	558	1,827	558	2,385	3,382
20	51	32	661	2,044	661	2,705	3,788
19	54	39	675	2,034	675	2,709	3,773
17	31	47	660	2,277	660	2,937	3,985
1909/10–1913/14 = 100)							
100	100	100	100	100	100	100	100
154	55	102	84	98	84	94	93
131	59	93	94	115	94	110	110
154	65	73	111	129	111	124	124
146	69	89	113	128	113	124	123
131	40	107	111	144	111	135	130

(Percentages)

(group B)				% of groups A plus B			% of world total		
				A	B	A plus B	A	B	A plus B
2	13	7	100	73	27	100	52	19	71
4	9	8	100	75	25	100	53	18	71
3	8	8	100	77	23	100	54	17	71
3	8	5	100	75	25	100	54	18	72
3	8	5	100	75	25	100	54	18	72
3	5	7	100	77	23	100	57	17	74

Sources of Table III:

(*a*) Australia, New Zealand, South Africa, Argentina, Uruguay, United States, United Kingdom Canada, France, Germany.

(*b*) Australia, New Zealand, South Africa: clip years, July 1st–June 30th. Argentina and Uruguay: October 1st–September 30th. Other countries: calendar years.

Sources: 1889/90–1908/9: H. Stoker, "World Production and Price of Wool," in *Farm Economics*, No. 67, 1930, published by New York State College of Agriculture, Cornell University.

1909/10–1923/24: Merino and crossbred percentages from H. Stoker, but actual figures adjusted on basis of Empire Marketing Board estimates. 1924/25–1930/31: Empire Marketing Board, *Wool Survey, 1932*. 1931/32–1938/39: Imperial Economic Committee, *Wool Production and Trade, 1938/39*. 1939/40–1943/44: Wool Conference Report, S.M.(B) 1302–1000 (28.8.45), R.M.5. (U.S.A. figures adjusted according to U.S.A. Department of Agriculture estimates.)

Sources of Table IV:

1909/10–1923/24:

Australia:
New Zealand:
South Africa:
Argentina:
Uruguay:
U.S.A.:
United Kingdom: Commonwealth Bureau of Census and Statistics, Canberra, quoted by H. Schueler, *Wolle*, Leipzig, 1936.

U.S. Department of Agriculture, *The Wool Situation*, March 1941.

1909/10 estimated. 1910/11–1923/24: Ministry of Agriculture and Fisheries, *Report on Wool Marketing in England and Wales*, Economic Series No. 7 (adjusted to exclude production in Eire).

Canada:
France: 1909/10–1914/15: estimated. 1915/16–1923/24: *Dalgety's Annual Review*, 1932/33. U.S. Department of Agriculture: *Agricultural Statistics*, 1923, 1924 and 1927.

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Sources of Table IV (contd.):

Germany: H. Behnsen and W. Genzmer, *Weltwirtschaft der Wolle*, Berlin, 1932.
World: 1909/10–1913/14: H. Behnsen and W. Genzmer, *Weltwirtschaft der Wolle*, Berlin, 1932.
1914/15–1923/24: U.S. Department of Agriculture, *Agricultural Statistics*, 1923, 1924 and 1927.
1924/25–1930/31: Empire Marketing Board, *Wool Survey*, H.M.S.O., London, 1932.
1931/32–1938/39: Imperial Economic Committee, *Wool Production and Trade*, H.M.S.O., London, 1939.
1939/40–1943/44: Wool Conference Report, S.M.(B) 1302–1000 (28.8.45), R.M.5 (U.S.A. figures adjusted according to estimates of U.S. Department of Agriculture quoted in *The National Association of Wool Manufacturers' Bulletin*, New York, August 1945).

TABLE

War-time Production of Apparel

Season	Group A: Five main surplus countries						Group B:	
	Australia	New Zealand	South Africa	Argentina	Uruguay	Total	U.S.A.	U.K.
								Section I (millions
1938/39 ...	985	328	264	394	114	2,085	458	110
1939/40 ...	1,121	302	271	375	134	2,203	450	111
1940/41 ...	1,077	333	260	409	139	2,218	457	109
1941/42 ...	1,108	336	260	473	115	2,292	478	92
1942/43 ...	1,120	331	250	511	120	2,332	481	91
1943/44 ...	1,132	320	250	518	120	2,340	471	89
1944/45 ...	948	364	210	518 (*b*)	120 (*b*)	2,160 (*a*)	442	89 (*b*)
1945/46 ...	847 (*a*)	364 (*b*)	195 (*a*)	518 (*b*)	120 (*b*)	2,044 (*a*)	412 (*a*)	89 (*b*)
								Section II (Index:
1938/39 ...	130	149	172	116	102	132	132	97
1939/40 ...	148	137	176	110	120	139	130	97
1940/41 ...	142	150	169	120	124	141	132	96
1941/42 ...	146	153	169	141	103	145	138	81
1942/43 ...	148	150	162	150	107	147	139	80
1943/44 ...	150	145	162	152	107	148	135	78
1944/45 ...	125	165	136	152 (*b*)	107 (*b*)	136 (*a*)	127	78 (*b*)
1945/46 ...	112 (*a*)	165 (*b*)	127 (*a*)	152 (*b*)	107 (*b*)	129 (*a*)	119 (*a*)	78 (*b*)

(*a*) Preliminary estimate.

Sources:

1938/39–1943/44: See footnotes to Table IV.
1944/45–1945/46:
Australia, 1944/45; *Dalgety's Annual Wool Review*; 1945/46, Dalgety & Co., provisional estimate.

A graphic illustration of these long-term changes in wool production is shown in Chart No. 1 (pp. 194–195). A comparison of the production curves shown in this chart with the curves of merino and crossbred price movements (Charts II–IV) gives a first approximation of the marked contrast between the moderate and fairly steady movement of apparel wool output and the violent and often erratic changes in price levels, especially during the two decades of the inter-war period. We shall consider this contrast in some more detail in a survey of wool price movements later in this paper (p. 215).

Apparel Wool Production Changes, 1909/10–1943/44

A more detailed survey of wool production developments from 1909/10 to 1943/44 is given in Table IV (pp. 184–185). As in Table II, the output of the ten most important apparel wool producing countries has again been divided into two main groups: (A) "Surplus Countries" and (B) "Deficit Countries." The first two sections of Parts I and II of Table IV show five-year averages of output (by weight) and production indices (based on 1909/13 = 100) for each of the countries in both Groups. Section III shows the individual countries' shares in the respective Groups' output.

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In Part III the indices and shares by groups are compared with each other and with world output.

Let us first look at the Summary Survey given in Part III of the table. This shows that world wool production, which before World War I averaged just over 3,000 million lb., was increased by 23 per cent. above the 1909/13 level in 1934/38, and by 30 per cent. in 1939/43. Output in the Deficit Countries' Group in 1939/43 was only 11 per cent. above the 1909/13 level; but the increase in world wool output during the inter-war period was strongly influenced by the more than pro-

IVA

Wool by Main Groups of Countries

Five main deficit countries				Apparel wool production (Groups A and B) compared with world production			
Canada	France	Germany	Total	Group A total	Group B total	Groups A and B total	World wool production
of lb., actual weight)							
19	54	45	686	2,085	686	2,771	3,991
16	39	42	658	2,203	658	2,861	3,910
16	30	43	655	2,218	655	2,873	3,932
16	28	45	659	2,292	659	2,951	3,994
18	28	48	666	2,332	666	2,998	3,969
19	28	56	663	2,340	663	3,003	3,920
19 (*b*)	28 (*b*)	56 (*b*)	634 (*a*)	2,160 (*a*)	634 (*a*)	2,794 (*a*)	3,958 (*a*)
19 (*b*)	28 (*b*)	56 (*b*)	604 (*a*)	2,044 (*a*)	604 (*a*)	2,648 (*a*)	3,760 (*a*)
1909/13 = 100)							
146	69	101	115	132	115	128	130
123	50	96	110	139	110	131	128
123	38	98	110	141	110	132	128
123	36	101	111	145	111	135	130
138	36	109	112	147	112	137	130
146	36	127	111	148	111	138	128
146 (*b*)	36 (*b*)	127 (*b*)	106 (*a*)	136 (*a*)	106 (*a*)	128 (*a*)	129 (*a*)
146 (*b*)	36 (*b*)	127 (*b*)	101 (*a*)	129 (*a*)	101 (*a*)	121 (*a*)	123 (*a*)

(*b*) Assumed same as previous year.

New Zealand, 1944/45; *Dalgety's Annual Wool Review.*
South Africa, 1944/45; *Report of British Wool Commission.*
U.S.A., 1944/45 and 1945/46: United States Department of Agriculture.
World, 1944/45 and 1945/46: United States Department of Agriculture.

portionate expansion of production in the Surplus Countries, which represent more than half of world output.

Looking at Parts I and II of Table IV, we find that the expansion of output in the Surplus Group was mainly due to increased production in the three Dominions, which in 1934/38 accounted for over 80 per cent. of the Group's total. From 1934/38 to 1939/43 there was a considerable increase in the production of all five Surplus Countries: especially marked in the Argentine, where output over this period rose by more than 20 per cent. In considering the 11 per cent. increase of Australian production over the same period, it must be remembered that this increase has since been wiped out by severe drought conditions, which reduced the Australian clip to 95 per cent. of the 1934/38 average in 1944/45 and to 85 per cent. in 1945/46. Drought losses during the 1944/45 season have also affected sheep numbers in South Africa and Argentina; but despite these downward changes, the estimated level of the Surplus Group's current output is still about one-third above the 1909/13 average (*cf.* Table IVA).

The Deficit Group's 11 per cent. increase from 1909/13 to 1939/43 was mainly due to the rise by one-third in the size of the American clip. America's share in the Deficit Group's total rose

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from 58 per cent. in 1909/13 to 70 per cent. in 1939/43, and thus more than counterbalanced the downward output changes in some of the European countries.

The peak of American wool production was passed in 1942, and it is possible that U.S. sheep numbers will continue to decline during the next decade.* The effect of such a possible future decline in American output on the Deficit Group's total would probably be offset to some extent by a period of re-stocking in European countries; but European expansion is not in itself likely to be large enough to lead to a future increase in the Deficit Group's total.

Summing up our examination of Table IV, we may say that the expansion of world wool production by about one-third during the last three decades was dominated by the more than proportionate rise in the output of the Surplus Group, which accounts for more than half of world wool supply.

In order to see the expansion of world wool production in proper perspective, it is, however, essential to compare it with changes in the world output of other main textile fibres used for clothing purposes, and with changes in the size of the world population. Such a comparison is given in Table V.

TABLE V

World Production of Four Main Textile Fibres Used for Clothing Purposes, and World Population, 1909/13–1939/43

Five-year averages	Cotton	Wool (clean)	Silk	Rayon and staple fibre	Clothing fibres total	World population
Section I	million lb.					Million inhabitants
1909/13	10,648	1,685	65 (*a*)	20 (*b*)	12,418	1,786 (*b*)
1924/28	12,452	1,860	112	240	14,664	1,925
1929/33	12,364	2,089	126	526	15,105	2,025
1934/38	14,186	2,075	122	1,398	17,781	2,112
1939/43	13,552	2,192	99	2,581	18,424	2,170 (*c*)
Section II	Index: 1909/13 = 100					
1909/13	100	100	100 (*a*)	100 (*b*)	100	100 (*b*)
1924/28	117	111	170	1,200	118	108
1929/33	116	124	194	2,630	121	113
1934/38	133	123	188	6,990	143	118
1939/43	128	130	174 (*c*)	12,965	148	122 (*c*)
Section III	Clothing Fibres: Percentage Shares.					
1909/13	85·6	13·6	0·5	0·2	100	
1924/28	85·0	12·7	0·7	1·6	100	
1929/33	81·1	14·5	0·9	3·5	100	
1934/38	79·8	11·6	0·7	7·9	100	
1939/43	73·6	11·9	0·5	14·0	100	

(*a*) Average 1911/15. (*b*) Average 1910/13. (*c*) 1939.

Sources:

Cotton (ginned; crop years): 1909/41: *League of Nations Yearbook*. 1942 and 1943: U.S. Department of Agriculture estimates.
Wool: clean basis; estimated on 60 per cent. of grease basis. Sources: *cf.* notes to Table III.
Silk: 1911/14: *League of Nations Yearbook*, 1926/28. 1924/43: *Rayon Organon*, New York.
Rayon and Staple Fibre: 1910/43: *Rayon Organon*, New York.
World Population: 1913: League of Nations, *Production and Trade*, 1926 and 1929/30. 1924/39: *League of Nations Yearbook*.

* Prices for American domestic wool are now subsidized by the U.S. Government. This official subsidy scheme is likely to continue for some time: but if it were reduced in scale and/or if a downward revision of the American wool tariff were agreed upon, the effects on U.S. output might be considerable.

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The figures shown in Section II of Table V indicate that the rate of increase in the world output of wool and cotton since 1909/13 was, on the whole, not very much ahead of the rate of growth of the world population; the 1939/43 averages of wool and cotton production having increased over the 1909/13 level by 30 and 28 per cent. respectively as compared with a world population increase of 22 per cent. by 1939. Silk output constitutes a very small fraction of total supply of all clothing fibres (less than 1 per cent.), and the influence of changes in the size of silk production on the index of combined "clothing fibres" output (by weight) is therefore negligible.*

Hence, the more than proportionate rise of the combined "clothing fibres" index (48 per cent. as compared with 28 per cent. for cotton and 30 per cent. for wool) was strongly influenced by the spectacular expansion of output of rayon and staple fibre over the period under consideration. This is also illustrated by the changes in percentage shares (*cf.* Table V, Section III). Cotton succeeded in maintaining a dominating proportion of over 70 per cent. in the total; but to some degree even the share of cotton, just like the shares of wool and silk, was affected by the broadening wedge of rayon's share, which rose from 0·2 per cent. in 1909/13 to 14 per cent. in 1939/43. The significance of this development will be considered in some more detail in our analysis of factors influencing the world consumption of wool (pp. 200–206). At this juncture itwas merely intended to show the expansion of world wool production in proper relief by setting it against a brief background sketch of clothing fibre developments in general.

Year-to-Year Changes in Wool Supply

Despite occasional sharp changes in the size of individual countries' clips, short-term variations in wool supply by groups of countries are, on the whole, relatively small. A survey of such year-to-year changes during the 35-year period from 1909/10 to 1943/44 is given in Table VI. The

TABLE VI

Wool Production: Year-to-Year Changes, 1909/10–1943/44

Groups of producing countries	Percentage changes from previous year's total, grouped according to size			Total number of year-to-year changes
	Less than 5%	5%–less than 10%	10%–less than 15%	
Group A: Five Main Surplus Countties ...	25	8	1	34
Group B: Five Main Deficit Countries	29	5	—	34
Groups A, plus B: Ten Selected Countries ...	25	8	1	34
Merino Section of Groups A plus B	23	6	5	34
Crossbred Section of Groups A plus B	29	4	1	34
World Total	29	4	1	34

figures show that even in the Surplus Group, where year-to-year changes were relatively more marked than in the Deficit Group, only 2 out of 34 year-to-year changes exceeded 10 per cent. of the previous year's total, whilst in 24 instances the changes were less than 5 per cent. In the Deficit Group 29 out of 34 year-to-year changes were less than 5 per cent. and none were more than 10 per cent.

Dividing the total output of both groups into merino and crossbred sections, we find that year-to-year fluctuations in merino production were, on the whole, more marked than in crossbred. This is partly to be explained by the more serious effects of drought in the arid and semi-arid merino-producing areas, causing sudden changes in the merino clips in certain years. In part, the relatively greater stability in the crossbred section is also caused by the influence of meat prices, which are more stable than wool prices, and which tend to make the supply of crossbred wool

* The term of an index of "clothing fibres" output (by weight) is found convenient for this rather general exposition; but it must, of course, be borne in mind that this term is based on an over-simplification. Apart from various uses of clothing fibres for the manufacture of household goods and furnishings, a share of the world's output of cotton, rayon and even "apparel wool" is used for industrial production (*e.g.*, insulating materials, tyres, industrial wool-felts, etc.). On the other hand, other fibres which are not included in Table V are also used for clothing purposes (*e.g.*, flax). However, the use of the rather loose term "clothing fibres output" as defined above, may be permitted for bringing out the main features of the general development with which we are here concerned.

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even less responsive to changes in wool values than the supply of merino wool.* The last line of Table VI shows that year-to-year changes in the size of total world wool production over the period considered correspond to the pattern of the more stable "crossbred section" of apparel wool output.

Factors Influencing Supply

In examining the main factors influencing changes in wool supply, it may be convenient to start on the assumption that these factors can be roughly divided into:

(i) factors beyond the producers' control; and
(ii) deliberate changes in wool supply.

However, recent scientific improvements in breeding and sheep-husbandry tend to blur the dividing line between (i) and (ii). Certain factors which, in earlier times, had to be classified as entirely "beyond the producers' control"—*e.g.*, the effects of certain pastoral pests—can now be kept in check by scientific methods. Even the effects of drought can be mitigated to some extent by improved methods of irrigation and artificial feeding. Given the technical limits of their effectiveness, the degree to which such labour-intensive and capital-intensive methods are applied depends on decisions which are within the producers' control and which can be influenced by their cost calculations and market expectations. Soil erosion is another important element on the border line between (i) and (ii). To the individual farmer, erosion may be a factor beyond his control, but to a larger group, or to the country as a whole, it may be a factor which can be influenced by deliberate policy.† On the whole, however, scientific development, whilst lessening to some extent the arbitrariness of factors beyond the producers' control, has not succeeded in eliminating the importance of these factors.

On the other hand, the scope for producers' deliberate decisions of policy based on their market expectations may be severely restricted in practice by the lack of alternative uses for the productive resources at their disposal. Where alternative uses of land do not exist—as in certain arid and semi-arid regions—the owners of the land are not really "free" in their decisions of production policy, and may find it worth while to hang on, or even attempt to increase output, during bad market periods. Indeed, the fall in land values during such periods may tend to make them even more reluctant to sell their property.

Again, an important section of wool output in more fertile areas is produced jointly with meat (crossbred wool, mutton and fat lamb). In this section, changes in wool output may be brought about as a mere by-product of deliberate decisions in regard to meat output which tends, at times, to make the supply of crossbred wool even less elastic than the supply of merino.‡

* *Cf.* below, pp. 34–37.

† The Australian Rural Reconstruction Commission, in their Third Report, on "Land Utilization and Farm Settlement" (1943), stress the essential need for concerted measures to combat the serious effects of soil erosion in Australia.

"... The Commission earnestly recommends that each State should establish a special soil conservation organization." It is further recommended that these central soil-conservation authorities should work in close co-operation with all other authorities concerned with land utilization; and that they should also take a special interest in the provision of adequate credit for approved soil-conservation work. "The Commission emphasizes that the authority dealing with erosion in a State will succeed only if Governments accept their full responsibility for the problem and ensure that the State's most vital heritage, its soil, is receiving adequate protection"; *cf.* also U.S. Dept. of Agriculture, 1938 Yearbook, *Soils and Men*; "The Long-Term Problem of Soil Conservation" by J. B. Abbott, in *The Future of the Wool Industry*, a paper included in *Australia's Post-War Economy*, Sydney, 1945; K. B. Cumberland, *Soil Erosion in New Zealand*, Soil Conservation and Rivers Control Council, Wellington, 1944.

‡ Whilst using the customary and convenient broad division between "merino" and "crossbred" production, it should be borne in mind that, in reality, the classification of types of sheep by returns is more gradual and more complex. Nichols, *op. cit.*, p. 8, states: "Sheep may be grouped into eight main types according to the nature of the return secured by the breeder; Australia furnishes examples of most of them. In general the progression from one type to another follows that of the zones of origin and is imposed upon the progression from pastoral to agricultural conditions. Thus in the Merino zone occur sheep, chiefly wether, which are kept as wool producers only (type 1), and breeding sheep which are wool producers and return profit by the sale of their castrated male offspring as potential wool producers (type 2). These are succeeded, largely within the 'comeback' and quarter-bred zone, by animals which are bred for the purpose of producing wethers' offspring as potential wool and mutton producers (type 3), and which have a greater rate of increase (or higher fertility, . . .) and so supply surplus stock, of both sexes, for sale for breeding and for wool and mutton production (type 4). In all these the production of wool is the main issue and the stock is predominantly of Merino kind, but successive crossing with Long-

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Over the short period the effectiveness of graziers' decisions, especially in merino production, is limited by the length of the production period (averaging from about 3 years in New Zealand to 5–6 years in the merino regions of New South Wales *), and by the rigid nature and relative importance of their overhead costs.

A reliable analysis of costs of wool production is a difficult matter, because costs cannot be easily ascertained, and vary widely as between different properties in different areas, different types of breeding and grazing management, and, of course, also to a very marked degree as between good and bad years. In the Australian Riverina district, for instance, the average of drought expenses calculated for a number of properties varied as between 0·09*d.*, or less than about 2 per cent. of total working expenses per lb. of wool in 1909/13 and 1931/32; 1·74*d.*, or 15 per cent. in 1929/30; and 9·61*d.*, or 41 per cent. in 1919/20.† In view of such wide variations, it is also difficult to differentiate as between "high-cost" and "low-cost" producers. The more labour-intensive and capital-intensive types of production may have to be classified as belonging to the "high-cost" strata in a climatically good year and to the relatively "low-cost" strata in a b ad year Despite all these difficulties, however, some indication of the structure of wool production costs can be gained from the survey given in Table VII, which shows "Average Cost Attributable to Wool Only" as estimated for Queensland in 1938.‡

For the purpose of our analysis, the different cost items have been roughly grouped under three main categories: (*a*) variable, (*b*) semi-variable, and (*c*) fixed. The cost of depreciation, interest on capital and rent, which must all be classified as "fixed" expenses (*i.e.*, independent of the volume

TABLE VII

Average Cost Attributable to Wool Only. Queensland, 1938

	Pence per lb. of wool	% of working expenses	% of total expenses
Working Expenses			
(*b*) Wages, salaries and rations	2·04	34	18
(*a*) Shearing and crutchings	1·57	26	14
(*a*) General road and rail carriage (excl. of rail freights on wool) ...	0·38	6	3
(*b*) Repairs and maintenance	0·45	8	4
(*c*) Rent	0·62	10	6
(*b*) Rates and assessments	0·28	5	3
(*b*) Miscellaneous expenses	0·64	11	6
Working expenses, total	5·98	100	54
(*a*) Drought expenditure	0·85	—	8
(*a*) Freight, handling and selling charges	1·11	—	10
(*c*) Depreciation	1·00	—	10
(*c*) Interest on borrowed capital	1·39	—	12
(*c*) Interest on owner's capital	0·77	—	6
Total	11·10	—	100

(*a*) = Variable expenditure (35 per cent. of total expenses).
(*b*) = Semi-variable expenditure (31 per cent. of total expenses).
(*c*) = Fixed expenditure (34 per cent. of total expenses).

Computed from figures quoted by the *Report of the Wool Advisory Commission Appointed to Enquire into the Economic Condition of the Wool Industry in Queensland*, Brisbane, 1939.

wool and then Down stock, leads gradually to the higher developed mutton forms, such as wool-producing 'half-bred' ewes, whose offspring are largely used as a basis for mutton production (type 5); wool-producing ewes, whose wether lambs are sold for early mutton and fat lamb, and whose ewe lambs are kept to breed fat lamb (type 6); and crossbred ewes themselves finished for mutton after their useful life as the mothers of early-maturing lambs is ended (type 7). The incidence, in Australia, of the eighth type of pure-bred mutton ewes producing crossbred lambs, is small. (It will be noted that the progression is also towards early maturity of offspring.)"

* Schueler, *op. cit.*, p. 74. The reasons for the shorter period in New Zealand are the higher lambing percentages and lamb slaughterings in crossbred production as compared with merino production.

† *The Australian Wool Industry*; Report by the Commonwealth Wool Inquiry Committee, Canberra, October 1932, p. 18.

‡ *Report of the Wool Advisory Commission Appointed to Enquire into the Economic Condition of the Wool Industry in Queensland*, Brisbane, 1939.

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of annual output on a given property, account for just over one third of the total. In presenting the figure for "interest on owner's capital," the authors of the survey point out that this estimate is based on current capital values of sheep lands and stock, and not on the actual amount of capital invested in the past, which, if ascertainable, would probably be found to be considerably higher than current values. About another third of total costs must be classified as "semi-variable"—*i.e.*, only partly related to the volume of annual output. Among the items falling within these categories are salaries and wages (other than the cost of shearing), repairs and maintenance and "miscellaneous expenses," which include a collection of fixed items (such as insurance, office charges and accountancy fees) and variable items (such as expenses for pest destruction). The composite share of "fixed" and "semi-fixed" expenses represents 65 per cent. of the total.

The sheep-carrying capacity of a given area is variable to a degree, but over the longer period the margin of variability is often much less than may be apparent in the short run. In other words, the higher proportion of fixed and semi-fixed expenses may induce farmers to make up for losses per unit of wool in slump periods by increasing the number of sheep on a given area; but if this process is carried far, the temporary gain due to larger numbers may be offset, or more than offset, by a very high rate of loss of flock numbers in over-stocked areas during the next drought period.

Indeed, the reader will have noted by now that the division between (i) "factors beyond the producers' control" and (ii) "deliberate changes in wool supply" has been introduced, for the sake of convenient argument, mainly in order to show that most of the important factors influencing supply do not, strictly speaking, belong to either one or the other of these groups, but to an intermediate position somewhere in between the two. The composite effect of these various and partly divergent influences cannot be accurately measured, but emphasis must be laid on the relative importance of rigidities in wool production which tend to lessen the responsiveness of supply to price changes, especially over the shorter period.

Wool Production Prospects

The delegates to the London Wool Conference, in their Report on *Facts of Accumulation and Prospects of Disposal*, estimated that it would take two years to make up for the drought losses of the 1944/45 season, and that from 1947/48 to 1957/58 the combined total of wool production in the three Dominions could be expected to remain fairly stationary at a level corresponding to the 1939/44 average.* No explicit statement was made in regard to the anticipated trend of wool production in all other countries; but an indirect clue is given in the Report's estimates of surplus stock disposals, which imply a stationary level of production in the rest of the world.

The two main components determining the trend of wool production are changes in sheep numbers and changes in the average yield per sheep. The London Wool Conference Report's estimates appear to be based on the assumption that neither of these two components is likely to increase over the next decade; or that further increases in the average yield will be offset by a reduction in numbers.

As regards sheep *numbers*, it is a view shared by many experts that the marked upward trend of the last 100 years is not likely to continue, especially not in the main merino areas. It is held that the optimum sheep-carrying capacity of grazing land has on the whole been reached or, in some instances, surpassed, and graziers are urged to take good care that scientific improvements are used for safeguarding the pasture and for feeding the stock already carried to economic capacity before any further increase in carrying capacity is attempted.

Much more optimistic forecasts have been made by some, however, in regard to long-term prospects of increasing the average yield per sheep. Dr. Clunies Ross, who is a well-known expert in this field in Australia, holds the view that "the great bulk of our sheep could produce 50 per cent. more wool per head if adequately fed."† Dr. Ross points out that the wool-growing capacity of sheep is determined basically by heredity. Hence great advances could be made by the wide adoption of progeny testing of rams—*i.e.*, the determination of the quality of breeding stock on the basis of their progeny. Modern scientific research has led to important improvements in the

* London Wool Conference Report, *op. cit.*, Annex A.

† *Pastoral Review*, December 1942, quoted by J. P. Abbott, "The Future of the Wool Industry," *op. cit.*, pp. 77–78.

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application of this system.* The "high-yield" breeds evolved in this way can, however, be maintained only by artificial pasture improvement and improved nutrition based on more labour-intensive and capital-intensive methods of production. Hence, in the final analysis the adoption of such methods will again depend to some extent on the ratio of the more scientifically informed farmers' cost calculations to their market expectations. In other words, the production forecasts of the London Wool Conference Report seem to imply that growers' returns during the next decade will not be such as to encourage a substantial increase in yields by the adoption of much more intensive methods.

Finally, it remains to consider the possible effects of the new Empire Marketing Scheme's guarantee of purchase of all wool produced in the Dominions during the next ten to fifteen years.† It has been argued that this new element of peace-time security offered to farmers under the new plan might, *cet. par.*, be reflected in an upward trend of production. The new scheme does not provide any guarantee in regard to the level of reserve *prices* at which unsold "new clip" wool is to be lifted from the market; but it is argued by some that farmers' security in regard to the *volume* of sales over a fairly long period might exercise a stimulating effect on output despite, or even because of, anticipated low values per unit. Others argue that the new guarantee in regard to the volume of sales is not, in fact, as novel as it might at first sight appear, because the past history of wool-marketing shows that even under completely free market conditions wool-producers managed to sell the bulk of their clips year by year; changes in the scale and intensity of demand having been reflected in market values rather than in the volume of sales. Nevertheless, it may well be that the express guarantee of a market for all new wool over a series of years, coupled with centrally established reserves and a higher degree of stability of returns, will, *cet. par.*, tend to stimulate output by increasing the feeling of security over the longer period. Much will depend, therefore, on the technical possibilities of expansion at reasonable cost.

At least another two seasons will probably be required to recover the losses of the recent severe drought in the main exporting countries. No definite forecasts can be made in regard to the longer period; but there is reason to assume that, despite the cumulative effects of erosion and other technical limitations, the pre-war trend of expansion in overseas wool production will not be stopped completely and abruptly; and that therefore the world's exportable surplus of "new clip" wool will not fall short of, but may exceed, the estimates given in the London Wool Conference Report.

II. Wool Consumption

In the past the growers' slogan has always been "Sell and repent but sell," and with the exception of abnormal conditions at the end of the last war, the size of carry-over stocks, expressed as percentages of annual clips of the chief exporting countries, was, on the whole, relatively small (*cf.* Table VIII). At the end of 11 out of the 14 seasons from 1925/26 to 1938/39, the combined total of stocks in the three Dominions and in the Argentine was below 7 per cent. of annual output, and in only one out of 14 instances was it more than 9 per cent. (9·7 per cent. at the end of the 1931/32 season). Stocks were relatively lowest in the two chief merino-producing countries; less than 3 per cent. of annual output at the end of 10 out of 14 seasons in Australia, and in South Africa. A survey of end-of-season stocks, in millions of lb. actual weight, is given in Appendix III.

The liquidation of the bulk of the world's exportable surplus of wool year by year must not be taken to imply that demand during the inter-war period showed a high degree of stability. On the contrary, the composition and intensity of world demand were subject to strong fluctuations, but these fluctuations were reflected in sharp changes in values rather than in major short-term changes in the volume of world sales.

Pre-War Distribution of Wool Exports

The pattern of the distribution of shipments from the five chief exporting countries during the inter-war period is shown in Table IX. The figures reveal the predominant share of exports sent to the United Kingdom and to the European continent which together took up 80 per cent.

* Cf. *Canterbury Agricultural College, Annual Report*, 1942, quoted by J. P. Abbott, *op. cit.*, p. 77.

† For a more detailed account of guarantees of the volume of sales under the new Empire Marketing Plan, *cf.* below, pp. 43–44.

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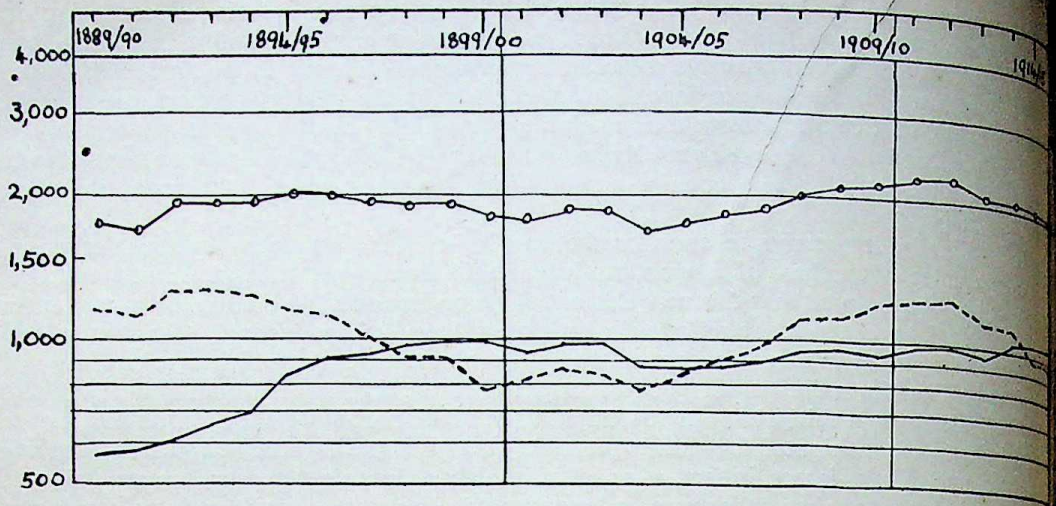
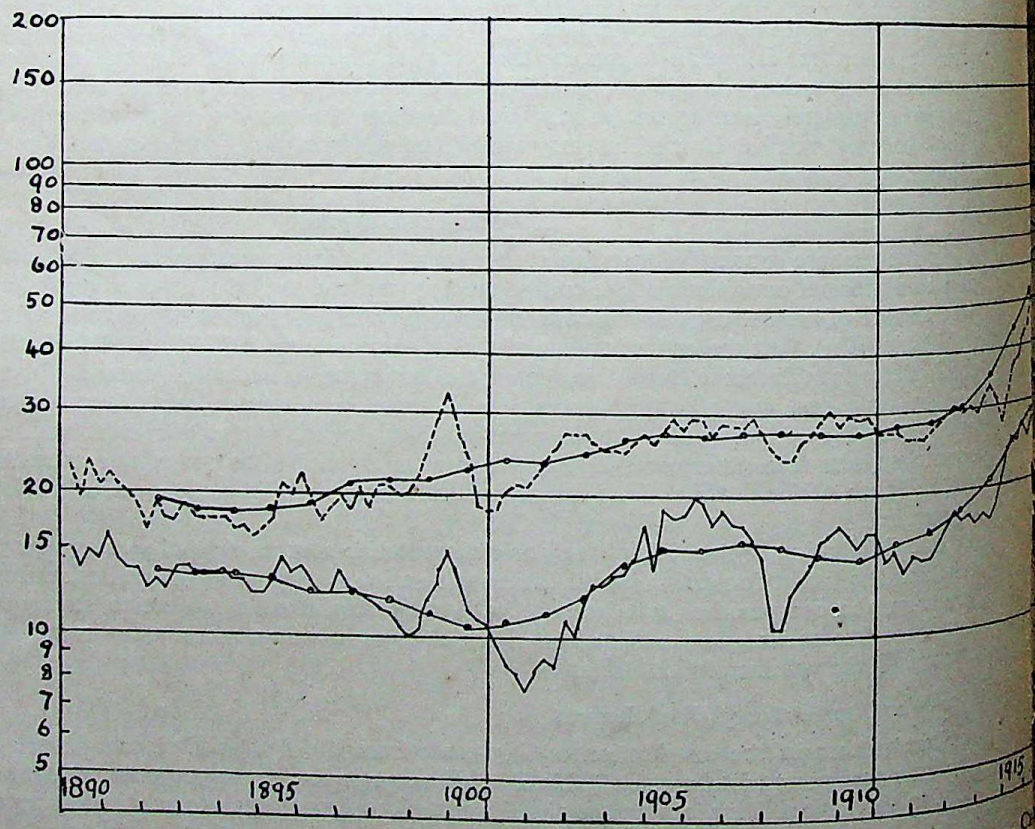


Production of Merino and Crossbred Wool (grease basis) in Ten Main Producing Countries, Austr

—o—o— Total
- - - - - - Merino
——— Crossbr



- - - - - - - - = Merino, 66's.

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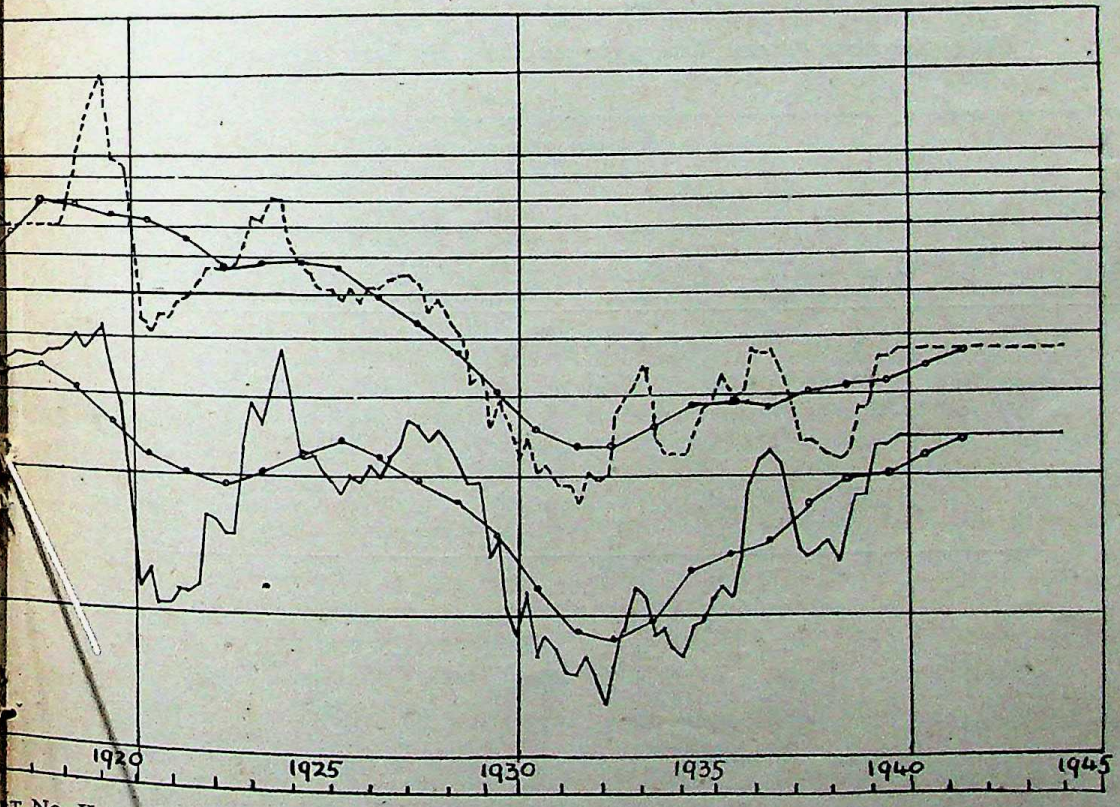
T No. I

ew Zealand, South Africa, Argentina, Uruguay, United States, United Kingdom, Canada, France, Germany,
–1943/44

Total
Merino
Crossbred
} Production.



T No. II

Raw Wool, 1890–1944
per lb., London)
ossbred, 46's.

— o — o — c — = Moving 5-year averages.

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TABLE VIII

End-of-Season Stocks, Expressed as Per Cent. of Annual Output (grouped according to size) During the Period 1925/26–1938/39

Country	End-of-season stocks as per cent. of annual output (grouped according to size)						Total number of seasons
	Less than 3%	3%–less than 5%	5%–less than 7%	7%–less than 9%	9%–less than 11%	11% and over	
Australia	10	2	2	—	—	—	14
South Africa	10	2	1	1	—	—	14
New Zealand	—	3	3	2	1	5	14
Argentine	4	2	4	1	2	1	14
Total, Four Countries ...	5	3	3	2	1	—	14

Sources: Wool Production, *cf.* notes to Table IV. End-of-season Stocks: 1925/26–1927/28: H. Schueler, *op. cit.*; 1928/29–1938/39: I.E.C., *Wool Production and Trade*, volumes 1928/34 and 1938/39.

(retained) in 1934/38, and more than 80 per cent. in the preceding periods. Notwithstanding this fact, however, the pre-war distribution of wool shipments was much more widely dispersed by individual countries of destination than by countries of provenance. The category "Rest of Europe," for instance, includes a large number of different countries of destination, the most important ones among them being Italy, Czechoslovakia, Poland, Switzerland and some of the Scandinavian countries.

The differences between the percentage figures shown in Sections (A) and (B) of Table IX indicate

TABLE IX

Distribution of Wool Shipments (Grease Basis) from the Five Chief Exporting Countries, 1909/13–1934/38

Exports	From five chief exporting countries							
	1909/13		1924/28		1929/33		1934/38	
	Million lb. greasy	% of total exports	Million lb. greasy	% of total exports	Million lb. greasy	% of total exports	Million lb. greasy	% of total exports
(A) Not adjusted for U.K. entrepôt trade								
To:								
U.K. (gross) ...	593	41·1	613	37·7	694	36·9	690	37·7
France	292	20·2	322	19·8	307	16·4	248	13·6
Belgium	135	9·3	133	8·2	192	10·2	188	10·3
Germany ...	300	20·8	245	15·1	271	14·4	182	9·9
Rest of Europe ...	36	2·4	98	6·0	161	8·5	177	9·7
Total Europe (excl. U.S.S.R.)	763	52·7	798	49·1	931	49·5	795	43·5
U.S.A.	56	3·9	109	6·7	60	3·2	106	5·8
Japan	11	0·8	79	4·9	174	9·2	187	10·2
Others	21	1·5	26	1·6	23	1·2	51	2·8
Total ...	1,444	100·0	1,625	100·0	1,882	100·0	1,829	100·0
(B) Adjusted for U.K. entrepôt trade								
To:								
U.K.	385	26·6	398	24·5	435	23·2	488	26·6
France	329	22·8	376	23·1	397	21·1	325	17·8
Belgium	176	12·1	167	10·3	225	11·9	225	12·3
Germany ...	345	23·9	303	18·6	365	19·4	206	11·3
Rest of Europe ...	46	3·2	118	7·3	184	9·8	219	12·0
Total Europe (excl. U.S.S.R.)	896	62·0	964	59·3	1,171	62·2	975	53·4
U.S.A.	128	8·9	152	9·3	77	4·1	115	6·3
Japan	11	0·8	79	4·9	174	9·2	187	10·2
Others	24	1·7	32	2·0	25	1·3	64	3·5
Total ...	1,444	100·0	1,625	100·0	1,882	100·0	1,829	100·0

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the importance of U.K. entrepôt trade in wool. Even on the "retained" basis, however, the United Kingdom remained the largest individual importer. Three other old industrial countries—France, Belgium and Germany—ranked next in importance. At the same time, the respective percentage shares of these three chief Continental consumers showed a declining tendency—especially marked in Germany during the period of economic totalitarianism from the early 'thirties onward—and this decline was only partly balanced by an increase in the share taken up by the "Rest of Europe."

After the end of the first World War, Japan began to take an increasingly important place in the general picture, until in 1934/38 the Japanese share (10 per cent.) was almost equal to the shares of Belgium and Germany, and considerably in excess of the portion taken by the United States. The effects of the rise in Japanese wool consumption during the inter-war period were felt most in the Australian market. Japan's share in Australian exports, which in 1909/13 merely averaged 2 per cent. of the total, rose to an average of 18 per cent. of all Australian shipments during the 1929/33 period, and maintained an average of 15 per cent. during the 1934/38 period. The expansion of the Japanese wool-textile industry during the inter-war period led to a change in her position from an importer of wool goods to a large-scale exporter of cheap yarns and tissues. At the same time, demand for wool fabrics in Japan's internal market was increased by the westernization of clothing habits and by the rise in living standards generally.

United States imports showed sharp fluctuations, which are brought into even stronger relief when U.S. wool imports for individual years and by individual countries of origin are considered separately. For instance, American imports of Australian wool rose from 9·5 million lb., actual weight, in 1935 to 32 million in 1936, to 69 million in 1937, and then fell again to 6·5 million in 1938. Total U.S. imports during the years 1932/38 were as follows:

U.S. Wool Imports

	Actual weight, million lb.	Clean basis, million lb.
1932	56·3	38
1933	174·3	119
1934	108·4	74
1935	200·5	140
1936	254·0	173
1937	322·3	216
1938	102·7	70

The often erratic changes in U.S. consumption of foreign wools during the inter-war period were partly due to the effects of the American wool tariff, which restricted imports to the varying margins between total demand and domestic supply. The pre-war American domestic wool clip showed a fairly steady average of about 450 million lb. per annum, and was generally absorbed in the internal market year by year. Additional demand was met by imports. Hence, an increase in American consumption from, say, 600 million lb. to 750 million lb.—*i.e.*, by 25 per cent.—meant an increase of imports from 150 million lb. to 300 million lb.—*i.e.*, by 100 per cent.; whilst a moderate reduction in total consumption could cause a drastic fall in imports.*

The pre-war level of U.S. wool consumption per capita was relatively low—averaging only 2½ lb., clean basis, as compared with over 5 lb. in the United Kingdom. The low American consumption levels were partly due to differences in living conditions (effects of central heating, motor-cars, etc.), but they were certainly also strongly influenced by the tariff of 34*c.* per clean lb., which widens the price spread between wool and other fibres in the American market. As the tariff is on a flat-rate basis, its effects tend to be specially marked during depressed periods, when the amount of duty expressed as a percentage of price is highest.

Finally, it remains to consider the last category of "Other Countries" in Section (B) of Table

* During the war years American wool consumption reached the unprecedented record of around 1,000 million lb. greasy per annum. At the same time, prices for home-grown wools which were bought in bulk and re-sold by the Government were fixed at levels ranging from 10 to 15 per cent. above values of comparable imported qualities on a duty-paid basis. The American market for domestic wools was protected to some extent by priorities established for the use of these wools for military supplies; but in the "free" section of the market the price spread between domestic and imported types led to a shift in manufacturers' demand towards the latter. This meant that the amount of foreign wool consumed was even higher than the difference between the American domestic wool clip and the vastly increased consumption total. In December last the Commodity Credit Corporation, acting on behalf of the American Government, scaled down prices for domestic wool to more competitive levels, the cost of the difference between selling prices and prices paid to growers being borne by the Government.

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IX. This category, which includes the U.S.S.R., India and China, represents nearly three-quarters of the world population, but merely absorbed from 2 to 4 per cent. of the world's exportable surplus of wool during the inter-war period. Some of these countries produce substantial clips of coarse carpet wool within their own borders; but even when this domestic supply is taken into account, the amounts of wool consumed per capita in these regions are found to be extremely low. We shall now proceed to consider per capita wool consumption levels in different countries a little more closely.

Average Net Wool Consumption Per Capita in Different Countries Compared with Average Per Capita Real Incomes

The demand for wool at given prices in different countries is obviously strongly influenced by differences in climatic conditions and social habits affecting clothing expenditure; by the relation

TABLE X

Average Net Wool Consumption Per Capita (lb. clean) Compared with Per Capita Income Levels in 20 Different Countries

	I	II	III		IV
	Average wool consumption per head	Average real income per head	Countries classified by order of magnitude of— Wool consumption per head	Income per head	Average wool consumption per 1000 international units
	lb. clean	International Units	Order of magnitude		lb. clean
United Kingdom ...	5·27	1,069	1	3	4·93
New Zealand ...	4·65	1,202	2	2	3·87
Australia	4·63	980	3	6	4·72
Belgium	4·55	600	4	10	7·58
Sweden	3·55	653	5	8	5·46
France	3·52	684	6	7	5·04
Switzerland	3·32	1,018	7	4	3·24
Argentina	3·00	1,000	8	5	3·00
Germany	2·87	646	9	9	4·44
U.S.A.	2·66	1,381	10	1	1·93
Czechoslovakia ...	2·04	455	11	11	4·48
Yugoslavia	1·74	330	12	16	5·27
Japan	1·26	353	13	13	3·57
Italy	1·22	343	14	15	3·56
Hungary	1·12	359	15	12	3·12
Poland	0·99	352	16	14	2·81
U.S.S.R.	0·87	320	17	17	2·72
South Africa ...	0·83	276	18	18	3·00
India	0·20	200	19	19	1·00
China	0·10	100–120	20	20	1·00–0·83

Coefficient of correlation: $r = 0{\cdot}797 \pm 0{\cdot}084$.
Coefficient of rank correlation: $p = 0{\cdot}86$.

of the cost of wool clothing to the cost of other budget items (which even at given world wool prices may differ widely in different countries); and by other specific factors. For all these reasons, a comparison of average net per capita wool consumption levels with average per capita real incomes in different countries cannot be expected to show a very high degree of correspondence by order of magnitude. The American market, for instance—for reasons just mentioned—illustrates the case of a country with high living standards and relatively low per capita levels of wool consumption. Nevertheless, despite all imperfections of such a comparison, the relation of average wool consumption to real income levels in different countries reveals some facts of interest.

Some relevant figures are shown in Table X. The 20 countries considered in this table have been selected more or less at random, but with a bias towards diversity of average income levels *and* diversity of other conditions affecting the demand for wool (*e.g.*, climate). The amounts of average wool consumption per capita are calculated on a "net" basis (*i.e.*, domestic production of raw wool plus imports and minus exports of wool, tops, yarns and tissues, all converted to a comparable clean-weight basis). The real-income estimates are taken from Mr. Colin Clark's *Conditions of*

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Economic Progress, where per capita income levels are calculated in "International Units" (defined as "the amount of goods and services which could be purchased for $1 in the U.S.A. over the average of the decade 1925–34, or an amount interchangeable with them, as shown by the Pigou formula").* The two series of "per capita" figures relate to different periods,† and there are also several other reasons limiting the value of a comparison of two sets of figures determined by such widely varying factors and both subject to wide margins of error. At the same time it is of some interest to note that a line dividing the "upper" 10 from the "lower" 10 countries, classified by order of magnitude of average "per capita" wool consumption, also happens to provide a correct division between the "upper" and "lower" 10, classified according to "per capita" real income levels. The last column to the right in Table X shows the average amounts of wool (in lb. clean) consumed per 1,000 international units of real income. Here again emphasis must be laid on the high degree of arbitrariness of any such estimates; but, despite all qualifications which must be kept in mind, it is of interest to see that the 5 countries with the lowest average real incomes per head are also the countries where average per capita wool consumption "per international unit" was lower than in all other countries, with the exception of the United States. These 5 "lowest" countries include regions with severe climates, such as Poland and the U.S.S.R.; and even in India there are wide tracts where winter temperatures are so low as to create a strong need for warm protective clothing and blankets. In other words, these figures suggest that in the lower income ranges the income elasticity for expenditure on wool goods is above unity, and that rising living standards in the poorer countries of the world, and of the poorer classes everywhere, would, *cet. par.*, lead to a more than proportionate increase in the demand for wool at given prices; whilst a reduction of real incomes must also be expected to exercise a more than proportionate adverse effect on the demand for wool.

Any such conclusions on the basis of the figures shown in Table X can be only rather vague assumptions; but these assumptions are supported by the results of more detailed enquiries into the income elasticity of clothing expenditure as calculated on the basis of various series of budget data. Such calculations were made by Allen and Bowley in their study on *Family Expenditure—A Study of its Variation*.‡ The authors calculated the income elasticity coefficients of clothing expenditure for 21 different collections of budgets of salaried employees and workers in different European countries and in the United States. The results of these calculations show that for 18 out of the 21 budget series considered, the income elasticity of clothing expenditure was above unity, ranging from 1·1 to 1·9; in 2 cases it was unity, and in 1 case only below unity (0·9).

Nor is it surprising that this should be so. Given a basic minimum, clothing needs are less essential than the needs of food, and during periods of falling incomes it is easier to postpone clothing expenditure than food expenditure. Moreover, purchases of the chief articles of clothing (suits, coats, etc.) require the expense of a relatively large lump sum at a time, whilst most food purchases are made at shorter intervals and in smaller amounts. Besides, clothing expenditure is often postponed for the sake of meeting other lump sum expenditure which must be met under rigid contractual liabilities (*e.g.*, rent, insurance); or in settlement of accounts for services consumed in variable small amounts, but payable subsequently in lump-sums at fixed dates (*e.g.*, gas and electricity bills). For all these reasons, clothing expenditure is more strongly influenced by changes in prosperity levels than expenditure on any of the other essential budget items (food, rent, fuel and light).§

* Colin Clark, *The Conditions of Economic Progress*, London, 1940, p. 2.

† 1924/35 for real-income levels and 1934/38 for wool consumption. There are no sufficiently accurate and detailed data for calculating net per capita wool-consumption figures for all countries considered for the earlier period.

‡ R. G. D. Allen and A. L. Bowley, *Family Expenditure—A Study of its Variation*, London, 1935, pp. 32–33.

§ A summary grouping of Allen and Bowley's income elasticity coefficients for main items of different budget series gives the following results:

	Number of budget series for which income elasticity coefficients are:			Total number of budget series considered
	Below unity	Unity	Above unity	
Food	21	None	None	21
Rent	14	5	2	21
Fuel and Light	18	1	2	21
Clothing	1	2	18	21
Other Items	None	None	21	21

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As a first approximation, it seems also reasonable to assume that within a given range of clothing expenditure, especially in the lower income groups, the income elasticity of demand is higher for higher-grade wool goods than for other fabrics made from cheaper fibres. This effect is, however, modified to some extent by the low ratio of raw material costs to the retail prices of finished products; and it may also be counteracted by the consumers' knowledge of wool's intrinsic wearing qualities. The anticipated durability rate, for instance, may have an even more important bearing on the poor man's choice than on the choice of the well-to-do. This last point raises the problem of adequate consumers' information, which must, however, be discussed within the framework of a more general examination of the influence of competition from other fibres.

Wool Consumption Seen in Relation to the Consumption of Other Fibres

Over the long period, changes in world production can be taken as fairly indicative of broad changes in world consumption. Hence in order to get an idea of the share of wool in the pattern of world consumption of all major clothing fibres, we can recall some of the salient features of changes in world production as given earlier in Table V (page 10). As shown then, the share of cotton remained of dominant importance throughout the period between the two wars (86 per cent. in 1909/13 and 74 per cent. in 1939/43). The share of wool (scoured basis) fell from 14 per cent.

TABLE XI

World Production of Rayon Filament Yarn and Staple Fibre, 1934–44

Year	Filament yarn	Staple fibre	Total	Staple fibre as % of total
	Million lb.			
1934	770	54	824	7
1935	940	141	1,081	13
1936	1,018	304	1,322	23
1937	1,201	618	1,819	34
1938	1,021	925	1,946	48
1939	1,145	1,082	2,227	48
1940	1,184	1,280	2,460	52
1941	1,267	1,492	2,759	54
1942	1,213	1,537	2,750	56
1943	1,166	1,467	2,633	56
1944	1,040	1,117	2,157	51

Source: *Rayon Organon*, New York.

in 1909/13 to 12 per cent. in 1939/43, whilst the share of rayon (filament yarn and staple fibre) showed a spectacular increase from 0·2 per cent. in 1909/13 to 14 per cent. during World War II. 1941 was the first year when the share of wool was surpassed by the share of rayon (14 per cent. for wool, scoured, as compared with 15 per cent. for rayon yarn plus staple fibre). The peak in world rayon production was reached in 1942, when the combined output of filament yarn and staple fibre totalled 2,759 million lb. as compared with 2,540 million lb. of wool, clean. During 1943 and 1944 the sharp upward development of world rayon output was arrested by accentuated war-time production difficulties in Germany and Japan; the respective shares of wool and rayon in the combined world output of the four main clothing fibres were about equal.*

"*Fabrics Competition.*"—In considering the changes in the proportion of wool in world production, it must be remembered that wool consumption is affected not only by "fibre competition" (*i.e.*, possibilities of using alternative fibres in the production of "wool" textiles), but also by "fabric competition" (*i.e.*, the possibilities of substitution which exist as between entirely different textile materials for clothing purposes).

Take, for instance, the case of rayon filament. The characteristics of rayon yarn are totally dissimilar from those of wool yarn, and the rayon filament yarn cannot be used as substitute in the production of fabrics or hosiery of a wool-like character. At the same time, filament rayon did succeed in displacing wool in certain types of women's wear, not by imitating wool fabrics, but by providing fabrics of quite different qualities, yet adaptable to the same purposes. This develop-

* *Rayon Organon*, December 1945.

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ment has also been influenced by the trend towards lighter clothing; and, especially in the field of women's wear, by the greater demand for variety which tends to lessen the importance of durability.

"*Fibre Competition.*"—Even before the advent of synthetic fibres, the amount of raw wool used in the wool textile industry could be varied to some extent in response to price changes by varying the admixture of re-used wool and cotton in certain types of fabrics. With the quantitative and qualitative development of synthetic fibres, these possibilities of fibre substitution have been widened.

Since the early 'thirties the share of rayon staple fibre came to represent an increasing percentage of world rayon output, rising from 7 per cent. in 1934 to 56 per cent. in 1942 and 1943. A slight drop was recorded in 1944, but the percentage of staple fibre in total rayon output was still over one-half.

No exact figures are available in regard to the output of "wool" types of rayon staple fibre (*i.e.*, staple fibre produced in denier comparable to the fineness of the wool fibre); but it is estimated that "wool" types represent about one fifth to one quarter of total staple fibre output, the other three-quarters being produced in "cotton" fineness. On the basis of this estimate, the world output of "wool type" staple fibre in 1940/44 would have amounted to about 500 to 600 million lb.—*i.e.*, about one fifth to one quarter of the clean weight of world apparel wool production during that period.*

Throughout the inter-war period, and during the earlier war years, the dominant share of world output of rayon staple fibre was produced by the three countries which first concentrated on the manufacture of these substitutes for reasons of economic policy—*i.e.*, Germany, Italy and Japan. Up to 1942 these three countries were responsible for well over 80 per cent. of world output. By 1944, however, their combined share in world output had fallen to 65 per cent. of the total.

The rate of increase in some of the other staple fibre producing countries was steep. In the United States, for instance, production of rayon staple fibre in 1944 (169 mn. lb.) was more than three times as high as in 1939 (51·3 mn. lb.) and 14 times as high as in 1936 (12·3 mn. lb.).

The bulk of synthetic fibres produced in the world to-day is still made from cellulose base—*i.e.*, from a raw material supplies of which are ample. Sulphite pulp made from spruce is used for the higher-grade qualities; but for inferior rayon types use can also be made of various other kinds of timber, as well as reeds, stalks and other plants which grow back quickly and are not bound to any particular climate. Cotton linters are another raw material used for cellulose-base fibres.

Compared with the quantitative importance of "cellulose-base" fibres, the volume of "protein-base" fibres, synthesized polymers and inorganic fibres which enter into direct competition with wool is as yet relatively small; but new types are being evolved all the time, and some of them must be taken seriously. The wide range of base materials used for the production of synthetic fibres to-day can be seen from the chart on page 202, showing the latest system of grouping all existing categories of "man-made fibres," as synthetics are now being called by American scientists. As can be seen from this chart, synthetics are produced from such various materials as, for instance, soya beans, peanuts, corn, chicken feathers, clam shells, seaweeds, glass and aluminium.

Some of these fibres, especially in the "inorganic" group, do not enter into direct competition with wool. In regard to the various protein fibres, further developments may largely depend on the importance of alternative uses of their base materials. Italian, French and Polish pre-war experiences with the manufacture of Lanital (a skim-milk product comparable to the American "Aralac" fibre) have shown that casein prices in these European countries were driven to levels which seriously affected all cost calculations. However, these pre-war experiences are not sufficiently conclusive to justify any forecast in regard to future prospects for the wide range of new protein-base materials which have since been evolved.

Production of nylon (which is a synthesized polymer made from coal, lime and nitrogen) was started on a commercial scale not long before the outbreak of war, and experiences of its utilization in the civilian market are as yet limited; but it is already clear that nylon will play a leading rôle in

* It should be noted in this connection, however, that staple fibre supplies used on cotton machinery cannot be entirely disregarded as competitors to wool. New methods have been evolved for spinning staple fibre yarn on a modified cotton system in such a way as to produce certain worsted effects. Advocates of this new system maintain: (i) that manufacture of "worsted" yarn spun on the cotton system from rayon tops is cheaper because the process of combing is eliminated; and (ii) that the denier of the cotton-spun yarn gives a higher count limit and smoother yarn (*cf.* W. Hardacre, "Recent Developments in Rayon Staple," *Journal of the Textile Institute*, April 1943).

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MAN-MADE FIBRES

I. REGENERATED NATURAL POLYMERS

- **Carbohydrate Base**
  - **Animal**
    - Chitin (clam shells)
  - **Vegetable**
    - Algin (seaweed)
    - Regenerated Cellulose
      - Viscose Rayon
      - Cuprammonium Rayon
      - Nitro Rayon
    - Cellulose Esters
      - Acetate Rayon
      - Miscellaneous Esters
    - Cellulose Ethers
      - Ethyl Rayon
- **Protein Base**
  - **Animal**
    - Casein (milk)
    - Albumin (eggs)
    - Keratin (feathers)
  - **Vegetable**
    - Soyabean
    - Peanuts
    - Maize

II. MINERAL BASE

- Glass Fibre
- Aluminium Fibre

III. SYNTHESIZED POLYMERS

- Polyamide Base — Nylon
- Vinyl Base — Vinyon
- Vinylidene Base — Saran

Compare: H. R. Mauersberger, *Textile Research*, 1943, **13** (6), 11.

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the post-war textile world, and indications are that the competition of nylon will not be confined to silk and rayon, but that "wool" types of nylon may also be produced. Indeed, pullovers made from nylon and coats made from nylon plush began to appear on the American market even during the war, despite drastic reductions of nylon supplies for civilian uses.

It is still true to say that no one synthetic fibre has as yet attained all the superior qualities of natural wool, especially in regard to warmth-giving and hygroscopic properties, reversible elasticity, water repellence, etc. Just before the war special types of synthetic fibres imitating certain specific qualities of the wool fibre were brought on the market. For instance, there were synthetic fibres with artificial air-pockets so as to increase the warmth-giving effect of air-insulation; artificial scales to improve felting properties; artificial crimp to increase elasticity, etc. But success was limited, and some of these types also proved too expensive for effective competition on a commercial scale. At the same time, the qualities of natural wool have been further improved in recent years by the development of new scientific processes for non-shrink treatment and anti-moth treatment, and by the results of far-reaching researches into the molecular structure of the wool fibre. Despite the superiority of the natural fibre, however, the production of synthetics on an increasing scale must be regarded as an important factor affecting the future of wool.

The comparison of prices of wool and synthetic fibres is complicated by various factors. In the first place, the variety of technical characteristics which have to be taken into consideration is so large that no one special type of staple fibre can be regarded as equivalent to any one special type of wool. Secondly, there is the complication arising from the fact that synthetic fibres have not at any time had a representative world market price level. Even before the war it was not possible to define a representative price for staple fibre because the bulk of these fibres was produced in Germany, Italy and Japan—*i.e.*, in economies based on price systems which were divorced from the world price level. With the outbreak of war the difficulty of defining a world price was even further accentuated. The only war-time quotations which could be regarded as fairly representative of price formation in a free market were the American quotations, but here again the price-spread between wool and synthetics was, and still is, artificially widened by the tariff which raises wool prices in the United States above the world price level.*

In the United Kingdom the price of "Fibro" tops—*i.e.*, tops made from Courtaulds' standard staple fibre—is now quoted at *22d.* per lb., compared with a price range from 28½*d.* to 48¾*d.* per lb. of tops made from Dominion wools.

Apart from the actual price-spread, a competitive advantage of synthetic textiles as against natural fibres lies in the higher degree of short term price stability of the former. This is partly due to the fact that output of an industrial product can be adjusted much more easily over the short period and planned in advance with precision, whilst output of an agricultural product like wool is less elastic and dependent on various factors beyond the producers' control. Moreover, quotations for synthetic products are generally based on deliberate policy decisions made by a relatively small number of large-scale manufacturers, whilst pre-war wool prices were the result of free, and at times erratic and excessive, market fluctuations.

The relatively high degree of stability in the prices of substitute fibres adds weight to the argument for the desirability of measures aiming at the evening-out of short term fluctuations

* When these qualifications are kept in mind, the following American quotations may be of interest:

Prices of Wool and Other Textile Fibres, United States, December, 1945

	Cents per lb.
Wool:	
Boston, Territory, clean, 64's–80's	103·5
" " 56's combings	92·5
Australian, 64's–70's	75·8
Montevideo, 56's	42·4
Cotton:	
Middling	24·7
Synthetic Fibres:	
Rayon Viscose, 1–½ denier	25·0
Rayon Acetate, 5 denier	38·0
Aralac, 3–7 denier	64·0

Source: U.S. Department of Agriculture, *The Livestock and Wool Situation* and *Rayon Textile Monthly*, New York.

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in raw wool prices. Manufacturers who need to contract ahead in regard to their finished products must clearly regard price stability as an important advantage which may under certain conditions influence their choice of raw materials quite apart from the actual price spread.

It is sometimes argued that the increased use of synthetic fibres in admixture with wool may ultimately help to increase the use of wool by cheapening textile products, and thus leading to increased total consumption at lower prices. Such reasoning may have some substance in regard to certain speciality lines, but not in regard to the level of wool consumption as a whole. It must be remembered in this connection that the cost of raw materials expressed as percentage of retail prices of finished textile goods is, on the whole, relatively very small; and that therefore the reduction in costs due to admixture cannot manifest itself in substantially lowered retail prices charged to ultimate consumers. Hence, an admixture of, say, 50 per cent. synthetics reducing consumption of raw wool per unit of cloth by one half cannot lead to a reduction in retail prices which would be so marked as to cause an increase in demand by 100 per cent. (which would be the increase required for maintaining, *cet. par.*, the level of raw wool consumption at the pre-admixture rate).

Take, for instance, the case of the average "utility" suit sold in the United Kingdom at a price of about 100 shillings. The length of cloth contained in a suit averages 3 to 3½ yards; and if the material is of a heavy all-wool quality of, say, 16 oz. to the yard, then the amount of wool contained in the suit is about 3 to 3½ lb. The price of a good quality merino combing wool is about 2*s.* 6*d.* per lb. It follows that even if we take high-grade wool, the cost of the raw material is not likely to exceed 7*s.* 6*d.* to 8*s.*, or 7½ to 8 per cent. of the retail price charged for the suit; whilst more than 90 per cent. goes to the costs of manufacture and distribution. Let us now assume that the material used for this suit were staple fibre at a price of, say, 1*s.* 3*d.*—*i.e.*, half the price of wool. The total cost of raw material used for the suit would then be 3*s.* 9*d.* to 4*s.* 6*d.* instead of 7*s.* 6*d.* to 8*s.*; but the effect of this economy, if expressed as a percentage of the retail price for the suit, would be relatively small. Indeed, if all "other costs" remained equal, the retail price of the suit made from synthetic fibres could be only 3½ to 4 per cent. below the price of the all-wool suit. In practice the price difference would be higher owing to the principle of "percentage mark-up," which, though open to serious criticism on economic grounds, is generally accepted as a widely practised trade usage. However, even when this specific factor of the "percentage mark-up" is taken into account, the resulting difference in retail prices will clearly be far from sufficient to increase the aggregate demand for suits on a substantial scale. Moreover, it must be remembered that by choosing a low-priced "utility" suit we have taken a moderate example. For higher-class suits, the cost of the raw material expressed as a percentage of retail prices would be even lower.

As we go up in the vertical chain of production, the effect of raw material substitution on cost calculations becomes more marked. It is generally estimated that the cost of raw wool constitutes about 30 to 40 per cent. of the manufacturer's total cost of the cloth as it leaves the loom, and about 70 per cent. of the spinner's total cost of his yarn. Hence, spinners and manufacturers may be induced to respond to changes in the price relationship of wool and substitutes by introducing corresponding variations in proportions of admixture rather than by raising the prices for their finished products. Such a development may be accentuated by a combination of the following two factors: (i) the growing element of price rigidity at the retail end, which may create a tendency to meet variations in raw material costs by variations of quality higher up in the vertical chain rather than by fractional changes in retail prices; and (ii) the absence of adequate "Truth in Fabrics" legislation (except in the U.S.A.), which makes it extremely difficult for retail customers to detect the fibre composition of the goods they buy, and which thus blunts the influence of their preference for certain fibres as against others.

This point is also of relevance in answering the important question of the effect of admixture on the ultimate consumer's interest. So far, we have merely shown that the use of lower-priced substitutes for admixture is not likely to cheapen retail prices of finished goods to an extent which could make up for the loss of wool consumption per unit by an increase in the number of units sold; and that, therefore, aggregate wool consumption would, *cet. par.*, be reduced. This may be bad for the primary producers who live by the sales of their wool at remunerative prices; but might it not be highly beneficial to the interests of consumers? The answer to this question depends on the consumers' relative valuations of different textile materials for various purposes. Hence it would appear essential in the interests of consumers that they should be given full opportunities to exercise their choice on the basis of adequate information in regard to the fibre composition

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of the goods they buy. The range of qualities ascertainable by the average retail customer at the time of purchase is generally confined to texture, colour and handle, which, to the non-expert, cannot give any reliable indication as to the fabric's durability, elasticity, warmth and other important wearing qualities. Hence, without full information in regard to fibre composition (which

TABLE XII

Indices of Exports of Tops, Yarns and Tissues, by Weight: World and Four Selected Countries, 1928–38

(1928 = 100)

Year	World			United Kingdom				France			
	Exports of—			Available supplies of raw wool (clean)	Exports of—			Available supplies of raw wool (clean)	Exports of—		
	Tops	Yarns	Tissues		Tops	Yarns	Tissues		Tops	Yarns	Tissues
1928	100	100	100	100	100	100	100	100	100	100	100
1929	92	97	96	109	97	96	91	107	97	94	92
1930	89	87	78	115	85	76	68	115	89	89	77
1931	95	82	65	131	82	72	51	92	93	81	62
1932	95	56	43	128	124	78	47	97	87	47	28
1933	108	63	46	126	135	88	54	111	109	55	26
1934	91	61	49	116	124	88	59	72	87	48	19
1935	103	58	49	119	165	84	63	81	91	37	13
1936	99	59	53	135	153	76	68	72	85	24	11
1937	93	59	62	124	117	65	71	69	95	24	11
1938	83	54	52	135	97	57	54	85	93	39	15

	Germany				Japan			
	Available supplies of raw wool (clean)	Exports of—			Available supplies of raw wool (clean)	Exports of—		
		Tops	Yarns	Tissues		Tops	Yarns	Tissues
1928	100	100	100	100	100	—	—	100
1929	97	78	113	100	93	—	100	200
1930	94	83	96	86	100	—	250	200
1931	88	87	96	83	164	—	250	100
1932	89	44	54	43	178	—	250	200
1933	102	39	50	40	207	—	750	500
1934	99	22	42	29	161	—	1,500	1,000
1935	88	9	46	26	214	—	1,250	1,400
1936	78	9	75	43	197	—	1,750	1,900
1937	78	4	62	51	234	—	1,750	1,800
1938	97	4	38	46	107	—	2,000	1,400

Main Exporting Countries: Percentage Shares in World Exports of Tops, Yarns and Tissues, Average 1928–38

From—	Per cent. of world exports of—		
	Tops	Yarns	Tissues
	%	%	%
U.K.	32	28	42
France	41	26	13
Germany	8	12	13
Japan	—	3	5
Four countries total	81	69	73

Source: Indices compiled from statistics published in Imperial Economic Committee, *World Wool Consumption, 1938*. Corresponding figures in millions of lb. are given in Appendix IV.

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cannot be adequately and comprehensively provided without "labelling" legislation), a large proportion of consumers may have no means of acting in full accordance with their true scale of preferences. However, even on the assumption of improved consumers' information, the influence of competition from synthetic fibres must be regarded as one of the major factors affecting the future of wool.

World Trade in Wool Goods

Up to the outbreak of war an important share of the world's available wool supplies was used for the manufacture of goods entering international trade. During the period 1928/38 the weight of world exports of tops, yarns and tissues averaged about 400 million lb., which represented nearly a quarter of the average of annual "available supplies" of wool in all consuming countries, and nearly one-half of the average weight of world exports of raw wool (clean basis).

At the same time, world exports of yarns and tissues during the last pre-war decade showed a declining tendency which was specially marked in the case of exports from France and Germany (*cf.* Table XII). The decline of shipments from the United Kingdom, which is by far the most important exporter of tissues, was roughly in proportion to the world decline. Japan's exports of wool yarns and tissues did not reach a substantial share of the world total, but the *rate* of increase of Japanese exports provided a marked contrast to developments in the older industrial countries.

"Available supplies" of raw wool in the consuming countries may be regarded as a very approximate indicator of wool manufacturing activity. It is of interest to note, therefore, that despite the drop in U.K. exports during the last pre-war decade, the index of "available raw wool supplies" retained in the United Kingdom showed a considerable increase (*cf.* Table XII). This must be taken to imply that an increasing share of total output was absorbed in the U.K. home market. In France the index of "available supplies" was declining, but the decline was far less than proportionate to the drastic reduction in French exports of wool tissues.

The decline in the flow of wool goods from the traditional exporting countries was accompanied by the expansion of wool textile industries in the newer industrial areas. This development has made further strides during the war, especially in Australia and Argentina. New Zealand's domestic consumption of wool has also been increased, and in South Africa large-scale plans for the development of a domestic wool manufacturing industry are now being put into effect.

TABLE XIII

Wool Consumption in Five Main Surplus Countries

Country	Consumption, 1934/38		Consumption, 1939/43		Consumption, 1939/43 % increase over 1934/38 level
	Mn. lb. actual weight	% of wool production	Mn. lb. actual weight	% of wool production	
Australia	67	7	128	12	91
New Zealand	7	2	16	4	129
South Africa	—	—	8	3	—
Argentina	60	16	100	22	67
Uruguay	5	5	10	8	100
Total	139	7	262	12	88

The marked increase of domestic wool consumption in the main Surplus Countries was fostered by war-time conditions which reduced possibilities of importing textiles, and, in the case of Australia, also caused a large expansion of manufacture for military requirements.

Wool Consumption Prospects

The London Wool Conference Report estimates that world consumption of wool from the five main Surplus Countries during the next decade might, on average, be increased by one fifth over the pre-war level. The Report states:

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"On the assumptions that:

(*a*) there is a substantial increase in world purchasing power compared with pre-war by reason of the successful prosecution of policies of full employment and expansion of world trade,

(*b*) the price of wool and wool textiles is a reasonable one which encourages consumption,

(*c*) use is made of every aid, such as long-term credits to needy countries, publicity and scientific research in order to facilitate sales and expand consumption,

it is not thought impossible that (from 1947/48 onward) total world consumption of apparel wool might increase to a level of 12 per cent. above pre-war. This will represent an increase of 20 per cent. in the consumption of Dominions and South American wools over pre-war levels.

"If this aim is achieved the period required to dispose of existing stocks alongside new clips, at the estimated rate of future production, would be, on average, 13 years from 30th June, 1945." *

In order to examine this estimate a little more closely, we must refer again to our earlier analysis of the pre-war percentage distribution of wool exports from the five main Surplus Countries (*cf.* page 196, Table IX, Section (B)).

As was shown there, exports to Germany and Japan during the 1934/38 period accounted for 22 per cent. of total exports. Wool imports of these two countries are likely to be considerably below the pre-war average for some years to come. Even on *very* liberal assumptions, it would not seem reasonable to expect that German and Japanese imports during the 1947/48–1957/58 period will, on average, be more than, say, 80 per cent. of the 1934/38 level.† This means that in order to increase world exports by 20 per cent. over the 1934/38 level, shipments to the sum-total of all other countries would, on average, need to be increased by about one third.

Wool textile machinery equipment in the United Kingdom and in France and Belgium has, on the whole, remained fairly intact but it has been run down under war conditions, and much leeway has to be made up in replacements and renewals. The immediate problem of the wool textile labour shortage in the United Kingdom will be gradually lessened but the process of adjustment has had a slow start. Future progress will depend to some extent on the adjustment of relative wage rates in the textile and other industries.

As regards longer-term prospects of U.K. consumption, an increase of retained wool imports by one third over the 1934/38 level could be effected only if the pre-war downward trend in British wool textile exports were drastically reversed and/or if domestic consumption could be raised con-

* London Wool Conference Report, *op. cit.*, Annex A. On these assumptions, the Report's estimate of the period required for the disposal of Dominion stocks is as follows:

	Million lb.
STOCKS, June 30th, 1945	3,245
Exportable surplus	1,310
	4,555
Consumption	1,390
STOCKS, June 30th, 1946	3,165
Exportable surplus	1,330
	4,495
Consumption	1,620
STOCKS, June 30th, 1947	2,875
Exportable surplus	1,330
	4,205
Consumption (pre-war + 20%) ...	1,730
STOCKS, June 30th, 1948	2,475

† The *domestic* wool textile requirements of Germany and Japan will probably be curtailed to considerably lower levels; but it is not impossible that the export side of their wool textile industries will be expanded at a quicker rate. In view of the severe restrictions on the size of heavy industries and the resulting loss of foreign trade, the expansion of exports of textiles, and products of other lighter industries, may have to be permitted as a means of payment for the most essential imports.

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siderably above the relatively high pre-war per capita average of 5·3 lb. clean. Such a favourable development would need to take place in the face of increasing competition from substitutes. No doubt the war-time depletion of traders' stocks and consumers' wardrobes will result in some considerable extra restocking demand for wool goods as soon as supplies are again more plentiful. Even when this specific factor is taken into account, however, an *average* increase of U.K. wool consumption over the next decade by, say, 15 per cent. above the pre-war level would seem to be a fairly optimistic assumption.*

Similar considerations apply to France (where exports of wool goods during the inter-war period declined sharply), and to Belgium (where per capita wool consumption before the war had already achieved a higher level than in any other country on the Continent).

As regards "Other Europe," exports to certain countries may conceivably be raised by as much as one third above pre-war, but, on balance, the assumption of a 20 per cent. increase for the groups as a whole would appear to be liberal.

Exports to the United States may—under favourable conditions—exceed the 1934/38 level considerably. It has been estimated by the U.S. Tariff Commission that the level of U.S. total consumption may settle down at about 750 million lb. greasy, or 25 per cent. above the pre-war average. This might lead to an average increase of imports by as much as 100 per cent. over the 1934/38 level; † and if a downward revision in the American tariff were effected, the increase might be even higher. If, however, the tariff is not only maintained at its present rate but even supplemented by imports, quotas and/or subsidies of home-grown wool, then the outlook for the wool-exporting countries will be much less promising. Besides, the estimate of a permanent increase in the overall rate of U.S. wool consumption by 25 per cent. above pre-war does not allow for the possible danger of another serious depression in the United States.

For the sake of illustration, let us consider the prospective volume of world wool exports on the very optimistic assumptions of a 20 per cent. increase above pre-war in all European countries other than Germany, an average of 80 per cent. of pre-war in Germany and Japan, and a 100 per cent. increase above pre-war in the United States. Even on the basis of these liberal assumptions, an overall increase of world wool exports to 20 per cent. above pre-war could be achieved only if shipments to all "Other countries" could be increased by as much as 100 per cent. above the 1934/38 level. ‡

* Before the war imported wools accounted for about 90 per cent. of total "retained available supplies of raw wool" in the U.K. If U.K. consumption of home-grown wool remained stationary, a 15 per cent. increase in total consumption would lead to a 17 per cent. increase in retained imports. It may well be, however, that in the case of an over-all increase of consumption, the use of home-grown wool will expand *pari passu* with the use of imported wools. During the war years the U.K. domestic clip has been reduced by about 20 per cent.; but, on the other hand, the pre-war consumption of U.K. home-grown wool only absorbed about one half of the domestic clip, whilst the other half was exported. There is thus a margin of supply for a moderate increase in home consumption.

† *Cf.* above, p. 17.

‡ If, on the other hand, we assume a 10 per cent. increase above pre-war in the U.K. and on the Continent; a reduction to 60 per cent. of the 1934/38 level in Germany and Japan; and a 30 per cent. increase above pre-war in the U.S.A. and in the group of "Other countries"; then the average rate of world wool exports would only be equal to pre-war.

These rather conjectural estimates are summed up in the table below:

	1934/38 average	1947/57 average: changes from 1934/38 level		Estimated 1947/57 average expressed as % of 1934/38 level	
		A,	B,	A,	B,
	%	%	%	%	%
U.K. (retained)	27	+10	+ 20	30	32
France	18	+10	+ 20	20	22
Belgium	11	+10	+ 20	12	13
Germany	12	—40	— 20	7	10
Other Europe	12	+10	+ 20	13	14
U.S.A.	6	+30	+100	8	12
Japan	10	—40	— 20	6	8
Other countries	4	+30	+100	5	8
	100			101	119

If exports to Germany and Japan during 1947/57 are assumed at one-half of pre-war and exports to all other countries as equal to pre-war, then the general export level would be 91 per cent. of pre-war.

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The group of "Other countries" includes the vast potential markets of India, China, and the U.S.S.R., where pre-war consumption of overseas wool was, on the whole, very low, so that even a relatively small increase in imports by actual weight would represent a considerable percentage increase. Progressive industrialization in India and China and a higher rate of output of consumers' goods in the U.S.S.R. might thus conceivably lead to a sharp percentage rise in wool imports; but any such expectations are largely conjectural, and account must also be taken of the possibility of further quantitative and qualitative development of the domestic wool supply of these countries.

III. PRICES

A comparison of long-term movements of wool prices with the general Sauerbeck index for the period 1850 to 1944 (Chart No. III) indicates that wool values were, on the whole, strongly influenced by the general trend of world economic developments. This one would expect. At the same time, the influence of specific factors determining wool values was far from negligible. This aspect is brought out more clearly in Chart No. IV, which shows ratios of merino and crossbred prices to the Sauerbeck index. Two outstanding features of the curves shown in Chart No. IV are: (i) the marked accentuation of fluctuations in the ratios of both merino and crossbred values to the general level of wholesale commodity values in the inter-war period as compared with the three decades before World War I; and (ii) the relative depression of the crossbred/Sauerbeck ratio during certain periods of the inter-war decades as compared with (*a*) the merino/Sauerbeck ratio during the same period, and (*b*) the crossbred/Sauerbeck ratio up to the beginning of World War I. The marked degree of intra-annual and year-to-year fluctuations during the inter-war period is also illustrated in Chart No. II, which shows quarterly end-of-month quotations in *d.* per lb. (clean basis) for merino, 66's, and crossbred, 46's. We shall now consider these price movements a little more closely.

Survey of Wool Price Movements, 1890–1939

1890–1913.—With just a few exceptions (1900/4, 1906 and 1908), the course of merino prices from 1890 to 1913 shows a gentle and fairly steady upward movement. In 13 out of 19 instances the moving five-year price averages during this period showed upward changes which varied from one-tenth of a penny to 1·1*d.*, or 0·4 to 6 per cent. of the previous average. The 1909/13 average of merino prices was about 50 per cent. above the 1890/94 level. This compares with an 18 per cent. increase in the general Sauerbeck index over the same period.

The increase of merino prices in terms of general wholesale commodity values was not equalled by crossbred prices, which in 1909/13 were merely 16 per cent. above the 1890/94 level. This was partly due to a recession in 1908, which, like all adverse periods, affected crossbred values much more strongly than those of merino. A more general reason for the relatively lower crossbred values is to be found in the increased profitability of the overseas meat trade (due to refrigeration) which brought forth much more plentiful supplies of crossbred wool; whilst merino supply had been decimated by heavy droughts during the years 1892–1902. The 1909/13 average of crossbred output was 77 per cent. above the 1890/94 level, whilst merino production had been reduced by 23 per cent. over the same period.

1914–1918.—In the first World War the necessity for Government intervention was only gradually realized, and during the first two years of the war prices and supply of raw wool remained uncontrolled. Mr. E. M. H. Lloyd, in his account of *Experiments in State Control at the War Office and at the Ministry of Food*, states that, in spite of weighty arguments in favour of Government bulk purchases of raw materials on grounds of economy and expenditure and stabilization of prices, this measure might never have been taken if by 1916 the Cabinet had not become afraid of an actual shortage of wool supplies required for clothing the Allied armies. This danger arose because of a serious Australian drought in the 1914/15 season, a reduction in the South American clip, and increasingly heavy American and Japanese purchases. In view of this world situation, the British Government decided in November 1916 to take over the Australian and New Zealand clips at prices averaging about 55 per cent. above the 1914 level. In mid-1917 South African wool was included in the bulk purchase scheme, which continued to operate until June 1919 for South Africa, and until June 1920 for Australia and New Zealand.

1919–1939.—London wool auctions, which had been suspended in December 1916, were resumed

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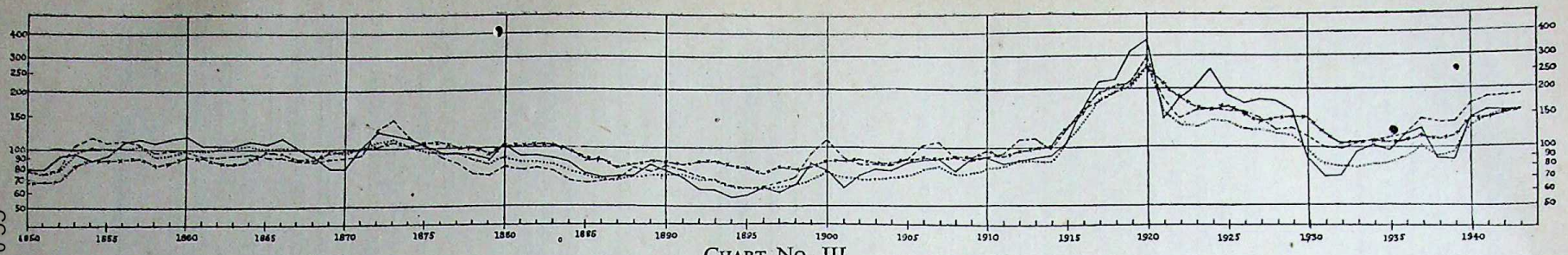

CHART NO. III

Sauerbeck/Statist *Indices of Merino Wool, Minerals. Animal Food and General Index*, 1850–1944

(1867/77 = 100)

———— = Merino wool.
– – – – – – – – – – = Minerals.
—×—×—×— = Animal food.
..... = Sauerbeck General Index.

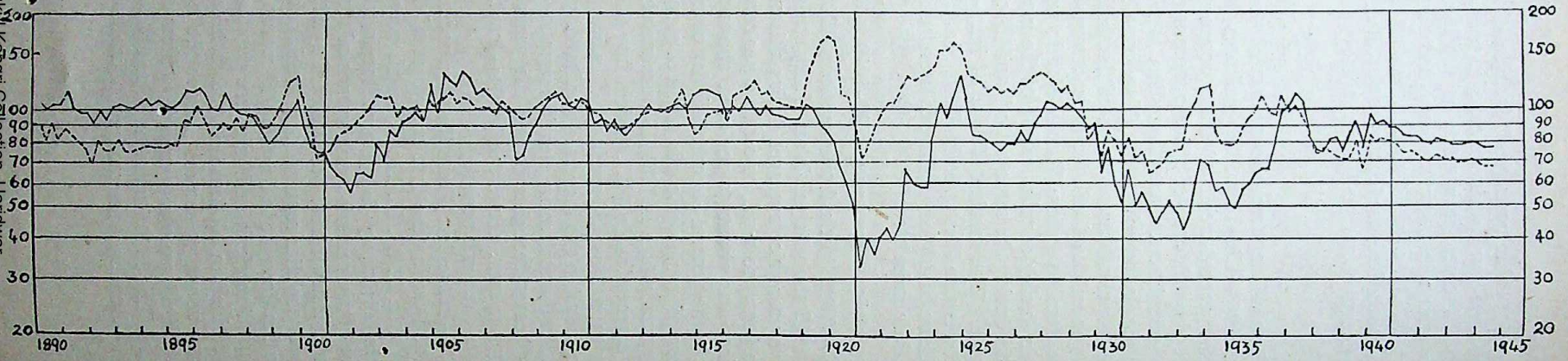

CHART NO. IV

Quarterly Ratios of Merino and Crossbred Price Indices to Sauerbeck General Index, 1890–1944

(1909/13 = 100)

– – – – – – – – – – = Merino/Sauerbeck ratio.
———— = Crossbred/Sauerbeck ratio.

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in April 1919, and supplies of Government-owned Australian wool were sold in free competition with wools from South Africa and South America. The first season of post-war auction sales was characterized by a spectacular rise in merino values. By March 1920 merino quotations had risen to 150*d.*, as compared with 70*d.* during the control period and 83*d.* at the first London auction sales. In view of these boom conditions, many Australian and New Zealand woolgrowers were anxiously awaiting the date when the Imperial Wool Purchase Scheme would be terminated, and they, like growers in other countries, could make the most of the free market.

It was not realized at the time that this unprecedented boom might be merely artificial, and that consumers' needs in the impoverished European countries were not sustained by adequate purchasing power. When the truth became apparent towards the end of the 1919/20 selling season, Australian and New Zealand growers were faced with the prospect of resuming free trade in a rapidly declining market, and no preparation had been made for co-ordinating sales of the new clips with the disposal of vast stocks which were still left in the hands of the Government.

Conditions in the other main wool-exporting countries were also far from bright,* and the world situation was further aggravated by the prospect of the introduction of the United States tariff on raw wool. The tariff had been removed shortly before the outbreak of the first World War, but it was re-imposed at the end of 1920.

The catastrophic fall in prices during the period March 1920 to March 1921 can be seen from the following figures:

End-of-Quarter Prices of Raw Wool

(In pence per lb., clean; London)

	Merino 66's	Crossbred 46's
1920, March	150	42
June	98	33
September	96	29
December	65	19
1921, March	44	11½

In view of these difficult market conditions, it was decided to set up an emergency scheme for the liquidation of surplus stocks of Dominion wool. In January 1921 the disposal of all stocks left in the hands of the British Government was taken over by the newly formed British Australian Wool Realization Association (BAWRA), which was a limited company owned by the Australian wool growers. The BAWRA scheme has come to be known as the only major example of "managed" wool-marketing in peace-time, and a more detailed account of the main features and effects of this scheme will therefore be given in a general survey of wool-marketing methods in the final section of this paper (pp. 40–41). Meanwhile, it may just be noted that during the first six months of BAWRA's marketing operations (January–June 1921), wool prices continued to fall; but from then on up to the termination of BAWRA's activities in 1924 a jerky but marked upward movement led to the recovery of merino and crossbred values to approximately the same levels which had been fixed during the years of war-time control.

A renewed fall in wool values set in in 1925 and, with the exception of some temporary breaks, the downward movement continued until, in mid-1932, merino values reached the lowest level experienced since the early 'nineties of the last century; whilst crossbred values fell even more sharply to levels lower than any realized during the previous four decades. Recovery during subsequent years was interrupted by renewed minor recessions, and even the peaks (1935 and 1937) remained below 1924 levels.

Wool Price Fluctuations Compared with Fluctuations in the Prices of Other Commodities

The drastic price changes during the inter-war period were so strongly influenced by general world economic developments that it is of little object to consider the wool market in isolation. Some indication of the specially severe effects of the Great Depression on wool values as compared with the general level of wholesale commodity prices is given by the ratios of merino and crossbred values to the general Sauerbeck index (1909/13 = 100). In 1932 these ratios had fallen to 0·64

* In South Africa the volume of sales was declining at falling prices, and the market was burdened with 100,000 bales of carry-over wool from the 1919/20 season. In Argentina mid-1920 stocks of crossbred wool amounted to 250 million lb., and growers had to be helped by State credits.

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for merino and to 0·45 for crossbred. For the purpose of our analysis, it is even more important, however, to compare the extent of wool price fluctuations during the inter-war period with fluctuations in the prices of other *individual* commodities. In the following we shall give a comparison of:

(*a*) intra-annual price fluctuations;
(*b*) year-to-year changes in annual price averages; and
(*c*) maximum price-spreads during consecutive five-year periods

for each of the following 10 commodities:

I. *Raw Wool, Merino*, 66's.
II. *Wool Tops, Merino*, 64's.
III. *Raw Wool, Crossbred*, 46's.
IV. *Mutton*, New Zealand, 1st Quality.
V. *Cotton*, American Middling.
VI. *Wheat*, English *Gazette*.
VII. *Butter*, New Zealand, 1st Quality.
VIII. *Rubber*, Smoked Sheet.
IX. *Copper*, Standard.
X. *Tin*, Standard.

(a) *Intra-Annual Price Fluctuations*.—The purpose of this analysis is to present a picture which can be of some assistance in the task of determining future marketing policy. For this reason we are here not so much concerned with outstanding instances of exceptionally violent changes, but mainly with the representative range of annual price changes. The figures given in Table XIV show that this representative range was wide for most commodities, but for the wool groups it was even wider than for all other commodities with the exception of rubber. In the case of merino, the year's high exceeded the low by more than 38 per cent. in 10 out of 20 years. This means, in other words, that in one year out of every two there was one date in the year when prices were at least one-third higher than at some other date in the same year. The merino range was nearly equalled by cotton and wheat, where the minimum excess of high over low in 10 out of 20 years was 32 per cent. For crossbred, however, the corresponding figure was 44 per cent., which was considerably higher than for any other commodity, with the exception of rubber. During the 1929/38 decade there were five instances when the year's highest crossbred quotations exceeded the lowest by more than 50 per cent., and four instances when the excess of high over low was more than 60 per cent. This means that in 5 out of 10 years the crossbred grower who sold his wool at the year's low point could have earned an extra sixpence to each shilling if instead he had concluded his sale at the year's high point. The average intra-annual price range for merino wool during the 1929/38 was less than for crossbred, but more than for all other commodities except rubber and copper.

It is easy to see that these wide intra-annual price fluctuations are in themselves a further element accentuating the instability of the market. The staggering extent of the annual price range for many staple commodities has been stressed by Keynes, who points out that the high degree of intra-annual fluctuations " is apt to be concealed from those who only watch the movements of index numbers and do not study individual commodities; since index numbers, partly by averaging and partly by including many commodities which are not marketed in fully competitive conditions mask the short-period price fluctuations of the sensitive commodities." Keynes illustrates his point with examples from various commodity markets, and concludes that " the ill effect of these truly frightful fluctuations on trade stability is great. But the ultimate results of the obstacles which they offer to the holding of stocks is even more injurious. . . . Even though fluctuations in the demand for many finished commodities owing to changes in fashion and in the direction of demand may be unavoidable, and though it is certainly the case that no radical remedy for fluctuations is possible, except through measures to stabilize the aggregate of effective demand, nevertheless some modification should be possible in the case of the great staple raw materials, most of which can be readily stored without serious deterioration, by direct measures affecting the individual commodity. Assuredly nothing can be more inefficient than the present system, by which the price is always too high or too low, and there are frequent meaningless fluctuations in the plant and labour force employed." *

All this is largely true in the case of wool, where the average annual price range before the war

* J. M. Keynes, " The Policy of Government Storage of Foodstuffs and Raw Materials," *Economic Journal*, September 1938.

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was even slightly wider than for wheat and cotton and considerably wider than for a number of other primary products.

(b) *Year-to-Year Changes of Annual Price Averages.*—A survey of year-to-year changes of annual price averages for wool and other commodities is given in Table XV. Each figure represents the mean of ten successive year-to-year changes expressed as per cent. of the price average for the corresponding 10-year period. Here again the figures for rubber are generally higher than for the other commodities shown in the table; but in 1929/38 rubber was equalled by the average percentage figure for crossbred. The merino percentage ranked next to crossbred, and exceeded the percentages for all other commodities.

TABLE XIV

Intra-Annual Price Fluctuations During the Inter-war Period: 10 *Selected Commodities*

	I Raw wool, Merino	II Wool tops, Merino	III Raw wool, Crossbred	IV Mutton	V Cotton	VI Wheat	VII Butter	VIII Rubber	IX Copper	X Tin
PART I. Mean of Maximum Intra-Annual Price-Spreads in 10 (20) Successive Years as Per Cent. of Corresponding 10 (20) Year Price Average.										
Period	%	%	%	%	%	%	%	%	%	%
1919/28	39	47	38	20	40	31	25	83	18	26
1929/38	31	33	35	22	24	30	25	55	35	26
1919/38	37	43	36	21	36	30	25	73	26	26
PART II. Number of Years During which Maximum Intra-Annual Price-Spread was More than 50 Per Cent. of Annual Price Average.										
Period	No.	No.	No.	No.	No.	No.	No.	No.	No.	No.
1919/28	3	3	3	—	1	2	—	8	—	1
1929/38	1	2	3	—	—	1	—	7	2	—
1919/38	4	5	6	—	1	3	—	15	2	1
PART III. Number of Years During which Maximum Intra-Annual Price-Spread Exceeded Year's Low Point by 50 Per Cent. or More.										
Period	No.	No.	No.	No.	No.	No.	No.	No.	No.	No.
1919/28	4	4	3	1	3	2	2	8	1	1
1929/38	2	2	5	1	—	2	2	8	3	2
1919/38	6	6	8	2	3	4	4	16	4	3

TABLE XV

Survey of Year-to-Year Changes of Annual Price Averages of 10 *Selected Commodities* 1904/13, 1919/28 *and* 1929/38

	I Raw wool, 66's	II Wool tops, 64's	III Raw wool, 46's	IV Mutton	V Cotton	VI Wheat	VII Butter	VIII Rubber	IX Copper	X Tin
Mean of 10 Successive Year-to-Year Changes (irrespective of sign) Expressed as Per Cent. of Price Average for Corresponding 10-Year Period.										
Period	%	%	%	%	%	%	%	%	%	%
1904/13	6	7	15	7	13	8	4	30 (*a*)	12	15
1919/28	28	25	30	11	30	12	10	38	13	18
1929/38	23	19	31	8	18	17	13	31	22	18

(*a*) Years 1906/13 only.

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Description of Grades and Sources of Price Quotations for 10 *Commodities Considered in Tables* XIV, XV *and* XVII

I. Raw Wool, Merino, 66's (clean basis); pence per lb., London.
Sources: Schwartze, Buchanan & Co., *Annual Reports*; *Weekly Wool Chart*; Imperial Economic Committee, *Wool Survey*, 1932, and *Wool Production and Trade*, 1938/39.

II. Wool Tops, 64's (clean basis); Bradford, spot.
Source: *Weekly Wool Chart*.

III. Raw Wool, Crossbred, 46's (clean basis); pence per lb., London.
Sources as under (I).

IV. Mutton, New Zealand, 1st Quality; pence per lb., London.
Source: Ministry of Agriculture and Fisheries, *Agricultural Statistics*, 1909/39.

V. Cotton, American Middling, Fair Staple ($\frac{7}{8}$-inch) Average; pence per lb., spot, Liverpool.
Source: U.S. Dept. of Agriculture; Bureau of Agricultural Economics, *Statistics of Cotton and Related Data*, December 1939.

VI. Wheat, English *Gazette*; shillings per cwt., U.K.
Source: Ministry of Agriculture and Fisheries, *Agricultural Statistics*, 1909/39.

VII. Butter, New Zealand, 1st Quality; shillings per cwt., average three markets: London, Liverpool, Bristol.
Source: Ministry of Agriculture and Fisheries, *Agricultural Statistics*, 1909/39.

VIII. Rubber, 1906/09: Fine Hard Para. 1910/27; Plantation Best Crêpe. 1928/38; Smoked Sheet; pence per lb., spot, London.
Source: W. H. Rickenson & Sons, *The World's Rubber Position*.

IX. Copper, Standard; £ per long ton, spot cash, London.
Sources: U.S. Bureau of Mines, *The Mineral Industry*, Vol. XIX, 1910 for years 1904/10. *The American Bureau of Metal Statistics* for years 1911/38.

X. Tin, Standard; £ per long ton, spot cash, London.
Sources: U.S. Bureau of Mines, *The Mineral Industry*, 1912, 1915, 1916 and 1918 for years 1904/18. *The American Bureau of Metal Statistics* for years 1919/38.

In order to arrive at a comparable basis of price fluctuations, all highs and lows are based on monthly averages except in the case of rubber, where the actual highest and lowest quotations in each month had to be used. Monthly averages of (I) Raw Wool, Merino, and (III) Raw Wool, Crossbred, for non-auction months are based on nearest interim market quotations.

The comparison of the degree of year-to-year changes in annual wool price averages during different periods shows the marked accentuation of these changes during the two inter-war decades as compared with 1904/13. It is also of interest to compare the year-to-year changes in wool price averages with year-to-year changes in wool production. The curves shown in Charts Nos. I–IV have already given us a preliminary indication of the marked contrast between the relative short-term stability of supply and the often violent movements in wool values. A more exact numerical illustration of this contrast is given in Table XVI. As can be seen from this table,

TABLE XVI

Year-to-Year Changes in Wool Production and Prices, 1904/13, 1919/28 *and* 1929/38

Mean of 10 successive year-to-year differences (irrespective of sign) as % of corresponding 10-year average

Period	Wool production in 10 selected countries			Annual price averages	
	Total, %	Merino, %	Crossbred, %	Merino, 66's, %	Crossbred, 46's, %
1904/13	3	6	4	7	15
1919/28	5	7	4	25	30
1929/38	2	2	3	19	31

the highest figure for average year-to-year production changes was 7 per cent. for merino in 1919/28. The average changes of total output in the 10 main producing countries did not exceed 5 per cent., and in crossbred supply the maximum average change was 4 per cent.; whilst year-to-year changes in annual wool price averages during the inter-war period were $3\frac{1}{2}$ to 10 times as high as corresponding changes in output.

(c) *Maximum Price-Spreads During* 5*-Year Periods*. The third summary table of price movements for 10 commodities shows the ratios of maximum price-spreads during 5-year periods

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to corresponding 5-year price averages (Table XVII). As would be expected, rubber has again the doubtful honour of leadership in the range of price fluctuations. During 1924/28 and 1929/33 the differences between high's and low's of rubber prices within each period were more than 6½ times as large as the corresponding 5-year price averages. Apart from rubber, however, the crossbred ratio comes out higher than for all other commodities in 1929/33 and 1934/38, and higher than all others except cotton in 1924/28. In three out of the four 5-year periods from 1919 to 1938 the spread between high's and low's of crossbred prices was three times as large as the corresponding 5-year price average. The merino ratio ranked next to rubber and crossbred in 1919/23; but in some of the other 5-year periods it was surpassed by the ratios for cotton, wheat, copper and tin. At the same time, the maximum merino price-spreads during consecutive 5-year periods from 1919/23 to 1934/38 were never less than 60 per cent. of the corresponding 5-year price averages.

TABLE XVII

Maximum Price-Spreads During 5-Year Periods: 10 Selected Commodities

Period	I Raw wool, Merino, 66's	II Wool tops, 64's	III Raw wool, crossbred, 46's	IV Mutton	V Cotton	VI Wheat	VII Butter	VIII Rubber	IX Copper	X Tin
PART I. Ratio of: Maximum Price-Spread During 5-Year Period/5-Year Price Average										
1909/13	0·14	0·14	0·31	0·47	0·47	0·42	0·27	1·73	0·39	0·62
1919/23	1·41	1·74	1·42	0·81	1·36	0·80	0·86	1·70	0·81	1·16
1924/28	0·55	0·64	0·78	0·40	0·91	0·40	0·42	2·40	0·25	0·38
1929/33	1·03	0·90	1·55	0·72	1·23	1·05	0·78	2·20	1·35	0·77
1934/38	0·60	0·58	1·11	0·28	0·53	0·71	0·86	1·31	1·16	0·55
PART II. Ratio of: Maximum Price-Spread During 5-Year Period/Lowest Monthly Average of 5-Year Period										
1909/13	0·16	0·16	0·38	0·63	0·67	0·48	0·31	5·00	0·45	0·87
1919/23	2·57	3·66	3·0	1·41	3·00	1·27	1·46	4·18	1·06	1·76
1924/28	0·70	0·80	1·0	0·50	1·43	0·51	0·50	6·59	0·28	0·47
1929/33	1·51	1·30	3·0	1·00	1·66	1·58	1·07	6·64	2·41	1·18
1934/38	0·83	0·75	2·0	0·33	0·71	1·09	0·97	2·27	1·70	0·73

Summing up, we can conclude that, with just a few exceptions, the range of wool price fluctuations during the inter-war period was generally at least equal to, and in the case of crossbred often much higher than, the very considerable range of price fluctuations for a varied selection of other staple commodities.

The accentuation of fluctuations in crossbred prices as compared with prices of other commodities was specially marked during depressed periods. At the same time, the adverse effects to crossbred producers were mitigated to some extent by the greater stability of mutton and lamb prices, which provided a limited hedge against disastrous drops in crossbred growers' overall returns. The majority of merino growers, on the other hand, are dependent on the wool market for the dominant portion of their returns, and their incomes are thus very strongly affected by fluctuations in merino values.

The importance of the effects of changes in wool prices on the economies of the main exporting countries can be gauged from the extent of variations in their annual "wool cheques." In Australia, for instance, the annual wool cheque was £A.52·2 million in 1933/34, as compared with £A.38·5 million in 1934/35 and £A.60 million in 1936/37. The New Zealand wool cheque was £N.Z.10·5 million in 1933/34, as compared with £N.Z.4·5 million in 1934/35 and £N.Z.15·3 million in 1936/37.

Main Causes of Wool Price Fluctuations

A full analysis of all the main causes of the high degree of wool price fluctuations during the inter-war period would entail a description of the conditions of *general* economic instability,

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which were, of course, of major importance in the wool market, just as in all other international commodity markets. Such a general analysis would, however, lead much beyond the scope of this paper, and in the following remarks we shall merely give a summary of some *specific* factors which tend to accentuate the degree of price fluctuations in the case of wool. Most of these specific factors have already been considered in some detail in our previous survey of conditions of supply and demand, and they can therefore be summed up very briefly under four main headings:

1. *Influence of Supply on Price.*—Despite occasional sharp changes in the size of individual countries' clips, short-term variations in world wool supply are generally low, averaging about 5 per cent. or less of annual output. In years of severe drought, reductions in merino output may be more marked; but during the inter-war period, year-to-year changes in merino production were generally quite insignificant in comparison with corresponding price changes.

2. *Influence of Price on Supply.*—In view of the length of the production period and the relative importance and rigidity of overhead costs in wool production, the responsiveness of supply to price changes is very low. In the case of crossbred wool, the price elasticity of supply is lessened further by the fact that crossbred wool is a joint product with meat.

3. *Influence of Demand on Price.*—In view of the low responsiveness of wool supply to price changes, short-term variations in demand are not balanced by short-term changes in output. Hence, the full effect of changes in the scale and intensity of demand is reflected in fluctuations in value. During the inter-war period the general effects of cyclical fluctuations in world income levels were accentuated by specific factors in individual major wool consuming countries; such as, for instance, the sharp and erratic changes in American imports (for reasons discussed earlier, page 197). Strategic influences also played an important part. For instance, the tendency towards self-sufficiency in Germany was largely responsible for a sharp fall in German wool imports from 1933 to 1936, whilst the urgent desire to build up strategic stocks in the period of "war preparation" caused a change in policy, and raised the level of German wool imports in 1938. Japanese imports also showed sharp and erratic year-to-year changes. In the "free" sector of the world market the anticipation of a high degree of price variability led to manufacturers' and traders' dynamic stocking and de-stocking operations, which again accentuated the effects of cyclical fluctuations of real demand on prices.

4. *Influence of Price on Demand.*—The ratio of the cost of raw wool expressed as percentage of the retail prices of finished goods is generally very low. At the same time the income elasticity of clothing expenditure is high, and this means that during depressed periods the adverse effect of falling incomes on the level of wool consumption is likely to be much stronger than the counteracting effect of lower raw wool prices; the more so as all other cost components of finished wool goods (wages and other costs of manufacture and distribution) tend to prove more rigid and to fall less during a depression than the price of raw wool. In short, *the influence of wool price changes on demand is much less marked than the influence of changes in demand on wool prices.* This is one of the wool industry's basic problems.

IV. The Marketing of Wool

Pre-War Marketing Methods

In peace-time the major share of the world's apparel wool entering international trade was sold by public auctions.

The auction system, just like the system of futures trading, attempts to centralize competition and to eliminate effects of the personal influence or bargaining power of individual buyers and sellers. The means by which these aims are achieved under these two most highly competitive marketing systems are, however, different in the extreme. Whilst the futures contract is based on the highest possible degree of standardization of the commodity, the auction method puts the special quality characteristics of each individual lot into the foreground. In fact, it may be said that each single lot is treated as a different commodity, all other lots influencing the process of price formation merely as very near substitutes.

Given conditions of competitive sales, the auction system is obviously suitable for a commodity like wool, which shows such a wide range of commercially important differences in quality and yield.

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To some extent, the development of auctions in the Empire wool trade can also be explained historically. Dominion wools were naturally shipped to the mother country, and the first London auction sales of Australian wools in 1835 marked the beginning of London's position as the world's leading international market for apparel wools. Up to the outbreak of the First World War, London was not only the leading, but also the largest raw wool auction market in the world. During the inter-war period auction sales in the Dominions developed rapidly, and the volume of London sales was surpassed by sales in the Sydney market. Other Australian, New Zealand and South African selling centres also gained in quantitative importance. In all three Dominions auction programmes were fixed each year by committees consisting of growers', brokers' and buyers' representatives, who determined the dates of sales as well as amounts to be offered by each different selling centre. In this way supplies were spread evenly over the year, and competition between the different centres in each country was minimized. In fact, the method of allocating dates of sales and quantities to the various centres had very much the same effect as full centralization, whilst at the same time minimizing freight costs (which in the case of actual centralization in one centre in Australia would have been increased to such an extent as to have been almost prohibitive).

The only important overseas markets where farmers still sell their wool individually and by private treaty are the South American countries. It is widely agreed that the system of private treaty sales is not advantageous to farmers, who, acting in isolation, are usually less well informed in regard to the market values of their clips than the buying agents with whom they deal. In view of the commercial disadvantages to producers, it may seem surprising that these methods have been retained. However, custom and tradition play a large part, and to some extent the explanation is probably again historical. The history of the evolution of wholesale auctions shows that it was closely bound up with the economic development of colonies, for which the mother country remained the main market for their goods. In the South American countries, on the other hand, the economic relations with the original mother country—Spain—had dwindled in importance a long time ago, and foreign markets became of dominating influence. As the overseas wool trade gained in importance, the buying of wool became concentrated in the hands of a number of foreign firms who established their own local large-scale buying agencies.

Futures markets in wool tops were established before the war in Antwerp, Roubaix-Tourcoing and New York. The New York market is still functioning and also provides a " grease-wool " contract; but the volume of transactions is relatively of very small importance.

The development of futures trading in raw wool has always been hampered by difficulties of standardizing quantity and quality. These difficulties are diminished in a futures market dealing in wool not in the raw state, but in the next stage of manufacture—*i.e.*, in clean combed wool or in wool tops. Standardization of quantities of wool tops does not present any special difficulties, and the grading of qualities is based on characteristics which are easier to define and to ascertain than characteristics of raw wool, so that inspection in bulk can be dispensed with and trading by samples, and even by description, is possible.*

The functions of futures trading on the Continental wool top exchanges during the inter-war period were often distorted by trade and exchange restrictions limiting the effectiveness of arbitrage; whilst at the same time the instability of foreign exchange rates induced some traders to use the futures market as a medium for foreign exchange speculation. On the whole, the use of futures as a means of risk insurance offered less scope and was *much* less widespread in the wool trade than in cotton, and consequently the full brunt of the risks of the sharp and erratic wool price fluctuations was borne by producers, manufacturers and traders without a hedge.

The peace time history of wool marketing does not include any record of bulk purchase agreements. The nearest approach to a bulk purchase contract was a barter agreement which was first concluded between the South African and German Governments in 1934, and subsequently renewed for later seasons up to South Africa's entry into the war. Under this agreement, Germany undertook to buy South African wool to a specified maximum value over a certain period, whilst

* In so far as the effectiveness of hedging is concerned, the situation of futures trading in a stage which is not too distant vertically from either the raw material stage or the later stage of manufacture is not without advantages. On the other hand, the substitution of tops for raw wool as standard grade accentuates the difficulties arising from deviations in the price movements of those wools which are not, as a rule, combed into tops. Besides, there is an important difference between the " dry-combed " tops used on the Continent and the " oil-combed tops " used in Britain, and this also tends to split the market.

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the value of purchases made in Germany by Union importers over the same period was offset against such purchases of wool.

With the exception of the organization of Dominions wool surplus disposals at the end of the last war, the peace-time wool market was not at any time subject to joint control by producers or by their respective Governments.

At a Conference held in Melbourne in January 1937 representatives of the wool growing industries in Australia, New Zealand and South Africa agreed to induce their Governments to impose statutory levies on all wool exported from the three countries, and to sanction the utilization of these funds for research and publicity. The Dominion Governments endorsed the scheme, and part of the funds raised in this way was devoted to the work of the International Wool Secretariat, which was established in London in August 1937. The aims of the Secretariat can be defined as the furtherance of the interests of the wool industry in its widest sense by means of (1) scientific and economic research, and (2) consumers' education, promotion and publicity. The constitution of the Secretariat does not, however, provide for any direct activities in regard to the marketing of wool.

BAWRA—Experiment in Peace-time Control, 1921–24

In the general survey of wool price movements during the inter-war period, reference has already been made to the emergency scheme set up for the orderly disposal of stocks left over at the end of the First World War. This scheme is often quoted as a precedent for solving the problems of the present world wool situation, and for this reason it is important to describe its scope and limitations.

The scheme was initiated during the very severe 1920 slump which had followed the spectacular but short-lived merino boom of the first post-war season of free auction sales in 1919. In view of the very difficult market conditions during the second half of 1920, the British and Australian Governments, after consultation with the Australian Wool Council, decided to sponsor the formation of BAWRA (the British Australian Wool Realization Association), which was a Limited Company owned by the Australian wool-growers and set up " for the purpose, *inter alia*, to control and dispose of all Australian carry-over wool."

Assets transferred to the Company through the British and Australian Governments included 1,836,000 bales of wool, half of which it took as owner from the Australian Government and the other half as selling agent for the British Government.*

The capital of the Company was set down at £25,000,000, including half the value of the wool which was established by adding the charges to the cost price and allowing 40 per cent. off this total for depreciation. The Company's entire capital was owned by the Australian wool-growers, and the basis of distribution was fixed in accordance with the amount of wool which each grower had sent in during the previous four seasons of control (1916/17–1919/20).

The selling policy framed by the BAWRA directorate was based on the regulation of quantities offered from stocks. Market allocations of quantities of " old clip " wool were fixed month by month in the light of the latest market reports, and disposals of BAWRA wool were co-ordinated with quantities of " new clip " wool released for sale by the Australian National Council of Wool-

* The total quantity of wool actually taken over by BAWRA early in 1921 was as follows:

	Wool (bales) owned by—			
	BAWRA		British Government	
Australian:				
Merino	452,090		452,090	
Crossbred	465,912		465,913	
		918,002		918,003
New Zealand:				
Merino			3,181	
Crossbred			769,762	
				772,943
Falkland Islands				2,329
		918,002		1,693,275

Grand Total: 2,611,277. Grand Total, Australia: 1,836,005.

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Selling Brokers. In order to avoid overloading the market in any one centre, stocks were disposed of in different places in the United Kingdom as well as on the Continent.

At the outset of operations the BAWRA directorate also attempted to obtain agreement in regard to fixed proportions and price limits at which both BAWRA wool and "new clip" wool were to be sold in the London market. Some agreement was reached on these points with the London wool brokers, but a number of merchants were strongly opposed to the imposition of limits, and some of them continued to meet the market below the agreed reserves.

During the initial period of BAWRA's selling operations, prices continued to fall and stocks were growing. In view of the serious problems arising from vast accumulations in a falling market, the directors of BAWRA felt that the voluntary agreement with regard to the allocation of quantities and price limits would have to be supplemented by Government action. In May 1921 the Commonwealth Government, in response to BAWRA's appeal, passed legislation laying down that wool was not to be sold below limits based on a flat rate of 8*d.* per lb., and these legally fixed reserves remained effective during a period of six months.

Actually, Australian prices never fell to the minimum level fixed by the Government. Notwithstanding this fact, however, it is held by some that the mere existence of a legal minimum reserve had a decisive psychological influence, and helped to restore confidence and to sustain the market in an extremely difficult trading period.

There is no means of measuring the influence of these legal minimum reserves, and opinions as to their effectiveness will always differ. It is a matter not of opinion, but of fact, however, that, with the exception of these—admittedly most critical—six months of their enforcement, wool prices remained uncontrolled. Price fixation cannot, therefore, be described as a feature inherent in the system of BAWRA's selling operations. Indeed, after the expiration of the act of price legislation in November 1921 the directors of BAWRA did not even succeed in re-establishing the earlier method of fixing limits by voluntary agreement.

At a meeting of shareholders in December 1922, the Chairman of the Australian Directorate of BAWRA proposed the conversion of BAWRA into a permanent central selling organization for Australian agricultural primary products, but the motion was defeated. BAWRA's financial liquidation was not completed until 1932, but its marketing activities ended much earlier; the last bale of BAWRA wool was sold in May 1924.

BAWRA's intervention must be seen as the strictly limited operation of planned disposals of carry-over wool through ordinary channels, and without any direct control over current supplies. This task was carried out effectively, but there is little evidence to show what the effect of BAWRA's limited operations might have been if the general world economic situation had continued adverse after the initial period of the company's activities. As it was, selling conditions from the beginning of 1922 onward throughout the remaining period of BAWRA's market operations were generally favourable. In considering the value of lessons to be learnt from BAWRA, it must also be borne in mind that stocks of wool held by BAWRA—though nearly the size of a year's Australian clip and of a magnitude unheard of in the previous history of marketing—were nevertheless only about one quarter of the present accumulation of Dominion wool stocks. Besides, it must also be remembered that the influence of synthetic fibres will constitute an important factor in the world wool market in the years to come, whilst at the time of BAWRA's operations this influence was still negligible.

Wool Control during the Second World War.

Control of Empire supplies of apparel wools during the Second World War was organized much more quickly and on more stringent lines than during the First World War

A shadow organization of Wool Control had been set up even before the outbreak of hostilities, and started operations during the first week after the declaration of war.*

Already, two weeks after the outbreak of war, the British Government announced the conclusion of agreements to purchase the entire Australian and New Zealand wool clips for the duration of the War and one wool year thereafter.

* An order was issued immediately bringing all business in wool, tops and noils to a temporary standstill. All stocks of raw materials in the U.K. (not in the hands of spinners and manufacturers) were taken over by the Control and stocks in private hands were checked by a Census. As soon as these preliminary measures were completed, a system of trade licences was put into operation for all sales from private stocks, and a schedule of official maximum prices for wool, tops and noils was announced.

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Indeed, it is now known that the British Government's policy of wool bulk purchase in the case of war had already been decided as early as 1938.* Under these bulk purchase contracts each Dominion was to share equally with the United Kingdom in any profit made on the re-sale of wool sold for use outside the United Kingdom. Six weeks later an agreement with the South African Government was concluded. †

Prices under the British Government's bulk purchase agreements were based on flat rates averaging about one third above the 1938 level. A 15 per cent. increase in May 1942 brought the flat rates up to just over 50 per cent. above the pre-war average.

The settlement concerning raw wool supplies enabled the Wool Control to determine raw material allocations and a rationing scheme was put into operation on November 1st, 1939. Priorities were established for military requirements and civil rations were based on ratios of manufacturers' machinery capacity. After the first year of the war it was found that control of raw material supplies for the home civil market had to be supplemented by quantitative restrictions imposed at the wholesale trading stage, and in June 1940 new severe restrictions of home civilian consumption were introduced under the Consumers' Rationing Orders. Cost and profit margins from the raw material stage down to retail prices of finished goods were also made subject to constantly tightening control.‡

With the purchase of the Dominions clips, the British Government had become the sole importer of these wools, and U.K. imports from South America were also brought under Government control. The London wool auctions were discontinued immediately after the outbreak of war, and the normal functions of private importers and brokers ceased.§ Practically all processing up to and including combing was done for the Wool Control on a commission basis. In the case of all Government contracts, qualities were specified by the Control. In regard to other requirements, manufacturers were given a limited right of selection, but the range of available raw materials for civilian uses was largely determined by military priorities and by the shipping situation, which made it essential to give preference to shipments of wools with high clean yields.‖

South American wools continued to be sold in a free market, with the United States being by far the largest buyer throughout the war period.

The American domestic clip was first guaranteed, and in 1943 actually taken over by the American Government.

The war-time rate of American wool consumption rose to the all-time record of around 1,000 million lb. greasy per annum. Consumption in Australia, Argentina and India was also consider-

* This was revealed by the Australian Secretary of Commerce, Mr. J. F. Murphy, speaking at a dinner held in Melbourne on November 14th, 1945, on the occasion of the termination of the Australian Central Wool Committee's administration. Mr. Murphy said: "In 1938 when I was in England with Sir Earle Page, we discussed with representatives of the British Government, about twelve months in advance of the outbreak of war, what the Empire would do in regard to its major commodities in the event of war breaking out, and it was on that occasion that an agreement was reached—a verbal agreement, it was true, but nevertheless a binding agreement amongst partners in the Empire—that in such an event Britain would buy the wool clips of Australia for the duration of the War and twelve months thereafter, just as she had done during the 1914–18 war."

† Under the first agreement concluded with the South African Government in December 1939, the U.K. Government undertook that "bidding shall take place on their behalf in the wool auctions in South Africa in such manner as to ensure that prices not below those in the schedule which has been agreed upon, shall be offered for the wool placed on sale there." As the U.K. was by far the dominant buyer in the South African wool market, this guarantee of purchases at schedule prices had much the same effect as a straightforward bulk purchase agreement. In the summer of 1940 the British agreement with South Africa was brought into line with the agreements concluded with Australia and New Zealand, and it was announced that for the remaining period of the war and one wool year thereafter the U.K. Government would buy the whole of the Union's exportable surplus of wool at prices in accordance with the schedule agreed in the previous year.

‡ Quantitative restrictions, together with control of costs and profit margins, had the combined effect of encouraging manufacturers to concentrate on the high-price lines, so as to make up for the loss in turnover by increased profits per unit (profits being fixed on a "percentage of cost" basis). In order to counteract these tendencies and to ensure adequate supplies of cheaper clothing for the lower income groups, the British Government introduced the "Utility" system, which was based on earmarking a certain share of raw materials for the production of medium- and low-priced goods manufactured according to guaranteed Government standards and sold at cost-plus or overriding maximum prices, whichever was the less.

§ A limited quantity of East India wool required for blanket manufacture was not bought by the Government, and continued to be imported through private channels against licence.

‖ London Wool Conference Report, *op. cit.*, Appendix A.

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ably increased. The higher rate of consumption of these countries was, however, far from sufficient to make up for the loss of markets on the European continent, and, from the end of 1941 onward, in Japan. War-time civilian supplies in the United Kingdom were reduced to about 35 per cent. of the pre-war level, and the overall rate of U.K. war-time consumption was about one quarter below pre-war.

On balance, world consumption during the war could absorb only about two-thirds of current supplies from the five main exporting countries; which meant that by the summer of 1945—*i.e.*, after nearly six years of war—the accumulation of stocks in the Dominions and South American countries had reached a total equal to about two years' supply.*

Empire Wool Disposals Scheme

In view of this vast accumulation of stocks, the Governments of the United Kingdom and of the Dominions of Australia, New Zealand and South Africa decided that the time had come for co-ordinated plans of post-war disposals. At an inter-Governmental conference held in London in April/May 1945, agreement was reached on a new Empire Marketing Scheme.†

It was resolved that the stock of all Dominion-grown wool owned by the U.K. Government at July 31st, 1945, should be transferred to the joint ownership of the U.K. Government and the three Dominions Governments concerned; and that a Joint Disposals Organization should be set up for the purpose of buying, holding and selling wool on behalf of the four Governments.

It was further agreed that the Joint Organization should be incorporated as a private registered Company. Its nominal capital is to consist of twenty shares, eight of which are to be issued. Four shares are to be held by nominees of the U.K. Government, two by nominees of the Government of Australia and one each by nominees of the New Zealand and South African Governments. The Company will have three active subsidiaries to act on its behalf, one in each of the three Dominions.‡

The main functions of the Joint Organization are defined as follows:

(*a*) to determine total quantities of wool to be offered from time to time concurrently in the Dominions and elsewhere, to be made up of all new-clip wool and an appropriate proportion of wool from the Organization's stocks,

(*b*) to prepare schedules of reserve prices at which the Organization will itself be willing to acquire wool;

(*c*) to lift from the market such quantities of new wool as cannot be sold at these reserve prices;

(*d*) to hold and dispose of stocks as the agent of the Governments concerned and, where necessary, to acquire and own stores, plant and equipment for this purpose;

(*e*) to facilitate the sale of wool in every possible way with a view to stimulating demand.

The functions of the new Joint Organization are thus to be much wider than those exercised

* *Cf.* Table I, p. 2.

† *Cf.* London Wool Conference Report, *op. cit.*

‡ *Constitution of the Joint Organization.*—The directors of the Joint Organization—the principal company—will consist of an independent chairman, appointed jointly by the four Governments. In accordance with the distribution of shares, four directors will be appointed by the U.K., two by Australia and one each by New Zealand and South Africa. The chairmen of the three subsidiary companies will be *ex officio* directors of the principal company but will have no voting powers.

Financial Plan.—The U.K. and the Dominions concerned will each take up 50 per cent. of the original capital represented by the opening stocks of wool grown in that Dominion to be handed over to the Joint Organization. The opening stock will be taken in by the Joint Organization at its original cost (f.o.b.), less the amounts accumulated in the divisible profit accounts under the war-time contracts. Each Dominion and the U.K. will share equally in the provision of any further capital required by the Joint Organization during the operation of the scheme for payment for bought-in new wool of that Dominion.

The operating expenses of J.O. (the Joint Organization) will be borne equally between the wool-growing industry and J.O. itself. The share of the industry will be paid by the Dominions Governments primarily from the proceeds of a contributory charge on all sales of new-clip wool; the share of J.O. will be met by deduction from the proceeds of sales by J.O. before application to capital repayment. Operating expenses will also include provisions to cover depreciation of wool bought in by J.O.

After deduction of one half of the operating costs, the proceeds of all sales by J.O., together with any sums added in accordance with paragraph 3 (*d*), will be used for repayment of capital equally between the U.K. and the Dominion Government concerned.

The *ultimate balance of profit or loss* arising from the transactions of J.O. in the wool of any Dominion will thus be shared equally between the U.K. and the Government of that Dominion.

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by BAWRA at the end of the last war; especially in regard to direct control over "new clip" wool disposals; the fixing of reserve prices; the responsibility for lifting the unsold surplus from the market; and the active interest in the stimulation of demand.

In considering methods of marketing, the delegates to the London Wool Conference decided that buyers must be afforded full opportunities for inspection and selection; and that a fair degree of elasticity in the price relationship between different types of wool must be allowed. It was concluded that these conditions could best be met by a return to the auction system, and that auctions should be resumed "as soon as practicable." All growers' wool from current clips is to be offered at auction, alongside such quantities from stock as may be determined by the Organization.

The Organization will prescribe minimum reserve prices at which it will be willing to take up any unsold "new clip" wool, and it will offer at auction wool from its own stock "in such quantities as will prevent prices from rising much above the desired level." In some circumstances the Organization may make standing offers to sell from stock outside the auctions at a fixed percentage above the auction reserve prices, and thus impose a ceiling on prices actually obtainable at auction. The Organization will be prepared, possibly at short notice, to increase or hold back its offerings from stock in some or all of the selling centres to meet fluctuations of demand and to check their influence upon prices.*

Representatives of the four Governments will meet prior to the opening of each wool year, and at such other times as may be required, to agree upon the general level of reserve prices in the light of the advice of the Board of the principal Company. The Board will have discretion to make changes in the general level from time to time. It will be the duty of the Board to fix the *relative* values of the different types of wool in accordance with the general level of reserve prices of wool, and to adjust these differentials as may be found necessary from time to time in the light of market reactions.

THE FUTURE OF WOOL MARKETING—CONCLUSIONS

Surplus Disposals

The London Wool Conference Report's tentative estimate of 13 years from June 1945, as possible period of Dominion wool disposals alongside new clips was based on the assumptions that (i) wool production during the 1947/48–1957/58 decade will remain stationary at the war-time level in the Dominions, and at the pre-war level in the rest of the world; that (ii) world consumption of wool from the five chief exporting countries can be increased by 20 per cent. above pre-war (assuming "reasonable" wool price levels, general world prosperity and certain other conditions favouring wool consumption); and that the share of Dominions wool in this overall increase of world trade could be at least proportionate to the South American share.

In our previous analysis of prospective wool supply and demand, we have reached the tentative conclusions that wool supply during the 1947/48–1957/58 decade is not likely to fall short of, but may possibly exceed, the pre-war level, and even the war-time level; whilst an average increase in the consumption of wool from the five chief exporting countries by 20 per cent. above pre-war cannot be expected to take place in all major consuming countries, but would probably require a much more than proportionate consumption increase in some of them; the more so as consumption levels in Germany and Japan (which in 1934/38 absorbed 22 per cent. of world wool exports) are likely to remain below the pre-war average for a number of years to come. Hence we must conclude that the Wool Conference Report's forecast of about 13 years as a possible period of Dominion wool disposals alongside new clips does not overstate the difficulties. Indeed, the widest possible interpretation will have to be given to function (*d*) of the Joint Organization: ". . . to facilitate the sale of wool in every way possible with a view to stimulating demand."†

A comprehensive programme for the further expansion of scientific research and promotion is under consideration; but the special proviso that "use is made of every aid such as long-term credits to needy countries" ‡ will probably also have to be put into effect on a substantial scale.

Whatever the period which may ultimately be required for achieving the aim set by the new

* Some extension of the selling seasons in the Dominions may be effected, and special sales of old wool, or sales comprising mostly old wool, may be held during the off-season.

† Wool Conference Report, Appendix C.

‡ Wool Conference Report, Appendix A.

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Empire Wool Disposals Scheme, the aim itself as it stands at present is clear. It can be defined as the clearance of stocks without disrupting the market and with the object of "getting back to normal"—*i.e.*, restoring a situation when world stocks will again be reduced to a negligible fraction of current supply. The Report's estimate of the period required for the liquidation of stocks is based on this implicit assumption, and no mention is made of the alternative of a permanent buffer pool scheme which might require a reserve of six months' to one year's supply, and would thus alter the basis of calculations in regard to the disposals period.* The plan does, however, provide for the revision of all arrangements after a period of five years, and this might allow for the consideration of a permanent pool.

The pro's and con's of permanent marketing control can best be considered briefly under the following two main headings:

(1) the long-term regulation of supply by means of quotas; and
(2) the evening out of short-term price fluctuations (without interference with the trend) by means of buffer stock operations.

(1) *Long-Term Regulation*

The long-term problems of wool producers may well be serious. Much will depend, of course, on general world economic conditions. If constructive and expansionist economic policies are pursued in all major consuming countries, and if living standards of the poorer sections everywhere, and in the poorer countries in particular, can be raised and made more stable at the higher levels, then the prospects for wool, just as for all other staple commodities, will, no doubt, look considerably brighter. Even in the best of cases, however, the growing importance of synthetic substitutes must be reckoned with as an important factor in the future wool market, possibly as a serious danger; whilst the limited possibilities of transferring productive resources from sheep-growing to other uses tend to lower the influence of depressed prices on supply. Under adverse conditions, many farmers, for lack of alternatives, are likely to continue production at a loss, and may well get into debt until—in view of the importance of wool in the economies of some of the main producing countries—Governments may find it necessary to subsidize production.

The task of finding the right safeguards against these possible dangers is a difficult one. The most important point to be kept in mind is the futility of any attempt to interfere with the trend of wool prices by restrictionist practices. In view of the development of new substitutes, such a long-term policy of restriction for the purpose of bolstering up prices would almost certainly defeat its own ends. Moreover, the operation of such a scheme on a world scale would need to be based on quotas of production and/or exports. Political considerations would probably make it very difficult to operate such a scheme on a basis favouring low-cost producers, and this would make any such plan even more objectionable on economic grounds.

* If the composition of the present accumulation of stocks were *balanced* in regard to qualities, then the decision to establish a permanent buffer reserve of one year's supply would reduce the market surplus by about one half. In fact, however, stocks of certain qualities are much more plentiful than others, and a period of relative adjustment would be required before a balanced reserve could be set up. This process of readjustment is not an easy one, because market disposals of the qualities in scarce world supply should obviously not be curtailed even further by Control policy. However, the concurrent turnover of the more balanced new clips may help to bring about a gradual readjustment of stocks.

Even under the scheme as it stands at present, the Joint Organization will have to give much attention to sectional markets for those qualities which are in relatively plentiful supply. The guidance derived by the Control from price differentials in spot auction sales might possibly be supplemented by the provision of facilities for forward trading, which could serve as a longer-range indicator of future market reactions. The possible de-stabilizing effect of speculative forward trading could be prevented by the fixation of limits for forward quotations. At the same time the "forward" limits could leave a fair margin for the indication of the market's valuation of the commodity in general and of the relative values of different qualities in particular. In the case of forward *sales*, the Control would have to provide facilities for selection from fairly homogeneous groups of graded central stocks. Standards for the Control's acceptance of delivery in fulfilment of central forward *purchases* would have to be governed—subject to arbitration—by grading tables similar to the Tables of Limits used during the war-time appraisement schemes in the Dominions. It is doubtful, however, whether the technical difficulties of grading could be overcome. In the past the development of futures trading has always been hampered by technical difficulties of standardizing quantity and quality. If facilities for forward trading were offered by the Control itself, the authority and reputation of the central Organization might help to mitigate these technical difficulties: but they could probably not be entirely eliminated. In any case these methods of forward sales by description could not replace, but merely supplement, the main system of auction sales by selection.

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International long-term schemes aiming at interference with the price trend by means of restriction should not be confused, however, with well thought-out plans for the gradual transfer of productive resources where alternatives for other uses of such resources *do* exist (and with special consideration for the encouragement of lower-cost producers); this transfer of resources to be encouraged *not* for the purpose of interfering with the trend of prices for the remaining output (which, owing to the influence of near substitutes, would be a largely futile attempt), but for the dual purpose of:

(*a*) mitigating the possible hardships of a situation when price is left as the sole long-term regulator of supply in an industry with low mobility of resources; and

(*b*) diminishing the dangers resulting for some of the main producing countries from a very high degree of dependence on the world market situation for one single commodity,

At the same time every possible effort will need to be made to achieve further perfection of efficiency and rationalization in the industry. Much valuable work has already been done in the three main wool-growing Dominions in regard to pastoral and biological research, the spread of information concerning the results of these researches, and other measures helping to improve production and to reduce its costs. This work will be continued on an even larger scale in the future. At the same time much attention will continue to be given to industrial research, improving the physical and chemical properties of the wool fibre, modernizing wool textile machinery equipment and creating new uses for wool; the publicizing of all uses and spread of consumers' education in regard to the still superior qualities of the wool fibre; and the rationalization of all "other costs" in the manufacture and distribution of wool goods. Adequate legal protection of the terms "wool" and "woollen" in all countries might also have an important influence on future wool consumption. A reduction in the American tariff would, of course, be a major element affecting the outlook for the wool industries in low-cost producing countries.

(2) *A Permanent Buffer Pool?*

Having reached the conclusion that attempts to interfere with the long-term trend of wool prices by means of restricting output should be discouraged, we can now give brief consideration to the possibility and desirability of a permanent buffer pool scheme for the purpose of smoothing out excessive short-term fluctuations

It is clear, of course, that the *basic* remedy for stability of prices over the short period—just like the basic remedy for long-term equilibrium prices at adequate levels—cannot be found in any innovations in marketing technique, but must be sought in far more general measures ensuring the stabilization of effective world demand for consumers' goods at adequate levels. At the same time, permanent plans for marketing operations aiming at the evening out of excessive and erratic short-term fluctuations may be of some benefit as such, and deserve serious consideration.

As has already been explained, the auction system, which before the war was the dominating method of wool-selling in the U.K. and in the Dominions, offers considerable advantages for a commodity like wool, which varies widely in quality and yield. Selling on pre-war auction markets was, on the whole, fair, and there was little concrete evidence of buyers' rings or other marketing manipulations. Selling charges were not unreasonable.

It would be futile to expect, however, that the auction system as such can be effective in securing a reasonable degree of price stability. On the contrary, it tends to accentuate the influence on prices of the slightest change in marketing conditions. It does so by reason of its essentially competitive character, which provides little incentive for the equalization of stockholdings at adequate levels.

This has been well explained by Keynes, who states: "It is an outstanding fault of the competitive system that there is no sufficient incentive to the individual enterprise to store surplus stocks of materials, so as to average, as far as possible, periods of high and of low demand. The competitive system abhors the existence of stocks, with as strong a reflex as nature abhors a vacuum, because stocks yield a negative return in terms of themselves. It is ready without remorse to tear the structure of output to pieces rather than admit them, and in the effort to rid itself of them. Its smooth and efficient working presumes in practice, as stringently as the static analysis presumes in theory, a steady rate, or a steady growth, of effective demand. If demand fluctuates, a divergence

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immediately ensues between the general interest and the course of action in respect of stocks which is most advantageous for each competitive enterprise acting independently." *

According to Keynes, the three main reasons for this state of affairs are: (1) the relatively high cost of storage and interest, especially in the case of surplus stocks, " which strain the capacity of normal accommodation "; (2) the non-speculative users' inclination to base their cost calculations on *current* prices, and the resulting " lack of incentive to the retailer or manufacturing consumer to purchase in advance "; and (3) speculative stocking and de-stocking operations, which tend to accentuate fluctuations in the general level of stockholdings.

Unfortunately, there is no adequate statistical information for obtaining a comprehensive world picture of the general level and fluctuations round the average of wool stocks " in all hands "; but the indirect evidence of the sharp and erratic wool price fluctuations during the inter-war period suggests that Keynes's argument, which relates to staple raw materials and foodstuffs generally, was also true in large measure in regard to the peace-time wool market; and that non-speculative stocks in all hands were, on the average, too low to act as an effective buffer against speculative variations in stockholdings and against short-term changes in demand due to other causes.

For all these reasons, a permanent buffer pool operated for the purpose of counteracting sharp short-term variations in demand, and not displacing but supplementing the auction market, may have a useful function to perform.

During the initial period of the new Joint Organization's activities, the decision concerning the permanency of buffer stock operations may not be of great practical importance. Whatever long-term policy may eventually be decided upon, the task of the new Organization will be to reduce the present excessive surplus by all means at its disposal. As time goes on, however, a decision on long-term policy will become increasingly important. As the plan stands at present, it can only be carried out by relying on a constant excess of current consumption of current supply over the next decade. Such an inherent disequilibrium situation will not be easy to achieve and to maintain, and the Joint Organization's heavy task of adequate marketing policy might be facilitated if a part of stocks could be maintained as a permanent reserve and disposals could again be directed with a view towards restoring long-term equilibrium between current supply and demand; whilst " open-market operations " could gradually begin to assume their permanent function of evening out short-term variations without interfering with the trend.†

Even if the specific difficulties of the inherent transitional disequilibrium can be diminished, the technical and human problems of current price policy will no doubt be great. It may be possible to ascertain in retrospect whether certain price movements were due to cycle or trend, but it is much more difficult to do so whilst the movement is still in progress. Besides, the steering committee responsible for stabilizing operations will not only have to be wise enough to differentiate between cycle and trend, but also strong enough and sufficiently detached to exercise its influence on prices, irrespective of political pressure from any side. The generally unsatisfactory results of some commodity equalization schemes initiated during the inter-war period give proof of the dangers inherent in such management.

The dangers of policies trying to maintain an artificial price level are generally lessened if provision is made for the balanced representation of producers' and consumers' interests on the controlling body. At present, the four Governments joined in the common enterprise of the new Disposals Scheme are the U.K. and the three Dominions. The South American countries and the United States are not included, but the possibility of a subsequent wider international agreement has been kept in mind by the delegates to the London Wool Conference. The Conference Report

* J. M. Keynes, " The Policy of Government Storage of Foodstuffs and Raw Materials," *Economic Journal*, September 1938.

† A few words need to be said about the cost of such a permanent reserve. Wool is a practically non-perishable commodity, and the cost resulting from deterioration of stocks may be regarded as negligible —especially if turnover of reserve stocks were made part of the controlling body's policy. The cost of interest, insurance and storage of wool would probably amount to 4–4½ per cent. per annum. On the basis of this assumption, the holding of extra reserve stocks averaging, say, nine months' supply would raise the price of wool by about 3 per cent. However, it must be remembered in this connection that buffer stocks held by a central control authority need not all be classified as " additional " to stocks held by the trade in the absence of a buffer scheme. Indeed, it should be one of the functions of such a scheme to counteract speculative stockholdings, which, though subject to wide variations, are extremely large at *some* periods.

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also suggests that the International Wool Textile Organization (which represents wool textile manufacturers' interests in a large number of consuming countries) should be invited to nominate representatives to a Committee which should act in a consultative capacity from a viewpoint of wool consumers. Representatives from certain other consuming countries who are not members of the Wool Textile Organization are also to be invited to join this Committee. But no explicit provision has as yet been made for the representation of Governments of consuming countries other than the U.K. on the Executive of the Joint Organization.*

Buffer pool schemes, in contrast to quota schemes, do not require the participation of all important producing and consuming countries as an absolutely essential constitutional premise for their proper functioning. Even if only some of all the major producing and consuming countries are partners to the scheme, it is theoretically possible that the arrangement may work provided that the central marketing organization is backed up by finance on the scale required for effective operations in the world market. On the other hand, even if representation by producing and consuming countries on the Central Board is comprehensive, the scheme cannot work satisfactorily if financial resources are insufficient for international marketing operations on the required scale. In short, buffer-pool schemes are more flexible and more closely related to the free market than quota schemes, and their functioning is therefore less dependent on constitutional factors. However, even though full representation of all interested countries is not an essential theoretical premise for the functioning of buffer pool schemes, the balance of producing and consuming interests is of great importance, and improves the chances of success.

But even if all constitutional aspects of a permanent scheme are solved to satisfaction, there still remains the intricate task of determining the "normal" price which is to be used as basis for differentiating between the permissible range of upward and downward variations as against "excessive" fluctuations. The method of defining the "normal" price by means of a "datum line" based on an average of, say, a period of ten to twenty years † seems arbitrary, because it does not make adequate provision for change in the conditions of supply and demand. Moreover, once the scheme has been in operation for some years, there arises the additional problem of determining what the "normal" price over these years would have been without intervention. Nor is it possible to peg the "normal" price to a general index of wholesale prices, because this general index may be influenced by notable price changes in other individual commodities, and these price changes may be due to specific circumstances which need not be of any influence on the "normal" price of the commodity in question.

At the same time, a combined index, even though it cannot be used as basis for direct "pegging," is a useful subsidiary instrument indicating certain general economic developments; and economic measures which may help to increase the stability of the general level of wholesale commodity values will also, *cet. par.*, improve the prospects of stability in the wool market, just as in other individual commodity markets. This is one of the reasons why the chances of success of permanent buffer stock operations for wool might also be considerably improved if these operations were not carried out in isolation, but linked with wider plans co-ordinating similar operations for a range of other commodities. In the *Proposals for an International Clearing Union*, the point was made that the control of stocks of a range of staple primary products might be shaped into "a powerful means of combating the evils of the Trade Cycle, but exercising contractionist or expansionist influences on the economic system as a whole or on particular sections." ‡ The establishment of a single international agency to co-ordinate buffer-stock operations for a range of primary products has also been advocated in the Report of the League of Nations Delegation on Economic Depressions. Plans for an international Commodity Commission to "investigate commodity

* The U.K. Government is, in fact, interested in J.O. in a dual capacity as (i) the Government of a major wool-consuming country, and (ii) the owner of a large share of the Dominion stocks. British woolgrowers are now also demanding the special representation of their interest on J.O. (cf. *Times*, 1.2.45). If this viewpoint is accepted, a third specific angle will be added to the complex task of the U.K. Government's representation on J.O.

† Method proposed by St. Clare Grondona, *National Reserves for Safety and Stabilization*, London, 1938. Even though this method of price fixation is subject to some criticism, some other parts of the book are very well argued, and deserve more detailed consideration than can be given in this condensed survey.

‡ White Paper, Cmd. 6437.

§ *Economic Stability in the Post-War World—The Conditions of Prosperity after the Transition from War to Peace*, Report of the Delegation on Economic Depressions, League of Nations, 1945.

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problems, including international buffer stocks or other arrangements to solve commodity problems" are on the agreed agenda for the coming World Trade Conference.*

In view of the relative importance of wool in the world trade of staple commodities, its inclusion in a permanent wider plan of co-ordinated buffer pool operations for a range of commodities might be of more than sectional interest by helping to bring about the degree of *general* world economic stability which is an essential pre-requisite for future prosperity.

APPENDIX I

Apparel Wool Production by Main Groups of Countries: Seasons 1909/10–1943/44

(Million lb., greasy)

Season	Group A: Five main Surplus Countries						Group B: Five main Deficit Countries						Total Groups A plus B	Total World Production
	Australia	New Zealand	South Africa	Argentine	Uruguay	Total Group A	United States	United Kingdom	Canada	France	Germany	Total Group B		
1909/10	740	221	133	354	103	1,551	364	118	↑	78	↑	617	2,168	↑
1910/11	788	212	132	369	113	1,614	359	119	↑	78	↑	613	2,227	↑
1911/12	798	219	162	356	113	1,648	357	117	13	78	44	609	2,257	3,063
1912/13	688	224	169	326	123	1,530	333	113	↓	78	↓	581	2,111	↓
1913/14	771	224	175	302	102	1,574	323	106	↓	80	↓	566	2,140	↓
1914/15	735	234	182	275	82	1,508	308	102	11	75	↑	534	2,042	2,872
1915/16	636	232	175	322	71	1,436	294	105	12	75	↑	524	1,960	2,836
1916/17	637	213	179	370	79	1,478	303	107	14	65	38	527	2,005	2,717
1917/18	654	230	175	380	87	1,526	290	108	16	65	↓	517	2,043	2,791
1918/19	736	255	180	418	109	1,698	310	99	18	50	↓	515	2,213	2,309
1919/20	762	237	177	355	118	1,649	335	96	20	50	40	541	2,190	2,394
1920/21	625	208	182	315	113	1,443	308	88	24	40	40	500	1,943	2,965
1921/22	723	219	188	316	108	1,554	307	85	21	40	43	496	2,050	3,003
1922/23	727	246	184	332	96	1,585	284	81	19	43	52	484	2,069	2,704
1923/24	663	235	198	312	106	1,514	287	87	15	44	49	482	1,996	2,720
1924/25	777	254	176	307	97	1,611	296	106	15	44	53	514	2,125	3,107
1925/26	834	238	234	312	116	1,734	315	112	16	45	44	532	2,266	3,267
1926/27	924	254	246	352	129	1,905	336	117	18	47	38	556	2,461	3,482
1927/28	888	262	276	332	131	1,889	357	123	19	48	35	582	2,471	3,543
1928/29	968	272	285	335	139	1,999	384	124	20	47	34	609	2,608	3,735
1929/30	938	272	309	311	150	1,980	400	122	20	46	32	620	2,600	3,737
1930/31	901	266	290	329	154	1,940	434	121	21	45	32	653	2,593	3,588
1931/32	1,008	270	313	383	106	2,080	464	113	19	54	32	682	2,762	3,308
1932/33	1,063	277	329	393	111	2,173	440	119	19	54	32	664	2,837	3,320
1933/34	996	290	283	387	93	2,049	460	120	19	54	31	684	2,733	3,702
1934/35	1,015	265	219	370	112	1,981	451	115	19	53	32	670	2,651	3,628
1935/36	971	304	250	359	110	1,994	453	109	19	53	36	670	2,664	3,680
1936/37	983	303	277	379	114	2,056	449	108	19	54	40	670	2,726	3,802
1937/38	1,023	297	242	389	107	2,058	455	107	19	55	43	679	2,737	3,893
1938/39	985	328	264	394	114	2,085	458	110	19	54	45	686	2,771	3,991
1939/40	1,121	302	271	375	134	2,203	450	111	16	39	42	658	2,861	3,910
1940/41	1,077	333	260	409	139	2,218	457	109	16	30	43	655	2,873	3,932
1941/42	1,108	336	260	473	115	2,292	478	92	16	28	45	659	2,951	3,994
1942/43	1,120	331	250	511	120	2,332	481	91	18	28	48	666	2,998	3,969
1943/44	1,132	320	250	518	120	2,340	471	89	19	28	56	663	3,003	3,920

* White Paper, Cmd. 6709.

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APPENDIX II

Production of Merino and Crossbred Wool in 10 *Selected Countries* (a)
Annual Output and 5*-Year Moving Averages,* 1889/90–1943/44

(Million lb., greasy)

Season	Merino		Crossbred		Total	
	Annual output	Five-year averages	Annual output	Five-year averages	Annual output	Five-year averages
1889/90	1,150	—	575	—	1,725	—
1890/91	1,110	—	584	—	1,694	—
1891/92	1,295	—	619	—	1,914	—
1892/93	1,269	—	667	—	1,936	—
1893/94	1,228	1,210	708	631	1,936	1,841
1894/95	1,165	1,213	855	687	2,020	1,901
1895/96	1,110	1,213	910	752	2,020	1,965
1896/97	1,012	1,157	934	815	1,946	1,972
1897/98	935	1,090	976	877	1,911	1,967
1898/99	926	1,030	991	933	1,917	1,963
1899/00	796	956	991	960	1,787	1,916
1900/01	836	901	957	970	1,793	1,871
1901/02	876	874	997	982	1,873	1,856
1902/03	852	857	983	984	1,835	1,841
1903/04	794	831	895	965	1,689	1,796
1904/05	853	842	897	946	1,750	1,788
1905/06	914	858	893	933	1,807	1,791
1906/07	993	881	904	914	1,897	1,795
1907/08	1,103	931	949	908	2,052	1,893
1908/09	1,133	999	973	923	2,106	1,922
1909/10	1,212	1,071	952	934	2,164	2,005
1910/11	1,243	1,137	981	952	2,224	2,089
1911/12	1,256	1,189	999	971	2,255	2,160
1912/13	1,148	1,198	963	974	2,111	2,172
1913/14	1,103	1,192	1,030	985	2,133	2,177
1914/15	983	1,147	1,060	1,007	2,043	2,154
1915/16	975	1,093	1,065	1,023	2,040	2,116
1916/17	928	1,027	1,077	1,039	2,005	2,066
1917/18	926	983	1,127	1,072	2,053	2,055
1918/19	963	955	1,240	1,114	2,203	2,069
1919/20	936	946	1,254	1,153	2,190	2,099
1920/21	839	918	1,103	1,160	1,942	2,078
1921/22	938	920	1,112	1,167	2,050	2,087
1922/23	976	930	1,093	1,160	2,069	2,090
1923/24	970	932	1,026	1,118	1,996	2,050
1924/25	1,068	958	1,057	1,078	2,125	2,036
1925/26	1,194	1,029	1,072	1,072	2,266	2,101
1926/27	1,320	1,106	1,141	1,078	2,461	2,184
1927/28	1,350	1,180	1,121	1,083	2,471	2,263
1928/29	1,424	1,271	1,184	1,115	2,608	2,386
1929/30	1,440	1,346	1,160	1,136	2,600	2,482
1930/31	1,422	1,391	1,171	1,155	2,593	2,546
1931/32	1,437	1,415	1,325	1,192	2,762	2,607
1932/33	1,484	1,441	1,353	1,239	2,837	2,680
1933/34	1,394	1,435	1,339	1,270	2,733	2,705
1934/35	1,350	1,417	1,301	1,298	2,651	2,715
1935/36	1,358	1,405	1,306	1,325	2,664	2,730
1936/37	1,397	1,397	1,329	1,326	2,726	2,723
1937/38	1,400	1,380	1,337	1,322	2,737	2,702
1938/39	1,395	1,380	1,376	1,330	2,771	2,710
1939/40	1,517	1,413	1,344	1,338	2,861	2,751
1940/41	1,450	1,432	1,423	1,362	2,873	2,794
1941/42	1,469	1,446	1,482	1,392	2,951	2,838
1942/43	1,482	1,463	1,516	1,428	2,998	2,891
1943/44	1,475	1,478	1,528	1,459	3,003	2,937

(*a*) Australia, New Zealand, South Africa, Argentina, Uruguay, United States, United Kingdom, Canada, France, Germany.

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APPENDIX III

End-of-Season Stocks as Per Cent. of Annual Output, 1925/26–1938/39

(Million lb., actual weight)

	1925–26			1926–27			1927–28			1928–29			1929–30		
	Clip	Stocks	%	Clip	Stocks	%	Clip	Stocks	%	Clip	Stocks	%	Clip	Stocks	%
Australia	834	10	1·2	924	8	0·9	888	10	1·1	968	14	1·4	938	33	3·5
South Africa	234	—	—	246	—	—	276	7	2·5	285	8	2·8	309	4	1·3
New Zealand ...	238	17	7·1	254	17	6·7	262	16	6·1	272	13	4·8	272	61	22·4
Argentina	312	9	2·9	352	4	1·1	332	18	5·4	335	24	7·2	311	20	6·4
Total	1,618	36	2·2	1,776	29	1·6	1,758	51	2·9	1,860	59	3·2	1,830	118	6·4

	1930–31			1931–32			1932–33			1933–34			1934–35		
	Clip	Stocks	%	Clip	Stocks	%	Clip	Stocks	%	Clip	Stocks	%	Clip	Stocks	%
Australia	901	17	1·9	1,008	46	4·6	1,063	13	1·2	996	65	6·5	1,015	26	2·6
South Africa	290	13	4·5	313	17	5·4	329	2	0·6	283	24	8·5	219	3	1·4
New Zealand ...	266	71	26·7	270	84	31·1	277	59	21·3	290	26	9·0	265	64	24·2
Argentina	329	13	4·0	383	44	11·5	393	9	2·3	387	42	10·9	370	13	3·5
Total	1,786	114	6·4	1,974	191	9·7	2,062	83	4·0	1,956	157	8·0	1,869	106	5·7

	1935–36			1936–37			1937–38			1938–39			1939–40		
	Clip	Stocks	%	Clip	Stocks	%	Clip	Stocks	%	Clip	Stocks	%	Clip	Stocks	%
Australia	971	23	2·4	983	14	1·4	1,023	71	6·9	985	29	2·9	1,121	6	0·5
South Africa	250	6	2·4	277	4	1·4	242	8	3·3	264	5	1·9	271	2	0·7
New Zealand ...	304	17	5·6	303	11	3·6	297	24	8·1	328	16	4·9	302	12	4·0
Argentina	359	22	6·1	379	24	6·3	389	41	10·5	394	8	2·0	375	96	25·6
Total	1,884	68	3·6	1,942	53	2·7	1,951	144	7·3	1,971	58	2·9	2,069	116	5·6

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APPENDIX IV

Exports of Tops, Yarns and Tissues, World and 4 Selected Countries, 1928–1938

(Million lb., clean weight)

Year	World			United Kingdom				France			
	Exports of—			Available supplies of raw wool	Exports of—			Available supplies of raw wool	Exports of—		
	Tops	Yarns	Tissues		Tops	Yarns	Tissues		Tops	Yarns	Tissues
1928	128	199	242	327	34	49	97	301	54	62	47
1929	118	194	233	357	33	47	88	321	52	58	43
1930	114	173	189	377	29	37	66	345	48	55	36
1931	122	163	158	427	28	35	49	277	50	50	29
1932	121	112	105	420	42	38	46	291	47	29	13
1933	138	125	112	411	46	43	52	333	59	34	12
1934	116	122	119	380	42	43	58	216	47	30	9
1935	132	115	119	388	56	41	61	243	49	23	6
1936	127	118	128	442	52	37	66	217	46	15	5
1937	119	117	149	404	40	32	69	209	51	15	5
1938	106	107	125	442	33	28	52	255	50	24	7

Year	Germany				Japan			
	Available supplies of raw wool	Exports of—			Available supplies of raw wool	Exports of—		
		Tops	Yarns	Tissues		Tops	Yarns	Tissues
1928	216	23	24	35	59	—	—	1
1929	211	18	27	35	55	—	0·4	2
1930	202	19	23	30	59	—	1	2
1931	189	20	23	29	97	—	1	1
1932	193	10	13	15	105	—	1	2
1933	220	9	12	14	122	—	3	5
1934	213	5	10	10	95	—	6	10
1935	189	2	11	9	126	—	5	14
1936	169	2	18	15	116	—	7	19
1937	169	1	15	18	138	—	7	18
1938	209	1	9	16	63	—	8	14

4 *Main Exporting Countries*

Average Exports of Tops, Yarns and Tissues, 1928–1938

From—	Share of world exports (million lb.)		
	Tops	Yarns	Tissues
U.K.	39·5	39·1	64·0
France	50·3	36·0	19·2
Germany	10·0	17·7	20·5
Japan	—	3·6	8·0
Four countries total	99·8	94·4	111·7
World total	121·9	140·5	152·6

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APPENDIX V

Annual Average Prices and Indices of Merino and Crossbred Wool With 5-Year Moving Averages, 1890–1944

	Merino 66's (clean basis)				Crossbred 46's (clean basis)			
	Price, *d.* per lb., London		Index, 1909/13 = 100		Price, *d.* per lb., London		Index, 1909/13 = 100	
	Annual average	Five-year moving average	Annual average	Five-year moving average	Annual average	Five-year moving average	Annual average	Five-year moving average
1890	21·60	—	77	—	14·50	—	91	—
1891	20·40	—	73	—	14·50	—	91	—
1892	17·80	—	63	—	12·90	—	81	—
1893	18·00	—	64	—	13·80	—	87	—
1894	16·90	18·9	60	67	13·00	13·7	82	86
1895	18·75	18·4	67	65	13·40	13·5	84	85
1896	19·50	18·2	69	65	12·90	13·4	81	83
1897	19·56	18·5	70	66	12·20	13·1	77	82
1898	20·50	19·0	73	68	10·80	12·5	68	78
1899	27·10	21·1	96	75	12·70	12·4	80	78
1900	22·20	21·8	79	77	11·60	12·0	73	76
1901	20·30	21·9	72	78	8·60	11·2	54	70
1902	24·50	22·9	87	81	9·40	10·6	59	67
1903	26·25	24·1	93	86	12·00	10·9	76	68
1904	25·75	23·8	92	85	14·90	11·3	94	71
1905	27·60	24·9	98	88	17·20	12·4	108	78
1906	28·25	26·4	101	94	18·90	14·5	119	91
1907	28·10	27·2	100	97	16·60	15·9	104	100
1908	24·75	26·9	88	96	11·60	15·8	73	100
1909	28·50	27·4	101	98	16·10	16·1	101	101
1910	28·50	27·6	101	98	16·25	15·9	102	100
1911	26·50	27·3	94	97	14·60	15·0	92	95
1912	27·50	27·2	98	97	15·25	14·7	96	93
1913	29·60	28·1	105	100	17·40	15·9	109	100
1914	30·50	28·5	109	101	18·20	16·4	114	103
1915	36·90	30·2	131	107	25·25	18·2	159	114
1916	52·50	35·4	187	126	28·25	20·9	178	131
1917	70·70	44·0	252	157	35·50	24·9	223	157
1918	70·00	52·1	249	186	36·50	28·8	230	181
1919	110·50	68·1	393	243	39·60	33·0	249	208
1920	102·25	81·2	364	289	30·75	34·1	193	215
1921	44·00	79·5	157	283	11·20	30·7	71	193
1922	52·00	75·7	185	270	12·56	26·1	79	164
1923	59·00	73·5	210	262	17·25	22·3	108	140
1924	76·75	66·8	273	238	30·90	20·5	194	129
1925	57·25	57·8	204	206	23·25	19·0	146	120
1926	50·25	59·0	179	210	19·50	20·7	123	130
1927	53·25	59·3	190	211	21·60	22·5	136	142
1928	50·75	57·6	181	205	25·50	24·1	160	152
1929	37·75	49·8	134	177	21·25	22·2	134	140
1930	25·50	43·5	91	155	11·80	19·9	74	125
1931	21·25	37·7	76	134	9·25	17·9	58	112
1932	19·50	31·0	69	110	7·69	15·1	48	95
1933	26·81	26·2	96	93	8·80	11·8	55	74
1934	25·90	23·8	92	85	9·40	9·4	59	59
1935	26·25	23·9	93	85	9·60	8·9	60	56
1936	32·75	26·2	117	93	13·90	9·9	87	62
1937	34·40	29·2	122	104	20·60	12·5	130	78
1938	23·40	28·5	83	102	13·90	13·5	87	87
1939(a)	25·10	28·4	89	101	16·10	14·8	111	95
1940(a)	36·75	30·5	131	108	23·50	17·6	148	113
1941(a)	37·50	31·4	134	112	24·00	19·6	151	125
1942(a)	37·50	32·1	134	114	24·00	20·3	151	130
1943(a)	37·50	34·9	134	124	24·00	22·3	151	142
1944(a)	37·50	37·4	134	133	24·00	23·9	151	150

(a) From September 1939, the averages are based on Wool Control Home Issue Prices which ranged from 7—20 per cent. below Export Prices.

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APPENDIX VI

End-of-Quarter Prices of Merino and Crossbred Wool and Ratios of the Price Indices to the General Sauerbeck Index, 1890–1944

Year		Merino 66's, clean scoured basis			Crossbred 46's, clean scoured basis		
		Price, *d.* per lb., London	Price index, 1909/13 = 100	Merino/ Sauerbeck index ratio, 1909/13 = 100	Price *d.* per lb., London	Price index, 1909/13 = 100	Crossbred/ Sauerbeck index ratio, 1909/13 = 100
1890	March	23	81·8	92	15	94·4	106
	June	19¾	70·2	80	13¾	86·5	99
	September	23	81·8	91	14¾	92·8	103
	December	20¾	73·8	83	14½	91·2	103
1891	March	21¾	77·3	87	16	100·6	113
	June	21	74·7	84	14½	91·2	102
	September	20	71·1	80	13¾	86·5	97
	December	19	67·6	76	13¾	86·5	97
1892	March	16¾	59·6	69	12½	78·6	91
	June	19	67·6	80	13	82·4	98
	September	17¾	63·1	76	12¼	77·1	93
	December	17¾	63·1	75	13¾	86·5	103
1893	March	19	67·6	80	14	88·1	104
	June	17¾	63·1	75	13¾	86·5	103
	September	17¾	63·1	74	13¾	86·5	102
	December	17¾	63·1	76	13¾	86·5	104
1894	March	17¼	61·3	77	13¾	86·5	108
	June	17	60·4	77	13	82·4	105
	September	17	60·4	77	13	82·4	106
	December	16¼	57·8	77	12¼	77·1	103
1895	March	16¾	59·6	79	12¼	77·1	102
	June	17¼	61·3	79	13	82·4	106
	September	21	74·7	95	14½	91·2	115
	December	20	71·1	93	13¾	86·5	114
1896	March	22	78·2	104	14	88·1	117
	June	19½	69·3	94	13	82·4	112
	September	17¾	63·1	83	12¼	77·1	101
	December	18¾	66·7	87	12¼	77·1	100
1897	March	19¾	70·2	91	13¾	86·5	112
	June	18¾	66·7	87	12¾	80·2	105
	September	21	74·7	95	12¼	77·1	98
	December	18¾	66·7	86	12	75·5	97
1898	March	21	74·7	98	11½	72·3	95
	June	21	74·7	95	11¼	70·8	90
	September	20	71·1	88	10½	66·0	82
	December	20	71·1	89	10	62·9	79
1899	March	22	78·2	96	10½	66·0	81
	June	25	88·9	107	12	75·5	91
	September	29	103·6	124	13	81·8	98
	December	32½	115·1	129	15¼	95·9	107
1900	March	27	96·0	102	13	81·8	87
	June	24	85·3	91	11½	72·3	77
	September	19	67·6	72	11	69·2	74
	December	18¾	66·7	73	10¾	67·6	74
1901	March	18¾	66·7	76	9½	59·8	68
	June	20½	72·9	84	8¾	55·0	64
	September	21	74·7	86	8½	53·5	62
	December	21	74·7	88	7¾	48·7	57
1902	March	22	78·2	91	8¾	55·0	64
	June	24	85·3	97	9	56·6	65
	September	25	88·9	103	8¾	55·0	64
	December	27	96·0	112	11	69·2	80
1903	March	27	96·0	110	10	62·9	72
	June	27	96·0	111	12	75·5	87
	September	26	92·4	96	13	81·8	85
	December	25	88·9	102	13	81·8	94

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APPENDIX VI (*continued*)

Year		Merino 66's, clean scoured basis			Crossbred 46's, clean scoured basis		
		Price, *d.* per lb., London	Price index, 1909/13 = 100	Merino/ Sauerbeck index ratio, 1909/13 = 100	Price, *d.* per lb., London	Price index, 1909/13 = 100	Crossbred/ Sauerbeck index ratio, 1909/13 = 100
1904	March	25	88·9	101	13½	84·9	96
	June	25	88·9	103	13¾	86·5	100
	September	26	92·4	94	15	94·4	96
	December	27	96·0	109	17¼	108·5	123
1905	March	26	92·4	103	14	88·1	99
	June	27½	97·8	109	18¾	117·9	132
	September	29	103·1	114	18	113·2	126
	December	28	99·6	107	18	113·2	121
1906	March	29	103·1	109	20	125·8	134
	June	29	103·1	108	19½	122·7	128
	September	27	96·0	100	17½	110·1	114
	December	28	99·6	101	18¾	117·9	119
1907	March	28	99·6	100	17¾	111·6	112
	June	28	99·6	98	17½	110·1	108
	September	29½	104·9	107	16	100·6	102
	December	27	96·0	101	15	94·4	100
1908	March	25	88·9	96	10½	66·0	72
	June	24	85·3	94	10½	66·0	73
	September	24	85·3	95	12¼	77·1	84
	December	26	92·4	103	13	81·8	91
1909	March	27	96·0	107	14½	91·2	101
	June	29	103·1	110	16	100·6	108
	September	30	106·7	115	16½	103·8	112
	December	28	99·6	105	17¼	108·5	114
1910	March	29	103·1	105	16½	103·8	105
	June	29	103·1	108	15½	97·5	102
	September	29	103·1	107	16½	103·8	108
	December	27	96·0	99	16½	103·8	107
1911	March	27	96·0	98	14½	91·2	93
	June	27	96·0	96	15	94·4	95
	September	26	92·4	92	13¾	86·5	87
	December	26	92·4	92	15	94·4	94
1912	March	26	92·4	88	14½	91·2	87
	June	27	96·0	90	14½	91·2	86
	September	28	99·6	92	15½	97·5	90
	December	29	103·1	96	16½	103·8	97
1913	March	30	106·7	99	18	113·2	105
	June	29½	104·9	100	17¼	108·5	104
	September	30	106·7	100	17¼	108·5	102
	December	29	103·1	99	17	106·9	103
1914	March	31	110·2	107	17¼	108·5	105
	June	33	117·3	116	17	106·9	106
	September	31	110·2	99	18	113·2	102
	December	27	96·0	84	20½	128·9	113
1915	March	32	113·8	88	24	151·0	117
	June	36½	129·8	98	25	157·3	119
	September	37½	133·3	99	25	157·3	117
	December	41½	147·6	100	27	170·9	115
1916	March	43	152·9	94	25	157·3	97
	June	51	181·3	111	27	170·0	104
	September	53	188·4	113	27	170·0	102
	December	63	224·0	117	34	213·9	111
1917	March	73	259·6	124	35	220·2	105
	June	70 (*a*)	248·9	111	35 (*a*)	220·2	98
	September	70 (*a*)	248·9	113	36 (*a*)	226·4	103
	December	70 (*a*)	248·9	108	36 (*a*)	226·4	98
1918	March	70 (*a*)	248·9	106	36 (*a*)	226·4	97
	June	70 (*a*)	248·9	104	36 (*a*)	226·4	95
	September	70 (*a*)	248·9	102	37 (*a*)	232·7	95
	December	70 (*a*)	248·9	102	37 (*a*)	232·7	95

(*a*) No auctions held, but limited quantities offered to licensed buyers at fixed prices.

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APPENDIX VI (*continued*)

Year		Merino 66's, clean scoured basis			Crossbred 46's, clean scoured basis		
		Price, *d.* per lb., London	Price index, 1909/13 = 100	Merino/ Sauerbeck index ratio, 1909/13 = 100	Price, *d.* per lb., London	Price index, 1909/13 = 100	Crossbred/ Sauerbeck index ratio, 1909/13 = 100
1919	March	83	295·1	128	38	239·0	104
	June	101	359·1	145	40	251·6	101
	September	120	426·7	160	38	239·0	89
	December	138	490·7	168	40	251·6	86
1920	March	150	533·3	164	42	264·2	81
	June	98	348·4	110	33	207·6	65
	September	96	341·3	110	29	182·4	59
	December	65	231·1	90	19	119·5	46
1921	March	44	156·4	71	11½	72·3	33
	June	42	149·3	77	12¼	77·1	40
	September	45	160·0	86	10½	66·0	36
	December	45	160·0	96	10⅝	66·0	40
1922	March	48	170·7	103	11¼	70·8	43
	June	50	177·8	105	11	69·2	41
	September	53	188·4	118	11½	72·3	45
	December	57	202·7	126	16½	103·8	65
1923	March	57	202·7	123	16	100·6	61
	June	57	202·7	127	15	94·4	59
	September	58	206·2	130	15	94·4	59
	December	64	227·6	137	23	144·7	87
1924	March	74	263·1	154	29	182·4	107
	June	73	259·6	153	26	163·5	96
	September	80	284·4	161	31	195·0	111
	December	80	284·4	155	37½	235·9	128
1925	March	63	224·0	129	28	176·1	101
	June	58	206·2	126	22	138·4	85
	September	56	199·1	121	22	138·4	84
	December	52	184·9	114	21	132·1	81
1926	March	52	184·9	118	19¾	124·2	79
	June	49	174·2	112	18¾	117·9	76
	September	52	184·9	116	20	125·8	79
	December	48	170·7	111	19½	122·7	80
1927	March	52	184·9	120	21	132·1	86
	June	52	184·9	121	20	125·8	82
	September	54	192·0	127	22	138·4	92
	December	55	195·6	130	23⅛	147·8	98
1928	March	55	196·6	128	26½	166·7	108
	June	53	188·4	124	26	163·5	107
	September	46	163·6	113	24	151·0	104
	December	49	174·2	119	25	157·3	107
1929	March	44	156·4	104	24	151·0	101
	June	42	149·3	106	22	138·4	98
	September	32	113·8	81	19½	122·7	88
	December	33	117·3	87	19½	122·7	91
1930	March	26	92·4	72	13¼	83·3	65
	June	29	103·1	86	14½	91·2	77
	September	25	88·9	79	10½	66·0	58
	December	22	78·2	72	9	56·6	52
1931	March	24	85·3	80	11¼	70·8	65
	June	20½	72·9	71	8¼	51·9	51
	September	21	74·7	74	9	56·6	56
	December	19½	69·3	65	8½	53·4	50
1932	March	19	67·6	65	7½	47·2	45
	June	18½	65·9	69	7⅛	47·2	49
	September	20½	73·0	73	8¼	51·9	52
	December	20	71·2	74	7½	47·2	49
1933	March	20	71·2	74·5	6½	40·9	43
	June	25	89·0	88	8	50·3	50
	September	29	103·1	103	9½	59·7	60
	December	33¼	118·0	119	11¼	70·7	71

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APPENDIX VI (*continued*)

Year	Merino 66's, clean scoured basis			Crossbred 46's, clean scoured basis		
	Price, *d.* per lb., London	Price index, 1909/13 = 100	Merino/ Sauerbeck index ratio, 1909/13 = 100	Price, *d.* per lb., London	Price index, 1909/13 = 100	Crossbred/ Sauerbeck index ratio, 1909/13 = 100
1934 March	34½	122·9	120	11	69·2	68
June	24	85·5	85	9	56·6	56
September	22½	80·0	78·5	9¼	58·1	57
December	22½	80·0	78	8½	53·5	52
1935 March	22½	80·0	78	8¼	51·9	50
June	25	89·0	85·5	9½	59·7	57
September	28	99·6	94	9¾	61·4	58
December	29½	105·0	97	11	69·2	64
1936 March	33	117·3	109	11½	72·3	66
June	29½	105·0	99·5	11	69·2	66
September	30½	108·3	96	13¾	87·2	77
December	38	135·1	110	19½	122·6	100
1937 March	37	131·9	99	22	138·4	104
June	37	131·9	101	23	144·9	111
September	34½	122·9	95·5	21½	135·1	105
December	29	103·2	85	16	100·7	83
1938 March	24	85·5	73	14	88·0	75
June	24	85·5	75	13½	85·0	75
September	23	81·9	74	14	88·0	80
December	22½	80·0	72	14¼	89·6	81
1939 March	22	78·4	71	13	81·9	74
June	22½	80·0	71	15¼	96·0	85
September (a)	28	99·6	80	18	113·2	91
December	28	99·6	67	18	113·2	76
1940 March	36	128·0	83	23	144·7	94
June	36	128·0	79	23	144·7	90
September	37½	133·5	81·5	24	150·9	92
December	37½	133·5	80	24	150·9	90
1941 March	37½	133·5	78	24	150·9	88
June	37½	133·5	74	24	150·9	84
September	37½	133·5	74	24	150·9	83·5
December	37½	133·5	73	24	150·9	83
1942 March	37½	133·5	70	24	150·9	79
June	37½	133·5	69·5	24	150·9	78·5
September	37½	133·5	72	24	150·9	81
December	37½	133·5	70·5	24	150·9	80
1943 March	37½	133·5	70	24	150·9	79
June	37½	133·5	69	24	150·9	78
September	37½	133·5	69	24	150·9	78·5
December	37½	133·5	70	24	150·9	79
1944 March	37½	133·5	69	24	150·9	78
June	37½	133·5	67	24	150·9	76
September	37½	133·5	67	24	150·9	76
December	37½	133·5	67	24	150·9	76

Source: 1890/31: Empire Marketing Board, *Wool Survey*, 1932.
1932–June 1939: Imperial Economic Committee, *Wool Intelligence Notes.*
September 1939/44: Wool Control.

(a) From September 1939 to December 1944 the quotations refer to Home Issue Prices which ranged from 7–20 per cent. below Export Prices.

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Discussion on Dr. Blau's Paper

Mr. B. H. Wilsdon: I regret Dr. Blau is not present in person to contribute to the discussion. At the same time we are all indebted to Dr. Cronje for the admirable way in which he has picked out the salient points of a paper covering so wide a field. My own qualifications are not those of an economist or a statistician who has any intimate knowledge of the data given in the paper. My direct interest is with wool as a raw material for manufacture—a very important textile which requires careful study of the physical and chemical processes by which it is converted into fabrics which are competitive commodities on the world market.

Workers in applied science, no less than those whose interests are described as pure, are prone to become exclusively concerned with technicalities and the fascination of the pursuit of scientific hares. They have recently been charged with immuring themselves in ivory laboratories. If there is any truth in the charge, the ivory laboratory must be of special construction if it is to keep out the everyday sounds of the market-place.

Research in an applied science can be most usefully directed if the economic factors are also studied. Here the scientific man must depend on the economist, and he will certainly profit if he can find an economist such as Dr. Blau who will collect and analyse data in the way she has done. The value of such a service will be enhanced for him when a full appreciation exists, as is evident with Dr. Blau, of what I like to call technical factors, including the range of variation of quality which is found in the bulk commodity. Frequently a knowledge of this range of variation of quality obscures for the ordinary man any appreciation of the broad statistical and economic arguments which may be applied. Therefore I hope I may be excused if in the few remarks I make I call attention more to the technical implications of this subject than to the economic or statistical core of the argument.

The author has drawn attention to the outstanding facts of the wide variations in prices of wool subject to trends which generally follow the Sauerbeck index. Despite this there has been a general increase in bulk production of some 30 per cent. over the last thirty years. There is, however, clear evidence of differential movements between the various classes such as merino and crossbred wool. It is this question of the quality of the product which introduces the first difficulty. There are some 1,000 different separately priced qualities in the Australian wool "Table of Limits," and these may be multiplied by a factor of 2 or 3 if the drawbacks on the prices due to defects, etc., are taken into account. In New Zealand there are 600 different qualities. The spinner himself recognizes only about a dozen different qualities in defining the spinning quality of his yarn, but in Bradford alone at the beginning of the war there were some 300 well-known commercial varieties of top for yarn-making. This great variety in the form in which the product is marketed effectively accounts for the absence of future markets of any considerable extent.

The fact that limited markets exist on the Continent and in America for tops is to be explained by the further fact that it is possible to standardize the product by selection, but we should also note that if the whole of the product is under one control it would be equally possible to reduce the great complexity of the types on the market. For instance, during the Second World War the Wool Control issued tops of only a limited number of designations, yet the industry managed to carry on. Divergent views may be held as to the necessity for the wide range of choice of equivalent qualities which has been the rule. Probably actual price differences are as important as the imagined differences in quality of the tops which the spinner hopes to blend to give a desired result. By purchasing types at several different prices and blending them, the spinner can hedge. Standardization must clearly tend to decrease the importance of the middleman. Perhaps the prevalence of the middleman is the real deterrent to greater standardization.

We must recognize that the complexity of the market with its rapid price fluctuations must obscure the position in the eyes of the wool producer. What guide can he take? He has learnt to distrust the auction prices. He has taken great pains in the breeding and selection of his sheep, and is ready to provide greatly increased research on genetics and general husbandry. Should he concentrate on breeding for wool yield of the coarser types, or put his faith in the less heavy yield of the fine-wool types? Can we do more than remind him that the wool-clip has always been readily absorbed? We must examine his fears as to the impact on his industry of artificial fibres. We may make a rough and provisional comparison by putting cotton at 6*d*., rayon at 1*s*., and wool at about 2*s*. 6*d*. The value of the wool produced annually would then not be very different from that of cotton, and the value of rayon would be very much less; but I do not think that such a comparison would tell the whole tale.

Apart from the actual value of the annual production of textile fibres, artificial or natural, surely the life of a commodity has to be taken into account in some way. In the case of such a commodity as tea, for example, once it has gone into the teapot we have done with it. Regulation in the tea industry has been successful. The relationship between supply and demand, giving proper consideration to stocks and substitution between qualities, would appear relatively simple in such a case. But not so with wool, the life of which may be considerable. As we have had occasion to learn during the war, the useful, if not the beautiful, life of a garment can be protracted considerably before it assumes a new avatar. For the life of the wool fibre is by no means over

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with its first use. It can be re-manufactured, and this re-manufacture represents a considerable proportion of the total amount of fibre in service at any time. Actually the amount of re-manufactured wool fibre on the machines of the industry in this country and in Continental countries may amount to 50 per cent.

We have thus to consider the reactions through quantitative changes brought about by artificial fibres and by the increased amounts of re-manufactured wool. This makes a very complicated picture. How do these buffer factors affecting demand compare in magnitude with the possible effect on supply of regulation through the buffer pool?

The determining factor, as Dr. Blau has pointed out, must eventually be found in the consumer. She has distinguished in the paper between fibre competition and fabric competition. Over considerable fields the use of a particular fibre may be practically unchallenged. In other cases, where there is both fibre and fabric competition, the preference of the consumer is subject to many influences. Fashion, advertising, distribution, and manufacture interfere with the free exercise of choice by the consumer.

The relation between broadly distinguishable economic factors and the more detailed interplay of technical factors is a subject already recognized but so far little explored. It will be realized that the main complications arise again from this variation in the quality of the commodity both in the raw material and in the finished product. The producer will have to seek comfort from what can be told him by the economist or by the scientific man. What faith can he place in the scientific man's explanation that the wool fibre has a combination of physical properties which has not been met in any other artificial fibre, and, as a matter of fact, is not likely to be met? What comfort can he find from what I understand is Dr. Blau's suggestion, that even when one takes into account the market value of the raw material in the process of manufacture, the price differential is scarcely likely to affect the substitution at such an amount as will produce wide undercutting? I think we can say that the spinner will only be influenced in a limited range in this process. The spinner may be interested in introducing to the market mixture yarns which can be used for knitting, but in the production of fabrics the spinner works to a specification, and it is the manufacturer and the fashion designers who will determine what fabrics are put before the public.

I have attempted in these brief remarks to show how widely one's economic conclusions as to movements of prices and production must be affected by the consumer's recognition of quality in goods. In closing, I do not wish to appear ungrateful when I say of this admirable paper that it makes one avid for more detailed information. At present I feel that we have little clear indication of what the future of wool consumption will be.

I must end by expressing my sympathy with the producer of wool. I feel that he would be wise not to put too much trust in price maintenance through the possible operation of a buffer pool, and his greatest insurance is to acquire knowledge of the technical factors which will determine the qualitative as well as the quantitative demand for his product. At the same time he must increase efficiency in the production and marketing of his clip. The research organized by the Wool Security under whose auspices Dr. Blau has made this valuable contribution should do much to provide information which will give the producer the confidence he requires.

Dr. C. Oswald George read the following letter from Mr. R. L. Davis, who had intended to second the vote of thanks, but was unable to be present.

"I very much regret that owing to indisposition I shall be unable to be present at the reading of Dr. Blau's paper. This is an extremely useful and valuable picture of the position up to the outbreak of war, with a hint of what has been happening in the war-years. I am interested mainly in the picture of the United Kingdom wool-textile industry and the contribution that this industry will be able to make to our export target. From this point of view Table XII is extremely valuable, although the changes in the world trade in yarns and tissues ought perhaps to be compared with world exports of other commodities over the same period. However, it is clear that the trend towards the growth of domestic wool-textile industries in the "newer" countries, that operated before the war and was partly responsible for the decrease in exports of yarns and tissues, has increased substantially in the war-years and will be encouraged to the extent that the United Kingdom industry, by reason of labour shortages, is currently unable to export on a major scale. Unfortunately, there are extremely meagre data by which it is possible to measure the extent of that further industrialization in the war-years in the Southern Hemisphere and elsewhere."

Dr. George continued: I should like to pay tribute to the author for her valuable and exhaustive paper, which offers no targets for serious criticism. There are, however, one or two small points which occurred to me as I read it. At one stage she suggests that the importance of the wool industry in the economies of the chief wool-exporting countries is indicated by the ratio of the values of wool exports to the values of total exports. That ratio, however, must be used with great caution, and is obviously no guide in the case of countries such as this, and it may be worth while to look at one or two other aspects of wool production in this country which indicate in some measure its importance.

Detailed figures of recent production are not available for the Untied Kingdom as a whole,

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but some interesting details of wool production in Great Britain were given in the recent Report of the Wool Marketing Committee. For the five years immediately preceding the war the average annual production of wool was 105 million lb., of which about 75 million lb. were fleece wool and 30 million lb. fell-mongered or skin wool. During the war-years there was a substantial decline in the annual production of fleece wool, which fell from the pre-war level of 75 million lb. to an average annual figure during the last three years of 56 million lb., a drop of about 25 per cent. This rather substantial fall was, of course, the result of the war-time decrease in the number of sheep. The production of skin wool remained steady at about 30 million lb. Of the 75 million lb. of fleece wool, about 10 per cent. was produced in Wales, 33 per cent. in Scotland, and the remaining 57 per cent. in England; these figures, after allowance has been made for the acreage and other differences, indicate the different degrees of inportance of wool in the agricultural economies of the three countries.

Another point worthy of consideration is the number of agriculturists in this country who are concerned in wool production; the above-mentioned 56 million lb. of wool were provided by about 94,000 producers, and, what is still more surprising, 42,000 of those producers produced only 40 fleeces or less a year. This shows that wool production, even if it is not of the first order of importance in value of output, is widely spread and of financial interest to many people in British agriculture.

Passing to the question of quality, which maybe is outside the scope of this paper but is obviously not unimportant, English wool is claimed to be the best in the world. Whether it is as good as it might be at the present time is perhaps doubtful. There are certain factors which have tended to lower its quality, such as the use of colouring in dips and blue powders for improving the look of sheep when they come to be sold for mutton. But there is an unsatisfied demand for English wool of the best type, and it is to be regretted that it is in short supply.

One final point: the author suggests, not entirely conclusively, that her assumptions concerning the high elasticity of the demand for wool are supported by Allen and Bowley's classic work on family expenditure. This is not the time or place to enter into a lengthy discussion of their linear preference scales or the validity of estimating personal elasticities from the respective demands of different income groups. One may have doubts on these points, even in a relatively stationary economy, but at a time like this they may seem still more dubious. And one may also wonder whether the most important factor affecting demand may be, for a time at least, not any hypothetical increase in post-war real incomes, but a less disputable factor, namely, the urgent need to replenish our personal stocks, which have fallen almost to zero, of "undies."

I have much pleasure in seconding the vote of thanks to the author—and the reader—of this important paper.

The vote of thanks was put to the meeting and carried unanimously.

The CHAIRMAN (DR. E. C. SNOW), after mentioning the circumstances of the origin of wool control in the First World War in 1915, referred to a plea in a recent issue of a weekly economic journal for lifting the statistical "black-out" which existed outside official quarters with regard to wool. He thought that Dr. Blau had gone a considerable way to lifting the "black-out," and that gratitude was due to her for providing a paper which was as informative as the White Paper which was being called for was likely to be.

A feature brought out by the paper was the position regarding world stocks of wool, representing something like 2½ years' consumption, in striking contrast to the situation of a number of other commodities. Wheat was one of the latter most prominently before the public eye at the moment. The situation was little better regarding leather. Before the war the granaries in the Western Hemisphere were frequently reported to be bursting with stocks of wheat, but now there was practically none. Stocks of wool before the war were never of inconvenient magnitude, but they now appeared almost embarrassing. How was it that such a great difference had come about in the circumstances of the two commodities? There were, no doubt, many factors, but it was worthy of note that the producers of wool themselves had had some voice in the policy which was adopted regarding wool purchases during the war. The chief wool-growers of the world were relatively well organized, at any rate compared with the complete lack of organization in the cases of many other commodities. The arrangement made that the British Government should buy the whole of the exportable surplus during the entire war and 12 months afterwards at a price originally fixed as 33 per cent. above the immediate pre-war figure, and in 1942 advanced to 50 per cent. above that figure, was an excellent one from the sellers' point of view, as well, as it turned out, for the world at large. The price might not have been fixed so high had there been no organization to advocate the producers' standpoint. In the case of other commodities where the buyer was in the position of control, prices were fixed at lower levels compared with pre-war. There was no similar organization representing the wheat-growers of the world, and in fixing prices of wheat, in the circumstances which ruled through the great part of the war, it could be said to be entirely a buyer's market. Prices appeared to have been fixed too low to have encouraged the maintenance of stocks in countries overseas, and this might well be a contributing factor in the tragic circumstances in which a large part of the world now found itself.

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Dr. Snow next referred to the figures provided by Dr. Blau showing the extraordinary fall in prices of wool from March 1920 to March 1921. For merino wool the controlled price during the war was 70*d.* per lb. It went up to 150*d.* in March 1920, and tumbled until it reached the figure of 44*d.* in March 1921. That experience of a rapid boom and an equally rapid slump and collapse was fairly common in other commodities, and indeed in a number the movements were even more marked. Prices of raw goatskins fell to one-sixth of their previous level in a period of three months. The experience of 1920–21 was still vivid in the memory of many traders, and it seemed probable that the psychology created by it was having an adverse effect at the present time. Everybody who suffered by that slump seemed quite determined that they would not repeat their experience, and this determination, in many cases, was causing them to go the other extreme. In order to avoid the troubles created by prices rising too high, they were using their endeavours to keep them at too low a level to ensure adequate production, and we were now accordingly suffering from the trouble of scarcity rather than of relative plenty experienced on the previous occasion. This, he thought, had a bearing on the wheat situation at the present time. There were sound reasons for keeping prices low during the period when transport was insufficient to ship the material readily available, and there would have been no point in stimulating production if this meant increasing stocks held overseas to inconvenient levels. Unfortunately, it did not seem possible to adjust policy at the speed which change in material circumstances required.

Dr. Snow further referred to the figures given towards the end of the paper showing the extraordinarily wide range of annual variation in prices of various commodities. The author indicated that the average variation of the maximum from the mean over a series of ten years in the case of merino wool was 47 per cent., while in the case of rubber it was as much as 83 per cent. This wide range of variation is a common experience with many commodities, and it is extremely difficult to explain. The author quoted twice from Lord Keynes in expressing the opinion that these rapid variations in prices resulted in producers or middlemen refusing to hold stocks. He (Dr. Snow) did not agree with that view. His experience had been the reverse. Numbers of people deliberately held stocks on account of the fact of these rapid variations in price. They thought it was essential to hold stocks (they believed they were generally bought at a relatively low price) in order that they might be in a position when prices rose to stand out of the market.

Dr. Snow concluded by congratulating the Society on having such a valuable set of facts placed before it relating to a single industry. It might be that some enterprising publisher would feel that there was scope for a series of handbooks dealing on similiar lines with other commodities, and the work of Dr. Blau would be an excellent example for others to follow.

Mr. J. C. Westoby said that he could not allow the Chairman's first point to pass unchallenged. The Chairman had suggested that the wool-growers, by the strength of their organization, saved themselves and saved the world, and the obvious inference was that if the producers of other raw materials had had the same strength of organization the world would have emerged from the war with ample stocks of all raw materials. But had there been that organization, with agreements favourable to the producers in all cases, the net effect would have been that among the United Nations there would not have been that transition from peace-time production to the manufacture of war-making materials that actually was achieved during the war. In other words, the effect would have been to prolong the war, if not to lead to Allied defeat. He made that point because he felt that the Chairman had given rather too much credit to the wool-growers concerning the agreement they reached early in the war.

One very valuable feature in this paper was the searchlight it had cast upon some of the deficiencies in their economic thinking. They had been very much concerned with the development of policies for full employment in what might be termed insulated economies, and they had developed theories which they hoped would succeed in achieving freedom from want in the future. But they had not to the same extent examined the repercussions of changes in terms of trade, and in particular of the effect of changing stocks of primary commodities.

It was evident that in an insulated economy in which there was complete or semi-planning, and in which there was to be found an accumulation of stocks of a certain raw material, with at the same time a large section of the population needing the consumer goods eventually to be made from that raw material, it would be possible by some adjustment to marry supply and demand. It might be that the seller's price and the buyer's price did not match, but they could be made to match, and the cost could be borne by the community. But in the case of wool they were confronted, not with a raw-material surplus and a demand for consumer goods in the same section of the world, but with a surplus in certain sections and a demand or potential demand in other sections.

It seemed to him that this indicated how important it was that planning should be extended into the international field, and he felt that they should look in the future, perhaps, to some sub-committee of the Economic and Social Council of the United Nations Organization to act as world economic planners in this respect because, unless the great variations in the trade in primary commodities could be adjusted in the future, their efforts in any particular country to achieve full employment, freedom from want, or economic security were bound to be frustrated.

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DR. VICTOR BLOCH pointed out the connection between wool production and meat production. Let it be supposed that an artificial fibre could be made having all the qualities of natural wool and could be offered at a lower price; then the grazing industry in Australia, New Zealand, and South Africa would tend to disappear, the stations would be abandoned, and no wool would come on the market. In other words, the same thing would happen as happened many years ago in the case of indigo. Where there was no wool there was also no mutton, and as the question of food was particularly important just now, an investigation into the importance of wool production should be accompanied by some consideration of its natural and inevitable connection with meat production.

MR. H. CAMPBELL drew attention to what he described as two minor points. He spoke as one who had been brought up in the West Riding. There the demand for wool was essentially tied up with the amount of spinning in the manufacturing industry. As this amount was not readily expansible, the disposal of the surplus accumulated over the war-years would necessarily be a long process: the manufacturing industry would not be able to cope with the supply available and it would be necessary to withdraw stocks from the market to be made available later when production was capable of taking it up.

His other point was that as the ultimate demand was for manufactured goods and not for wool as a raw material, the raw material represented a small proportion of the cost of the final manufactured goods. For cheap suits, for instance, he understood that the total cost of the raw material was of the order of 10 per cent., and in other cases it might be less than 5 per cent. In consequence, the substitution of artificial fibres would be considerably retarded by the fact of the capital invested in the wool industry and also by the slowness of the change in tastes of consumers.

Even if the synthetic industry became capable of producing a fibre with all the qualities—particularly the hygroscopic qualities—of wool, and at a competitive price with wool, it would not immediately be a very severe competitor with the wool industry because of the small percentage which the raw material represented in the finished article and also because of the capital already invested in the industry.

MRS. TAYLOR said that adequate figures for wool-textile production were in existence and would, she thought, be made available to the public very shortly. There was a general feeling that this statistical effort should be continued for the benefit of trading people as well as for Government requirements. There was some reluctance on the part of manufacturers to undertake the work involved in providing the figures, but there was a real demand in the trade for information, and this seemed particularly necessary in view of the dependence of the consumption of raw wool on the machinery position and the activity of the textile industry generally. She thought that in this respect we lagged very much behind the United States, where they had the fullest information as to the stocks of raw wool and textile production at all stages. It should be the aim to have equally extensive figures over here.

DR. J. A. KRONSTEN wished to make what he thought was a rather obvious point. In the present demand for wool a factor which had to be borne in mind was the present shortage in many parts of the world of food, fuel, and shelter which affected very large sections of humanity. The victims of war, direct and indirect, were deprived of these things and accordingly were all the more anxious to get garments the highest protective quality rather than of other textile material which did not give the same amount of warmth and protection. With the return of normal conditions a great deal of this purely temporary demand was likely to fall off. He did not believe that it would be justifiable to take too optimistic a view of the future of the world's demand for wool, unless now one anticipated a considerable rise in the standard of living. This, however, could only be visualized as a long-term development, and thus came outside the range of Dr. Blau's brilliant analysis of the world wool economy.

MR. HAMILTON-RUSSELL referred to the table comparing national income in different countries with per capita wool consumption. He pointed out that in the United States, for example, per capita consumption was relatively low owing to the excessive tariff rate, which resulted in American wool prices being far above those elsewhere. Similar phenomena could be observed in other countries which imposed high rates of duty to protect their domestic wool or synthetic fibres industries.

He had hoped that this question of tariffs and their effects might have been referred to more specifically in the paper.

DR. CRONJE said he had no authority to reply for Dr. Blau, but there was one point raised by the Chairman on which he would like to say a word. When war broke out wool was regarded as an essential war material, and the general policy was to try to keep down the price of raw materials under long-term contracts. The surpluses developed because the war at first took a

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different turn from what was expected. France fell, the Continent was cut off, and Japan came into the war so that a larger percentage of wool than of any other material went to enemy countries on the Continent. One had to bear that in mind in considering the point which the Chairman had brought forward.

DR. BLAU subsequently wrote as follows:—

First, I should like to express my sincere thanks to Dr. Cronje for his kind help in reading my paper.

I was very sorry that I was not able to attend the meeting of the Society; and for the second time this year I am now again in Washington, and must therefore reply from a distance to the various points in the record of the discussion which has just reached me and which I have read with great interest.

I am very glad that Mr. Wilsdon could find time to come to London and take part in the discussion because I share his belief in the need for the closest possible relations between scientists and economists in this field of research. Mr. Wilsdon points to the influence of certain types of advertising and other distribution methods which tend to interfere with the free exercise of choice by the consumers. These dangers are greatly accentuated by the lack of adequate consumer education, and of impartial scientific information. This is a field where scientific research organizations can play a great part not only by the direct dissemination of knowledge, but also by giving their authoritative support to plans for adequate legal protection of quality standards. The scientists are aware of their responsibilities in this field; and the more they can help in educating consumers into rational beings, the better will be their chances of getting reliable answers from the economists in regard to questions about the market behaviour in general, and the relationship between quantities and prices in particular. This is not a plea for evolving a universal type of the legendary *homo oeconomicus* by means of scientific education. There may be good reasons for the behaviour of the "foolish" people who do not always do what economic textbooks think (or thought upon a time) they ought to do: but my plea is for a world where all foolish consumers are *deliberately* foolish, not trapped into foolishness by ignorance about the quality characteristics of the goods they buy.

I certainly agree with Dr. George's point that the ratio of a country's wool exports to total exports is not the only figure indicating the importance of wool production in that country's economy. No doubt, domestic wool-growing may be an important factor in the agricultural economies of major wool-importing countries—as, for instance, the United Kingdom or the United States. At the same time, I think Dr. George will agree that a high ratio of wool export values to a country's total exports *is* a strong indication, though not, of course, the only possible one, of the importance of wool in that country's economy—which was all I had meant to illustrate with the aid of the figures to which Dr. George refers.

As to the importance of re-stocking demand, I certainly agree that this is a major factor influencing the current market situation and the outlook in the near future. Indeed, the importance of re-stocking demand has been amply illustrated by the scale of 1945–46 purchases of Dominion wool which are estimated at nearly twice the rate of probable mill consumption during that season. At the same time, it would be dangerous to allow this temporary phenomenon to overshadow the long-term reflections on the influence of income changes on clothing consumption once the world's pipelines have been re-filled.

Dr. Snow's remarks touch upon various points of interest. In regard to the relationship between wool prices and the accumulation of wool stocks during the war-years, this point has already been taken up by Dr. Cronje in his reply which points to the sudden loss of chief wool markets in France and other Continental markets, and later in Japan. In regard to other markets, chiefly the United Kingdom, wool consumption was curtailed by the labour shortage and shipping difficulties and these physical factors were more important than the influence of prices on the level of raw wool consumption and on the resulting size of stocks.

I note that Dr. Snow does not agree with Lord Keynes's views on the influence of variations of traders' stocks on prices. As outlined in my paper, I believe in Keynes's argument on this point. At the same time, I agree that it is difficult to prove this argument in quantitative terms for the wool market, and in this connection I think that Dr. Snow's remark brings out an important point which has also been mentioned by Mrs. Taylor later in the discussion. I refer to the paramount need for statistical returns showing stocks of raw wool "in all hands," not only those held by or on behalf of Governments. Before the war, the United States was the only country publishing such figures. A complete periodic survey of traders' stocks in all major consuming countries must be regarded as an essential prerequisite for the better understanding of market prospects. The success of any planning in the international field (as supported by Mr. Westby) will also largely depend on the accuracy, comprehensiveness and timing of such factual information.

I should like to end by expressing my sincere thanks to the Chairman and to all those who took part in the discussion, for their helpful comments and for the kind interest which they have shown in my paper.

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As a result of the ballot taken during the meeting, the candidates named below were elected Fellows of the Society:—

Arthur Harold Aston.
Walter Reginald Bett, M.R.C.S., L.R.C.P.
Om Prakash Bhatnagar, B.Sc.
John Hugh Butler.
Hubert Campbell.
Kenneth Leon Cohen.
George Metcalfe Collier.
Detlev Bruno Halpern.
William Potter Kempsell.
Tom Stewart Lodge.
Harold French Lydall.
John Ratcliffe MacLeod.
Edward Samuel Medlicott.
Lawrence Nevard.
Phyllis Mary Nye.
Nadim Pachachi, Ph.D.
Hans Wolfgang Singer.
Patrick Slater.
Frederic Gordon Smith, M.A., F.I.A.
Jack Taylor.
Henry Cooper Thompson.
Leslie William Godfrey Tutt.
Gwilym Ogwen Williams.
Basil Young.

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Mutual Aid between the U.S. and the British Empire, 1941–45 *

By R. G. D. Allen

[Read before the Royal Statistical Society, May 29, 1946, Dr. E. C. Snow, C.B.E., in the Chair.]

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The Mutual Aid Settlement. Lend-lease aid began in March 1941, nine months before the U.S. entered the war, as the result of an Act passed by the U.S. Congress " to promote the defense of the U.S." Aid was furnished to countries " whose defense the President deems vital to the defense of the U.S." Lend-lease operations were developed and extended after Pearl Harbor, and the system of reciprocal aid from the British and other Allies was introduced, as essential pieces of the machinery for the combined planning of the war. However, the terms of the Lend-lease Act were never changed to adjust them to the new conditions of full U.S. participation in the war. In particular, the terms remained under which the U.S. was to receive, in return for lend-lease aid, " payment or repayment in kind or property, or any other direct or indirect benefit which the President deems satisfactory." The phrasing of the original Act makes it clear that the U.S. intention from the beginning was that lend-lease should be a purely war-time operation. At subsequent hearings in Congress on Lend-lease Appropriations and on extensions of the Lend-lease Act—including hearings as late as March–April 1945—it was repeatedly stated by the U.S. Administration that lend-lease would cease at the close of hostilities. Apart from minor exceptions, straight lend-lease and reciprocal aid did in fact terminate on the official V-J Day—*i.e.*, from 12.01 a.m. on September 2, 1945—though the machinery itself continued to operate for

* The Allies developed during the war a multilateral system under which goods were supplied and services rendered to each other without commercial payment. This is mutual aid in the widest sense. In this paper, however, the term is restricted to comprise only the dual system of lend-lease aid from the U.S. to the British Empire and of reciprocal aid to the U.S. from the various governments within the British Commonwealth. Only incidental reference is made to aid to and from third countries (Russia, China, France, etc.) and to aid between the governments of the Empire. The position of Canada in the mutual-aid system needs to be noted. Canada did not receive lend-lease aid from the U.S., nor did she provide reciprocal aid to the U.S.; purely U.S.–Canadian transactions remained throughout on a cash basis. The large contributions of Canada to the Empire's war potential—in men, munitions and materials—were made in part in the form of " mutual aid " similar to lend-lease from the U.S. This Canadian form of mutual aid was also extended to Russia and other countries. The fact that Canada receives only passing reference in this paper is not to be interpreted as a lack of recognition of the important part she played in mutual aid in the wider sense.

The data used in this paper are obtained in the main from published sources. Material prepared by the Treasury for publication in the White Papers on Mutual Aid (Cmd. 6483, 1943 and Cmd. 6570, 1944) and certain data subsequently made available from the same source are the basis for the tabulations of reciprocal aid from the U.K. Data on lend-lease aid, and also on reciprocal aid from Empire countries other than the U.K., are taken from the records of the U.S. agencies responsible for lend-lease—now the State Department (Office of the Foreign Liquidation Commissioner), previously the Foreign Economic Administration. The main source is the Report to Congress on Lend-lease Operations made quarterly by the President, and additional analyses of the same basic material are taken from Congressional Hearings on the Lend-lease Act and Appropriations, from the reports of the Senate Committee investigating the National Defense Programme and from other Congressional documents. The adjustments and manipulations of the basic data, and all comparisons and conclusions derived from the data, are made on my own responsibility. These draw on my experience as a British official concerned with the operation of mutual aid from 1941 to 1945, during which period I became familiar with the detailed statistical material available internally to British and U.S. agencies. Since this responsibility is mine, I wish to make it clear that neither the Treasury nor the U.S. State Department is in any way committed by my handling of their data.

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several months thereafter to clear the "pipeline" against a later cash settlement. The bitter feeling engendered at the time, now happily forgotten, was mainly a reaction to the abrupt manner in which the end was announced in Washington.

The mutual-aid account between the U.S. and the U.K. is not only closed; it is also settled finally and completely. The settlement was arranged so harmoniously as to pass almost unnoticed amidst the turmoil of the Loan Agreement. The mutual aid settlement and an understanding on commercial policy were two joint statements by the U.S. and the U.K. agreed and issued at the same time as the Financial Agreement of December 6, 1945 (Cmd. 6708–09). Though the settlement was closely associated with the Loan, it was an independent arrangement not subject to implementation by Congress of the Financial Agreement itself. As we have seen, the President had sufficient powers under the Lend-lease Act itself to agree the terms of the final "benefit" to the U.S. in the winding up of the mutual aid account.*

The settlement of the U.S.–U.K. mutual aid account is expressed in terms of hard cash. The details of the specific agreements making up the settlement are set out in a joint memorandum (Cmd. 6778) signed in Washington on March 26, 1946, following the more general statement of December 6, 1945. An exposition of the agreements and further details of the dollar sums involved are given in a letter dated March 7, 1946, from the U.S. Secretary of State to Senator Mead (Senate Document, 79th Congress, 2nd Session, Report No. 110, Part 5, Appendix XVIII).

A total sum of $650 millions is given as due from the U.K. to the U.S. to be paid on the same terms as the proposed loan. Of the total, $118 millions is a first estimate, to be adjusted by closer accounting later, of the net amount due to the U.S. in the off-setting arrangements under mutual aid after V-J Day. The clearing of the lend-lease "pipeline" gives the U.S. an estimated credit of $301 millions, against which are set $130 millions for clearing the reciprocal-aid "pipeline" and $53 millions for the net amount due to the U.K. on other claims in connection with mutual aid.

The U.S.–U.K. agreements (Cmd. 6778) do not specify further dollar figures, but the U.S. Administration has released additional details (in the Senate Document quoted), presumably without committing the U.K. to agreement on every figure. According to these U.S. estimates, some $60 millions represents a payment by the U.K. for the acquisition of surplus property owned by the U.S. and of the U.S. interest in installations in the U.K. For this sum, the U.K. takes over tangible assets previously the property of the U.S. and valued at about $350 millions at original cost to the U.S.

There remains a sum of $472 millions due to the U.S. in final settlement of the mutual aid account proper in the whole period of operation from March 11, 1941, to V-J Day. The method of calculation of this figure is complex, but the broad principles are clear. First, there are no financial obligations whatever for mutual aid goods destroyed or consumed during the war. Such goods "were used to defeat our enemies, thus achieving the primary purpose of lend-lease and reciprocal aid. Neither country profits financially at the expense of the other as a result of such mutual action, since both were devoting maximum shares of their national output to war production" (Senate Document, p. 87). Secondly, all naval vessels and all merchant ships of 500 gross tons and over provided under mutual aid and existing at V-J Day are returnable to the supplying government (Cmd. 6778, Memorandum, para. 3). Similarly, installations constructed under reciprocal aid for U.S. forces in the U.K. and colonies are transferred back to the U.K., and lend-lease installations in the U.S. are acquired by the U.S. (Cmd. 6778, Agreement VII, para. 5). Thirdly, the title to lend-lease munitions and other supplies held by U.K. forces at V-J Day remains with the U.S., and is not transferred to the U.K. The U.S. retains the right to recapture such equipment, though the right is not generally to be exercised. The value of the equipment "did not enter the financial settlement and should not be considered in appraising the financial results" (Senate Document, p. 88). The right of the U.K. to recapture reciprocal-aid stores in the hands of U.S. forces at V-J Day is equally protected. Fourthly, mutual aid stocks of petroleum held by military and civilian agencies throughout the world at V-J Day are subject to a re-distribution between the two countries as described in a specific agreement on petroleum (Cmd. 6778, Agreement VI, particularly Annex IX). In round figures, the re-distribution is as follows:

* Similar settlements between the U.S. and the Dominions and India can be made within the framework of the Lend-lease Act, and these are now under discussion.

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	Million tons	$ millions *
Lend-lease petroleum held by U.K.	4·1	145
Reciprocal-aid petroleum held by U.S.	0·8	25
Total mutual-aid petroleum	4·9	170
Acquired by: U.K.	3·3	115
U.S.	1·6	55

* Cost plus full ocean freight.

On balance, therefore, the U.K. acquires and the U.S. provides a net amount of 2½ millions tons of petroleum at an original cost of $90 millions, including ocean freights. The figure of $472 millions includes a payment for this net acquisition of petroleum stocks by the U.K.

Finally, the remainder of the $472 millions is intended to cover stocks of lend-lease goods held at V-J Day for civilian use in the U.K. and colonies with a deduction for reciprocal-aid stocks of civilian types held by the U.S. (Cmd. 6778, Agreement III). These goods include agricultural, mining and other machinery and a great range of foodstuffs, raw materials and manufactures. Machine tools, however, are not included, since they were subject to separate arrangements for acquisition by the U.K. after they were made ineligible for lend-lease in 1944. In addition, the sum is extended to cover certain non-combat aircraft of lend-lease origin transferred to the U.K. for civil or military use,* a few small ships of under 100 gross tons and the lend-lease constituent of installations in the U.K. and colonies (Cmd. 6778, Agreements V and VII).

Hence, as a result of the mutual aid settlement, considerable stocks of lend-lease goods of civilian types, held by the U.K. on V-J Day, are acquired outright by the U.K. The U.S. Administration estimates these stocks at $690 millions at original cost (Senate Document, pp. 88–90):

	$ millions	
Undistributed civilian stocks of mutual-aid origin:		
Held by U.K.: Food	240	
Raw materials	180	
Manufactures	20	
Less held by U.S. (raw materials)	—40	400 *
Lend-lease machinery held by U.K.		85 *
Net acquisition of petroleum by U.K.		90
Non-combat aircraft and small ships		80
Lend-lease interest in installations		35
Net total		690

* At f.a.s. cost plus 10 per cent. for mutual-aid share of ocean freights.

The payment of $472 millions by the U.K. is for these stocks and for nothing else.† The U.S. therefore obtains a return of nearly 70 cents on the dollar for the goods which are sold to the U.K. in the mutual aid settlement.

The calculations summarised in Table 11 below show that, in round figures, total lend-lease aid from the U.S. to the U.K. amounted to $27,000 millions—$2,000 millions for ships, nearly $14,000 millions for munitions, nearly $2,000 millions for petroleum, over $6,000 millions for other goods and $3,000 millions for services. At roughly comparable prices, the U.K. provided reciprocal aid to the U.S. to the value of nearly $6,000 millions—$900 millions for construction, $2,250 millions for military stores, $1,200 millions for petroleum, $400 millions

* The aircraft transferred comprise 72 Dakota (DC-3 type) aircraft for civil air lines, 600 Dakotas and 43 other types for military transport use. All other non-combat aircraft of lend-lease origin held by the U.K. at V-J Day are returnable to the U.S., but 671 Dakotas are to be leased for a limited period and certain Liberator transports and Harvard trainers are to continue for a time in military use.

† It would appear that the payment "is deemed to include an adequate compensation for the value of such (lend-lease supplies held by U.K. forces) as may be released for civilian use in the U.K. and British Colonial Dependencies." These releases, however, will be made in practice only "to a very limited extent" (Cmd. 6778, Agreement IV, para. 7).

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for other goods and $1,200 millions for services. The item for services on each side of the account is very largely composed of shipping freights and other charges concerned with transportation and maintenance of mutual aid goods. It can be treated best as an addition to the recorded value of transfers under mutual aid to convert the valuation from ex-works to delivered. With services distributed in this way, the balance in favour of the U.S. in the mutual aid books can be analysed in very round figures:

	$ millions
Ships and construction *	1,000
Military stores	13,000
Petroleum	500
Other goods	6,500
Total	21,000

* Including maintenance of buildings.

The closing of the mutual aid account can be freely interpreted in the following terms. Naval and merchant ships provided under lend-lease and construction under reciprocal aid can be ignored in the settlement. The ships not lost are returnable to the U.S. The installations constructed under reciprocal aid are handed back to the U.K., including the U.S. contribution to their cost. The use of the ships and installations during the war was obtained without payment in the settlement. The account for military stores is wiped out in the settlement without payment both as to munitions consumed and as to those remaining at V-J Day, estimated at over $5,000 millions in U.K. hands. The U.S. retains title to, and the right to recapture, military stocks of lend-lease equipment, and limits are imposed on the power of the U.K. to dispose of such equipment An exception is made of non-combat aircraft (and some small ships), which are either transferred to the U.K. against a payment or returnable to the U.S. Petroleum supplies were largely pooled between the U.S. and the U.K. in the interests of saving shipping space. This mutual aid account is not far out of balance in dollar terms and a re-distribution of closing stocks is made against a small payment from the U.K. The remaining account in goods other than munitions and petroleum shows a balance of $6,500 millions on the U.S. side. Food and other supplies for British civilians and materials for British war industry dominate this group, but it also includes food and other material directly or indirectly for the use of the forces. Undistributed civilian stocks and machinery held by the U.K. from this account are valued by the U.S. at around $500 millions, and the major part of the sum of $472 millions due to the U.S., perhaps $300–350 millions, can be regarded as in payment for the acquisition of these stocks at V-J Day.

A very large dollar sum standing to the credit of the U.S. on the mutual aid books, therefore, is wiped out in one sweep, and a possible source of post-war disharmony removed. The cancellation of more than $20,000 millions of lend-lease credit is a recognition of the combined effort in the defeat of Germany and Japan. The U.S. also acknowledges the contributions made by the British outside the mutual aid account in dollars and pounds. These contributions, though not measurable, are sufficiently obvious. There was the extra mobilization of British men and women for war service and war production, greater and longer sustained than in the U.S. There was the invaluable contribution to the build-up of U.S. armament production in the form of British cash contracts for munitions awarded to private U.S. firms before lend-lease, contracts on which deliveries and payments were made until 1943, and which were never taken over into lend-lease. There was the exchange of military and technical information and the pooling of research experience, all important to the U.S., particularly in the early days after Pearl Harbor.

The mutual aid settlement was expected to have, and in fact does have, strings attached. It was no accident that the Loan Agreement of December 6, 1945, the joint statement on the mutual aid settlement and the second joint statement on commercial policy were all agreed and issued together. Between them, they define the "general obligations" assumed by the U.S. and the

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U.K. on commercial policy, and in particular on the development of multilateral trade in the post-war world. These obligations were first expressed explicitly in Article VII of the Mutual Aid Agreement of February 23, 1942 (Cmd. 6341), and reaffirmed in the joint statement on commercial policy of December 6, 1945. The mutual aid settlement states that the two governments take "full cognizance" of these obligations. They are more specifically defined, particularly as to exchange arrangements, in the Financial Agreement; indeed, one of the essential purposes of the loan is to assist the U.K. in assuming the obligations on the promotion of multilateral trade.

With all such imponderables in mind, we can reasonably conclude that a fair balance has been struck between accounting in terms of dollars and pounds and benefits not assessable in value terms. So what has been described by Mr. Churchill as "the most unsordid act in history" passes into history through a quiet but equally unsordid agreement. The main purposes of this paper are to offer a partial and preliminary estimate of the importance of mutual aid in the conduct of the war, to indicate what has been achieved by this, one of the many examples of combined planning under the stress of war, and to hold out the hope that the experience will bear fruit in the development of peace-time co-operation.

Mutual Aid and Combined Planning. After Pearl Harbor, the war could have been fought with independent contributions from each of the Allies. To be equitable, each contribution would need to be in proportion to the capacity of the country—*e.g.*, armed forces furnished in proportion to the male population of fighting age and munitioned to the extent permitted by industrial capacity. This was clearly not a practicable proposition in 1941–2, when the Russians were in dire straits, and when U.S. strength was potential rather than actual. In any case, even if feasible, the method would not have been the best way of mobilizing the strength of the Allies and of maximizing the impact against the enemy.

In fact, the conduct of the war was planned very differently. Though always imperfect and sometimes halting, the governing idea was to plan on a combined basis to take advantage of the potentialities of each of the Allies. Many limitations were never overcome, such as those imposed by the different types and calibres of weapons used by the various national forces. Even so, the conduct of the war provided a striking example of the "division of labour" between countries. We can see, in broad outline, the division between the U.S., the U.K. and Canada from facts recently made available (*The Impact of the War on Civilian Consumption in the U.K., the U.S. and Canada*, H.M.S.O., 1945).

The U.K. maintained a larger proportion of her man-power in the armed forces than the U.S. Though the differential diminished as the war progressed, it was never quite eliminated. Canada's position was broadly similar to that of the U.S., while other sections of the Empire, and particularly the two southern Dominions, followed the U.K. pattern. The mobilization in the armed forces in the U.K.—and in Australia, New Zealand and India—was far in excess of what could be supplied from domestic capacity for production of munitions. The gap was bridged by the receipt of munitions supplies from the U.S. and Canada, since 1942 almost entirely

TABLE 1

Strength of the Armed Forces

At June	U.K.			U.S.		
	Total labour force, millions	Armed forces		Total labour force, millions	Armed forces	
		Strength, millions	% of labour force		Strength, millions	% of labour force
1939						
1941	21·0	0·56	3	51·8	0·36	0·7
1943	22·7	3·76	16½	54·0	1·75	3
1944	23·8	5·08	21	60·8	9·17	15
... ...	23·5	5·24	22	62·2	11·53	18½

Notes.—From *The Impact of the War on Civilian Consumption* (H.M.S.O., 1945), Table 3. Total labour force includes unemployed but (in the U.S.) excludes domestic servants. U.K. armed forces include full-time civil defence. All figures relate to June except U.S. armed forces and U.S. agricultural employment which are yearly averages.

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under lend-lease and mutual aid. As part of the combined plan, the U.S. (and Canada) put smaller armed forces in the field, but provided more munitions.

However, even with this large and relatively assured supply of military stores from North America, the U.K. mobilized a greater proportion of civilians for war work than did the U.S. One reason was that the U.K. was in the front line of the war and needed to devote some of her resources for purposes (*e.g.*, A.R.P. and repair of bomb damage) which the U.S. was spared. Another factor was the lower productivity in munitions production, as in manufacturing as a whole, in the U.K., requiring a large labour force to supply armed forces on the same scale.

TABLE 2

War Mobilization of the Labour Force at June 1944

	U.K.		U.S.	
	Millions	Per cent.	Millions	Per cent.
Armed forces	5·2	22	11·5	18½
War employment	7·8	33	13·4	21½
Other employment	10·4	45	36·3	58
Unemployed	0·1	*	1·0	2
Total labour force	23·5	100	62·2	100

Notes.—From *The Impact of the War on Civilian Consumption*, Table 4 and Appendix XIII. Total civilian employment is allocated between direct and indirect war work and other work. Other notes as in Table 1.

* Less than 0·5 per cent.

Even this does not tell the whole story. By D Day in mid-1944, the U.K. had 55 per cent. of her labour force on war duties, while the U.S. had 40 per cent. To achieve this the U.K. had far less resources of man-power to draw upon than had the U.S., and hence the diversion from employment for the civilian home market (and non-war export) was correspondingly higher in the U.K. The war-induced increase in the occupied population was of comparable size in the two countries. The U.S., however, had a much larger pre-war pool of unemployed to draw upon, and could count upon a natural increase in the labour force due to population growth which was not available to the U.K. Consequently, as Table 3 shows, the U.S. mobilization was achieved by diverting only 6¼ millions from civilian employment (including non-war exports) to the war sector. Over 7 millions were so diverted in the U.K. between 1939 and 1944, in addition to about 1½ millions already switched from peace to war production in 1939.

TABLE 3

Sources of Man-power Mobilized for War

(Millions)

	U.K.	U.S.
Peace-time defence employment	0·5	0·5
1939: Diversion from home civilian and export employment	1·5	—
1939–44: War-induced increase in labour force	2·5	7·0
Natural increase in labour force	—	3·4
Decrease in unemployed	1·3	7·75
Diversion from home civilian and export employment *	7·2	6·25
Total on war work at June 1944	13·0	24·9

Notes.—From *The Impact of the War on Civilian Consumption*, p. 9 and Appendix XIII. Peace-time defence employment is a round figure. Total war employment in the U.K. at mid-1939 is estimated at 2 millions, 0·56 million in the armed forces and civil defence and the balance on production of military supplies. See also Saunders, "Manpower Distribution, 1939–45: Some International Comparisons," *Manchester Statistical Society*, February, 1946.

* Residual item.

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The high degree of British mobilization for war was made possible only by the importation into the U.K. of food and other consumption goods, and of raw materials for the industrial machine, on as large a scale as shipping would permit. This is the second way in which the U.S. and Canada helped, very largely after 1941 under lend-lease and mutual aid. Other parts of the Empire added their contributions of food and raw materials, but the extra supplies under lend-lease and mutual aid were the marginal and perhaps decisive factor without which the degree of mobilization could not have been maintained.

The "division of labour" was not all in one direction. The U.K. and other Empire countries have supplied the U.S. with all kinds of goods and services under reciprocal aid. The supplies were mainly for U.S. troops in the European and Pacific theatres, supplies more economically furnished locally than by shipment from the U.S. Other goods were sent under reciprocal aid to the U.S. as a contribution to the American war effort at home.

All that can be entered in a mutual-aid account, expressed in value terms, is the transfer of goods and of ordinary services such as shipping. This is the account which will be considered in the following analysis, and which inevitably shows a large balance in favour of the U.S. To complete the account, we have to set the extra munitions and other supplies provided by the U.S. against the services of British fighters and war workers maintained by the extra supplies—and to take into consideration the balance of other mutual services in the conduct of the war. As we have indicated, all this has been assessed, not unfairly, in the lend-lease settlement.

U.S. Lend-lease Aid. Between March 11, 1941, and August 31, 1945, total lend-lease aid furnished by the U.S. to all countries amounted to $43,600 millions. This total comprises the value of all goods transferred and of all services rendered which could be accounted for in the period. It excludes $700 millions for production facilities constructed in the U.S. with lend-lease funds and $1,300 millions for other expenditures out of lend-lease moneys not charged to foreign governments. The estimate is on a revised basis of reporting by the U.S. State Department (Office of the Foreign Liquidation Commissioner), successor to the Foreign Economic Administration. The changed basis is described in the *21st Report to Congress on Lend-lease Operations* (January 1946), which, together with the earlier reports in this quarterly series, is the main source of the data used in the following account.

The total of $43,600 millions approximates quite closely to the value of all lend-lease aid during the complete period of operation, excluding only what was currently or subsequently paid for in cash. On the one hand, a small part of the total, amounting to around $1,000 millions, consisted of goods and services provided on a cash reimbursible basis to Canada, France, the Netherlands and other countries. This should properly be subtracted from the figure of $43,600 millions. On the other hand, certain additions should be made to the total. Lend-lease aid was continued to V-J Day, so that the period used here falls one day short of the full length of lend-lease operations. All goods transferred, though not necessarily exported, and all services rendered by V-J Day are counted as "straight" lend-lease. Goods in process of production out of lend-lease funds at V-J Day, or awaiting transfer on that day, constitute the lend-lease "pipeline," and subsequent transfers of such goods are against cash payment from the foreign government concerned, usually under an off-setting arrangement with reciprocal aid to the U.S. Certain exceptions have been made to this rule, straight lend-lease being continued to China and Belgium and for some shipping charges for a limited time after V-J Day. All this amounts to a small addition to the figure of $43,600 millions, an addition which may be taken as counterbalancing approximately the deduction for cash reimbursement of lend-lease.

An analysis of the total is given in Table 4 which shows that lend-lease aid to the British Empire amounted to a little over $30,000 millions as compared with less than $11,000 millions to Russia and less than $3,000 millions to all other countries together. A very small part of the British Empire total represents munitions and other goods paid for in cash by Canada. However, a significant proportion, perhaps more than one-quarter, of the figure recorded for countries other than the British Empire and Russia is lend-lease aid on a cash reimbursible basis. As a round figure for lend-lease proper during the whole period of operation, 70 per cent. of all lend-lease aid from the U.S. was rendered to the British Empire.

Of the total of $30,000 millions of lend-lease aid to the British Empire, $2,100 millions represents naval and merchant ships transferred to the British flag and not properly allocable to individual countries within the Empire. Some of these vessels were lost during the course of the war. The others are, generally speaking, returnable to the U.S., under the conditions of the

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original transfer, now that the war is over. The figure recorded for all the ships in the lend-lease account is the full cost of construction (if new) or the current value at transfer (if used); it is neither a valuation of the services rendered by the ships during their war service under the British flag, nor the original cost less depreciated value on return. To attempt to estimate "value received" more closely is not worth while for present purposes, and the recorded figure can be left as it is for what it is—namely, the original cost or value to the U.S. of all vessels transferred under lend-lease terms to the British flag for war service.

A total of $24,600 millions of munitions, raw materials, food and manufactured goods was transferred to the British Empire. Most of these goods will appear in the records of U.S. lend-lease exports as shipped from the U.S. to the U.K. and other British countries or as taken direct to war theatres overseas. There remain, however, some transfers under lend-lease which do not get recorded as U.S. exports. Munitions and other stores sent from the U.S. on government vessels—*e.g.*, lend-lease aircraft taken aboard R.N. carriers in U.S. ports—are not shown as exports. Other lend-lease goods are used or otherwise retained in the U.S. and lend-lease transfers include some material of foreign origin handed over without passing through the U.S. at all.

The official records of lend-lease transfers, unlike those of lend-lease exports, do not differentiate between countries within the Empire, and it is quite impossible to unscramble them because of the complications of transfer and re-transfer between one Empire country and another. An approximate, and necessarily rather rough, method is therefore adopted in Table 4 to distribute transfers according to country of destination or intended destination. The method is explained in the notes to the Table, but consists essentially of using the proportionate distribution of exports each year to give the distribution of transfers, thus sweeping in *pro rata* those transfers which do not appear as U.S. exports, and also allowing for changing amounts held in the U.S. awaiting export.

Distribution by country of destination, however, does not necessarily imply that the goods should be accounted to the corresponding Empire government. The three Dominions can be

TABLE 4

U.S. Lend-lease Aid to August 31, 1945

($ millions)

	1941, March–Dec.	1942	1943	1944	1945		Total
					Jan.–June	July–Aug.	
Aid to British Empire:							
Ships (sail-away)	65	195	1,078	540	160	69	2,107
Munitions destined for:							
U.K.	86	987	2,797	3,807	822	149	8,648
Australia	8	152	280	225	180	54	899
New Zealand	—	52	58	21	8	5	144
South Africa	—	40	88	55	10	1	194
India	8	230	371	555	227	31	1,422
Colonies	8	74	129	89	23	2	325
Other war theatres	76	610	1,205	1,349	493	169	3,902
Other goods destined for:							
U.K.	576	1,404	1,782	2,405	1,094	181	7,442
Australia	6	83	165	167	52	10	483
New Zealand	1	17	35	28	11	3	95
South Africa	—	20	29	18	—	—	67
India	1	87	175	295	157	51	766
Colonies	2	20	32	75	97	9	235
Services	245	786	807	1,137	270	99	3,344
Total aid to British Empire	1,082	4,757	9,031	10,766	3,604	833	30,073
Aid to Russia	20	1,376	2,436	4,074	2,169	595	10,670
Aid to other countries	...	...	...	...	...	...	2,872
Total lend-lease aid	...	...	...	...	...	...	43,615

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Notes.—Data relate to goods transferred and services rendered under lend-lease, as recorded by F.E.A. and published in the Reports to Congress on Lend-lease Operations made quarterly by the President. Certain major revisions were made in the cumulative returns after June, 1945. Aid to the British Empire shown in this table agrees with the revised F.E.A. figures cumulatively to June 30, 1945, and to August 31, 1945. The analysis by periods to mid-1945 is obtained from differences in F.E.A. cumulative figures on the unrevised basis, except for adjustments in services and in industrial products (metals and machinery) arising from the revision. All the F.E.A. revisions in the cumulative figure for aid to Russia (the net effect being not larger than $50 millions) are carried in the July–August 1945 entry in this table. The analysis by periods must therefore be accepted with caution. No attempt is made to allocate aid to other countries by periods since the F.E.A. revisions are mainly in this category.

The classification adopted is as follows:—

Munitions: aircraft, ordnance, combat and motor vehicles, and related equipment; small watercraft and naval equipment; signal, engineer, quartermaster, medical, chemical warfare and other military stores.

Ships (sail-away): naval and merchant vessels delivered under their own power.

Other goods: agricultural products, raw materials, petroleum and manufactures.

Services: ship repair, freights, ships' stores and other shipping services; ferrying of aircraft; training of personnel; storage and transportation; materials and charges for construction of bases; miscellaneous expenses.

This follows the F.E.A. classification, except that miscellaneous military stores (signal, engineer, etc.) are estimated and transferred from "other goods" to "munitions." Ships (sail-away) are separated from other items in the F.E.A. category of "watercraft" by the use of export records.

Allocation of transfers of munitions and other goods by country of actual or intended destination is based on export records (U.S. Department of Commerce for F.E.A. as published in the President's quarterly Reports to Congress on Lend-lease Operations). Transfers of munitions, except for the category of miscellaneous military stores (signal, engineer, etc.) are allocated to Empire countries and to Egypt and Sudan according to exports each year with allowance for the amounts held in the U.S. awaiting export (assumed to be one month's supply). The balance of the transfers is thrown, with those allocated to Egypt and Sudan, into the category of "other war theatres." This includes munitions on British account for direct export to N. Africa, Italy, N.W. Europe and other theatres, and also munitions exported on government vessels such as aircraft put aboard R.N. carriers in U.S. ports. Total transfers of miscellaneous military stores are distributed by countries in proportion to other munitions. Transfers of other goods are allocated by countries in proportion to exports, agricultural produce and industrial products being handled separately. The first allocation of industrial products includes miscellaneous military stores, which are then switched (distributed as indicated) from this category to munitions.

Lend-lease exports to Canada are included with the U.K. throughout. Since Canada did not receive lend-lease aid, these exports are of four kinds:—

(*a*) goods exported from the U.S. for shipment through Canada to other destinations, mainly the U.K.;

(*b*) supplies for use by the U.K. in Canada;

(*c*) components sent for incorporation in Canadian production on U.K. account, or material in substitution for such components;

(*d*) stores purchased by Canada through lend-lease channels on a cash reimbursement basis.

The last category, which is small, should properly be excluded from lend-lease transfers; it is here swept in with the other categories in transfers to the U.K.

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charged, as a first approximation, with the amounts shown as destined for export to them. This ignores such lend-lease supplies on their account as went direct to their forces serving abroad. Against this, lend-lease goods retained in the U.S. or shipped on government vessels are probably largely on U.K. account, but are here allotted *pro rata* to all Empire countries. The approximation may represent a slight understatement of the indebtedness of the Dominions under lend-lease, but the error must be so small as to be safely ignored in the present rough allocation. India raises more troublesome problems which cannot be gone into here in any detail; indeed, the accounting between the U.K. and Indian governments for war stores, including lend-lease, is a complicated process not yet completed. As a rough shot, simply to round off the picture, we have taken here one-third of munitions and of industrial products (including petroleum) and two-thirds of agricultural produce sent to India under lend-lease as on Indian Government account with the remainder charged to the U.K.

There remains a sum of rather more than $3,500 millions charged to the British Empire for services rendered under lend-lease. Almost $3,000 millions of this total consists of shipping services of one kind or another, mainly freight charges, but also including ship repairs and ships' stores. The balance is divided not unevenly among four other categories of services: the ferrying of aircraft, other transportation and storage charges, the training of personnel and charges in connection with the construction of bases. Practically the whole of the total for

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services, therefore, can be regarded as an addition to the value of goods transferred to cover the cost of transportation from the point of origin within the U.S. to various parts of the world. As a first approximation we can take a flat percentage addition, which comes to about 13½ per cent., to the value of goods destined for each area. There is, indeed, a longer haul to Australia, New Zealand or India, and more exports to these areas were carried in U.S. bottoms than to the U.K. Against this, some services, such as the ferrying of aircraft, should be much more largely attributed to the U.K.

TABLE 5

U.S. Lend-lease Aid to the British Empire to August 31, 1945

($ millions)

Government account	Ships (sail-away)	Munitions	Other goods	Services	Total
U.K.	2,107	13,823	8,113	2,980	27,023
Australia	—	899	483	188	1,570
New Zealand	—	144	95	32	271
South Africa	—	194	67	35	296
India	—	474	330	109	913
Total	2,107	15,534	9,088	3,344	30,073

Notes.—From Table 4, with allocations by countries as described in the text. Services are allocated in proportion to the total of goods (munitions and other goods) attributed to the countries. In particular, the figure of $913 millions for India is derived on the rough basis indicated in the text from an overall figure of $2,486 millions of goods destined for India (including $298 millions for shipping services). What portion of this should ultimately be taken on Indian or U.K. government account is an issue which this paper does not attempt to prejudge.

The results of these allocations of lend-lease aid to various Empire countries are assembled in Table 5. The Indian figures are rough estimates, but the others are sufficiently reliable for present purposes. The U.K. is charged with $27,000 millions,* or about 90 per cent., of the total lend-lease aid to the British Empire. The U.K. figure comprises the value of all ships, at original cost to the U.S., transferred to the British flag, the value of all goods destined for the U.K., the colonies and war theatres overseas, a part of the value of goods destined for India and the appropriate proportion of services rendered under lend-lease.

The total of lend-lease aid to the British Empire and its allocation by countries, as given in Table 5, are very close approximations to the amounts of aid received during the whole period of lend-lease. A small amount should be deducted from the U.K. figures for munitions and other stores purchased by Canada but procured out of lend-lease funds (see notes to Table 4). On the other hand, small amounts should be added throughout for transfers on September 1, 1945, to complete the period to V-J Day, and for shipping freights continuing for a short time after V-J Day.

Table 4 also shows lend-lease aid by years. As estimates of actual transfers made year by year, these figures are less accurate than the totals for the whole period. They are obtained as differences of cumulative totals reported at various dates, and so represent transfers reported in the year, together with the net adjustments made in the year on the value of transfers previously reported. The revised form of the cumulative totals after June 30, 1945, has also necessitated some small adjustments in the previous figures for aid to the British Empire; these revisions are confined to the figures for industrial products and for services. The corresponding revisions in the data for aid to Russia are quite small, and are carried in the entry for July–August 1945. The main effect of the changed basis of reporting is in aid to other countries, and no attempt is made in Table 4 to allocate this lend-lease figure by years. The yearly data of Table 4, therefore, can be relied on for a broad picture, but should not be subject to very fine analysis.

The timing of lend-lease aid to the British Empire clearly corresponds to the development

* An estimate of $26,000 millions has been given for lend-lease aid to the U.K. in a U.S. Senate report (Senate Document, 79th Congress, 2nd Session, Report No. 110, Part 5, p. 33). No indication is given of how this rather lower figure is derived.

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of war production in the U.S. and to the general course of the war. After small beginnings in 1941, when the U.S. indeed had little to send even to embattled Britain, the flow of lend-lease goods and services mounted rapidly to $9,000 millions in 1943 and to a peak rate of a little over $12,000 millions a year just before D Day in June 1944. A year later the flow had decreased to scarcely more than half the peak rate. The supply of munitions and other forms of lend-lease aid were well timed to bring maximum force to bear against Germany in the months following the invasion of Europe. The three Dominions received only a small proportion, a little over 7 per cent., of total lend-lease aid to the British Empire. It is significant, however, that the greatest flow of munitions to Australia occurred during the last months of lend-lease, when Australia was a base from which operations against Japan were conducted.

In contrast, lend-lease aid from the U.S. to Russia was built up more slowly in 1942 and 1943 when a good deal of the burden of supplying Russian forces fell on Britain. U.S. assistance, however, was longer sustained. Lend-lease supplies to Russia were at the rate of $4,000 millions per year by mid-1944 and did not fall below this level until V-E Day in May, 1945. A peak rate of nearly $5,000 millions a year was reached early in 1945 to lend force to the final Russian blows against Germany. Altogether, lend-lease aid to Russia amounted to more than one-third of aid to the British Empire.

Lend-lease was never intended to sweep in all British dollar transactions. In the first place, it did not take over contracts for munitions placed by the British government in the U.S. before March, 1941, payments and deliveries on which were not completed until the end of 1943. In round figures, deliveries actually taken by the British on these contracts came to $1,400 millions in 1940–1 and to $700 millions in 1942–3.* Secondly, there were numerous items not eligible for lend-lease or too small and troublesome to be obtained through the machinery of lend-lease procurement. The extent to which dollar payments for U.S. goods were reduced to a "hard core" of items ineligible or unsuitable for lend-lease is seen in Table 6. This shows "cash" exports from the U.S. to the British Empire, excluding Canada and Newfoundland, provisionally obtained as the difference between two sets of figures, only approximately comparable, for total and lend-lease exports. The bulk of the deliveries from British munitions contracts in the U.S. is included in these figures, but not all, since some deliveries were retained in the U.S., sent to

TABLE 6

U.S. "Cash" Exports to the British Empire, 1939–45
(excluding Canada and Newfoundland)

($ millions)

Exports to:	1939	1940	1941	1942	1943	1944	First half of 1945 (annual rate)
U.K.	505·4	1,010·8	1,064·7	*	*	143·7	263·8
Australia	61·6	75·5	78·0	73·1	27·4	25·3	35·6
New Zealand	16·5	18·2	29·2	12·8	11·4	13·4	13·6
South Africa	70·1	105·4	188·4	49·7	48·8	68·9	119·0
India	42·8	68·4	89·6	90·6	29·5	47·9	69·2
Colonies	75·2	82·6	156·6	71·5	49·4	78·0	87·0
Total	771·6	1,360·9	1,606·5	*	*	377·2	588·2

Notes.—Provisional estimates obtained by difference from U.S. Department of Commerce returns of total exports and of lend-lease exports, which are approximately but not exactly comparable. Australia includes New Guinea, South Africa includes Southern Rhodesia and Colonies comprise Burma, colonies, protectorates and mandates.

* In 1942–43, total exports data do not incorporate corrections made in lend-lease returns—*e.g.*, charges on British copper refined in the U.S. The corrections are significant only for exports to the U.K. The recorded difference between total and lend-lease exports to the U.K., about $450 millions in the two years, cannot be taken as an accurate measure of cash exports in these years.

* Including capital payments ($100 millions) but excluding deliveries diverted to the U.S. ($400 millions). These are munitions contracts only; there were additional government contracts for steel, machine-tools, etc. The rough estimates given here are based on the records of the British Supply Missions in U.S.

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Canada or to non-Empire destinations. A rough calculation indicates that the following (in $ millions):

1939—750; 1940—1,050; 1941—1,050; 1942—250; 1943—150; 1944—350

can be taken for cash exports exclusive of munitions from British government contracts.

It is clear that British purchases in the U.S. increased in 1940–1 over 1939. Excluding munitions, and allowing for increased prices, we can estimate the rise at around one-quarter. The largest increases were in exports to South Africa, West Africa, Burma and Malaya. After 1941 the switch to lend-lease was rapid in its effect on cash exports to the U.K., but slower for other exports, particularly to Australia, India and the colonies. (The fall in cash exports to the colonies is the result of the Japanese conquest of Hong-Kong, Malaya and Burma.) The hard core was reached in 1943, and then, as the end of the war approached, more items were made ineligible for lend-lease, and cash exports rose steadily throughout 1944 and 1945. Even so, in the first half of 1945 U.S. exports to the U.K. paid for in dollars were only one-half the 1939 rate in value, and perhaps nearly 40 per cent. in volume. The two southern Dominions were also taking, for cash, less from the U.S. than before the war. By V-E Day, U.S. cash exports to other British countries were well above the 1939 rates in value, and somewhat above in volume. The normal sources of imports into these areas had been greatly cut down and, outside the U.K. and the southern Dominions, shipping was less of a limiting factor.

Table 7 summarizes the position in round figures. The two years 1940–1, between the end of the Neutrality Act limitations and the entry of the U.S. into the war, represent the period of British cash purchasing in the U.S. and the beginnings of lend-lease. The following three and

TABLE 7

U.S. Supplies of Munitions and Other Goods to the British Empire, 1940–45
(excluding Canada and Newfoundland)

($ millions)

	1940–41	1942–44 and first half 1945
Munitions: for cash	1,400	700
under lend-lease	200	14,900
Other goods: for cash	2,100	1,000
under lend-lease	600	8,200

Notes.—Munitions for cash are deliveries on British cash contracts in the U.S. Other goods for cash are exports. Munitions and other goods under lend-lease are transfers, as recorded by F.E.A. The figures for munitions are those of Table 17, excluding Canadian purchases and ship construction and repair under lend-lease.

a half years, from the beginning of 1942 to mid-1945, saw the U.S. at war and lend-lease operating at full blast. In this period all but 5 per cent. of munitions supplies and all but 11 per cent. of other goods obtained by the British from the U.S. were under lend-lease.

British Reciprocal Aid. Public imagination has been caught by the original idea of U.S. lend-lease and by the sweeping nature of its operations. Reciprocal aid from the British Empire came later, and has been largely overlooked by the public, particularly in the U.S. Reciprocal aid accounts, by the very nature of the constituents, are more difficult to maintain, and there are gaps and some overlapping in the records of the various British countries furnishing aid.

Reciprocal aid was provided by the U.K. Government to the U.S. following the Mutual Aid Agreement of February 23, 1942 (Cmd. 6341). This agreement provides that, in contributing to the defence of the U.S., the U.K. government "will provide such articles, services, facilities or information as it may be in a position to supply." The flow of reciprocal aid had begun by the middle of 1942. It consisted of supplies and services of all kinds to U.S. forces stationed in the U.K. and in overseas theatres, and of the construction of military installations for use by these forces. At first, apart from small shipments (*e.g.*, of benzol) to the U.S., reciprocal aid was confined to the local servicing of U.S. troops. Later on, before the end of 1943, reciprocal aid was extended to the furnishing of raw materials and foodstuffs, mainly from colonial territories, for

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shipment to the U.S. An analysis of U.K. reciprocal aid provided to the U.S. up to V-J Day is given in Table 8, based on data assembled and issued by H.M. Treasury.

A total of around £1,200 millions was supplied to the U.S. under reciprocal aid, both from the resources of the U.K. and from abroad, but all at the expense of the U.K. government. As with U.S. lend-lease, the analysis of total reciprocal aid into various periods (Table 8) depends on differences of cumulative totals, and so represents aid reported in each period together with adjustments in amounts previously reported. However, several major adjustments have been

TABLE 8

U.K. Reciprocal Aid to September 1, 1945

(£ millions)

	To June 30, 1943	Year ending June 30, 1944	Year ending June 30, 1945	July 1–Sept. 1, 1945	Total
Aid to U.S.:					
In U.K.:					
Military stores	46·3	61·6	74·1	7·6	189·6
Petroleum	5·4	58·9	135·1	14·2	213·6
Food	0·1	8·1	8·9	2·0	19·1
Services	41·9	108·3	128·8	18·5	297·5
Construction	92·0	100·9	19·6	6·4	218·9
In U.S.:					
Military stores	17·6	5·6	1·5	0·5	25·2
Food and materials	2·3	19·4	37·1	6·4	65·2
Miscellaneous and Services ...	4·5	2·5	0·2	0·1	7·3
In Overseas Theatres:					
Military stores	16·7	29·2	24·5	2·5 *	72·9
Petroleum, India	2·4	12·0	38·3	8·0 *	60·7
Petroleum, Other	—	6·5	13·2	2·8	22·5
Construction	0·5	7·9	0·3	—	8·7
Total aid to U.S.	229·7	420·9	481·6	69·0	1,201·2
Aid to Russia	187·7	93·3	27·0	4·0	312·0
Aid to other countries	...	...	...	...	382·8 †
Total reciprocal aid	...	...	...	...	1,896·0

Notes.—From White Papers on Mutual Aid (Cmd. 6483, 1943 and Cmd. 6570, 1944) and cumulative data subsequently made available by H.M. Treasury. Figures of petroleum provided in India are from U.S. Army records supplied to F.E.A., not from Treasury returns; two-thirds of the U.S. Army's valuations are taken as landed cost of petroleum met on U.K. Government account, the remaining one-third representing handling charges on Indian government account. Additions, deductions and revisions were made in the Treasury cumulative returns from time to time. The most important of these are here carried back or spread over the periods to which they properly relate. For example, £27·5 millions for the Mulberry Harbour transferred to the U.S. was first reported in the cumulative returns to September 1, 1945, but is here included under construction in the year ending June 30, 1944. Other major additions carried back include construction and building maintenance by the War Office and military stores transferred in the U.S. Deductions for previous incorrect reporting, here carried back, include aircraft reported transferred in the U.K. but actually transferred in N. Africa, clothing and other stores for combined operations first classed as reciprocal aid and an over-valuation of petroleum transfers in the U.K. Less important revisions are carried in the periods in which they were reported.

The classification follows closely that for U.S. lend-lease (Table 4). Military stores include all munitions and other stores for direct use by foreign forces, with the exception of petroleum and food. Services are mainly transportation, but also include building maintenance. The construction figure represents the cost to the Service Departments of the construction of military installations, and includes the whole cost of the Mulberry Harbour.

Aid to the U.S. in the U.K. comprises all transfers of goods and services to U.S. forces in the U.K., but shipping services and petroleum bunkers are on a world-wide basis. Aid in the U.S. includes both goods transferred in the U.S. (*e.g.*, from British cash contracts) and food and materials exported from U.K. and the colonies to U.S. Aid in overseas theatres includes transfers and construction in the colonies, as well as in N. Africa, Egypt, Italy, N.W. Europe and other war theatres.

* Estimated in the absence of full returns.

† Incomplete, obtained by summing returns given cumulatively for different countries to various dates from March 31 to September 1, 1945.

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made in the figures for various periods. The more important additions, deductions and revisions to the reported cumulative totals of aid to the U.S. have been carried back to the periods to which they properly relate. For example, the figure of £100·9 millions for construction in the U.K. in the year ending mid-1944 includes £27·5 millions for the Mulberry Harbour transferred to the U.S., though this item was not reported until later.

It is clear that reciprocal aid was timed to meet the needs of U.S. forces and to maximize the combined impact against the German enemy in Europe. Goods and services rendered to the U.S. were built up to a rate of about £500 millions a year by the middle of 1944, and sustained at this rate until early in 1945 before falling off slowly. The nature of reciprocal aid gradually shifted, construction work declining and being replaced increasingly by current military stores, petroleum and shipping services as the military campaigns developed.

Reciprocal aid was also extended by the U.K. to Russia, China, European Allies, Portugal and Turkey. The formal agreements and arrangements with these countries were made later than that with the U.S., but they were retroactive to include credits granted earlier, in some instances before the Lend-lease Act itself. Supplies to Russia, dating back to the autumn of 1941, amounted to £310 millions by V-J Day in 1945. The equipment and maintenance of the forces of Allied countries over-run by the enemy were undertaken from the summer of 1940 onwards, at British expense where the governments concerned (*e.g.*, Czechoslovakia, Poland, Greece) were not able to finance themselves. Credits were advanced at first for this purpose, but these were later taken over and extended under mutual aid arrangements. A total of more than £380 millions was provided by V-J Day as reciprocal aid to governments other than the U.S. and Russia. Altogether, reciprocal aid from the U.K. to countries other than the U.S. was at least as large a part of total reciprocal aid as lend-lease from the U.S. to non-British countries was to total lend-lease. In round figures, U.S. lend-lease aid was divided 70 per cent. to the British and 30 per cent. to other countries, while U.K. reciprocal aid was split 65 per cent. to the U.S. and 35 per cent. to other countries.

The U.S. has also received reciprocal aid from the governments of Australia, New Zealand and India. Estimates of the amounts involved, converted to £ sterling at official rates of exchange, are shown in Table 9. The returns on which the figures for the Dominions are based were furnished by the respective governments to F.E.A. for incorporation in the President's quarterly reports to Congress. The government of India has not released data on Indian reciprocal aid and the figures shown in Table 9 are assembled from other sources. Estimates of the value of exports from India to the U.S. under reciprocal aid are made from the records of British Supply Missions in Washington and the valuations of reciprocal aid in India are taken from U.S. Army returns to F.E.A., as published in the President's quarterly reports. Since they are not derived from Indian government records, the estimates for India in Table 9 must be accepted with some caution.

As for the U.K., the assistance consisted primarily of supplies, services and construction for U.S. forces serving locally—here, in the Pacific and India–Burma–China theatres. In 1944, however, India began to ship raw materials and foodstuffs direct to the U.S. under reciprocal aid and continued shipments until the middle of 1945. In three years, from mid-1942 to mid-1945, the three governments supplied aid to the U.S. to a total amount of approximately £380 millions sterling.

Reciprocal aid from the British Empire to the U.S. in the whole period up to V-J Day is summarized in Table 10. In addition to the data shown in Tables 8 and 9, the totals of Table 10 include estimates for reciprocal aid from Empire countries other than the U.K. in the last two months before V-J Day. Total reciprocal aid to the U.S. amounted to £1,600 millions in the whole period. The U.K. financed £1,200 millions of this total, or about three-quarters. As we have seen (Table 5), the U.K. can be appropriately charged with a larger proportion, about 90 per cent., of total lend-lease aid from the U.S. to the British Empire.

Comparisons of Lend-lease and Reciprocal Aid. Lend-lease aid has been shown as valued by the U.S. in dollars, and reciprocal aid as valued by Empire countries (except in the case of India) and converted to £ sterling at official rates of exchange. Any direct comparison raises the awkward, almost insoluble, question of the appropriate relation between the dollar and the £ sterling. The official rate of exchange has no relevance to this problem. What is required is *either* an estimate of what lend-lease aid to the British Empire would have cost if the goods had been pro-

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TABLE 9

Other Empire Reciprocal Aid to the U.S. to June 30, 1945

(£ millions sterling)

	Year ending June 30,			Total
	1943	1944	1945	
Australia: Military stores	15·1	22·7	19·2	57·0
Food	9·7	19·8	26·1	55·6
Services	7·4	30·0	24·4	61·8
Construction	16·3	14·0	2·3	32·6
Total	48·5	86·5	72·0	207·0
New Zealand: Military stores	1·3	4·1	4·0	9·4
Food	3·1	9·9	14·8	27·8
Services	3·0	2·5	1·5	7·0
Construction	5·2	1·1	0·3	6·6
Total	12·6	17·6	20·6	50·8
South Africa: Total	—	0·1	0·1	0·2
India: Military stores	1·0	4·6	14·8	20·4
Petroleum	1·2	6·0	19·2	26·4
Food	1·0	4·4	5·0	10·4
Services	0·7	5·0	10·5	16·2
Exports to U.S.	—	3·0	12·4	15·4
Construction	7·8	11·6	13·1	32·5
Total	11·7	34·6	75·0	121·3

Notes.—Australia, New Zealand and South Africa—from Dominion records supplied to F.E.A. and reported in the President's Reports to Congress on Lend-lease Operations. India—exports to U.S. (raw materials and food) estimated from records of British Supply Missions in the U.S.; other estimates from U.S. Army records supplied to F.E.A. Transfers of petroleum in India were on U.K. government account as to landed cost and on Indian government account as to handling charges in India. The figures given here represent the latter only, taken as one-third of the total valuation of petroleum transfers recorded by the U.S. Army. Conversions from local currencies to £ sterling at official exchange rates.

duced and the services rendered in British countries, *or* an evaluation of what reciprocal aid to the U.S. would have cost at the U.S. prices used in the accounting of U.S. lend-lease aid.

For munitions and military stores (excluding food), an item-by-item comparison of unit costs in the U.S. and the U.K. indicates that an appropriate average rate of conversion is around $7

TABLE 10

British Reciprocal Aid to the U.S. to September 1, 1945

(£ millions sterling)

Government account	Construction	Military stores	Petroleum	Other goods	Services	Total
U.K.	227	288	297	90	299	1,201
Australia	33	59	*	60	64	216
New Zealand	7	10	*	30	7	54
South Africa	*	*	*	*	*	*
India	34	24	30	27	19	134
Total	301	381	327	207	389	1,605

Notes.—From Tables 8 and 9 with the addition of estimates for reciprocal aid from Australia, New Zealand and India in the period July 1 to September 1, 1945.

* Less than £0·5 millions.

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to £1. The rate is high because, quite apart from differences in the real costs of munitions production, the method of valuing output of munitions is very different in the two countries. In general, strict cost-accounting methods in the U.K. kept down the prices paid for munitions and other stores on government contract. In the U.S., the Service Departments used "standard" dollar costs, changed from time to time, in the evaluation of their military procurement, and hence in their estimates of lend-lease aid. These costs are not the same as, and generally are slightly in excess of, actual expenditures. More important, U.S. munitions costs and expenditures are inflated in comparison with those in the U.K. because of the high level of profits in the munitions trades and of the "cost plus" methods of contracting often adopted in the U.S. Throughout most of the war the main job in the U.S. was to get munitions quickly without regard to expense. It was only relatively late in the war that re-negotiation of contracts had any significant effect on U.S. munitions costs.

The rate of $7 to £1 for munitions and other military stores is an average of a very wide range of individual rates obtained from the relation of U.S. to British unit costs for comparable

TABLE 11

Comparison of Lend-lease Aid to the British Empire and Reciprocal Aid to the U.S. to V-J Day

Government account	In $ millions		In £ millions sterling	
	Lend-lease aid from U.S.	Reciprocal aid to U.S.	Lend-lease aid from U.S.	Reciprocal aid to U.S.
U.K.: Ships and construction	2,107	910	301	227
Military stores	13,823	2,014	1,975	288
Petroleum	1,850 *	1,187	462 *	297
Other goods	6,263 *	361	1,566 *	90
Services	2,980	1,195	745	299
Total	27,023	5,667	5,049	1,201
Australia	1,570	1,041	296	216
New Zealand	271	248	52	54
South Africa	296	1	53	†
India	913	610	178	134
Total	30,073	7,567	5,628	1,605

Notes.—From Tables 5 and 10. Conversion from dollars to £ sterling and conversely at $7 to £1 for Military Stores (including Ships) and at $4 to £1 for all other goods and services.
* Approximate division between petroleum and other goods.
† Less than £0·5 million.

items. The individual rates range from some as low as $3 to £1—for example, for many important types of military motor vehicles—to others as high as $10 or more to £1, as in the case of certain ship construction costs. An average of around $7 to £1 is obtained by weighting with the distribution of total production of munitions in the U.S. or with the distribution of supplies made available under lend-lease, the latter being a very fair representative selection from the whole of U.S. production. The same round average is obtained if the weighting is done with U.K. production of munitions. However, military stores provided by the U.K. under reciprocal aid differ in composition from total production in the U.K., being lightly weighted with combat types and more heavily with other forms of equipment. But there appears to be little correlation between the individual conversion rates and the types of stores most frequently provided under reciprocal aid and, at least as a first approximation, the average rate of $7 to £1 is appropriate to military stores in the reciprocal-aid account. We shall therefore take this rate for our conversion of mutual aid valuations of munitions and other military items from dollars to £ sterling, and conversely.

For other goods and for services only a fragmentary comparison of the U.S. and U.K. prices used in mutual-aid valuations is possible. The appropriate conversion rate, however, would seem to be not far from the official rate of exchange. We shall take here a round average rate

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of $4 to £1 for converting mutual aid values for goods other than military stores and for services. It will then be seen (Table 11) that the average conversion rate works out at a little over $5.3 to £1 for all lend-lease aid and at about $4.7 to £1 for all reciprocal aid. The latter figure is the lower because the proportion of munitions—relatively highly priced in U.S. lend-lease—is higher in lend-lease than in reciprocal aid. The first White Paper on Mutual Aid (Cmd. 6483, para. 30) assessed U.S.–U.K. costs at around $6 to £1 on the average, but this would appear to be an over-estimate of the rate. It must be stressed that, in using the conversion rates we have suggested, we are not rejecting the official exchange rate as a false rate, for example, in any comparison of present U.S.–U.K. costs of consumers' goods, but simply as an inappropriate rate for the particular valuations adopted in lend-lease and reciprocal aid.

Table 11 shows comparisons of lend-lease and reciprocal aid obtained both by translating lend-lease aid into comparable British prices and by evaluating reciprocal aid in comparable U.S. prices. It must be remembered that this table gives only an approximate allocation of U.S. lend-lease aid among British countries. Though there may be a slight under-statement of aid properly attributed to the Dominions, the greatest uncertainty is in the division between the U.K. and Indian governments. This does not greatly affect the total for the U.K., but it does make the amount attributed to India subject to a large percentage range of error. It must also be remembered that the figures for reciprocal aid from India are not obtained from or with the agreement of the supplying government.

The comparisons of Table 11, however, suffice for the broad conclusion that the British Empire provided reciprocal aid to the U.S. to the value of between 25 and 30 per cent. of lend-lease aid received from the U.S. The rate for the U.K. alone, however, was lower—between 20 and 25 per cent. In the two other main areas of mutual aid, the proportion of reciprocal aid was higher—about 70 per cent. in Australia and a very uncertain figure for India, which appears here as nearly 75 per cent. and may be greater. South Africa received relatively little lend-lease aid, but provided practically no reciprocal aid. Mutual aid between the U.S. and New Zealand was also on a small scale—New Zealand being a small country—but here there was an almost exact balance between the two sides of the account.

Apart from the U.K., therefore, lend-lease aid to the Empire was not very much larger than the reciprocal aid provided to the U.S. in return. An important factor, after Pearl Harbor, was the difficulty of shipping to and from India, Australia and New Zealand. It was never easy to get large shipments of lend-lease or other cargoes through to these areas or to move their normal exports. For the same reasons, it was reasonable and practical for these countries to supply U.S. forces on the spot, saving valuable shipping space from the U.S.

The balance in favour of the U.S. in the mutual aid account is primarily in the account with the U.K., and some further analysis of this is needed. Total lend-lease aid to the U.K. in nearly four and a half years of operation was around $27,000 millions, of which $2,100 millions was for ships, $13,800 millions for munitions, $8,100 millions for other goods (including nearly $2,000 millions for petroleum) and $3,000 millions for services. Against this, the U.K. provided about $5,700 millions worth of reciprocal aid in three and a quarter years—$900 millions for construction, $2,000 millions for munitions (in the broad sense of military stores), $1,200 millions for petroleum, $400 millions for other goods and $1,200 millions for services. In this comparison, however, we should make allowance on one side or the other for the value of certain aircraft originally paid for in dollars by the U.K. and later diverted to the U.S. An amount of around $250 millions was claimed by the U.K. on this score in the mutual aid settlement. The claim was rejected and the amount was "regarded as taken into account in the general settlement" (Cmd. 6778, Agreement II, para. 3). For our present purpose, the amount can be appropriately added to reciprocal aid, making the munitions figure $2,250 millions and the total almost $6,000 millions.

Reciprocal aid was furnished in a shorter period—roughly 3¼ instead of 4½ years—and by a country very much smaller in population and resources than the U.S. The relative strengths of the two countries is 2¾ to 1 by population and 3 or 4 to 1 in terms of industrial power. Reciprocal aid of about $6,000 millions is equivalent to perhaps $20,000 millions for a country the size of the U.S., and this comes within hailing distance of the $27,000 millions of lend-lease aid.

We can refine such measures of reciprocal aid in relation to lend-lease aid, allowing for the

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differing resources of the two countries without a conversion of dollars into pounds sterling or conversely.

U.S. lend-lease aid to the British Empire can be related to each of a number of more comprehensive U.S. figures, of which total war expenditures, national income and gross national product are three of the more readily available totals. Against a relation of this kind, we should

TABLE 12

Mutual Aid in Relation to War Expenditure and National Income

United States ($ millions)	1942	1943	1944	First half 1945	Total (3½ years)
War expenditure	47,600	81,100	86,400	43,300	258,400
National income	137,400	173,500	184,500	95,000	590,400
Plus depreciation, etc.	11,800	12,000	12,300	6,200	42,300
Minus net income from abroad	200	100	100	100	500
Gross national product (home produced)	149,000	185,400	196,700	101,100	632,200
Lend-lease aid to British Empire:					
Total	4,760	9,030	10,770	3,600	28,160
Home produced	4,400	8,500	9,890	3,010	25,800
Lend-lease aid in per cent. of:					
War expenditure	10·0	11·1	12·5	8·3	10·9
National income	3·5	5·2	5·8	3·8	4·8
Gross national product	3·0	4·6	5·0	3·0	4·1

United Kingdom (£ millions)	Year ending June 30,			Total (3 years)
	1943	1944	1945	
War expenditure	4,275	4,480	4,495	13,250
National income	7,935	8,315	8,425	24,675
Plus depreciation, etc.	475	475	475	1,425
Minus net income from abroad	130	120	105	355
Gross national product (home produced)	8,280	8,670	8,795	25,745
Reciprocal aid to U.S.: Total	230	421	482	1,133
Home produced	212	311	257	780
Reciprocal aid in per cent. of:				
War expenditure	5·4	9·6	10·7	8·6
National income	2·9	5·1	5·7	4·6
Gross national product	2·6	3·6	2·9	3·0

Notes.—Lend-lease aid from Table 4. Home produced lend-lease aid is total *less* all petroleum transferred and estimated foreign purchases transferred under lend-lease (*e.g.*, munitions from Canada and sugar from Cuba).

Reciprocal aid from Table 8, assuming (with only a small error) that aid to the U.S. began on July 1, 1942. Home produced reciprocal aid is total *less* all petroleum transferred, colonial produce exported to U.S., overseas construction and the overseas-produced part (estimated at one-quarter) of stores transferred by the War Office and Air Ministry in the U.S. and overseas war theatres.

War expenditure, national income and gross national product from *The Impact of the War on Civilian Consumption*, Appendix XII, where these concepts are adjusted to a broadly comparable basis for the U.S. and U.K. Estimates are added for the first half of 1945, and U.K. estimates for years ending June 30 are obtained by interpolation.

War expenditure is total government expenditure for war purposes *ess* indirect taxes; changes in privately-held inventories on government contracts are not included.

National income and gross national product are at factor cost, excluding indirect taxes, and exclude national debt interest. U.S. national income differs from the U.S. Department of Commerce series by the addition of corporate income taxes, net income from owner-occupied houses and subsistence of armed forces. U.K. national income differs from the official series (Cmd. 6623, 1945) only in minor respects—*e.g.*, by the addition of employers' social security contributions.

Gross national product is national income, as defined here, *plus* depreciation and similar allowances, *minus* net income from abroad. U.K. net income from abroad in 1945 is taken from *Statistical Material Presented During the Washington Negotiations* (Cmd. 6707, 1945), and estimated for other years on the basis of available data on overseas disinvestment and debt increase.

In the percentage figures, home-produced lend-lease and reciprocal aid are related to gross national product, while total lend-lease and reciprocal aid are used for the other relations.

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set British reciprocal aid to the U.S. similarly related to the corresponding war expenditures, national income or gross national product of the whole of the British Empire (excluding Canada and Newfoundland). This is not a practical proposition, since it is scarcely possible to define, still less to measure, such concepts as the national income for the colonies or India. It is better to concentrate on the U.K. alone, and relate reciprocal aid from the U.K. to the U.S. to war expenditures, national income or gross national product of the U.K. It should be noticed that it is not then appropriate to reduce the U.S. relation by taking lend-lease aid to the U.K. alone. The U.K. relation we aim at is, in fact, an indication of the relation for the Empire as a whole. Broadening the scope would increase both the numerator (reciprocal aid) and the denominator (*e.g.*, national income), and one such extension, to include Australia and New Zealand, is perfectly possible. It may be argued, however, that all lend-lease aid—to Russia and other countries as well as to the British—and similarly all reciprocal aid, should be taken into relations of the kind we have in mind. There is point to this, but it is in fact of little consequence. We have seen that the proportions of lend-lease and of reciprocal aid to third countries are roughly the same. If we establish a rough equivalence in mutual aid between the U.S. and the British Empire, then we have also established a similar equation for all lend-lease and reciprocal aid, at least within the margins of error with which we are inevitably confronted.

The data for the comparisons are set out in Table 12 for the period of $3\frac{1}{2}$ years between the entry of the U.S. into the war and the middle of 1945, ignoring only the small amounts of mutual aid provided in 1941 and in the last two months before V-J Day. It must be stressed that the relations we seek are not cut and dried. There is a large number of possible relations, all more or less appropriate; indeed, there are almost as many different definitions of (*e.g.*) national income as there are statisticians willing and able to provide estimates.

First, mutual aid can be related to total war expenditures, the latter being taken net of indirect taxes and the former being almost exclusive (by its method of estimation) of such taxes. For the whole period under review, U.S. lend-lease aid to the British Empire represented 11 per cent. of U.S. war expenditures, and the corresponding proportion was nearly 9 per cent. for the U.K. At the peak rate of flow of mutual aid, the U.S. was devoting not far short of 15 per cent. of all her war expenditures to lend-lease aid to the British Empire (first half of 1944), while the U.K. spent about 12 per cent. of the total cost of the war on reciprocal aid to the U.S. (late in 1944).

The second relation is between total mutual aid and national income, the latter being defined on a comparable basis for the two countries and at factor cost, excluding indirect taxes and interest on the national debt. The notes appended to Table 12 explain in more detail the national income concept involved. In the whole period of mutual aid, almost identical proportions of national income, about $4\frac{3}{4}$ per cent., were devoted to lend-lease in the U.S. and to reciprocal aid in the U.K. At peak, late in 1944, the proportions were nearly 7 per cent. in each country.

These two relations are, in a sense, financial concepts, and are based on all types of mutual aid financed by the respective governments. In each country some mutual aid was provided in the form of goods produced, or services rendered, abroad by other than the country's own nationals. Similarly, some part of the national income is derived from abroad. A different type of relation can be devised to set the home-produced part of mutual aid against domestic national output or product. The latter can now be measured conveniently in gross terms, including depreciation and similar charges, but net income from abroad must be excluded. Table 9 shows the derivation of this gross national product concept from the national income figures already used. Comparability between the two countries is preserved. The only difficulty is in the series for net income from abroad for the U.K., no official estimates being yet available for the war years. An estimate for the year 1945 has been published in one of the White Papers on the Loan Agreement (*Statistical Material presented during the Washington Negotiations*, Cmd. 6707). This values net income from abroad at £97 millions, apparently excluding insurance transactions, but including the net foreign income of British oil companies:

	£ millions
Gross U.K. receipts from abroad (excluding insurance and oil) ...	136
Plus net receipts of oil companies	+34
Minus gross U.K. payments abroad (excluding insurance and oil)	−73
Net income from abroad (excluding insurance)	97

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From what is known of overseas disinvestment and of the increase in overseas liabilities, we can offer the following figures as indicative of net income from abroad in earlier years:

	1942	1943	1944	1945
£ millions	135	125	115	97

The estimates in Table 12 are then interpolations for years ending at June 30th.

It is rather difficult to estimate the home-produced content of mutual aid. First, much of the petroleum supplied under both lend-lease and reciprocal aid comes from non-domestic sources. Further, as we have already said, petroleum supplies during the war were largely pooled and derived from the most convenient sources for all the various uses. For example, a good proportion of the petroleum originally lend-leased by the U.S. found its way back under reciprocal-aid disguises to U.S. forces in the U.K., India and elsewhere. For these reasons, it is better to leave petroleum out altogether in estimating home-produced mutual aid. Secondly, estimates can be made (as described in the notes to Table 12) for the value of stores in lend-lease and reciprocal aid which were produced outside the U.S. and the U.K. respectively. These values are deducted from the totals of mutual aid less petroleum. There remains the question of how to handle that part of mutual-aid stores produced domestically which consists of imported goods. We have ignored this element on the assumption that, to the extent that it cannot be offset by exports, it can be roughly cancelled by the home-produced element in mutual-aid stores produced

TABLE 13

Reciprocal Aid from Australia and New Zealand in Relation to National Income

	Year ending June 30			Total (3 years)
	1943	1944	1945	
Australia (£ million Aus.):				
Net national income	1,180	1,300	1,350	3,830
Reciprocal aid to U.S.	60·8	108·4	90·3	259·5
Per cent. of net national income	5·2	8·3	6·7	6·8
New Zealand (£ millions N.Z.):				
Private income	230	240	246	716
Net national income (estimated)	214	223	228	665
Reciprocal aid to U.S.	15·7	21·9	25·7	63·3
Per cent. of net national income	7·3	9·8	11·3	9·5

Notes.—Net national income of Australia and private income of New Zealand from semi-official sources. Reciprocal aid as in Table 9, but in local currency.

abroad and in net petroleum transfers. This calculation of home-produced mutual aid may work out slightly in favour of the U.S.

The results of the comparison of home-produced mutual aid with home-produced gross output are given in the last lines of the two parts of Table 12. About 4 per cent. of total domestic output in the U.S. was directed to the British Empire under lend-lease in the whole period of operation, and at peak the proportion was nearly 6 per cent. The corresponding proportion for the U.K. was about 3 per cent. in the three years from mid-1942 to mid-1945, with a peak of about 4 per cent. in the spring of 1944 before D Day.

The conclusion, therefore, is that all measures point to the same general result. Relative to her resources, the U.K. contribution in the form of reciprocal aid to the U.S. may have been rather less, but certainly it was not much less, than the U.S. contribution in the form of lend-lease aid to the British Empire.

We have already seen that reciprocal aid from Australia and New Zealand to the U.S. was large relative to lend-lease from the U.S. to these countries. Reciprocal aid was also a larger proportion of the national income of Australia and New Zealand than lend-lease was of the U.S. national income. As the data of Table 13 indicate, Australia contributed about 7 per cent. and New Zealand not far short of 10 per cent. of national income in the form of reciprocal aid to the U.S. It has been stated, moreover, that Australian reciprocal aid to the U.S. represented

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approximately 18 per cent. of her total war expenditures (*21st Report to Congress on Lend-lease Operations*, January 1946, p. 33). A comparable figure for the U.K. would be 9 per cent. for reciprocal aid to the U.S. and about 14 per cent. for all reciprocal aid in the three years ending mid-1945. Further, in relation to U.S. war expenditures in the three and a half years from Pearl Harbor to the middle of 1945, the proportion was 11 per cent. for lend-lease to the British Empire and about 16 per cent. for lend-lease to all countries. Reciprocal aid from the two southern Dominions to U.S. forces in the Pacific was, in a sense, the practical substitute for the assistance which could no longer be provided to the U.K. But, whichever way it is regarded, there can be no doubt of the size of the contributions of these two Dominions to the combined war effort.

The Composition of Mutual Aid. The following tables are designed to show the broad composition of the items making up lend-lease and reciprocal aid and the variation in the composition in the course of the war. Petroleum transferred under mutual aid is omitted altogether from the calculations for the reasons already adduced—namely, that dealings in petroleum products were so multilateral as to amount to an almost complete pooling of resources.

TABLE 14

Composition of U.S. Lend-lease Aid to the British Empire

	March–Dec. 1941	1942	1943	1944	Jan.–Aug. 1945	Total
$ millions:						
Total lend-lease aid	1,082	4,757	9,031	10,766	4,437	30,073
Less petroleum	83	232	372	799	656	2,142
Total, excluding petroleum	999	4,525	8,659	9,967	3,781	27,931
Per cent.:						
Aircraft and equipment	2·0	17·8	18·8	23·6	27·7	21·0
Ships, equipment and repairs	14·1	8·5	17·9	9·3	9·2	12·0
Ordnance and ammunition	7·8	15·4	12·1	9·0	7·8	10·8
Vehicles and equipment	6·7	9·5	17·0	14·6	9·4	13·5
Other munitions	1·1	2·3	4·5	11·0	10·2	7·1
Total munitions	31·7	53·6	70·3	67·5	64·3	64·4
Foodstuffs	29·1	14·3	9·5	11·7	12·7	12·2
Other agricultural produce	8·0	3·2	2·4	2·4	3·7	2·9
Metals	9·3	6·4	4·9	3·5	5·4	4·8
Machinery	2·4	4·2	3·4	2·7	2·6	3·1
Other manufactures	1·5	2·7	1·1	1·8	2·4	1·8
Services, excluding ship repairs	17·9	15·5	8·4	10·6	9·0	10·8

Notes.—Goods transferred and services rendered under lend-lease classified as in Table 4, but with ship construction and ship repairs included under munitions.

Other munitions comprise signal, engineer, quartermaster, chemical warfare and medical stores; radio, radar and other electrical equipment included here and not with the equipment of aircraft, ships and vehicles. Other agricultural produce includes cotton, tobacco, lumber and many less important items. Other manufactures is a residual group of which the main items are chemicals, rubber, textiles, civilian clothing and medical supplies.

Of all lend-lease aid to the British Empire between March 11, 1941, and V-J Day, excluding petroleum, munitions in the broad sense accounted for 65 per cent. and war services, mainly in connection with shipping, represented a further 10 per cent. Of the remaining 25 per cent., half was food and the other half was distributed over a wide range of goods—cotton, tobacco, lumber, metals, machinery, and all kinds of manufactures. Details are shown in Table 14.

We have seen, from the data of Table 12, that the U.S. provided lend-lease aid to the British Empire to the extent of between 4 and 5 per cent. of her total production. It is instructive to follow the variation of such a percentage from one category of lend-lease aid to another. The available data are summarized in Table 15.

The dominant position of munitions in lend-lease aid—to supply the relatively larger strength of the British armed forces—is clear from these two tables. Until the beginning of 1944, the supply of lend-lease munitions to the British Empire was built up, not only in total dollar terms,

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but also in proportion to all lend-lease. At the peak rate of flow over 70 per cent. of lend-lease aid, exclusive of petroleum, consisted of munitions, and the British Empire was then receiving no less than 12 per cent. of all U.S. munitions production under lend-lease. In fact, over a two-year period from the beginning of 1943 to the end of 1944 the U.S. sent to the British under lend-lease terms about 11½ per cent. of all the munitions she produced. In terms of value, aircraft and related equipment were the most important of all lend-lease munitions; allocations to the British kept pace with the rapidly expanding production in the U.S., and ran consistently around 12½ per cent. of total output after Pearl Harbor. But, relative to U.S. production, tanks and

TABLE 15

U.S. Lend-lease Aid to the British Empire in Relation to Total U.S. Production

	1942	1943	1944	First half 1945
Lend-lease as per cent. of total production:				
Aircraft and equipment	12·4	11·9	13·5	11·8
Ships, equipment and repairs	5·5	11·8	6·7	5·4
Ordnance and ammunition	10·4	10·0	8·8	4·6
Vehicles and equipment	9·8	26·7	29·4	12·1
Other munitions	1·4	3·4	9·9	5·5
Total munitions	7·6	11·2	11·7	7·6
Foodstuffs	4·3	4·4	5·4	3·9
Other agricultural produce	4·3	5·6	4·4	5·0
Metals	3·9	4·2	3·4	3·5
Machinery	2·6	5·7	7·1	4·2
Other manufactures	0·7	0·6	1·1	0·7

Notes.—Goods transferred under lend-lease classified as in Table 14. Total U.S. production from unpublished valuations by the War Production Board. Total munitions production is valued at "standard" costs at different dates and the values used here are approximations to current costs as follows:

Year 1942—standard costs of August 1942.
Year 1943—standard costs of August 1943.
Year 1944—average of standard costs of August 1943 and of 1945.
First half 1945—standard costs of 1945.

Other munitions in total production comprise, as in lend-lease, signal, engineer, quartermaster, chemical warfare and medical stores; petroleum, raw materials, industrial equipment, food and subsistence are all omitted. Total production other than munitions is as estimated by W.P.B. at August 1945.

motor vehicles were easily the largest category of munitions, with some 28 per cent. of all vehicles made in the U.S. allotted to the British Empire under lend-lease in the two years 1943–44. Ordnance and ammunition were more important lend-lease items in the early months than they were later, when the U.S. had large forces in the field. Later on, radio and other signal stores, engineer equipment, clothing and other quartermaster supplies became an increasingly important part of lend-lease munitions—*e.g.*, for the re-equipment of British forces for operation against the Japanese.

Food and manufactures for civilians, raw materials and equipment for industry, were supplied under lend-lease, and were important, and even decisive, in maintaining the high mobilization of the British labour force for war purposes. The supply of lend-lease food, beginning in 1941 at the same time as rationing on the points system, certainly added variety to the British diet. The contribution of the U.S. should, however, be kept in proper perspective. Not only were the lend-lease supplies from the U.S. marginal to Britain, they were also marginal from the point of view of U.S. agriculture and industry. They represented well under 5 per cent. of total U.S. production of such supplies, even in the peak years of 1943–44. The largest contribution was in machinery, over 6 per cent. of total U.S. output being sent to the British Empire under lend-lease in 1943–44, before most of the items were made ineligible for lend-lease in the last few months of the war. In the same two years, foodstuffs, cotton, tobacco, lumber and other agricultural produce supplied to the British represented about 5 per cent. of U.S. agricultural output. The proportion for iron, steel and other metals was under 4 per cent., and much lower figures were recorded for other manufactures.

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A broad picture of the composition of reciprocal aid from the British Empire to the U.S. is given in Table 10. Further details are only available for reciprocal aid from the U.K., and even here estimates are needed to fill out some gaps in the data, as explained in the notes to Table 16. Very broadly, U.K. reciprocal aid consisted of 30 per cent. in each of the three main categories—construction including building maintenance, military stores, shipping and other services. The remaining 10 per cent. consisted of bulk exports of foodstuffs and raw materials to the U.S., together with food provided to U.S. forces in the U.K. This is for the whole of the period, but the composition of reciprocal aid shifted greatly as the war progressed. The construction of military installations and their maintenance dominated reciprocal aid in the early days. At one time nearly one-third of the total building labour force in the U.K. was employed on the construction and maintenance of airfields, barracks, hospitals and other projects for U.S. troops.

TABLE 16

Composition of U.K. Reciprocal Aid to the U.S.

	Year ending June 30			July 1–Sept. 1, 1945	Total
	1943	1944	1945		
£ millions:					
Total reciprocal aid	230	421	482	69	1,201
Less petroleum	8	77	187	25	297
Total, excluding petroleum	222	344	295	44	904
Per cent.:					
Aircraft and equipment	7·0	6·5	3·1	1·8	5·3
Ordnance and ammunition	8·4	4·0	4·1	2·5	5·0
Vehicles and equipment	3·0	1·1	1·6	2·0	1·8
Other military stores	18·0	16·4	25·1	17·7	19·7
Total military stores	36·3	28·1	33·9	24·0	31·8
Foodstuffs	—	4·1	7·7	11·1	4·6
Raw materials	2·8	4·5	7·9	8·4	5·4
Construction	41·7	31·7	6·7	14·5	25·2
Building maintenance	1·2	6·2	5·9	3·9	4·8
Other services	18·0	25·5	37·8	38·1	28·3

Notes.—Goods transferred and services rendered under reciprocal aid as in Table 8. Classification by type of stores as shown in the Treasury returns. Aircraft and equipment includes radio, radar and electrical equipment. Other military stores include engineer stores, scientific instruments, transportation stores, clothing, camp and barrack equipment, medical supplies, together with materials of all kinds for use by U.S. forces. Foodstuffs comprise both food for U.S. forces and bulk exports from the colonies. Raw materials include only bulk exports and materials transferred in the U.S.; materials such as timber, chemicals and permanent way supplies for U.S. forces are included under military stores. Other services comprise shipping, inland transport, postal, broadcasting, printing, laundries and many others.

Some categories of reciprocal aid are not given in detail in the Treasury returns, and estimated distributions are used here. Transfers of "ordnance and R.E. stores" by the War Office in the U.K. before July 1943 are divided by categories according to the proportions found in the two years ending June 30, 1945, and the same method is used in allocating War Office transfers in overseas theatres in the whole period. Air Ministry transfers in overseas theatres are classified in proportion to the transfers by this Ministry in the U.K. in the whole period. Other transfers of stores in overseas theatres are lumped together under "other military stores."

Later on current military stores in great variety were a large proportion of reciprocal aid, particularly in the months after D Day, and the provision of shipping and other services and the export of bulk food and materials to the U.S. continued to increase up to V-J Day.

Military stores, or munitions in the broadest sense, formed about 30 per cent. of all reciprocal aid furnished to the U.S. by the British Empire. This is a much lower proportion than found in lend-lease aid, of which 65 per cent. was munitions if the construction, equipment and repair of ships are included—or over 50 per cent. if these are excluded as more analogous to construction and maintenance on the reciprocal aid side.* Further, whereas combat munitions are the

* The gap between the proportions of munitions in lend-lease and reciprocal aid is wider in these value terms than in volume since munitions are relatively dearer in the U.S.

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bulk of the military stores provided under lend-lease, the reciprocal aid items are a much more miscellaneous group.

Important contributions to the combat equipment of U.S. forces were indeed made under reciprocal aid. Amongst aircraft transferred to the U.S. air forces were 125 photographic-reconnaissance Mosquitos, over 400 Spitfires in the U.K. and 250 more in North Africa, and about 550 Horsa gliders for airborne operations. American aircraft were fitted with radar equipment of British manufacture and design, equipment for blind bombing, for identification and for other purposes. Ten corvettes were handed over to the U.S. Navy and about 4,500 barrage balloons were sent to the U.S. for the defence of her cities in the early days after Pearl Harbour. Transfers of ordance included 25-pounders and ammunition, a variety of bombs, including some 12,000-pounders, and many types of small arms ammunition. Altogether, however, little more than one-third of the value of military stores provided under reciprocal aid was combat material—ordnance, ammunition, aircraft, tanks and related equipment.

The main contribution of reciprocal aid in the direct military sense was probably the saving of shipping space through the local provision of a great range of expendable stores for the use of U.S. forces in all parts of the world. In the U.K. alone in the two years to mid-1944 it has been estimated that the military supplies provided under reciprocal aid would have required the use of 1,000 ships if they had been brought in from the U.S. (*17th Report to Congress on Lend-lease Operations*, November 1944). In the same two-year period the estimated proportion of current U.S. Army requirements in the European theatre met by the U.K. under reciprocal aid was as follows in terms of shipping tons:

Quartermaster Corps	63 per cent.
Engineer Corps	58
Medical Corps	49
Chemical Warfare	25
Signal Corps	22
Air Forces	21

U.S. forces in this theatre were supplied under reciprocal aid with practically all their requirements of such varied stores as Bailey bridges, auxiliary gas-tanks, jerricans for the distribution of motor spirit and spark-plugs for aircraft engines. Tyres, both new and recapped, were provided to meet a large proportion of U.S. service requirements in the U.K. and in Australia. Army clothing of all types was a large proportion of reciprocal aid in Australia, New Zealand and India.

Equally important was the local provision of food for U.S. forces, since this not only saved shipping space, but also gave U.S. troops a diet of fresh fruit, vegetables and other foods. Reciprocal aid in the U.K. included nearly £20 millions worth of food, much of it bulky—for example, 70,000 tons of flour, nearly 100,000 tons of vegetables and 20,000 tons of corned beef. The contribution was greater, however, in the Pacific and South-East Asia theatres. Australia supplied food to the value of about £60 millions sterling, New Zealand £30 millions sterling and India over £10 millions sterling. Together these three countries provided almost 2 million tons of food under reciprocal aid, meeting a large proportion of the total needs of U.S. forces in these combat areas.

Towards the end of 1943 certain exports of bulk foodstuffs and raw materials to the U.S. were brought under reciprocal aid. These came from the U.K. and the colonial Empire on U.K. account and, later on, from India at the expense of the Indian Government. Such exports grew in volume in 1944–45, and the shipments amounted to £80 millions in less than two years, £65 millions on U.K. account and £15 millions from the Government of India. The major items provided by the U.K. included nearly 150,000 tons of copra and cocoanut oil from Ceylon and the Pacific islands, over 250,000 tons of cocoa and 50,000 tons of palm oil and kernels from West Africa, £5 millions worth of tea from India and Ceylon, well over 100,000 tons of rubber from Ceylon and the African colonies, and nearly 350,000 tons of benzol from the U.K. There were numerous other exports, such as steel plates from the U.K., sisal from East Africa, copper from Cyprus, hides and goatskins from East Africa and other sources and raw sugar from Fiji and Queensland. In the same way, from the spring of 1944, India made available and shipped to the U.S. large quantities of materials on which the U.S. was heavily dependent. Jute and jute

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products comprised more than half the total value, and other large exports included mica, shellac, rubber and tea.

Services under reciprocal aid, around 30 per cent. of all aid to the U.S. from the Empire, also increased in importance as the U.S. forces were deployed against Germany and Japan. As in lend-lease aid, the main services were charges in connection with the movement of freight and troops, for shipping, rail and road transport of all kinds. Building maintenance, including the provision of fuel and light, was a major service item, particularly in the U.K. But services of many other types were rendered to U.S. forces in various parts of the world—postal and telegraph, broadcasting, printing, laundries, dry cleaning, boot repair and others in great variety. In addition, the U.K. devised and carried out numerous projects in the joint war effort for British and American forces alike. The Pluto oil pipeline to the Continent and the Fido fog dispersal system were two of the more publicized examples.

British Empire Supplies of Munitions. Munitions production in the U.S. was quite small even at the time of the entry of the U.S. into the war. What expansion had been realized was

TABLE 17

British Empire Supplies of Munitions from the U.S.

	Empire supplies ($ millions)			U.S. production	
	Purchases	Lend-lease	Total	$ millions	% to Empire
All munitions:					
1940 (second half)	400	—	400	2,000	19·1
1941	1,200	300	1,500	8,600	17·3
1942	900	2,400	3,300	32,000	10·5
1943	600	6,100	6,700	54,400	12·3
1944	400	6,700	7,100	57,700	12·3
1945 (first half)	100	2,000	2,100	25,500	8·1
Total (five years)	3,600	17,500	21,100	180,200	11·7
Of which:					
Aircraft and equipment	2,100	5,600	7,700	47,300	16·3
Ships, equipment and repairs	200	3,300	3,500	41,100	8·5
Ordnance and ammunition	700	3,000	3,700	34,200	10·8
Vehicles and equipment	400	3,700	4,100	19,300	21·2
Other munitions	200	1,900	2,100	38,300	5·5

Notes.—Empire purchases comprise deliveries on cash contracts, from the records of the British Supply Missions in the U.S., and Canadian purchases of finished munitions and of munitions components in the U.S., from estimates prepared by the Department of Munitions and Supply, Ottawa. Lend-lease to the Empire as in Table 14, including ship construction and repairs.

U.S. production from unpublished valuations by W.P.B. as in Table 15, with the addition of Canadian purchases of munitions components in the U.S. The latter, almost entirely by private contractors in Canada, are excluded from the W.P.B. valuations, though munitions components provided under lend-lease are included.

in part the legacy of cash orders placed by the U.K. and other Empire governments with individual U.S. firms and with the encouragement of the U.S. Administration. From the middle of 1940 to the end of 1941 deliveries on these contracts were coming forward in growing volume, and the British were receiving between 17 and 20 per cent. of all munitions turned out in the U.S. In the first year of U.S. participation in the war lend-lease deliveries gradually replaced those on cash orders, a lend-lease contract often being introduced as a follow-up of a cash order for the same item. In this year the proportion of U.S. output of munitions coming to the British declined to a little over 10 per cent. The proportion rose again to 12 per cent. in 1943–44, when the machinery of lend-lease and of the assignment of munitions was working smoothly and well (Table 17).

Over the period of five years from the middle of 1940 to the middle of 1945, during which U.S. armament production was first built up and then began to taper off, the British Empire obtained nearly 12 per cent. of all munitions produced in the U.S.—almost 10 per cent. under lend-lease and 2 per cent. for cash. The proportion was 21 per cent. for tanks and motor vehicles,

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16 per cent. for aircraft and related equipment, 11 per cent. for ordnance and ammunition and smaller figures for other munitions. British cash contracts placed before the introduction of lend-lease were very largely for aircraft, engines and other air equipment. Deliveries on these orders in the eighteen months up to the end of 1941 were 40 per cent. of all aircraft production in the U.S., and even then the contracts were by no means fully delivered. This is a good indication of the contribution made by British contracts towards the development of the American aircraft industry for war. It was British ordering which established U.S. production of such important types as the Mustang fighter (developed from the Spitfire design), the Hudson and Ventura bombers and the Harvard trainer. Cash contracts from the British for ordnance and ammunition were

TABLE 18

British Empire Supplies of Munitions from all Sources

	Sept.–Dec. 1939 and 1940	1941	1942	1943	1944	First half 1945	Total
Total supplies ($ millions) ...	9,200	13,000	19,900	24,800	24,700	9,300	100,900
Per cent. from:							
U.K.	90·7	81·8	72·6	62·4	61·2	66·1	69·5
Canada	2·6	5·2	8·6	8·8	8·9	10·0	7·9
Eastern Group	1·1	1·5	1·9	1·9	1·2	1·7	1·6
Purchases in U.S.	5·6	9·1	4·7	2·4	1·5	1·2	3·7
U.S. lend-lease	—	2·4	12·2	24·5	27·2	21·0	17·3

Notes.—Total supplies of munitions becoming available to Empire countries from domestic production and from the U.S., including munitions later transferred to Allies. The coverage of munitions is as in Tables 14, 15 and 17. Components are included in country of production, not in that of incorporation in end-products. Valuations are as far as possible at comparable U.S. costs.

U.K. production from estimates by Ministry of Production of home expenditures on munitions 1939-43, extended to 1944–45 by index-numbers of munitions production (*The Impact of the War on Civilian Consumption*, pp. 11 and 145). Conversion to dollars by rates appropriate to different categories of munitions as obtained from comparisons of unit costs of comparable stores. The average rate of conversion is nearly $7 to £1. The conversion has been checked by a more detailed comparison for 1943 on the lines described in *The Impact of the War on Civilian Consumption*, pp. 11 and 143.

Canadian production based on estimates, substantially at constant 1944 costs, by the Department of Munitions and Supply, Ottawa (*The Impact of the War on Civilian Consumption*, p. 153). These figures are here adjusted to a current cost basis, reduced by the exclusion of freight and inspection costs and of components purchased from abroad by contractors, and increased by the addition of war plant expansion amortised over five years.

Eastern Group production is almost entirely in Australia, New Zealand and India. Estimates of expenditures on munitions compiled by Ministry of Production are converted to dollars as for the U.K. U.S. purchases and lend-lease are as in Table 17 with the addition of purchases before July 1940.

also substantial, and deliveries in the eighteen months from the middle of 1940 were over one-quarter of all U.S. output of these items. The U.S. remained a large producer of weapons and ammunition of British design until quite late in the war, but her contribution might have been larger had there been a greater standardization of types and calibres.

We can now complete the picture by looking at deliveries of munitions from the U.S. from the other angle, that of British Empire supplies from all sources. Table 18 is designed for this purpose, and shows all munitions becoming available to the Empire, including what was later transferred to the U.S., Russia and other Allies. To obtain the total and distribution of Table 18 we value all munitions produced or delivered as far as possible at comparable U.S. costs. Further, to avoid duplication in components such as aircraft engines or empty shells made in one country for incorporation in end-products in another, we allot components to the country of their production, with corresponding deductions from the value of the end-products made in other countries. No allowance is made, or can be made, for trade in raw materials and semi-fabricated goods between countries. This means, for example, that the Canadian contribution to Empire supplies of munitions shows up as a relatively low figure, since Canada is heavily dependent on the U.S. for components. Table 18 is a distribution of supplies of munitions proper; it is not designed to show the proportionate shares of the various countries in the over-all war effort.

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In a more comprehensive story we should need to include the very large contributions from Canada in the form of foodstuffs, aluminium, copper, nickel, lumber and other materials.

At the U.S. costs used here, just over $100,000 millions of munitions became available to the British Empire from the beginning of the war to the middle of 1945. Of this total, 69½ per cent. came from domestic production in the U.K. and another 9½ per cent. from Canada and other parts of the Empire. The remaining 21 per cent. was drawn from the U.S., both for cash (over one-sixth) and under lend-lease (nearly five-sixths). These are approximately the same proportions as quoted for a rather shorter period in *Statistics Relating to the War Effort of the U.K.* (Cmd. 6564, para. 34). The main change during the war was the gradual shift from dependence on production in the U.K. to supplies drawn from North America. The U.S. and Canada together provided under 10 per cent. of all Empire supplies of munitions in 1940, but the proportion then rose to 36 per cent. in 1943 and to nearly 38 per cent. in 1944. By D Day in 1944, the U.S. alone was supplying 30 per cent. of all the Empire's munitions, practically all on lend-lease terms. Towards the end of 1944, however, assignments of munitions from U.S. production began to fall off, and the decline continued until V-J Day.

Conclusion. Lend-lease aid from the U.S. to the British Empire is valued at $30,000 millions. Reciprocal aid from the whole of the Empire to the U.S. amounts to £1,600 millions. There is therefore a large sum standing to the credit of the U.S. in the mutual-aid account, however the translation between pounds sterling and dollars is made. The credit arises very largely in the account between the U.S. and the U.K., in which about $27,000 millions of lend-lease aid stands against £1,200 of reciprocal aid. At comparable U.S. dollar costs, the latter amount certainly cannot be put at more than $6,000 millions. The whole of this mutual-aid account between the U.S. and the U.K. has been cancelled in a final settlement negotiated in December 1945 at the same time as the Loan Agreement. The U.K. is to pay, on the same terms as the proposed loan, a relatively small sum stated by the U.S. Administration as $472 millions. In return, the U.K. acquires stocks of lend-lease goods on hand at V-J Day—all undistributed civilian stocks of foodstuffs, raw materials and manufactures, together with machinery and installations and certain transport aircraft, small ships and petroleum supplies—estimated at nearly $700 millions at original cost to the U.S. In the official phrasing (Cmd. 6708, para. 2 of the Joint Statement):

> "In arriving at this settlement both governments have taken full cognizance of the benefits already received by them in the defeat of their common enemies. They have also taken full cognizance of the general obligations assumed by them in Article VII of the Mutual Aid Agreement of 23rd February, 1942, and the understandings agreed upon this day with regard to commercial policy."

In relation to the resources of the respective countries, however, reciprocal aid may be a little less, but certainly not much less, than lend-lease aid. Between 4 and 5 per cent. of total U.S. national income or output (according to the method of calculation) was diverted to the British Empire in the form of lend-lease goods and services. Reciprocal aid from the U.K. to the U.S. was about the same proportion of national income if all aid financed by the U.K. is included, but the percentage is rather lower if only those goods and services domestically produced are counted. Higher proportions are found for Australia and New Zealand; nearly 7 per cent. and 10 per cent. respectively of the national incomes of these countries was given to the U.S. as reciprocal aid.

The contribution of reciprocal aid to the combined war effort is less fully appreciated than that of lend-lease. When troops invade enemy territory, it is quite usual for them to "live off" the country—but the situation is not quite as simple when forces are operating in or from friendly countries. The U.S. was fortunate in the war in that her forces for the invasion of German and Japanese territories were based, not on the U.S., but on Allied countries and, in particular, on the U.K., Australia, New Zealand and India. The problem was how to "live off" these countries without arousing ill-feeling. The solution was found in the system of reciprocal aid. Under reciprocal-aid arrangements, the U.S. obtained without cash payment the military installations and the host of services which, in any case and for the most part, would have been provided locally. In addition, the U.S. received, from local resources under reciprocal aid, large quantities of military and other stores which could have been brought from the U.S. if shipping had been available for use on an extravagant scale. The contribution of reciprocal aid, in brief, was the

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better and quicker equipping and servicing of U.S. troops in overseas theatres and the saving of valuable shipping space.

The composition and contribution of lend-lease aid were quite different. Food and other supplies for British civilians, materials and equipment for British industry and services of various kinds played their part in lend-lease aid and helped to maintain the British war effort at a high level for so long. But the main purpose of the lend-lease system, at the outset and throughout the war, was to provide munitions, and particularly combat equipment, directly to the armed forces of the Allies of the U.S. This purpose was undoubtedly realized. By sending 65 per cent. of all lend-lease aid in the form of munitions, mainly of combat types, the U.S. enabled the Empire to maintain a very large proportion of her man-power in the armed forces. There are two facts which demonstrate the importance of U.S. munitions, whether bought for cash or received under lend-lease, to the British war effort. One is that the Empire obtained nearly 12 per cent. of all the munitions produced in the U.S. from mid-1940 to the end of the war. The other is that these munitions from the U.S. represented over 20 per cent. of British supplies from all sources. There can be no doubt that the course of the war was greatly influenced, and its length shortened, by the inclusion of mutual aid in the wider arrangements for the combined conduct of the war.

Indeed, mutual aid—lend-lease from the U.S. and reciprocal aid from the British—can only be assessed adequately in the context of the whole combined planning of the war. The content and direction of mutual aid were constantly changed to fit in with the demands of military operations and to help break open the bottlenecks which developed in the production and distribution of war supplies. One of the most important of the parts played by mutual aid was in the attack on the most persistent bottleneck of all, the losses and shortages of shipping. One example suffices to illustrate. Under lend-lease arrangements, foodstuffs in concentrated form—dried eggs, dried milk, cheese, canned meat and others—were produced in specially built plants in the U.S. for quick shipment to the people of Britain. At the same time, the food supplies of the southern Domunions and India which would have been exported to the U.K. but for shipping difficulties were used to maintain U.S. forces in the nearby areas of the Pacific and S.E. Asia. How obvious and easy are such arrangements under the system of mutual aid, and how difficult it might otherwise have been to achieve the same results in the economy of shipping.

Mutual aid was a military operation always integrated with the wider military plans of the Allies. In the winter of 1941–42 the Russians needed immediate supplies of munitions, and as much as could be spared and delivered was sent under mutual aid, largely from the U.K. as the nearest source of supply. Meanwhile, lend-lease and reciprocal-aid money was expended on building up supply routes to Russia, together with the related assembly, repair and storage facilities, so that later on an augmented flow of mutual aid goods could reach Russia by the sea lanes, through Persia and by the airways across Africa and over the top of the world. Mutual aid supplies to Russia were then built up steadily to a peak rate of $5,000 millions a year early in 1945, as the Russian armies swept from Stalingrad to the gates of Berlin.

The bulk of lend-lease supplies from the U.S., despite all the assistance to Russia, continued to go to the British Empire, and practically all reciprocal aid received by the U.S. was from British countries. In 1941 British troops were already in the field, and British war workers were at their machines, standing practically alone against German domination. It was the deliberate and combined policy to maintain and to increase British war mobilization, which was never quite equalled by the U.S. even in the last year of war. Lend-lease aid to the British, rising to a peak flow of over $12,000 millions a year just before D Day in 1944, was one of the means whereby this policy was implemented. At the same time, reciprocal aid prepared the way for fighting forces of the U.S., first by the construction of base installations, and then through the provision of an increasingly wide range of stores and services. It was the enemy who dictated that the opportunity for this return service should fall to the U.K., and not to France, Belgium or Holland, and to Australia, New Zealand and India rather than to China. But it was through mutual aid that the opportunity was exploited to the full.

The mutual aid account in value terms is incomplete. We must look at reciprocal aid, not only as so many pounds, shillings and pence, but as the means of servicing U.S. troops most economically and of releasing ships better employed on carrying munitions to the battle-fronts. We must look at lend-lease aid, not only as so many millions of dollars, but as one of the means of maintaining the fighting strength of the Allies of the U.S., and hence to be considered in relation

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to the sacrifices particularly of the British and Russian peoples.' We can value lend-lease supplies of food, materials and munitions but not the services of British soldiers and war workers maintained by these supplies. There is no rate of exchange between dollars and the currency of "blood, sweat and tears." The mutual aid settlement between the U.S. and the U.K., and the others still to be negotiated, take this wide view of war-time achievements, and look ahead to continued co-operation in the critical years of peace to come.

Discussion on Professor Allen's Paper

Mr. A. T. K. Grant: It is a pleasure to propose a vote of thanks to a former colleague and a friend on so comprehensive a paper. His subject is one on which he is especially qualified to speak. Roy Allen—he was not Professor Allen then—went out to Washington late in 1940, before lend-lease had come into being, and when we were rather anxiously counting the coins in our pockets. The next time I saw him was two years later, in Washington. Throughout the period of lend-lease he was either with the Treasury Delegation there or with the Combined Production and Resources Board. He has thus had, as it were, a ringside seat from which he has been able to watch the development of lend-lease and reciprocal aid from the very beginning, and he has done so with a special insight into its implications from the point of view both of finance and of the productive machinery of both countries. It is well, therefore, that we should have this review before the process of post-war reconversion—by which the expert is transmogrified into the Professor—is quite complete.

It is, after all, a matter of some importance that, after such a period as we have passed through, those who are qualified to speak with special knowledge should set down the basic facts on matters with which they have been concerned which may be of interest in future years. Wars give rise to innumerable statistics concerned with men or money or things, which are overwhelming to the point of becoming meaningless without the guidance of someone who knows why they were collected and how they fit in. In particular, it is always rather a mystery how wars are in fact financed. The present paper, by pulling together the facts, does make available for the future a coherent and comprehensive account of one aspect of this particular process.

It is worth remembering the origin of mutual aid statistics. The figures on lend-lease take the form they do largely because Congress, in passing the Lend-Lease Act, inserted a provision that "The President, from time to time, but not less frequently than once every 90 days, should transmit to the Congress a report of operations under this Act." Our figures of reciprocal aid might not have come into existence at all but for insistence by the American authorities on the desirability of some record of what was passing the other way. It was relatively easy to keep lend-lease statistics since they were very largely records of stuff ordered from central agencies and passing outwards to the points where it would be needed. In the case of reciprocal aid, a very large proportion consisted of goods and services rendered on the spot through the normal local agencies of the Service and Supply Departments. Much that we gave on our side of the account was assistance to American units operating in this country and to a lesser extent outside it. We had feared at the beginning that if we insisted on too complicated a reporting system, the process by which supplies ranging from bombs to pianos were made available on demand would be impeded and hampered. In the end, the Americans got both the supplies and the statistics of which Professor Allen is a residuary legatee.

On the text of the paper there are three specific points I would like to discuss.

First, in Table IV and elsewhere we have figures of lend-lease split up into years or even shorter periods. The analysis by period is obtained from differences in F.E.A. cumulative figures. Surely these differences are accounted for not only by lend-lease aid in the period itself but also by errors and omissions which have come to light in the interval. I do not suggest that this is a point of particular importance, but it would be interesting to have Professor Allen's view on how far this is likely to affect the figures he gives.

The second point which I should like to mention is that of lend-lease to India. A lend-lease/reciprocal aid settlement was recently made between India and the United States, and the *New York Times* of May 17th has the following:

> "After six weeks of discussions it was decided that the aid granted each other about balances and so there should be no dollar settlement. The aid given through the Lend-Lease operations during the war was estimated to aggregate between $500 million and $600 million on each side."

It is of course a difficult matter to assign the lend-lease which went to India between India proper, and our own commitments in India and in connection with the building up of forces for the attack on the Japanese. Here at any rate is something to go on.

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The third point concerns Canada. I am sorry that the scope of Professor Allen's paper does not cover Canada, from whom the United Kingdom alone received over Can. $3,000 million by way of gift and mutual aid, the Canadians getting nothing in return, as their forces paid their own way abroad throughout. An analysis on the lines of Table XII for Canada would have been of interest. Incidentally, had the figures been available in an appropriate form, it would seem more logical to make the lend-lease and reciprocal aid items in that table cover free-of-charge supplies to all countries and not only those between the United Kingdom and United States.

That brings me to a final point. What is the significance of these figures of mutual aid? When all is said and done, they derive from a piece of administrative financing, which in its turn is almost incidental to a great common enterprise in which we were all engaged. Their significance, I suppose, is that they show that Great Powers can palatably conduct their financial relations with each other in time of war and avoid creating an indebtedness which the debtor cannot meet. As a financial device, it is an advance. But the realities behind these particular figures are the tremendous productive output which the United States so unreservedly threw into the fray, and the untiring pertinacity with which we laboured near the front line to provide supplies and facilities. Even then this is only a fragment of the story. If the Americans provided us with a Liberator, it gets into the figures; if they provided the crew as well, it does not. If we provide a Mosquito to the Americans, it gets into the figures; if we provide a Mosquito and crew, it does not. Somehow, when peoples merge their all in great common undertakings, it becomes difficult for the book-keeper to keep comprehensive accounts.

DR. HERON: I have much pleasure in seconding the vote of thanks to Professor Allen and in congratulating him on this worthy addition to the *Transactions* of the Society. Professor Allen has to some extent disarmed criticism by stating that one of his objects was to offer a partial and preliminary estimate of the importance of mutual aid in the conduct of the war. I emphasize this only to express the hope that this paper may be one of a series of papers dealing with the contributions of the United Nations, if I may still use that term, to the conduct of the war. I hope that Professor Allen will be encouraged to give us other papers in this series whenever additional data become available.

It is rather unfortunate that this first paper, as I like to think of it, springing as it does from the settlement of the mutual aid account, has to be expressed in dollars and pounds, because I feel that this method of approach does not enable us to realize to the full our debt of gratitude to the United States for this great charter of mutual assistance. For this we need quantities as well as values. We need a record of our stocks of munitions and food—and of our stocks of dollars—during the whole course of the war, and a record of the arrivals of the lend-lease supplies, before we can get a true picture of what has happened. I hope that the necessary data will in time be released. I well remember the day when I was able to report to our President, Lord Woolton, then Minister of Food, that the arrivals of lend-lease foods in this country had reached a total of one million tons. That was a striking milestone in that long journey. It was possible to state at that time how many thousand tons had been lost on the journey; on the other side of the picture, it was impossible to assess in dollars or pounds the value of the British lives lost in bringing these supplies here, a not inconsiderable contribution to the joint war effort.

I have one suggestion to make and one point of detail for Professor Allen's consideration. Some of the documents from which Professor Allen has quoted are not readily available in this country. I hope Professor Allen will assist us in obtaining copies of those documents for the Society's library in order that our Fellows and others interested may consult them whenever they wish to do so.

The point of detail is rather a small one, but it should be mentioned. It is with regard to Professor Allen's use of the terms "munitions" and "military stores." The term "munitions" in Table V and "military stores" in Table XI are clearly intended to cover the same group of supplies, but a considerable number of variants of these terms is used throughout the paper and it is not always easy to see what they are intended to cover. The following examples may be cited: "Munitions and military stores," "Munitions in the broad sense of military stores," "Military stores or munitions in the broadest sense," "Total military stores," "Military stores including ships and munitions proper." It would be helpful if those terms could be more clearly defined.

The vote of thanks was then put to the meeting and carried unanimously.

MR. DAVID D. LLOYD (U.S. Embassy) said, as one who worked in the United States from the lend-lease side, how much he admired Professor Allen's paper. It was a great contribution to the understanding of the whole arrangement, and was a real explanation of several statistical mysteries which he himself was never able to comprehend from the figures available to Washington.

The great achievement of lend-lease was really the lend-lease settlement. He did not know whether the British public sufficiently realized how much the American administration had in mind from the very beginning the necessity of getting a war-time financial device which would not

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create post-war debts whereby they would get into great difficulties such as they had after the last war.

Recognizing the traditional tendency of the people in the United States to think in terms of debt and of obligation and payment, the lend-lease Act from the very beginning had in mind the necessity of avoiding repayment for what was consumed, lost, and destroyed; and it was interesting to note that the settlement, based on that principle, had gone through with a great deal less opposition than would ordinarily have been expected from those who represented the nation and the people. There had been, it was true, quite a number of criticisms, but he did not think they had a strength behind them sufficient to overturn what had been acomplished. He was interested to glance through the recent comments made on the terms of the settlement by an investigating Committee of the Senate. The Committee thought there was a balance of $16 thousand millions which had been wiped out. He saw that Professor Allen made it $20 thousand millions, which showed the advantage of the statistical approach.

MR. CAMPION wished to support the vote of thanks to Professor Allen. The paper he had given the Society was a most complete account of the extent of mutual aid between the United States and the British Empire during the war. Mr. Campion said perhaps he was in a position to realize more fully than other Fellows present how much work had gone into the preparation of the paper. The statistics which Professor Allen used were drawn from a maze of different sources, and only Professor Allen, with his detailed knowledge of the war-time statistics of the United States and the British Empire, could, in fact, have so ably brought the figures together and presented such a coherent story.

Professor Allen had preferred in his paper to present an overall survey of the extent of mutual aid as a whole between 1941 and 1945. Although in various sections of the paper he did give figures indicative of the rate at which mutual aid was taking place between 1939 and 1945, it was probably not so easy for those who saw the material in his paper for the first time to relate the flow of aid between the two countries to the strategic phases of the war. It might have been helpful if some analysis could have been made to the amount of mutual aid by the operational phases of the war.

Mr. Campion agreed with previous speakers that it would be unfortunate if the impression was given that the intermingling of the war efforts of the British Empire and the Americans was dictated entirely by financial considerations. Nothing could be farther from the truth. Most of the actual planning and programming of requirements and supplies was done in practice in terms of actual units of equipment and items of food and raw materials.

The figures given in Table IV of Lend-Lease aid related to goods transferred and services rendered under Lend-Lease for use by the United Kingdom. They related mainly to deliveries from North America and not to actual arrivals in the United Kingdom or other theatres of war. During the periods of intensive attacks by U-boats a proportion of these stores was lost at sea and, therefore, the figures in Table IV overstated the actual amount of stores received by the United Kingdom, as distinct from delivered by the United States. (The figures of reciprocal aid afforded by the United Kingdom to the United States were not subject to such a deduction since most of the aid provided by the United Kingdom, as Professor Allen showed in Table VIII, was transferred within the United Kingdom itself.)

MR. THOMAS C. BLAISDELL, Jr., desired to add his own personal word of appreciation of this paper. He first learned to know Professor Allen's sterling qualities by working with him in Washington, where they did some combined planning together. With regard to the paper itself, he considered it as of great importance. First, the bringing together of statistical material from such a wide variety of sources was a great achievement which only a person who had a wide acquaintance with the actual material could have accomplished. Professor Allen's work was a contribution which all who worked on those matters would be glad to have as a means of reference ready to hand.

The second thing he would comment on was the purpose for which this statistical material was compiled. Originally, there was considerable discussion as to the wisdom of keeping any statistics or financial records with regard to lend-lease and reciprocal aid activities. He knew that at the time when lend-lease was first suggested it was in the minds of everyone that these might be a series of transactions for which there would be no recompense. That thought was undoubtedly wise, in his judgment. Records were kept, therefore, not because there was any thought of submitting a bill or receiving credit, but because the maintenance of a record was essential in planning the operation itself. The submission of requisitions, the checking of allocations, the filling of specific requirements, called for records as a very important part of the proceedings. If the records had not been in existence, the statistical information used in the present paper could not have been obtained.

His third point was with regard to the utilization of this statistical material. It was possible in any case for people to misinterpret what had occurred during the operation of lend-lease, but where there was a record misinterpretation was more difficult. The statistical records had been admirably used in Professor Allen's paper to clarify possible misinterpretations about the relative contributions.

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Finally, he desired to mention one other source of "assets" which did not appear in the account. To his mind, an outstanding contribution, which could never be measured, consisted of the "assets" which grew out of the mutual exchange of information in the technical field. Those "assets" to-day were subject to capitalization. As one example he mentioned the building of aeroplane bases. What was the commercial value of those airfields either to the individual countries or to the private corporations which used them for commercial purposes? He cited that example, but there were many others in the technological field, where the earnings or dividends of lend-lease and mutual aid were going to be collected over the years to come.

In closing he would repeat what he said at the beginning, that to the statisticians concerned who had been working together on this job, the work was going to yield a high dividend in the course of time.

DR. WALKER said that Professor Allen had embarked upon an heroic enterprise. He had had a good deal to do with these figures during his official career, both in London and in Washington, and he knew the quality of Professor Allen's basic material. He himself would have quailed before the task. He liked particularly the comparison between lease-lend and reciprocal aid and the respective national incomes of the two countries given in Table XII; this confirmed the opinion they had always had in Washington that, given the relative size of the United Kingdom and the United States, the British contribution toward the common pool was not greatly inferior to that made by the United States.

He wondered, however, whether Professor Allen had ever pursued in more detail the apparent discrepancy which, together, they had discovered between the F.E.A. valuation of lease-lend exports and the Missions' valuation of deliveries by assignment and otherwise. The question had not been pursued at the time for the very good reason that the British side wanted to keep the dollar sign out of the negotiations as much as they could. The difference between the two valuations was now happily academic—but it might in less fortunate circumstances have been an important and disturbing discrepancy.

Referring to Dr. Heron's remarks, Dr. Walker thought that he would be interested to know that Missions in Washington were often forced to push departments in London, hard-pressed as they were, for the most detailed statistics of British requirements, output and distribution, particularly of munitions. At one point indeed, it seemed almost as if the Americans, lend-leasing stores without return either in cash or in kind, were insisting the more upon statistics. As Professor Allen had shown, the value of stores delivered was immense—but often, when Missions had not been provided with a well-documented story to tell in exchange, it had been most difficult to convince the Americans that we really could not supply the statistics required in answer to their questions, except at the cost of a wholly disproportionate amount of work.

MR. HAWTREY said that Professor Allen had asked and answered nearly all the questions that would have occurred to them. The generosity of the provision made through lease-lend and mutual aid in the recent war was in striking contrast with the arrangements which obtained in the war of 1914–18. The underlying theory at that time had been that every country should be debited with the cost of the fighting that it actually undertook. He did not think that the fallacy of that principle had ever been adequately brought out. But the departure from it in the recent war was certainly a very great improvement. At the same time the departure was only partial. Professor Allen gave some interesting statistics of the relative efforts of this country and the United States and the Dominions, but he was not sure that the statistics as supplied in Washington really brought out the fundamental point. The British Empire produced $100,000 millions of munitions, the United States $180,000 millions, of which it might be said that between $150,000 millions and $160,000 millions were used or, at any rate, retained by the United States. The proportion of material used by the United States was a good deal less than the proportion of the numbers mobilized.

A reliable estimate of the relative amount of actual warlike material used by the Forces of the several combatant nations would be a valuable indicator. The munitions currently provided for troops kept in reserve represented only a proportion of what the same forces used when engaged in active operations, and he would not be surprised to find that the proportion used up by the British Forces was actually more than the figures just given would suggest.

There were, of course, various reasons why one combatant might supply material resources to another. For instance the receiving country might be more closely engaged in the war than the supplying country; that occurred when one country entered the war much earlier than the others. In the war of 1914–18 the United States hardly got its overwhelming forces ready in time to participate in the war until the very end. The position was very different in the recent war, but if casualties suffered were any measure of the amount of fighting done, they were inevitably much higher in proportion to numbers mobilized in Great Britain.

The greater part of the Colonial Empire consisted of comparatively primitive populations having very little economic surplus to mobilize for war equipment, but a fine reserve of manpower. When these forces were mobilized it devolved upon the United Kingdom to supply the requisite resources and equipment. That, in addition to the very high proportion of our own inhabitants

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who were mobilized, went far to explain the disproportionately heavy economic burden which this country had to sustain during the war. The upshot of it was that our losses of capital resources at home and abroad had been so serious that it was all we could do to keep our heads above water. The grant of lend-lease was generous in comparison with the practice which had been previously followed in the present century; nevertheless Great Britain, in spite of all the assistance she had received from her Allies and friends, was in an extremely impoverished condition.

DR. STERN asked whether it would have been technically possible to adopt instead of lend-lease, an even less "sordid" procedure, for instance, the distribution over the two countries of all war expenditure in relation to some measure of the economic capacity of the countries, as, for example, in relation to the national income or to manpower or otherwise? Would it have been statistically possible to apply such a principle? Would available statistics have allowed the proper allocations to be made?

DR. BARNA said that Professor Allen's aim in expressing lend-lease as a percentage of national income in Table XII was to give some indication of the proportion of real national resources devoted to lend-lease. He suggested that, perhaps, a better indication might be obtained if Professor Allen converted dollars into pounds, or pounds into dollars, using a different rate of conversion for lend-lease expenditure and for total national income, as Professor Allen had made it clear that munitions were comparatively expensive in the United States. This correction would be likely to turn the percentage slightly in favour of the United Kingdom instead of the United States. This impression was confirmed if one looked at the figures indicating the distribution of manpower between war and non-war purposes which were given in Table II. The correction suggested was not intended to detach from the magnitude of the American effort, or from the value of the paper.

MR. LEPPARD suggested that in amplification of the point made by Dr. Stern that the idea of apportioning the total cost of the war amongst participating countries was thought of at the beginning of the war in the agreement made between this country and France. He did not remember much about it but he thought there was an agreement that of the total cost the United Kingdom would pay three parts and France two parts.

The following comment was received in writing :—

MR. W. R. BUCKLAND : A recurring feature of the discussion on Professor Allen's paper was the desire to penetrate the monetary veil and examine the quantities represented thereby and also the interchange of technique which could not enter into the accounting picture. Some of these can be set down without delay in so far as they refer to the Royal Air Force and in particular to Bomber Command. It must be recalled that from the summer of 1940 to the summer of 1945 the Heavy Bomber Offensive was the only sustained series of military operations waged from these islands. Hence the story of the maintenance of this offensive provides examples of mutual aid of a very intensive character. There is the case of the part played, along with the Dominions, in the training of aircrew personnel : this was better achieved in the overseas skies untroubled by the turmoil of battle. The partially trained personnel could then be brought home at a time when it was better for the concluding stages of their training to be carried out married to the noise and atmosphere of battle.

But it is probably on the Communications and Radar sides that the best stories are to be found. Here the broad balance is that of British inventive genius being brought to operational fruition by American industrial capacity. In some cases, such as types of ground transmitters and receivers, the American commercial instruments were the only ones available. In other cases the elegance of the manufactured article exerted great stimulus in the British field when the American goods were made available under mutual aid.

The American authorities have stated that one of the outstanding achievements of the Royal Air Force during the War was the very high-frequency control system. The British equipment was closely followed by the Americans, and large quantities of the American replicas were supplied for Bomber Command use.

Several of the senior officers of the Luftwaffe have testified to the excellence of the "jamming" campaign waged with such tenacity against the Fighter and Radar screens operated by the German Air Force. This particular phase of the battle owed a great deal to American-made apparatus, even to the supply of completely equipped aircraft.

With an eye to the air transport of the future, mention may be made of the ground-control approach apparatus. This is essentially of American origin, and is now a standard R.A.F. system of instrument approach for use in conditions of bad visibility.

Finally, reference may be made to Professor Allen's comment on p. 266 about the transfer of a variety of bombs. A vital service was afforded by the Americans when they contributed a little over half the essential 1,000-lb.-type bombs dropped by Bomber Command during the year 1944 when the offensive was at its peak levels.

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PROFESSOR R. G. D. ALLEN, in reply, said that he would take advantage of the permission to respond in writing to the many excellent points raised in the discussion. But there were one or two rather general points which he might deal with at once. He was extremely sorry that limitations of length compelled him in this paper to keep to rather mundane details. He would have liked to have had much more liberty to dilate on the wider aspects of mutual aid, but if he had done so his paper would have run to twice the length. He had hoped and expected that these wider aspects of mutual aid would come up in the discussion and his expectation had been amply fulfilled.

He was very much aware, in particular, of the benefits derived from the close Anglo-American co-operation and collaboration which had resulted from the early attempt to plan the war on a combined basis, an attempt which included the whole system of mutual aid. Personally he had benefited beyond any possible calculation from his own association with Americans and Canadians during his official career in the war. He was able to express his debt that evening through the very able representatives of the United States, Mr. Blaisdell and Mr. David Lloyd of the United States Embassy, who were present.

From the nature of his paper it did look as if he was setting reciprocal aid against lend-lease which, of course, was not his real intention at all. He regarded it as a combined operation. It also appeared from his paper that he was arguing entirely in terms of dollars and pounds which, again, was not his intention. President Roosevelt initiated the idea of lend-lease in 1940–41 and his intention was to avoid the wretched business of war debts and to keep the dollar sign out of the assistance provided under lend-lease. It was, of course, not possible to keep the dollar sign out completely; operations of this kind required reports and records of what was being done and such reports and records could proceed only in terms of a common unit, which was the dollar or the pound. But he thought he could say in conclusion that, by the mutual aid settlement President Roosevelt's original intention was in the end amply fulfilled in an arrangement reached so harmoniously and easily between British and American officials towards the end of last year.

Professor Allen subsequently wrote as follows:

As Mr. Campion points out, my analysis in this paper is designed to survey the operation of mutual aid as a whole rather than by phases. I would like to say more about the incidence of mutual aid at different periods in the war but the statistical material will not stand up to such an analysis. The footnotes to Tables IV and VIII explain that figures for years are differences of cumulative totals and so include, not only mutual aid rendered in the year, but also the corrections made in the interval. To answer Mr. Grant's question, however, I can say quite safely that the errors in the yearly figures I have quoted are not large since I have attempted to apportion major corrections to the years to which they properly relate. Reference to the footnotes to the various tables is also needed to clear up the ambiguity in my use of the terms "munitions" and "military stores" in the text, a point on which Dr. Heron has put his unerring finger. I must admit to loose employment of terms in the text, in an attempt to add variety to a bald narrative. However, I hope that the footnotes explain sufficiently that the terms are in fact used in rather different senses in various tables, according to the nature of the data, and that a precise definition of coverage is given in each case.

Dr. Walker and Mr. Campion raise points which I have had much in mind. Throughout the operation of mutual aid there were discrepancies and disagreements between the American and our valuation of stores. What I have done is to accept the U.S. valuation for lend-lease, which is, I believe, in line with other U.S. valuations (*e.g.*, of national income), and our valuation of reciprocal aid. The relation between the two is made by use of special "exchange rates" which in fact allow for differences in method of valuation as well as for real variations in cost. Mr. Campion is perfectly correct in stressing that the amount of lend-lease aid shown in Table IV overstates the value of stores actually received because of losses at sea. The extent of the overstatement cannot be precisely estimated but it must be well under 5 per cent. Generally speaking, however, the U.K. took over lend-lease stores in the U.S. and the subsequent loss was ours. In this sense, I think I am right in keeping total transfers in my analysis.

Mr. Hawtrey raised an important and fascinating point in his reference to the relation between the military mobilization in each country and the amount of munitions produced and retained. I had hoped to include a calculation on these lines but I was defeated in the end by lack of data. Very roughly, from Tables XVII and XVIII with deduction for munitions sent to Russia and other allies, I estimate that gross supplies of munitions produced for U.S. forces was $150,000 millions and for the forces of the Empire $95,000 millions in the whole period from September 1939 to June 1945. Data on the armed strength of the U.S. and U.K. are available but not of other parts of the Empire. At peak, the number of men in the services can be put at 12 millions in the U.S. and 9 millions in the Empire. As a rough shot, I estimate the number of man-years at just over 32 millions in the U.S. and 21 millions in the U.K., say 37½ millions in the whole Empire, in the period from September 1939 to June 1945. Hence the munitions produced represented a little over $4,500 worth per man per year in the U.S. and all supplies to the Empire were $2,500 per man per year.

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The figures confirm the general impression we have that U.S. forces were better provided for than the British. In this calculation, however, all munitions produced are counted whether they actually reached forces in the field or not and all troops are included whether engaged in operations or not. The relation is quite possibly reversed if we take only munitions used by forces actively engaged during the war. It is here that data are almost completely lacking.

Following up Dr. Barna's suggestion, I have re-calculated the percentage figures in Table XII, allowing for the facts that munitions were relatively dear in the U.S. and that they bulked larger in lend-lease than in total production. I have converted the figures of U.S. national income and product, and of lend-lease aid, from $ to £ by use of the conversion rates of $7 to £1 for munitions and $4 to £1 for other items. The revised percentages in the period of 3½ years are then 4·1 per cent. for total lend-lease in relation to national income and 3·4 per cent. for home-produced lend-lease in relation to gross national product (home-produced). These compare with the corresponding U.K. figures of 4·6 per cent. and 3·0 per cent. respectively. As Dr. Barna says, the correction turns the percentages slightly in favour of the U.K.

In conclusion, I am able to report that Dr. Heron's suggestion has been followed up and that the U.S. Embassy has kindly agreed to supply the library of the Society with copies of many of the U.S. documents which I have used in this paper.

As a result of the ballot taken during the meeting, the candidates named below were elected Fellows of the Society :—

James Arthur Bailey.
Denis Guy Bluett.
Alan Clement Barber Doncaster.
Clifford Walter Emmens.
Alan James Farmer.
Ronald Westlake Gloyne.
Muriel Gore.
Muriel Macintosh Johnstone, M.A.
Merlin Stephens Jones, M.Sc., D.I.C., A.R.C.S.
George Donald Kaye.
Norman Grace Keller.
Guru Datta Mehra.
John Robert Mitchell.
A. H. Pollard, M.Sc., F.I.A.
John Hubert Ray, A.C.I.S.
Arthur Sharp, M.A.
Alec Albert Sorrell, B.Sc.
James William Vernon.
Frederick Clement Watts, M.A.
C. L. Wheble, B.Com., M.Sc.(Econ.).
George Arthur Woods.
Ian David Phimester Wootton.

Corporate Representatives.

C. B. de Launay, *representing* The Daily Mirror Newspapers, Ltd.
Geoffrey E. Tewson, *representing* The Economist Newspaper.

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MISCELLANEA

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DERIVATION OF TOTAL FERTILITY, GROSS AND NET REPRODUCTION RATES FROM CENSUS STATISTICS OF MARRIAGE FERTILITY

By J. R. H. SHAUL

1. A COUNTRY which is unable to introduce compulsory registration of births and deaths must rely on census enquiries into marriage fertility for indications of the direction of movements in the natural increase of the population. These conditions apply to a large area of the continent of Africa inhabited mainly by indigenous Africans. In such areas the introduction of compulsory registration of births and deaths awaits the time when every inhabitant receives a medical practitioner's attention at death who will issue the relevant death certificate—a position which cannot be expected to be established in Africa for at least a generation. In the meantime the world is urgently requiring statistical determination of the direction and magnitude of the movements in the natural increase of the different communities and races of this vast continent. During the present century most branches of knowledge of the African continent are rapidly emerging from the deep obscurity which it was fashionable to attribute to the dark continent some decades ago, into the brightening light of a new day, but information on the population movements of the indigenous African still remains deeply obscured.

2. In view of these facts the demographic statistician is thrown back on the census of population for an assessment of the real rate of growth of the African population. Among the most useful measures of the factors determining the growth of population which have been devised in modern times are the gross and net reproduction rates originally advocated by Dr. Kuczynski. Since total fertility rates provide a useful basis for estimating these rates, and in view of their importance and the restrictions surrounding the study of African demographic statistics, a special study is made in this article of the problem of obtaining total fertility estimates from census statistics of marriage fertility.

3. The compilation of fertility statistics from a census of population demands only a limited number of very simple questions to be put to the African. The questions to be asked are:—

(1) whether the person is a male or female;
(2) whether below or above the age of puberty;
(3) if a woman above the age of puberty, the number of children she has had born alive and how many are still alive;
(4) whether born in the country or an immigrant from a neighbouring territory.

The attainment of the age of puberty in the African tribe is almost universally accompanied by certain initiation and religious rites which serve to separate the period of childhood from that of adult life, and which form a sufficiently good dividing line for separating the commencement of child-bearing ages from the immature age. Every African accordingly is normally in a position to give the information called for in (2), and there should be little practical difficulty encountered in securing the remaining information. From the fertility statistics obtained by the census it is suggested that total fertility can be obtained arbitrarily by the following method.

4. For the time being, assuming age at marriage and age at census date are known, write ${}_1F_n$ for the number of females who married in the first year of puberty and who are now n years of age and over, n being the limit of the child-bearing ages; ${}_2F_n$ for the women married in the second year who are n years of age and over at the census date, etc., up to ${}_nF_n$ for the women married in the nth year who are now n years of age and over. Similarly, ${}_1F_{n-1}$ represents the number of

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females married in year 1 who are now $n-1$ years old, etc. Write r_1, for the specific fertility rate of the women in the first year of puberty, r_2 for the specific fertility rate in the second year and so on up to r_n for the rate in the nth year. Then $\sum_1^n r_s = R =$ total fertility. Similarly write ${}_1A_n$ for the total number of children born to the ${}_1F_n$ mothers, ${}_2A_n$ for the number born to the ${}_2F_n$ mothers, etc. Finally, let $\Sigma_s F_t = M$ or total number of married women above the age of puberty in the country and $\Sigma_s A_t = C$ the total number of children born to these mothers.

5. Since the ${}_1F_n$ mothers are those who have completed their years of fertility, they have experienced all the different specific fertility rates for each age during the complete period of fertility. As a result, ${}_1F_n \times$ each of the r's in turn and the products added gives the total number of children born to this group, ${}_1A_n$. Similarly, ${}_2F_n$ multiplied by each r other than r_1 and the answers added gives the total number of children—namely, ${}_2A_n$—born to the ${}_2F_n$ mothers. In this way it is possible to draw up the following table:—

$$
\begin{aligned}
{}_1A_n &= {}_1F_n\,(r_1 + \ldots + r_n) = {}_1F_nR \text{ and} \\
{}_2A_n &= {}_2F_n\,(r_2 + \ldots + r_n) = {}_2F_n(R - r_1), \text{ up to} \\
{}_nA_n &= {}_nF_n\,(r_n) = {}_nF_n(R - r_1 - \ldots r_{n-1}) \text{ and similarly} \\
{}_1A_{n-1} &= {}_1F_{n-1}\,(r_1 + \ldots + r_{n-1}) = {}_1F_{n-1}(R - r_n) \text{ up to} \\
{}_{n-1}A_{n-1} &= {}_{n-1}F_{n-1}\,(r_{n-1}) = {}_{n-1}F_{n-1}(R - r_1 - \ldots - r_{n-2} - r_n) \text{ and in general} \\
{}_sA_t &= {}_sF_t\,(r_s + \ldots + r_t) = {}_sF_t(R - r_1 - \ldots - r_{s-1} - r_{t+1} - \ldots - r_n).
\end{aligned}
$$

Summing these terms gives the expression:—

$$
\begin{aligned}
\Sigma_s A_t = C &= \sum_1^n {}_sF_tR - \sum_1^n r_s \text{ (Number of Terms in } {}_sF_t) \\
&= RM - \sum_1^n r_s \text{ (Number of Terms in } {}_sF_t).
\end{aligned}
$$

The analysis shows that $C = RM -$ a number of terms in r and F, and therefore $\frac{C}{M} = R -$ a number of terms in r and $F \div M$ (1.0). Now C is the total number of children born to all the mothers and $\frac{C}{M}$ is therefore less than the true total fertility rate.

6. The terms in r and $F \div M$ can be further analysed as follows:—

For simplicity put $n = 5$

The term containing $r_1 = r_1({}_2F_5 + {}_3F_5 + {}_4F_5 + {}_5F_5 + {}_2F_4 + {}_3F_4 + {}_4F_4 + {}_2F_3 + {}_3F_3 + {}_2F_2) \div M$
$= r_1(M - {}_1F_5 - {}_1F_4 - {}_1F_3 - {}_1F_2 - {}_1F_1) \div M.$

Similarly
$r_2 = r_2(M - {}_1F_5 - {}_2F_5 - {}_1F_4 - {}_2F_4 - {}_1F_3 - {}_2F_3 - {}_1F_2 - {}_2F_2) \div M$
$r_3 = r_3(M - {}_1F_5 - {}_2F_5 - {}_3F_5 - {}_1F_4 - {}_2F_4 - {}_3F_4 - {}_1F_3 - {}_2F_3 - {}_3F_3) \div M$
$r_4 = r_4(M - {}_1F_5 - {}_2F_5 - {}_3F_5 - {}_4F_5 - {}_1F_4 - {}_2F_4 - {}_3F_4 - {}_4F_4) \div M$
$r_5 = r_5(M - {}_1F_5 - {}_2F_5 - {}_3F_5 - {}_4F_5 - {}_5F_5) \div M.$

Adding these coefficients

$$\Sigma \text{ terms} = \frac{M\Sigma r}{M} - r_1({}_1F_5 + {}_1F_4 + {}_1F_3 + {}_1F_2 + {}_1F_1) \div M$$

$$
\begin{aligned}
&- r_2({}_1F_5 + {}_2F_5 + {}_1F_4 + {}_2F_4 + {}_1F_3 + {}_2F_3 + {}_1F_2 + {}_2F_2) \div M \\
&- r_3({}_1F_5 + {}_2F_5 + {}_3F_5 + {}_1F_4 + {}_2F_4 + {}_3F_4 + {}_1F_3 + {}_2F_3 + {}_3F_3) \div M \\
&- r_4({}_1F_5 + {}_2F_5 + {}_3F_5 + {}_4F_5 + {}_1F_4 + {}_2F_4 + {}_3F_4 + {}_4F_1) \div M \\
&- r_5({}_1F_5 + {}_2F_5 + {}_3F_5 + {}_4F_5 + {}_5F_5) \div M.
\end{aligned}
$$

Examination of a large number of coefficients of r show that the values rise to a peak at or about 27 years of age and then gradually decline from 27 to 45 or 50 years of age. In other words, the shape of a function giving the values of the coefficients as ordinates is hump-backed, and may be described (very approximately) as triangular. The general term for the expression (1.0) is

$$
\begin{aligned}
\frac{C}{M} = R - \frac{M\Sigma r}{M} + r_1 \frac{({}_1F_n + \ldots + {}_1F_{n-1} \ldots {}_1F_1)}{M} + \ldots \\
r_s \frac{({}_1F_n + \ldots + {}_sF_n + {}_1F_{n-1} \ldots {}_sF_{n-1} + \ldots {}_sF_s)}{M} \ldots + r_n \frac{({}_1F_n + \ldots + {}_nF_n)}{M}. \quad (1.1)
\end{aligned}
$$

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7. Since

$$R - \frac{M\Sigma r}{M} = R - R = O, \frac{C}{M} = \sum_{s=1}^{s} {}^{n} r_s \frac{({}_1F_n + \ldots + {}_sF_n + {}_1F_{n-1} + \ldots + {}_sF_{n-1} + \ldots + {}_sF_s)}{M}$$

$$= \text{say } \sum_{s=1}^{s=n} R_s T_s.$$

But $\sum_{s=1}^{s=n} R_s T_s = n(\overline{RT} + \rho\sigma_R\sigma_T)$ where $\bar{R} = \Sigma\frac{r}{n}$; $\bar{T} = \Sigma\frac{T}{n}$ and ρ is the coefficient of correlation between R and T. The two series R and T cover the range 15 to 49 years inclusive, and in general move in sympathy, and both series normally reach a peak at the age of 26 or 27. The correlation coefficient ρ is therefore approximately equal to unity.

8. The frequency distribution of values of R and T is roughly rectangular in shape. Professor Bowley has shown that taking the average as the central value of a rectangular distribution, the extreme values of the distribution are at $\sqrt{3}\sigma$ on either side of the central value. Since the extreme values of R and T to the left of the central value are approximately 0, the distances from these extremes to the central value are $\bar{R}$ and $\bar{T}$, respectively, where $\bar{R}$ and $\bar{T}$ are the average values of R and T. These values are equal to $\sqrt{3}\sigma$, so that $\sqrt{3}\sigma_R = \bar{R}$ and $\sigma_R = \frac{\bar{R}}{\sqrt{3}}$, and similarly $\sigma_T = \frac{\bar{T}}{\sqrt{3}}$.

9. For general purposes where the value of $\bar{T}$ is unknown it is necessary to obtain some estimate of $\bar{T}$. It has been shown that the curve of the function of T is approximately triangular in shape. Values of T therefore roughly increase in arithmetical progression and then decline in a similar manner. The sum of the total values of T is therefore $\frac{n_1}{2}(t_1 + l_1) + \frac{n_2}{2}(t_n + l_2)$, where n_1 = the number of values of T in the part of the triangle which is increasing in value and n_2 the number of terms in the part which is decreasing. The sum of n_1 and $n_2 = N$ or the total number of values of T. It follows that $\bar{T}$ is approximately $\frac{n_1}{2}(T_1 + l_1) + \frac{n_2}{2}(T_n + l_2) \div N = \frac{1}{2}(\bar{T}_w + \bar{l}_w)$ where $\bar{T}_w$ is a weighted average of T_1 and T_n and $\bar{l}_w$ the weighted average of l_1 and l_2. The term l is the value of the last term in each series. In the first series it represents the maximum value of T and in the second the value of T next to the maximum.

10. It is only necessary to obtain the values of T, T_n and l, and l_2 in order to obtain an approximate value of $\bar{T}$. Taking the periods 15 to 45 and 15–50 as the child-bearing period, $N = 31$ in the first and 36 in the second case. It is usual for T to reach a maximum at the age of 27 for either period, so that in both cases $n_1 = 13$ and $n_2 = 18$ and 23 respectively. Once the values of T_{15}, T_{27}, T_{28} and T_{45} or T_{50} are known the approximate value of $\bar{T}$ can be easily estimated.

11. The following table gives the values of T_{15}, $T_{45 \text{ or } 50}$ for the Union of South Africa and Britain for the certain census years when fertility questions were asked. The maximum value of T and the value for the year after the maximum are also given, together with an estimate of $\bar{T}$.

	Union of S. Africa		England and Wales
	1921	1926	1911
T_{15}	0·013	0·01	0·002
T_{27} (l_1)	0·656	0·653	—
T_{28} (l_2)	0·651	0·653	—
T_{29} (l_1)	—	—	0·692
T_{30} (l_2)	—	—	0·692
T_{45}	0·25	0·29	0·33
T_{50}	0·16	0·20	—
$\bar{T}_{45}$	0·40	0·41	0·42
$\bar{T}_{50}$	0·38	0·39	—

The symbol $\bar{T}_{45}$ represents the average value of t when fertility is taken up to 45 years of age nd $\bar{T}_{50}$ the average up to 50 years.

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The five values for $\bar{T}$ vary between 0·38 and 0·42, and for working purposes it will be assumed that $\bar{T} = 0{\cdot}40$ is a good all-round average. Unfortunately, owing to the limited nature of the reference libraries in Rhodesia, it has not been possible to work out more examples, and it is suggested that more refined results will be obtainable when a greater number of values of $\bar{T}$ have been evaluated.

12. By substituting the assumed values of $\bar{T}_{15}$ and $\bar{T}_{50}$ in equation 1.2(*a*) the following general

(*a*)
$$\begin{aligned}\frac{C}{M} &= \Sigma R_s T_s \\ &= n(\overline{RT} + \rho\sigma_R\sigma_T) \\ &= n\left(\overline{RT} + 1 \times \frac{\overline{RT}}{3}\right) \\ &= n\overline{RT}(1 + \tfrac{1}{3}) \\ R = \frac{C}{M} \div \left(\frac{4}{3} \times \bar{T}\right) &= \frac{3C}{4\bar{T}M} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1.2)\end{aligned}$$

formulæ are obtained, which may be used for deriving the total fertility of married women in an African census where accurate figures of the age distribution of African mothers are unattainable owing to the low standard of education of the African.

$$R = \frac{C}{M} \cdot \frac{3}{4 \times 0{\cdot}40} = 1{\cdot}88\,\frac{C}{M} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1.3)$$

13. In employing this formula it is important to bear in mind the meaning of the term R. The individual r's employed in the mathematical analysis apply to a period of 30 or more years. This means that the specific fertility rates are assumed to remain unchanged over this period. The resulting estimate of R is an average total fertility over this period, and not the current total fertility. In cases where the birth-rate is falling, R is greater than the present total fertility, and vice versa. Over a series of successive census fertility enquiries the R's estimated on the basis advocated would represent a type of moving average, indicating the trend of total fertility over the period covered. Total fertility rates derived from the married population alone must obviously be greater than those obtained from the whole population. Specific fertility rates applied to all women, whether single or married, between 15 and 50 must necessarily be lower than those of married women even when the illegitimate birth rate is high. To relate them to the total population of women between 15 and 50 they may be corrected by the ratio of married to unmarried persons in the population. A conservative estimate of total fertility is then obtained because some of the unmarried women have illegitimate births. The true rate is of course somewhere between these two figures. Finally, the assumption that the functions of r and t are triangular in shape tends to raise the estimate of R because in practice they tend to be concave downwards with the result that $\bar{T}$ is larger than the estimate assumes.

14. The following table shows a number of estimates of total fertility rates derived from the formula. The rates have been used to obtain gross and net reproduction rates, but the latter have not been based on the masculinity at birth in the case of countries other than Portuguese East Africa, but on the sex composition of the total population at the date of the census. Probably it would be more accurate to base them on a straight 50 : 50 sex ratio. It can be shown mathematically that the correction of the gross reproduction rate by the average of the stationary population figures of the life-table for the years 15–45 or 50, as the case may be, gives a reasonable estimate of the net reproduction rate. For Portuguese East Africa the Life Table for Portugal was employed for Europeans, and the Union of South Africa Life Table for Coloured Persons (No. C.I. 1936) for all other races.

Remembering that the method gives average rates for over 30 years and also tends to overestimate the true figure, the results are not unreasonable. It is well known that there has been a downward trend in fertility in the Union of South Africa, New Zealand and England and Wales, and consequently the census results, being averages, give higher figures than the actual published estimate for the year concerned. The great difference in the net-reproduction rate of Europeans in Portuguese East Africa and Portugal is due to the difference in the ratio of the sexes at birth. The published figure gives a very high female ratio. Many of the differences in the estimates of the remaining races have been influenced by marked differences in the ratio of mothers to the

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Country	Census date	Total fertility		Gross reproduction rate		Net reproduction rate	
		Census result	Published	Census result	Published	Census result	Published
S. Rhodesia:							
15–50	1931	2,870	2,790[a]	1·43	1·35[a]	1·23	1·18[a]
Union of S. Africa:							
15–50	1921[d]	4,110	—	1·99	—	1·62	—
15–50	1926	3,760	3,450	1·84	1·67	1·54	1·42
New Zealand							
15–50	1921	3,410	3,000	1·67	1·44	1·45	1·29
England and Wales							
15–50	1911	3,430	2,750[b]	1·68	1·35[b]	1·33	1·11[b]
Port. E. Africa:							
15–50	1940						
Europeans		3,200	3,190[c]	1·51	1·87[c]	0·97	1·33[c]
Yellow races		4,830		2·02		1·22	
Brit. Indian		5,310		2·48		1·49	
Portuguese		4,680		2·22		1·33	
Coloured Persons ...		3,700		1·76		1·06	
Africans in Municipalities		3,720		1·67		1·00	

[a] Estimate for 1936; [b] 1921; [c] Portuguese in Portugal; [d] Includes still births.

NOTE.—The source of the published estimates is the *Statistical Year Book of the League of Nations*.

total number of women over 15 enumerated. This ratio is particularly high in the case of the Yellow races, and British Indians and relatively low in the case of Europeans, Coloured Persons and Africans residing in Municipalities.

15. The results of the census of population in Portuguese East Africa are particularly interesting, because, so far as I am aware, they represent the first attempt to enquire into the fertility of the African by the census method. For this reason the statistics of African fertility outside of the municipalities have been dealt with separately. The questions relating to fertility were put to every female of 10 years and over, whether married or not. It may be argued that native marriage customs are such that from the date of puberty all women are as liable to have children as married women in other countries. Thus, instead of having a formula for T involving a number of different ages of marriage, as in European countries, the age of child-bearing in the great bulk of the cases would start at 10 to 15. Examination of the distribution of mothers by age at the census date suggests that the actual position does not differ markedly from that of European communities, and that a large number of African females become mothers for the first time between the years of 25 and 30 and the bulk between the years of 20 and 30.

16. The subject of mortality conditions is difficult to determine. Out of 1,753,000 African women enumerated at the ages of 10 and over, 698 were 95 years and over and 2 between 120 and 125 years of age. The number of surviving children was 70 per cent. of the total born, as compared with figures varying between 76 and 93 for the other races. There is therefore no indication that mortality rates are markedly inferior to those of other races in the Colony, and it is therefore considered that the South African Life Table for Coloured Persons (No. C.1. 1936), if applied to the gross reproduction rates, will not be unduly favourable to the estimates of African net reproduction rates. The total fertility rate obtained by applying the formula is 4,658. The gross reproduction rate is 2·231 and the net reproduction rate 1·350.

17. Official estimates of the African population in Southern Rhodesia are made by applying certain arbitrary coefficients to the number of native taxpayers, and show the population as doubling in 35 years. The net reproduction rate given above shows an increase of 35 per cent. in a generation and indicates a doubling of the population every 58 years approximately (assuming a generation to be 25 years and fertility to remain constant). The demographic characteristics of the Southern Rhodesian African population cannot differ very markedly from those of the Portuguese East African natives, particularly in the areas bordering the Eastern Districts of Southern Rhodesia where the political boundary is entirely artificial. Examination of Portuguese

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East African census figures in the districts bordering Southern Rhodesia should yield some very interesting results. The estimates for Southern Rhodesia are of course further influenced by the large immigration from other territories which has taken place in the 35 years, but even allowing for this it would appear that the rate of growth of the African population in that country is possibly lower than is usually believed. An alternative explanation is that the census fertility figures are understated, but in view of the magnitude of the total fertility rate this explanation seems improbable.

18. The methods of arriving at total fertility rates proposed in this article are admittedly arbitrary. It will probably be found that after examination of the relationships between R and T and the evaluation of ρ, σ_K, σ_T, $\bar{T}$ by reference to a much larger range of census statistics than is available locally, greater refinements can be made in the formula. It is, however, claimed that a theoretical basis has been laid down for arriving at a practicable statistical method of utilizing very simple statistics of census fertility investigations for estimating gross reproduction rates where detailed age questions cannot be asked, and that the discovery of such a method is of pressing importance in determining the factors governing the rates of increase or decrease in the indigenous races of Africa. Finally, the organization of census enquiries into fertility in every State in the African Continent is also of pressing urgency if the true facts regarding the growth of population in Africa are to be reliably ascertained.

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The Use of a Punched-Card System for the Analysis of Survey Data, with Special Reference to the Analysis of the National Farm Survey

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Introduction

The analysis of a large mass of data presents computing problems which can be solved only by the aid of punched cards and mechanical tabulation, and it is important that the main principles of punched-card operations should be understood before an analysis by this means is undertaken, especially as the large amount of material to be handled necessitates careful planning from the start. This paper describes the methods which were used in the analysis of the National Farm Survey, which was carried out on Hollerith machines, and was supervised by the Statistical Department of Rothamsted for the Ministry of Agriculture. These methods provide a good illustration of the principles and potentialities of the Hollerith system.

The National Farm Survey

The Farm Survey of England and Wales was carried out between 1941 and 1943, and a full report of the results has been published by the Ministry of Agriculture.(5) All farms, or more strictly "holdings," with 5 acres or more of crops and grass were surveyed. The qualitative information consisted of type of tenure—whether the farm was rented or not—class of farmer—full-time, part-time, etc.—condition of the permanent equipment of the farm—farmhouse, buildings, roads, fences, etc.—presence of pests and weeds, information on water and electricity supply and grade of management of the farmer. Information was also obtained on the number of cottages, acreage of derelict land, length of occupation and rent.

The purpose of the analysis was not only to tabulate the results according to particular classifications such as counties or type of farming areas, but also to examine the inter-relationships of various attributes of farms. It was decided that the information on crops and stock in the return made by the occupier of each surveyed farm on June 4th, 1941, should also be related to the information obtained from the survey. It was obvious, as soon as an analysis on a national basis was contemplated, that the only possible way of analysing the data expeditiously and thoroughly was by the use of punched cards.

The total number of farms covered by the Survey was of the order of 300,000, but it was decided that results of sufficient accuracy could be obtained from the analysis of a sample of about 40,000 records. The sampling fractions used varied according to size of farm as follows :

Acreage, crops and grass	Sampling fraction
5 to 24·9	1/20
25 to 99·9	1/10
100 to 299·9	1/4
300 to 699·9	1/2
700 and over	1

The reason for the use of a variable sampling fraction was that, while small farms are relatively more numerous in the country, the large farms cover the greater part of the total agricultural area. The sampling fractions, while being of convenient magnitudes for sampling from a complete list of the holdings of the country, yield nearly maximum accuracy on the proportion by area of farms having a given attribute, for a given expenditure of labour in the analysis (this being roughly proportional to the number of farms for which the data were analysed). These sampling fractions are also such that sample totals for each size group may be integrated by the reciprocals on a Hollerith tabulator.

Hollerith machines

Accounts have been given elsewhere of Hollerith machines, (1, 2, 3, 4) and of their application to various problems. As far as possible, the present description will be brief, emphasizing the particular functions of the machines of which use was made.

(*a*) *The sorter.* This machine can sort cards according to the twelve possible positions of a hole

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in any one column of the card. It is possible to prevent the sorter reading any desired positions on the card by means of a commutator having twelve segments : when the segment corresponding to a position is pulled out, the machine ignores a hole which may exist in that position. Counters may be attached to the sorter in order to count the number of cards with holes in each position in the column, but the counts must be recorded by hand. A further development is the printing sorting-counter, which can sort cards to one column while counts are made on one, two or three columns at the same time. The group-selection device may also be attached to the sorter. This enables all cards with a particular combination of holes in up to eight columns to be segregated from the main pack in one passage of the cards through the machine, leaving the main pack in the same order as before.

(*b*) *The reproducing summary punch.* The function of the reproducer is to reproduce information from one card to another card. Briefly, such reproduction may be one of two types—gang punching or reproducing from interspersed master cards. In the former case the same information is required on a number of cards, and the machine obtains it from the card previously punched, the procedure being to insert a card at the beginning of the pack which contains the relevant information. Frequently, however, the cards may be classified into a large number of small groups, each requiring different information. In many cases this situation may be met, though rather laboriously, by feeding each group of cards with its master card into the reproducer separately. Such a procedure, however, may easily lead to mistakes, and the method of gang punching with interspersed master cards is used. For each group a master card is punched containing the designation of the group and the information to be punched on all cards of the group. The detail cards with the master cards leading are then sorted into the required groups. The master card belonging to a particular group of detail cards will then be immediately in front of that group, and the whole pack of cards is fed through the reproducer. The machine is plugged to read the information to be reproduced from the card in front of the card being punched at a particular moment. When the latter is a master card the punching is suspended by what are known as "X" brushes; each machine has six of these brushes which are permanently set on the columns most frequently used for this type of punching. (The positions may vary from machine to machine and may be altered to any desired columns by raising the cover of the machine and manual setting.) The master cards must be punched in the "X" position of one of these columns, which the machine is then plugged to read.

The reproducing summary punch may also be coupled to a tabulator, so that the results of tabulations may be punched on cards.

(*c*) *The tabulator.* The tabulator has often been described in other contexts, (1, 2, 3, 4) and only points of special interest in connection with the analysis of the Farm Survey data will be mentioned as they arise. The process of summary multiplication in correlation work was not used in the analysis of the Farm Survey. A description of this important method has been given by Hartley (3).

(*d*) *Other machines.* Other machines which were not used in the analysis of the Farm Survey, but which should be mentioned, as they are often useful in connection with the analysis of survey data, are the multiplying punch and the collator. The former needs little description.(1, 4) The latter is used for a variety of collating purposes, the simplest of which is the merging of two packs: in this instance the two packs are sorted into increasing order and are merged into one pack in the correct order. When the collator is used for matching two packs, cards of the same designation in up to fifteen columns in the two packs may be merged into one pack, the other classes—present in pack one and not present in pack two, present in pack two and not in pack one—being kept separate. This machine has two feeds and two card-sensing stations with a mechanism for sensing from which pack a card is to be cleared first, and four stackers for the possible groups into which cards may be sorted.

The coding of information

Considerable importance must be attached to the coding of information for punched-card analysis, as unless care and foresight are exercised the coding may be inaccurate. Furthermore, with bad initial planning it may be difficult, if not impossible, to carry out some of the required analyses without a large amount of machining and reproduction of cards. As an example, the writer some time ago was asked to supervise the analysis of data which involved approximately

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100,000 cards. The data were coded in such a way that the analysis could not be started until the cards had been modified, and this required the use of two to four sorters and a reproducer for two to three weeks. This work could have been entirely avoided by considering the potentialities of the system and the analysis required, when the coding scheme was being devised.

The coding scheme gives the meaning of each hole in any column: for instance, in the final coding of the Farm Survey the information of tenure was punched in column 7 of the first card for each farm, a "1" denoting that the farm was rented to at least three-quarters of its extent, a "2" that it was owned to at least three-quarters of its extent, and a "3" that it was partly rented and partly owned by the occupier. The principal problem which arises is that the card—say an eighty-column card—is not sufficient to contain all the information if one column is given over to each qualitative fact and the numerical data are not compressed. The use of two or more cards for the information relating to one farm or individual will often be inconvenient, because information on one card cannot be compared with that on another card. This and some other methods of accommodating the information on cards are described in turn.

(*a*) *The combination of qualitative information.* As a simple example from the Farm Survey, answers were given on the questionnaire to the questions, "Is there infestation with rabbits, rats, wood pigeons, other birds, insect pests, weeds?" To each question the answer was "Yes" or "No." It would obviously be wasteful to allocate one column for each of these questions, using six columns in all: as there are in all sixty-four possibilities, two columns only could be used, combining the first three answers to give eight possibilities, and likewise the second three answers. With increasing complexity, however, the accuracy of coding falls, and it cannot be expected that clerical labour would combine the three answers into single code with absolute accuracy. As a compromise, therefore, the six answers were grouped into three sets of two, requiring three columns in all, the coding for each pair of answers being Yes/Yes=1, No/No=2, Yes/No=3, No/Yes=4. This procedure is quite feasible when there are only two possible answers to each question, and may be worth while following if there are three possible answers and it is necessary to save a column. When the tabulator is used to evaluate the number of farms with each attribute, all the factors can be segregated immediately by its distributors in one run. A limitation to this procedure arises when it is desired to form totals of several numerical fields for the groups of farms with the possible answers to any one question: control of the tabulator on the particular column will always act when the position of the hole on successive cards is different, and all the totals will be split for the answers to the other questions. The tabulation can of course be made by dividing the pack into sub-packs according to the answers to the question, and tabulating each separately. If the totals are being made according to a further classification given by another column, the sorting should be made first to this column, and control on this one column will suffice generally.

(*b*) *The use of double punching.* Double punching implies the use of two or more holes in the columns of the cards, one hole referring to one piece of information and the other to the same or different information. In some types of survey work the answers to a question may not be mutually exclusive: a card may then contain more than one hole in the relevant column. The representation of the alphabet in one column of the card is accomplished by double punching using the *X* and *Y* positions (or "zones"). The letter *A*, for instance, is represented by "*X*," *B* by "*XO*," and so on, except that *G* is represented by "6" and *I* by "1," to *M*, which is "*X*9"; *N* is given by "*YO*," *O* by "*O*," *P* by "*Y*1," *Q* by "*Y*2," *R* by "*Y*3," *S* by "8," *T* by "*Y*4," and so on to *Y*, which is given by "*Y*9" and *Z* by "*Y*." Most tabulators are fitted with alphabetical bars and can interpret this code, printing the correct letter of the alphabet for as many as fifteen columns. Double punching is often useful in the representation of card reference numbers which will be used only for picking out or matching individual cards—*e.g.*, any number between 0 and 2,999 can be represented by a three-column field using the *X* or *Y* position of one of the three columns. The use of the *X* and *Y* positions to increase the range of a numerical field of which totals may be required may be used in exceptional cases, but is in general inconvenient, as additional counter capacity is required to accumulate the *X*'s when tabulating. The *X* and *Y* positions of columns occupied by numerical fields may be used for qualitative information in order to economize in card space, as they are ignored during addition.

The more straightforward type of double punching is the use of one column for two independent pieces of information. For example, the question "condition of farmhouse" required

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four classes—good, fair, bad and no farmhouse—and the question "condition of buildings" required the same number of answers. It would have been possible, therefore, to place both answers in one column, using positions "1" to "4" for the first answer and "5" to "8" for the second. As long as this information is only to be summarized or to be related to other qualitative information, card space can be usefully saved. If it is desired to form totals according to the groups given by the answers to one of the questions, the device is not to be advised, since the same difficulty arises as in the combination of qualitative information. This difficulty can be avoided to some extent, however, by the use of distributors or control cards which are inserted during sorting. A further disadvantage of this type of coding is that coding errors—for example, transposition of the answers and punching errors—are likely to be made relatively more frequently without being discovered at an early stage. Complete accuracy in the verification of the punching of cards which are double punched in some columns is rather difficult to attain.

(*c*) *The use of two or more cards.* The advantage of the use of two or more cards is that the coding can be made very simple, it being unnecessary to combine any of the information and not imperative to compress quantitative information. The main disadvantage has already been mentioned—namely, that information on one card cannot be related to information on another card. Against this may be balanced the fact that information can be reproduced from one card to its corresponding card rapidly and economically. The required information may be reproduced from one card to the other or from the two cards to a third card quite easily with the reproducing punch. The extension of this method is obviously to divide the information into groups combining the groups two at a time on cards—*e.g.*, to divide the information into three groups, *A*, *B* and *C*, and to put *A* and *B*, *A* and *C*, and *B* and *C* on cards together. All possible inter-relationships can then be examined at will.

(*d*) *The condensation of quantitative data.* In the Farm Survey the proportion of land which was heavy, medium, light, peaty bog and peaty fen, and which was naturally of good, fair and bad fertility was recorded. Since it was necessary to add these proportions, which were usually given to the nearest 10 per cent., it would appear at first glance that 16 ($=8\times2$) columns were required to place the information on cards. A total of eight columns, however, were saved by translating the percentages to a scale of eight with little loss of information: on this scale proportions between 0 and 6·25 per cent. were coded "0," between 6·25 and 18·75 per cent. were coded "1," and so on.

A different type of condensation is the following: farms were classified according to size, and for farms of under 100 acres only two columns were required to represent the acreage of any crop. On farms of over 100 acres, however, the acreage of individual crops might well exceed 99 acres, and three columns would be required. In order to save this extra column for each acreage it was decided to use the rule that acreages on farms of over 100 acres should be divided by 10. For some purposes it would be inadvisable to use such a rule, since all acreages of 5 or less on farms of over 100 acres are coded as zero, and in the case of some crops the distribution of acreages is markedly skew, with a preponderance of small acreages. Totals of such items would then be seriously under-estimated.

Apart from such devices, which sacrifice only a little of the information, in a few cases a grouping of a numerical item with variable intervals may be used: for example, the acreage of derelict land on each farm was given in the Farm Survey, but since little was present on the great bulk of farms, the following code, using only the twelve positions of one column, was used: 0 acres coded "0," 1–5 acres coded "1," 6–10 acres "2," 11–20 acres "3," 21–40 acres "4," and so on. When such grouping is used the total acreage is not easily obtained, but in this case the information of greater value was not the total acreage, but the proportion of farms with the various amounts of derelict land.

(*e*) *The use of trailers.* When dealing with quantitative data the position is often that the bulk of the individuals contain magnitudes of a certain order—say, less than a hundred—while a small proportion contain magnitudes of a higher order—say up to a thousand. If all the numbers are coded as they occur, one column is wasted on almost all the cards for each numerical field, and it may be necessary to eliminate this wastage. A simple method of doing so without seriously increasing the number of cards is the use of trailers. Suppose the bulk of the farms have acreages between 0 and 99 acres, but that some have acreages of 100 or more. Two columns are allocated to the acreage, and additional cards are introduced for those farms with more than 99 acres: such

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cards are called "trailers," the first card being called the main card. When a particular item for an individual is greater than the capacity on one card, the maximum possible is put on the main card and the surplus put on trailers; *e.g.*, if the number of cattle on a farm is 211, and two columns only are allowed for the item, the main card will contain ninety-nine, and two trailers will be necessary, one containing ninety-nine and one thirteen. All the qualitative information will be repeated on the trailers. It will be necessary to distinguish between main cards and trailers when relating attributes of individuals, since trailers must not be counted. This may be done by using the *X* and *Y* positions in columns given over to numerical data, or, as in the Farm Survey, where double punching was deliberately avoided, by the use of an extra column, and distribution of card counts on this column. If a large number of counts is being made, the trailers are removed by needling.

Further points in connection with coding

It is advisable to avoid as far as possible any blank columns interspersed with punched columns on any of the cards, because it is then easy to see that the coding is complete and codes will not be punched erroneously in blank columns. Usually the coding scheme may be easily modified to bring this about.

Care should be taken to avoid the unnecessary use of two columns for qualitative information when only one is necessary, as this may result in two sorting operations being necessary when only one would have sufficed. A typical example of this type of error is the coding of a "Yes or No" answer, of which the "No" answers classified into further groups. On the other hand, if the secondary classification is of minor importance, and it is necessary to break down numerical data according to the "Yes or No" answer, the combination of the two codes may be a definite disadvantage.

It is essential to avoid carrying unnecessary digits in quantitative information, as this may increase the number of times the cards have to be run through the tabulator to obtain the required totals. This is particularly important when the number of cards is very large and there are several numerical fields. The number of digits carried should be related to the accuracy required in the final results, the amount of card space available and the total counter capacity required in the making of tabulations.

Description of the coding used in the Farm Survey

When devising a questionnaire for this type of work, it is worth considering the possibility of using the same sheet as a coding sheet. In the case of the Farm Survey the original information was obtained from two distinct forms, and could not easily be coded on them. Specially printed coding sheets were therefore used, and the sheet for one farm is reproduced below. When the bulk of the analysis is being done by punched cards, but more detailed analyses on a proportion of the farms may be carried out by hand, it will be found that compact coding sheets similar to that below make such analyses easy to carry out.

Unless a farm required trailers for some of the quantitative information, two cards were used for each farm, card I being on the front and card II on the back of the sheet.

In general, these sheets need little explanation, the only device mentioned above which was not used being double punching. The first six columns on card I (which are also punched on card II by the punch operator) give the reference of the farm. There follows all the Farm Survey information; for example, the farm referred to being tenant-occupied by a full time farmer; the farm is half of heavy soil and half of medium soil; the layout is good, and so on. The codes for the condition of equipment are on a scale of good = 1, fair = 2, bad = 3, and absent = 4. It will be noted that all columns have one position punched: in the case of column 47, which gives the reason for bad (grade B or C) management, the cards of farms which had good management ("1" in column 46) had "*Y*" in column 47, this code also being used for unclassified reasons. Card II is largely self-explanatory: the acreages of all farms of over 100 acres (size group 3, 4 or 5—column 7) were divided by ten to save space. The column denoted by "no. of farm" was used to differentiate between main cards (punched "1") and trailers (punched "0"). Eventually much of the information of card I was reproduced on to card II and vice versa.

A certain number of routine checks on the coding could be made immediately the data were punched: for example, "condition of farmhouse," coded as 4 (= no farmhouse), should be

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FARM SURVEY CODING SHEET

Card I (Survey Record)

No. of Column	Item	Code
1, 2	County	15
3	C.W.A.E.C. District	1
4, 5	Parish	19
6	Farm	1
	Tenure	
7	A1 Tenure	1
8	A3 Class of Farmer	1
9	Type of Holding	3
10	A4 & A5 Other Land & Grazing Rights	2
	Condition of Farm	
11	B1 (Types of Soil) Heavy	4
12	Medium	4
13	Light	0
14	Peaty Bog	0
15	Peaty Fen	0
16	B2 Lay Out	1
17	B3 Natural Fertility Good	0
18	Fair	8
19	Bad	0

No. of Column	Item	Code
20	B4 Sit. Road	1
21	B5 Sit. Railway	2
22	B6 Cond. Farmhouse	3
23	B6 Cond. Buildings	3
24	B7 Cond. Farm Roads	3
25	B8 Cond. Fences	2
26	B9 Cond. Ditches	2
27	B10 Field Drainage	2
28	B11 Cond. Cottages	4
	Cottages	
29, 30	B12 (Parts 1 & 2)	00
31, 32	B12 (Part 2)	00
33, 34	B13	00
35	B14 Rabbits & Rats	2
36	B14 Rooks & Other Birds	2
37	B14 & B15 Insects & Weeds	2
38	B16 Derelict Fields	0

No. of Column	Item	Code
	Water Supply	
39	C1 To Farmhouse	6
40	C2 To Buildings	6
41	C3 To Fields	4
42	C4 Seasonal Shortage	1
	Electricity	
43	C5 Public Light and Power	2
44	C5 Private Light and Power	2
45	C6 Used in House and Farm	2
	Grade of Farmer	
46	D1 Grade	1
47	D2 Reason for B or C	Y
	Pasture & Arable	
48	D3 Condition Arable	1
49	D4 Condition Pasture	1
50	D5 Fertilisers Arable	1
51	D5 Fertilisers Pasture	3
52, 53, 54	Type of Farm Area	345
55	No. of Farm	1

Card II (4th June Return)

No. of Column	Crops	No. on June 4th Return	Code
7	Size Group		3
8, 9	Wheat Barley Rye	1, 2, 6	09
10, 11	Other Cereals	3, 4, 5, 7, 8	08
12, 13	Potatoes Sugar Beet	9, 10, 13	02
14, 15	Fodder Etc.	11, 12, 14–21	02
16, 17	Hops, Orchards	22, 23	00
18, 19	Small Fruit and Vegetables	24–26	00
20, 21	Fallow	28	00
22, 23	Temporary Grass	29, 30	00
24, 25, 26	Permanent Grass	31, 32	005
27, 28, 29	Acreage C. & G.	33	026
30, 31, 32	Rough Grazing	34	000
	Labour		
33, 34	Labour	35–38	04

No. of Column	Livestock	No. on June 4th Return	Code
35, 36, 37	Cows	43, 44	000
38, 39	Cattle/10	54	02
40, 41, 42	Sheep/10	63	029
43, 44	Pigs/10	72	00
45, 46, 47	Poultry/10	79	000
48, 49	AG Horses	81, 82	06
	Page 3		
50	Engines	133, 135–9 141	0
51, 52	Tractors	140, 142	00
53, 54, 55	Rent/10	143–146	026
56	Length of Occupation	147, 148	2
57	No. of Farm		1

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accompanied by "water supply to farmhouse," code 0 (= no farmhouse), and such checks were carried out on sorters. On card II the total acreage of crops and grass should equal the sum of the acreages above it: in the case of two counties this was checked by the tabulator, but the operation was slow and expensive compared to checking by hand, and was not used.

Supplementary coding by the machines

Supplementary coding by Hollerith machines was in general made by the use of master cards and the reproducer. The master cards were prepared by hand, except in one instance, when they were prepared by the tabulator and summary punch.

(*a*) *Simple combination of qualitative information.* As an example, the combination of the information on the soil classification may be quoted. As stated above, the proportion of each of the five soil types was punched in five columns using a scale of eight. For many purposes, however, a simple classification on one column was required: the ten classes chosen in the order in which they were to be obtained were as follows:

	Code
Predominantly peaty bog (75% or more)	0
Other peaty bog	1
Predominantly peaty fen (75% or more)	2
Other peaty fen	3
Predominantly heavy (75% or more)	4
medium ,, ...	5
light ,, ...	6
Mixed heavy and medium	7
Mixed medium and light	8
Remainder	9

In order to carry out this classification, master cards were prepared, one master card for each possible combination of codes in the five columns, *i.e.*, for each possible combination of amounts of the five soil types. These cards were punched with the above code and "*X*" in a column corresponding to one of the "*X*" reading brush positions, and were placed at the beginning of the main pack. By sorting to the five columns in order, the master cards were placed in front of the group of cards to which they refer. The whole pack was then run through the reproducer, using the procedure for gang punching with interspersed master cards.

(*b*) *Combination of quantitative information.* For each farm, in addition to the acreages of other crops, the acreage of permanent grass and the total acreage of crops and grass were available, and it was desired to classify farms according to the percentage of farm arable into the following twelve groups: 0% (*X*), 0·1 to 9·9% (0), 10 to 19·9% (1), 20 to 29·9% (2), 30 to 39·9% (3), 40 to 49·9% (4), 50 to 59·9% (5), 60 to 69·9% (6), 70 to 79·9% (7), 80 to 89·9% (8), 90 to 99·9% (9), 100% (*Y*). It would have been possible to perform divisions by hand or by the multiplying punch using reciprocal cards, and then to code the percentages. A more economical method using only sorters and the reproducer is the following. For any value of the total acreage of crops and grass the classes of percentage arable are determined by twelve values for the acreage of permanent grass—*e.g.*, if the total acreage is 100 acres the lower limits of the acreage of permanent grass in the twelve groups of percentage arable given above are respectively 100, 91, 81, 71, 61, 51, 41, 31, 21, 11, 1 and 0. Twelve master cards are therefore punched for each value of the total acreage, one for each lower limit of the permanent grass, and the master cards are sorted into the pack according to the columns giving the acreage of permanent grass and the total acreage, the master cards leading. After the sort has been carried out the master cards are in front of the detail cards belonging to the same group of percentage arable as the master card. The master cards with an "*X*" in a chosen column contain the code of the group of percentage arable which is reproduced on all cards between it and the next master card. In gang punching of this type there is control by the machine only on the total acreage, which is compared card by card in each group. It is therefore necessary that the sorting operations be checked with extreme care. To gang punch this information on all cards with a range of 5 to 99 in the total acreage required just over 1,100 master cards. It was not economical to evaluate the index on farms of acreages of 1,000 or more by this method, since each acreage requires twelve master cards and the number of such farms was small: the index for these was obtained by hand.

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In the same way, farms were coded for rent per acre (total rent divided by total acreage of crops and grass), and for percentage of farm which was rough grazing (acreage of rough grazing divided by the acreage of crops and grass plus the acreage of rough grazing).

(*c*) *The combination of qualitative information on the basis of scores into indices.* It was decided that the separate condition factors should be combined with chosen weights into an index giving an overall picture of the condition of the farm. The procedure was to prepare master cards covering every possible combination of the factors to be combined. Since one of the indices required a very large number of master cards, it was desirable to find some method of producing these master cards by the use of machines. This index combined water to farmhouse (twelve possible codes), water to buildings (twelve codes), water to fields (eight codes), seasonal shortage (two codes) and electricity (four codes), so that in all $12 \times 12 \times 8 \times 2 \times 4 = 9{,}216$ master cards were required.

A card for each possible combination of codes in the five columns was obtained by the reproducer: thus twelve cards were hand-punched for the possible codes in the first column; these were reproduced twelve times gang-punching, first a " 0 " in the second column, then a " 1," and so on. At the same time the score for each factor was punched on the card. When all the possible combinations were on cards (set I), a second set was obtained on the reproducer (set II). The cards of set I were then run through the tabulator, adding the scores for each card and the total for each card was punched on the corresponding card of set II. The cards of set II were sorted according to total score, and the code corresponding to each score punched on the card. These cards were then used as master cards to index the individual cards by the method of gang punching with interspersed master cards.

The analysis

The results obtained in survey work may be roughly divided into two classes—counts which indicate the number of individuals with particular attributes and tabulations giving the totals of numerical fields for groups of cards.

(*a*) *Counts.* There are two methods of obtaining numbers of individuals with particular attributes, *i.e.*, of cards with holes in the various positions. The simple method involves the use of the sorting counter. The cards are counted as they are sorted, and the counters count the total number of holes punched in each position. This is useful when a card may be punched in more than one position of the column—*i.e.*, the classes corresponding to the possible positions are not mutually exclusive, and it is desired to find out how many individuals fall into each class.

The other method requires the use of the tabulator. The tabulator can count the cards passing through it, and this count can be led through what are known as distributors into counters. Distributors enable a numerical field to be led to one of several counters according to information punched on the card. The path through the distributor is determined by a reading of the card one cycle before it is tabulated, this reading being taken to a commutator which determines whether the distributor is to be in the " on " or the " off " position—the numerical field going into the distributor in the centre position and coming out either in the " on " or the " off " position. (Actually the " on " position is also the " centre " position.) By the time the card is being tabulated, the distributors are in the correct position for that card. As an example, to split a card count on the twelve positions in one column, twelve counters (*e.g.*, by splitting each of six counters into halves) and four distributors will be necessary. The distributor set up is the same for counts on all columns, except for a simple plugging change which determines on which particular column distribution is to take place. The mechanism is illustrated in the diagram on p. 292.

Each distributor is able to distribute a numerical field of eleven columns, but in this case the field distributed consists of the card count of unity for each card. The diagram shows the path of the card count through the distributors, going into them by the centre position and out by the " on " or " off " position either to another distributor or to a counter. This method of obtaining counts is of great value when the count must be broken down according to other information on the card. In the Farm Survey, for example, all counts had to be broken down by size of farm groups, since a variable sampling fraction was used. By sorting the cards first according to county and size of farm, and then tabulating the card count in the above way, the numbers of farms with particular attributes in each size of farm group within each county were obtained without splitting the pack. To obtain the same results by the counting sorter it would be necessary to split the cards into the required groups and to feed each group separately, recording the counts by hand. Against

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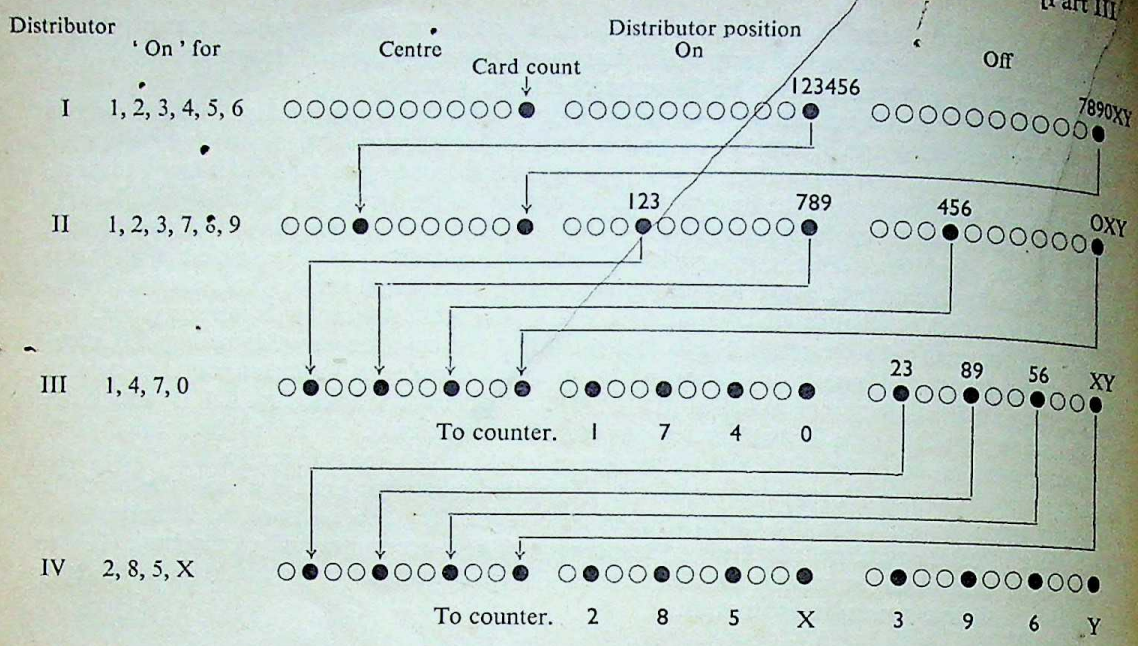

this advantage must be balanced the fact that the maximum speed of the tabulator is only 9,000 cards per hour compared to 24,000 cards per hour for the sorter. Table 1 contains a section of the distribution of coded rent per acre (column 78) for the farm size groups (column 58) of farming areas (columns 52, 72 and 73). This distribution was obtained on a tabulator by distribution of the card count as described above.

TABLE 1

Distribution of rent per acre

Cols. 52	72	73	58	1	2	3	4	5	6	7	8	9	0	X	Y
1	2	7	1	1		1	7	4	9	6	5	7	5	9	15
1	2	7	2	1		3	17	32	23	27	14	15	4	1	2
1	2	7	3		4	18	55	59	32	10	7	3	2	1	
1	2	7	4			5	15	11	7	4	3				
1	2	7	5			1		5	1						
1	4	1	1				1			1		2	2	3	2
1	4	1	2		1		3	2	3	2	3	2		1	
1	4	1	3		2	7	15	24	20	7	9	2			
1	4	1	4	1		5	15	29	21	9	3	2			
1	4	1	5			2	2	7	3		1				

(*b*) *Tabulations.* All the models of the Senior Rolling Total Tabulator have six counters of either nine or eleven wheels. They have varying numbers of banks for listing and total printing. Some models have a bank which lists alphabetical information given by double punching on the card. Their function is to obtain totals of numerical fields on the cards, and the totals can be broken down according to information given in other columns of the card. When totals are obtained at the end of card groups they may be printed or rolled into other counters to form progressive or grand totals. A description of the actual mechanism of control which enables the machine to decide when totals must be taken and disposed of either by rolling or printing is given by Comrie (2). These functions of the machine may be modified by distributors which enable a numerical field to be taken from the card to one of several counters (as above in the case of card counts), or enable the total in a counter to be rolled into one of several counters or to be printed by one of the printing banks according to its designation. The six counters may be split into parts which accumulate totals independently, though care must be exercised to avoid the accumulation in a counter of a

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total greater than its capacity, in which case the first figure of the total is added into the units position of the total of the other part of the counter.

It is not essential for the individual worker who is supervising the analysis of data to be familiar with all the details of the plugging of tabulators, as these are known by the operators. An important step in the analysis, however, is the planning of the exact form of the tabulations and the preparation of a scheme of tabulation which minimizes the amount of work, particularly of preliminary sorting.

The rolling feature of the Rolling Total Tabulator proved very useful in the analysis of the Farm Survey, as it was necessary to break down all totals according to size of farm groups, and to integrate the size group totals by the reciprocals of the sampling fractions. These were in fact 20, 10, 4, 2 and 1, and the integrated totals were obtained from the unintegrated totals on the tabulator directly. The multiplications were made by combinations of repeated rolling of a counter into itself, together with transposition by one place (by distributors). It was possible to call for the particular combinations by the sensing of the column containing the size group designation.

TABLE 2

Tabulation of acreage according to county, size, group and type of tenure

Cty.	Size group	Tenure	57	64	8–9	10–11	12–13	14–15	16–17	18–19	20–21	22–23
12	1	1	260	400		26	6	6	6	4		2
12	1	2	540	840	18	36	20	24	18	12		
12	1	3	40	40		10			10			
12	2	1	250	430	164	251	6	68	4			21
12	2	2	190	310	90	120	12	33	35	8	37	21
12	2	3	140	270	33	125	14	37	6		1	27
12	3	1	344	540	924	912	44	256	8	68	104	212
12	3	2	156	216	576	488	64	168	8	48	84	156
12	3	3	40	64	116	84	12	44	4	8	20	44
12	4	1	152	216	1390	966	74	350	4	20	206	400
12	4	2	60	76	664	418	40	142	58	54	102	178
12	4	3	34	40	424	222	18	100	8	6	74	94
12	5	1	26	28	585	349	29	160	2	3	127	243
12	5	2	25	29	934	422	21	285	11	1	231	315
12	5	3	6	7	139	81	13	39		2	16	48

The American tabulator does not have the rolling feature, but is capable of carrying out direct addition or subtraction according to card designation. In contrast to the Rolling Total Tabulator, the counting wheels may be grouped in counters entirely at will, and this enables much more efficient use of counter capacity in tabulating work. This tabulator, however, has no distributors, and this will often detract from its usefulness in the analysis of survey data.

Table 2 contains part of the tabulation of crop acreages according to type of tenure, size of farm group and county. In this case the totals for each size group are multiplied by the reciprocals of the sampling fractions by rolling and distributing.

The speed of Hollerith work

(*a*) *Coding*. The speed with which coding of raw data is accomplished depends on the actual state of the data and the complexity of the coding scheme. In some cases the answers on the questionnaire are so simple that coding can be done at sight, and in other cases special coding sheets must be designed.

(*b*) *Punching*. The rate of punching depends on the state of the documents which the puncher must read. In the case of the Farm Survey, when two-thirds of the card was to be punched, a speed of about 100 cards an hour was realized. The speed of verifying was slightly higher.

(*c*) *Sorting*. The theoretical speed of the sorter is 24,000 cards per hour. The extent to which this speed is realized in practice depends on the number of cards which must be sorted to each particular column, since before a change may be made the machine must be stopped and all boxes emptied. The actual changing of the column to which the cards are sorted is a trivial operation.

(*d*) *Reproducer*. Gang punching and reproducing are carried out at a speed of 6,000 cards per hour.

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(*e*) *Tabulator*. The tabulator lists cards at 6,000 cards per hour and tabulates at 9,000 cards per hour. This latter speed is realized only if there are a very large number of cards per group. This is often not the case, and the speed may fall to less than 1,000 cards per hour if the number of cards per group is small and several rolling and printing operations have to be made for each group. It is therefore of great importance to estimate roughly the output of the machine on particular jobs. The essential points to be borne in mind when doing so, as far as numerical tabulators are concerned, are that

(1) listing of the designation of first card of a group takes 0·6 second (this includes the tabulation time of 0·4 second);

(2) all the remaining cards of the group will be tabulated at a speed of 0·4 second each;

(3) all the other functions to be carried out require a cycle each, though two or more functions can be carried out at the same time, provided that no counter is used in more than one function at the same time. Cycles involving printing take 0·6 second each and all other cycles—*e.g.*, for rolling—take 0·4 second. If summary cards are being punched, a further 1·2 seconds must be added for each summary card punched.

As an example, if there are twenty cards per group and these are totalled without rolling, the time taken for each group in the printing of the total is

0·6 second for listing and tabulating the first card
7·6 seconds for tabulating the remaining nineteen cards
0·6 second for printing totals
0·4 second for zeroising the counters

9·2

On the average each card takes 0·46 second and the speed realized will be about 7,800 cards per hour. If, however, there were only two cards per group, the speed would be $\frac{3,600}{2·0} = 1,800$ cards per hour.

As a more complicated example, it was necessary in the Farm Survey to check that the sum of nine separate acreages was equal to the coded total acreage. The method of checking by the tabulator is the following. The nine fields are put into five counters, four of which are split into two parts:

i.e.,	A/B,	C/D,	E/F,	G/H,	I,	Coded Total
counter	1	2	3	4	5	6

Counter 1 is rolled into 2, while 3 is rolled into 4: then counter 2 is rolled into counter 4 giving $A + C + E + G$ in the first half and $B + D + F + H$ in the second half of this counter; each of these two parts is rolled successively into counter 5; the resulting number in counter 5 is then rolled negatively into counter 6, and the result (which should be zero) printed and all the counters zeroised. In all, therefore, the time per card is

0·6 second for listing
2·0 seconds for five rolling cycles
0·6 second for one subtracting and printing cycle
0·4 second for zeroising

3·6

A speed of 1,000 cards per hour is therefore realized. In this particular case, when the proportion of errors was very small, a more reasonable method would have been to sort the cards to sufficient columns to give groups of approximately twenty cards, say, and to carry out the check on these groups. The speed would then be $\frac{3,600}{11·2} \times 20$ cards per hour—that is, approximately 6,500. Groups which did not check could then be tabulated individually. The location of coding and punching errors is most important in Hollerith work, as endless trouble results when errors are found after the analysis has been started, and every effort should be made to impose checks by machine.

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Summary

A description is given of the use of punched cards in the analysis of survey data with particular reference to the National Farm Survey. Information on many aspects of the condition of farm equipment had been obtained in this survey, and it was necessary to tabulate the results and examine relationships between various farm attributes and between attributes and the cropping and stocking of the farms. While the actual computations involved were of a simple character, the only practicable method of carrying out the analysis was by the use of punched cards and Hollerith machines. The technique of punched card analysis for this type of data is described.

References

(1) L. J. Comrie. *The Hollerith and Powers Tabulating Machines.* Private circulation, 1933.
(2) L. J. Comrie, G. B. Hey and H. G. Hudson. "The Application of Hollerith Equipment to an Agricultural Investigation." *J. Roy. Stat. Soc.* (*Suppl.*), 1937, **2**, 210–24.
(3) H. O. Hartley. "The Application of some Commercial Calculating Machines to Certain Statistical Calculations." *J. Roy. Stat. Soc.* (*Suppl.*). (In the press.)
(4) O. Kempthorne. "The Analysis of a Series of Experiments by the Use of Punched Cards." *J. Roy. Stat. Soc.* (*Suppl.*). (In the press.)
(5) *National Farm Survey of England and Wales.* A Summary Report. Ministry of Agriculture and Fisheries. H.M.S.O. 1946.

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The Frequency Distribution of the Difference Between Two Poisson Variates Belonging to Different Populations

By J. G. Skellam

The coefficient of t^r in the expansion of $G_1(t) = e^{m_1(t-1)}$ is the probability of r successes in a Poisson population (1) whose mean is m_1.

Similarly, the coefficient of t^{-r} in the expansion of $G_2(t) = e^{m_2(t^{-1}-1)}$ is the probability of r successes in a Poisson population (2) whose mean is m_2.

It follows that the coefficient of t^r in $G(t) = G_1(t)G_2(t)$ is the probability that a variate value from population (1) will exceed a variate value from population (2) by exactly r

$$G(t) = e^{-(m_1+m_2)+m_1t+m_2t^{-1}}$$

The function $L(t)$ obtained by writing e^t for t in $G(t)$ and taking the logarithm generates the cumulants. K_r is given as the coefficient of $\frac{t^r}{r!}$.

We have $L(t) = -(m_1 + m_2) + m_1e^t + m_2e^{-t}$.

Hence for $n \geqslant 1$,

$$K_{2n-1} = m_1 - m_2 = D$$
$$K_{2n} = m_1 + m_2 = S$$

and from these we obtain

$$\mu_2 = S,\ \mu_3 = D,\ \mu_4 = S + 3S^2, \text{ etc.},$$
$$\beta_1 = \frac{D^2}{S^3}, \qquad \beta_2 = 3 + \frac{1}{S}.$$

A transformation:

write $$m_1 = \frac{z}{2}\phi \quad \text{and} \quad m_2 = \frac{z}{2}\phi^{-1} \text{ in } G(t)$$

then $$G(t) = exp\left\{-\frac{z}{2}\left(\phi + \frac{1}{\phi}\right) + \frac{z}{2}\left(\phi t + \frac{1}{\phi t}\right)\right\}$$

Now it is well known that

$$e^{\frac{z}{2}\left(t+\frac{1}{t}\right)} = \sum_{-\infty}^{\infty} t^n I_n(z)$$

where $I_n(z)$ is a Bessel Function which has been extensively tabulated.

Note.—$I_n(z) = i^{-n}J_n(iz)$ defined by Basset, see *Proc. Camb. Phil. Soc.*, vi (1889), p. 11, tabulated in *British Association Reports*, and by others—*e.g.*, E. Anding, *Sechsstellige Tafeln der Bessel'schen Functionen imaginären Argumentes* (Leipzig, 1911).

Hence $G(t)$ becomes

$$exp\left\{-\frac{z}{2}\left(\phi + \frac{1}{\phi}\right)\right\} \sum_{r=-\infty}^{+\infty} \phi^r t^r I_r(z)$$

and the probability of a variate difference of exactly r is the coefficient of t^r:—

$$e^{-\frac{z}{2}\left(\phi+\frac{1}{\phi}\right)} \phi^r I_r(z)$$
$$= e^{-(m_1+m_2)} \left(\frac{m_1}{m_2}\right)^{\frac{r}{2}} I_r(2\sqrt{m_1m_2})$$

If we consider the particular case $m_1 = m_2 = m$ we obtain the result

$$e^{-2m} I_r(2m)$$

[see Irwin (1937), *Journ. Roy. Statist. Soc.*, 100, p. 415.]

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REVIEWS OF STATISTICAL AND ECONOMIC BOOKS

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1.—*Rudimentary Mathematics for Economists and Statisticians.* By W. L. Crum and Joseph A. Schumpeter. McGraw Hill Publishing Co. Ltd. 1946. $8\frac{1}{4}'' \times 5''$. xi + 183 pp. 12*s.* 6*d.*

This volume is a revision and expansion of a text by Professor Crum issued under the same title as a supplement to the *Quarterly Journal of Economics* in March, 1938. The additions are largely due to Professor Schumpeter; they include the introduction of such useful methods as the Lagrange multiplier, an extension of the section on differential equations to higher order and partial equations and a new chapter on determinants. There are also some additional economic applications such as the homogeneous production function.

Two comments only are needed in this review of a text which has already been widely used. The first is to underline the statements made by the authors on their objects in writing this brief exposition. The emphasis is all on the word " rudimentary." The economist with no mathematical equipment is at a disadvantage in reading modern economic theory. The book is designed to supply such a student with the simplest ideas of mathematical reasoning, arranged in the form most comprehensible to him as an economist. It is not a substitute for a serious study of mathematics which is, in fact, recommended to all who can reach the last page of this introduction without flinching. The content is very simply ordered to proceed from curves and limits, through derivatives to problems of maxima and minima, with a final dip into differential equations and determinants. Some non-mathematical economists, however, may find that the final dip leads to complete immersion.

The second comment derives from the fact that the title of the volume addresses itself to the statistician as well as to the economist. In fact, apart from the derivation of lines of regression by least squares as an example of extreme values, there is very little indeed in the text of interest or assistance to the statistician as such.

R. G. D. A.

2.—*Sampling Inspection Tables : single and doubling Sampling.* By H. F. Dodge and H. G. Romig. New York: John Wiley, 1945 (London: Chapman and Hall). $8\frac{1}{2}'' \times 5\frac{1}{2}''$. vi + 106 pp. Charts. 9*s.*

Sampling Inspection Tables, by H. F. Dodge and H. G. Romig, is a reprint in book form of three articles which appeared in the *Bell System Technical Journal* durung 1929, 1941 and 1942, plus an Introduction.

Dodge and Romig were the pioneers in the introduction of sampling schemes of inspection on a statistical basis, and their system has been, and is, very widely used both in America and in Great Britain. Its special characteristics are the following:—

(i) It is applicable in the main to the inspection of " things " as distinct from materials.

(ii) It is intended for use with inspection by attributes, or, as it has come to be called in this country, by " number defectives."

(iii) It is intended for use in the main where the alternatives as a result of inspection are " accept " or " 100 per cent. inspect."

It can also be used in theory where the alternatives as a result of inspection are " accept " or " reject outright," but loses some of its special advantages of economy when so used, and competing systems have special advantages to offer for that type of inspection.

(iv) It is designed to give information on the quality of single lots, but uses as part of the information on which the number in the sample and rejection number are based, the Process Average, or estimated average level of per cent. defective as found from records of previous lots submitted.

Other systems have been developed which compete with the Dodge-Romig plan when information about the Process Average is not obtainable, as would happen when a manufacturer is frequently changing his source of supply, or is buying from a number of sources of supply.

(v) The system employs a fixed number for the sample (or double sample), with a corresponding fixed arbitrary number of defectives for rejection. The lot is submitted for 100 per cent. inspection if the number of defectives in the sample exceeds the rejection number.

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(vi) The size of the sample and the rejection number are calculated on a total Consumer's Risk of 10 per cent. The term Producer's Risk as employed by Dodge and Romig means the risk that a good-quality lot may be rejected as a result of a decision from a sample, and the Consumer's Risk means that a bad-quality lot may be accepted as a result of a decision from a sample.

(vii) The theoretical background in calculating the size of sample and rejection number to a total Consumer's Risk of 10 per cent. is the Process Average, and either Lot Tolerance Per cent. Defective (*i.e.*, maximum allowable), or A.O.Q.L., Average Outgoing Quality Limit, the maximum average level for the product after submission to both sampling and 100 per cent. inspection. Tables are provided for both bases.

It is to be noted that with sampling alternative to 100% inspection as distinct from sampling alternative to rejection, A.O.Q. will rise with rising incoming percentage defectives to a maximum A.O.Q.L., and then slowly decline to a limiting value of zero as more and more of the product is turned over to 100 per cent. inspection as a result of the sampling.

The book suffers considerably from the fact that it is a reprint of three articles. The subject of statistical sampling is so important, and Dodge and Romig's contributions to the general thought have been so well worth while, that one would suggest that the book be rewritten as a single entity, and that in the near future. When this is done there are suggestions which can be made as to improvements and additions. In the first place, although the tables are admirable in their practical lay-out, they are not completely consistent in themselves, and show signs of rather too much smoothing out. Revision of the tables from this point of view is definitely indicated. The graphs from which sample size and rejection numbers can be determined are not sufficiently simple to use, and should be entirely re-designed.

Recognition should be made of the limitations of the system; and some of the practical consequences which cannot be avoided, and must be accepted by the user, should be weighed against the very great advantages. For instance, with sample-100 per cent. inspection, the number of inspection staff is bound to vary, and so must the inspection facilities (interim storage, etc.), and the inspection time-lag. Variations of this type will depend on the closeness with which the incoming percentage of defectives corresponds to the Process Average. The authors might well devote a section to problems of this type which are of great importance to the executive who is to use a sampling scheme of inspection.

The practical notes are very good, but it is suggested that they could be amplified to include: (*a*) a comparison of the use of "screening" sampling for single lots of purchased material with quality control charts, (*b*) notes on applications in other types of businesses other than those manufacturing electrical apparatus, with information on A.O.Q.L. found acceptable in practice in different industries.

A. W. S.

3.—*Measuring Business Cycles.* By Arthur F. Burns and Wesley C. Mitchell. National Bureau of Economic Research, New York, 1946. Pp. x × 560.

Wesley Mitchell's earlier book on *Business Cycles: The Problem and its Setting* has remained for nearly twenty years perhaps the most objective published study of cyclical behaviour in economic time-series. The present book is the result of many years' work along the lines foreshadowed in the earlier volume, in which the senior author recognised that the primary problem was to describe the phenomena, and not to try to explain them until it was clear what was to be explained. Notwithstanding its size, this book does not go beyond the descriptive stage; it is a discussion of methods, not of results. We are promised a further series of monographs from the National Bureau examining in more detail the application of the technique to various groups of economic activities, and even a final volume to weave these individual studies into a theoretical account of how business cycles run their course. This Napoleonic scheme compels the profoundest admiration for the perseverance of its directors and the fortitude of the staff who have to carry out the detailed work. It will earn the respect of all statisticians who are interested in time-series or in economic behaviour. Even those economists who seem more interested in accounting for the trade cycle than in studying its statistical manifestations will profit by browsing through some of the book and reading the summaries at the end of each chapter.

The study covers about 1,300 time series in four major industrial countries, the U.S.A. (76 per cent.), Great Britain (11 per cent.), France (7 per cent.) and Germany (6 per cent.). Most phases of economic life are represented, among them production, transport, prices, foreign trade, personal incomes, savings and investments, security markets and interest rates. The majority of the series are less than 30 years in extent, but a substantial number run for a good deal longer, the greatest length being about 145 years. Monthly and quarterly data are employed where available.

The methods used for measuring cyclical behaviour are described and illustrated at great length; and as they are novel in many ways, this is a most desirable feature, though it makes rather heavy reading. I was reminded of the comment of one famous statistician on another; he is like a small boy who insists on showing you *the whole* of his stamp collection. Mitchell and Burns, influenced no doubt by the knowledge that in the study of cyclical movements the

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results depend very largely on the analytical technique, describe their own procedure exhaustively. Moreover, they do not shirk difficulties. Sometimes they have to ask a question without being able to answer it; sometimes they admit that their procedure is rough-and-ready and capable of refinement; sometimes their choice of possibilities is determined experimentally by trying them all and seeing which gives the most acceptable results; but the reader is rarely in doubt about what they have done or why they have done it.

The authors start from the fact that, in nations which organise their work in business enterprises, there is a sequence of changes affecting nearly all branches of the economy: times of expansion followed by recession, contraction and revivals leading to new expansions. This sequence is recurrent *but not periodic*. Rather arbitrarily, they do not normally admit as "cycles" in this sense recurrent changes of more than 12 years duration. The longer waves of economic life they appear to regard as cycles of cycles.

To the extreme sceptic it might appear at an early stage as if the authors beg an important question by speaking of *the* business cycle. They recognize, of course, that the fluctuations in different branches of the economy do not vary together, that they get out of step, lead or lag behind the main movement and have little sub-cycles of their own. But the authors hold that amidst all this variation there is a sufficient communality to call into existence the notion of a cycle in business generally. This is certainly the common notion of a "trade cycle" and it may be accepted as a correct representation of the facts. Whether it is due to some common cause, to reactions between one business and another or is accidental in the sense that a number of independent fluctuating movements will occasionally get into step by sheer chance, is another story. There are dozens of theories to account for the phenomenon and most of them are right up to a point. Their weakness is usually that they concentrate on only one feature of the oscillations and attempt to explain too much with too little. However that may be, in the sense of Mitchell and Burns the business cycle does exist.

The main use made of the business cycle in this book is as a reference-pattern against which to measure the specific cycles of individual economic series. The reference-cycle, so to speak, provides a measure of location; the comparison of a specific cycle with the reference cycle enables a study to be made of its deviations from the norm, its comparative amplitudes and its leads or lags. In addition to this the specific cycles are subjected to a routine analysis to show their duration, amplitude, secular movements, and to exhibit their "patterns" in three stages of expansion and three of contraction. Numerical work on those lines requires a decision as to what are peaks and troughs in the various series and the authors give considerable space to a description of their adventures in these treacherous waters.

There are also some interesting chapters on the treatment of various difficulties: random movements, the effect of time-units, the effect of trend adjustments and smoothing. The authors' view on trend elimination is best given in their own words: "Business cycles are a development of late modern times. . . . Cyclical fluctuations are so closely interwoven with these secular changes in economic life that important clues to the understanding of the former may be lost by mechanically eliminating the latter." They go on to examine what would happen to their measures of cyclical behaviour in certain series if trend were eliminated by fitting lines in the customary manner and conclude that "cyclical measures of different series, as well as cyclical measures of the same series, tend to be more alike when made from trend-adjusted data than when made from unadjusted data. . . . It may be legitimate for students concerned with secular trends to put cyclical fluctuations out of sight but students of cyclical behaviour cannot take similar liberties with secular trends." There seems, indeed, to be a growing feeling that any form of trend-elimination is likely to produce misleading results in the study of oscillatory movements. The method of Burns and Mitchell is less likely to be affected than some, but even so their conclusions are of great interest.

Altogether this is a remarkable piece of intensive study which, though requiring the closest attention throughout, should be read by all students of time-series. M. G. K.

4.—*Changes in Income Distribution during the Great Depression.* By Horst Mendershausen. New York; National Bureau of Economic Research. 1946. 9½″ × 6″. xviii + 173 pp. $2.50.

The fundamental basis of this study was a sample enquiry respecting family incomes for each of the years 1929 and 1933 in 33 large and medium-sized cities in the United States, while its purpose was to discover, as far as possible, the effect of the decline in national income after 1929 on various income groups and classes. No one has ever supposed that the decline in national income during a depression affects all income recipients proportionately; indeed, assumptions to the contrary based on the rigidity of salaries and wage rates are commonly made, but little if anything seems to have been done to verify these or other assumptions inductively. Accordingly, this investigation of the underlying pattern of income changes during an industrial fluctuation is particularly interesting, representing work in an entirely new field.

Somewhat surprisingly, it seems to be established that in the United States depression is not associated with a distribution of incomes more equal than that obtaining in conditions of prosperity. While average income declined between 1929 and 1933 in each of the 33 city samples, the higher a

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city's income in 1929, the lower its relative income decline between 1929 and 1933 appears to be. Furthermore, in 27 of the cities relative income dispersion measured in terms of the coefficient of variation actually shows an increase in the latter year compared with the former. In order to carry this analysis a stage farther, income-earners were divided into an upper and a lower group, and it is shown that the relative differences in average income level between the two groups increases in all cities, but in this case the relative increase in income dispersion is confined to the lower-income group. In the upper group the gap between top incomes and those below narrowed in all cities.

Explanations of these facts are offered. The impact of increased unemployment in the depression is chiefly borne by members of the lower-income group, so that the income gap between employed and unemployed would lower the average and tend to increase the income dispersion. Regarding the upper group, it is tentatively suggested that income from property, forming the greater portion of the top incomes, fluctuates more than salaries which form the bulk of the incomes below. In a depression, therefore, the top incomes would tend to drop more sharply than those immediately below, though not so far, it would seem, as to drag the whole of the incomes in the upper group down in proportion with those in the lower. Another factor tending to widen the difference between average incomes in the upper and the lower groups during years of depression may be that at the higher levels salaries and wages are more resistant to reductions than at the lower ones.

The final aspect of change in income distribution between the two years with which the study deals is the extent of changes in ranking among the individual families. Three measures were used to bring out this phenomenon; the percentage of families changing position with respect to the median income, the percentage moving across the dividing point between the upper and the lower income group, and the coefficient of correlation between 1929 and 1933 incomes. Unfortunately, none of these measures is really adequate to describe the pattern of movement. It is, however, established that roughly 25 per cent. of the families cross the median line, while some 20 per cent. move from one group to the other. Further investigation shows that those families at the extremes of the income scale in 1929 experienced most scattering over the income scale in 1933. In tracing the net effects of the depression on broad income groups, it was found that the recipients of very low and of moderately high incomes in 1929 fared relatively well in 1933, while those with moderately low and extremely high incomes fared relatively ill.

Following the practice usual in National Bureau of Economic Research publications, the nature and shortcomings of the data used are dealt with in detail in appendices. In connection with the shortcomings, the most important point is the acknowledged lack of precision in the definition of both "income" and "family" in the original enquiry. As for the basic data itself, its nature is such that Mr. Mendershausen is able to compare at every stage in the study the experience of house-owners and of tenants. Furthermore, the author has not hesitated to use data from other sources in endeavours to check and broaden his conclusions whenever this course was possible. Nevertheless, some of the conclusions are indicated rather than definitely established, and perhaps more of interest is to be found in the method than in the results of the study. The methodological basis as revealed in the text and amplified in another appendix shows a resourceful and ingenious use of statistical tools. The patient application of these varied tools to the raw material comprising the basic and supplementary data must have involved a formidable amount of arithmetical labour.

T. M. R.

5.—*Price Flexibility and Employment.* By Oscar Lange. Cowles Commission for Research Economics. Monograph No. 8. Principia Press, 1944. $9\frac{3}{4}'' \times 6\frac{1}{2}''$. 144 pp. $2.

It has been maintained by the classical economists and their descendants that unemployment could be cured by cutting money wage-rates. Against this the Keynes theory held that a cut in money wages leads (in a closed system) to a proportional fall in prices and to no increase in employment; in fact some economists recommend a rise and not a cut in money wages as a remedy for unemployment.

Dr. Lange sets out to discuss this important problem of economics. He discusses it in the most general form: if any factor of production is "under-employed" or if excessive demand for that factor creates "bottlenecks," is a flexible price for that factor sufficient to restore equilibrium (that is, a balance at a certain price between the supply of and demand for that factor)? Further, does price flexibility of factors lead to more or less stable employment?

Price flexibility is sufficient to make "under-employment" or "bottlenecks" disappear in the case of a given factor, if the prices of all other factors and of all commodities remain unchanged. In a realistic case, however, this is very rarely so. The general problem is first discussed under three limiting assumptions: that current prices are expected to continue, that perfect competition rules and that there is no international trade; these assumptions are later one by one removed.

In order that the flexibility of prices of factors should lead to equilibrium (that is, full employment in the case of under-employed factors), certain special conditions must be satisfied. If these conditions are not satisfied, price flexibility will fail to eliminate under-employment or bottlenecks; in fact, it may even aggravate unemployment or scarcity. In the first place, there must be a positive monetary effect, which is defined as a rise in the real quantity of money as prices fall, or a fall as prices rise. Even so, the effect depends on the type of goods towards which the increased demand

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is directed. It might lead to a rise in demand for securities and not for commodities which, in so far as new investment is not sensitive to interest rates, might cause a stock-exchange boom but have no effect on employment; the direction of increased demand depends to a great extent on the distribution of cash balances. The existence of friction, when the demand for the under-employed (or bottleneck) factor is indirect, might also prevent the operation of equilibrating conditions.

The analysis becomes more complex when the limiting assumptions mentioned are removed. With price expectations taken into account, equilibrium depends on the "responsiveness" of the monetary system (which is defined as the change in the real quantity of money to the change in real demand for cash balances). Elastic price expectations require a responsive, inelastic expectations an irresponsive monetary system. The existence of uncertainty further weakens the monetary effect.

Under conditions of imperfect competition it is even more difficult to restore equilibrium; a fall in factor price might lead to higher profits instead of to an expansion of output. International trade has a stabilizing effect, as it enlarges the market in relation to a given factor; thus price flexibility might restore equilibrium more easily by an expansion of exports (if the factor was under-employed). On the other hand, automatic international monetary effects (which are not worked out fully at all by Dr. Lange) might have a destabilizing effect.

Dr. Lange's monograph is an admirable and stimulating piece of writing in its building up of a completely generalized analysis. Both classical theory and modern theory are exhibited as special cases of the generalized theory, though it is admitted that the Keynesian model was a very good description of the American (and perhaps also the British?) economy of the 'thirties. But it is shown that the classical dogma, that over-saving is impossible, has only very limited validity. The orthodox belief in unlimited investment opportunities is also the description of a special case only; though there is no doubt that a diminution in monopolistic tendencies would facilitate the absorption of the consequences of capital accumulation. Innovations might, but do not necessarily, cause readjustment.

After the development of his thesis, which (though one might not accept some of its arguments) is certainly an advance in economic science, Dr. Lange concludes his monograph much too rashly. In a few lines, not quite adequate for the purpose, he explains how price flexibility operated in achieving equilibrium between 1840 and 1914 when the necessary conditions were present, but has failed to operate in the different conditions of the world since then. As to economic policy, the socialization of certain industries is suggested too indiscriminately and on, what seem to the reviewer, inadequate grounds; the fact that a firm is not able to predict the reaction of other firms to a change in its price is insufficient reason for socialization, as a Government fixing price policy would overcome the difficulty. The linch-pin in the system of stability conditions is then the stability of the price of one important commodity. Gold is not suitable; the stability of the general level of money-wages is recommended, even in preference to a commodity-standard. The stability of the general level of wages implies that when readjustments in relative wages take place some rates are raised and some cut; the latter would raise difficult practical problems which are not discussed in the book.

Dr. Lange has successfully advanced modern economic theory by building on the foundations laid by Lord Keynes and by perfecting the tools given in *Value and Capital* by Dr. Hicks. The main disadvantage of the present monograph is that it is unbalanced in so far as important points are not adequately developed or explained; doubling the length of the book would have certainly improved it. Perhaps it would have then been possible to do without the concentrated technical jargon which is not always necessary, and to have made the mathematical appendix of some twenty pages, on the conditions of stability of economic equilibrium, an integrated part of the exposition.

T. B.

6.—*Price Control and Business*. By George Katona. Principia Press, Indiana. 1945. $9\frac{3}{4}'' \times 6\frac{1}{2}''$. xi + 264 pp. $3.00.

This book was published under the auspices of the University of Chicago; it is the ninth volume of the series of monographs issued by the Cowles Research Commission of the university. Although the subject-matter of this study is more limited than the preceding volumes of the series, the book deals with a highly topical matter in a new and experimental way, and should therefore be of interest to a wide circle of students.

How do business-men behave under price control? This question cannot be answered by deductive reasoning, the problem cannot even be gauged by observation and measurement. Like the whole structure of pricing procedures and other business decisions, it baffles the theoretical economist. Our ignorance can only be alleviated by what business-men are prepared—and able—to disclose. To a certain extent, the subject of price control helps the more general investigation, for it is easier to ask business-men why they object to certain Government regulations than to find out what determines prices. On the other hand, effective regulations are bound to distort the price structure, so that not even price evasions can show the true forces which are at work in unregulated circumstances. Nevertheless, the analysis of any kind of business reaction

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reveals a great amount of "commercial psychology," and the author and his associates have rightly emphasized the important psychological aspects of their investigations.

A small sample was used to obtain data for the present work: 528 Chicago firms, covering mostly the furniture, clothing and food trades, both manufacturing and distribution. Confidential and detailed interviews were the main sources of information, but detailed "case studies" were also made in 61 instances. It would appear that, on the whole, business-men proved co-operative; this success was primarily due to the shrewd yet tactful way in which the interviews were conducted. Many of the questions were "open questions", the interviewers were free to alter the order of the questionnaires or to ask supplementary questions. Awkward matters like the contravention of the controls were first approached from the angle: "To what extent do your competitors or suppliers seem to comply with the regulations?"

The study covers a period of two years from April 1942 to April 1944. Though mainly concerned with price policies, the enquiry necessarily incorporated many questions relating to rationing, scarcity of supplies, and other war-time conditions. The individual conclusions reached are too numerous to be stated here; their significance would, in any case, be greatly diminished by the limited sample. A few tentative attempts were made to link up the tables with published U.S. national statistics, but disparities here are, naturally, of little importance. The methods adopted in this enquiry are of much greater interest than the conclusions reached, one should therefore refrain from stressing the fact that the material presented in numerous tables is generally left in its raw state and no attempt is made to evaluate the results numerically in the form of correlations or even percentages.

The effects of war-time conditions on prices were divided into the following headings: (1) Legal direct price increases, (2) illegal direct price increases, (3) indirect price increases: quality deterioration, (4) reduction in bargain sales, (5) "uptrading": shift to grades of higher quality. The data obtained under each heading are significant, but the attempted measurement of these shifts in prices must be treated with great reserve. How can, for instance, the change in the advertised prices of goods be taken as indicating the extent of the shift in demand when the change in the price level of the goods had to be ignored?

Perhaps the most interesting part of the book is Part Three, dealing with the merits and demerits of various types of price controls used in the United States. This section is highly controversial, but contains a fair amount of interesting material. Most difficult of all, there is the problem of rationing as an aid to price stability: while the curtailment of demand has a damping effect on prices, the pressure on black-market prices is increased. The author's preoccupation with psychological factors leads to an interesting analysis of the consumers' reaction to price control, and his criticism of the tactics adopted by the authorities is well borne out by the statistics of national consumption figures. During the war price expectations played a more important part in determining consumers' demand in the United States than in any other country's war economy, but if the practice of "tinkering with the thermometer" of prices is to continue, greater attention would certainly have to be paid to these psychological factors everywhere.

Detailed interviews of the kind adopted in this survey are probably our only means of filling in the gaps in our knowledge about how business-men behave. Unfortunately, this method is too elaborate to form the basis of enquiries on a national scale. We find in this book that a number of business-men believed it was essential for them to charge a certain percentage above direct costs if they were not to be driven into bankruptcy; they felt, therefore, that they were morally justified to raise their prices above the legal maximum. Others decided to charge below the legal price, in order to maintain their goodwill and trade connections for after the war. Information of this nature can often be linked with the peculiarities of market structure in different industries or with the characteristics of the locality. But it can only be obtained through long and intimate interviews, and unless there is a reasonable continuity in business conditions the analysed results of a widespread enquiry are bound to be out of date and of little practical value when published. Needless to say, this would prejudice the willingness of business-men to co-operate with the scheme, and would, in any case, make it impossible to find the finance required for an enquiry on the national scale. Confined, however, to a more limited field, as in the present case, it can provide a most interesting contribution to the study of "economic psychology."

G. A. B.

7.—*Other New Publications* *

Fisher, Irving. *100% Money*. New Haven, City Printing Co., 1945. 3rd Ed. 8 × 5¼. ix + 257 pp. $1.

[The first edition of this book appeared in 1935 and was reviewed in the *Journal*, Part II, 1935, p. 388. A second (revised) edition was published in 1936. This present edition, the author tells us in his preface, is the result of a new interest in the subject created by the Voorhis Bill H.R. 3648 for 100 per cent money ("a bill to prevent inflation and deflation and answer America's debt problem"). It is an exact reproduction of the second edition plus the author's preface and an addendum to the bibliography.]

* *See also* "Additions to Library," pp. 317 *et seq.*

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Public Investment and Capital Formation. A Study of Public and Private Investment Outlay. Canada, 1926–41. Dominion-Provincial Conference on Reconstruction, Ottawa, 1925. 11½ × 9¼. 128 pp.

[The preface states that this study was prepared under the direction of O. J. Firestone and M. C. Urquhart, of the Department of Reconstruction, for the Dominion-Provincial Conference on Reconstruction. It shows for the period under review: (1) the volume of public investment and maintenance expenditures; (2) the rôle played by public investment as a segment of total gross investment in Canada for selected years; (3) the types of public investment and maintenance expenditure; (4) the division between the Dominion Government and the various provincial and municipal governments, and (5) the importance of public investment as a segment in total public expenditures. Gross investment and gross capital formation, public and private, have been estimated to provide a background to public investment in order that the part played by the latter in the whole may be seen. The work is divided into four parts —concepts, definitions and measurement; factual summary; tables; and explanatory notes on the tables.]

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STATISTICAL NOTES

(I) British Official Statistics

During the six months April to September 1946 the general course of wholesale prices as measured by the Board of Trade index-number was steadily in an upward direction, and at the end of the period general prices showed an advance of 3·1 per cent. Prices of food and tobacco showed no change, owing chiefly to subsidization, although the price of meat invoiced to retailers was raised 2½ per cent. in August. On the other hand, prices of industrial materials and manufactures rose 4·8 per cent. The rise was appreciable in all groups of materials except that of coal, where it only amounted to 0·5 per cent. It was most considerable in the group of non-ferrous metals (25·2 per cent.), and in that of building materials (12·2 per cent.). For manufactured articles the increase in price amounted to 6·4 per cent. and for iron and steel and the textile groups the increase amounted to from 4 to 5 per cent. The serious rise in non-ferrous metals is the only one of any considerable size since the end of 1939, when the index-number was 114·1 (August 1939 = 100·7). During the period 1940 to 1945 the number fluctuated only between 121·9 and 129·3, and at September 1946 it was 163·3. These metals were, however, subject to control both as to price and supply throughout the war. Prices of iron and steel have advanced 13·1 per cent. since December 1945, when subsidies were withdrawn. Compared with August 1939 wholesale prices of food and tobacco have advanced 75 per cent., of industrial materials and manufactures 83·8 per cent., and prices generally 80·8 per cent. Prices of basic materials (excluding fuel) have risen 105·5 per cent., and building materials 77 per cent.

The following table gives the Board of Trade index-numbers from March to September 1946.

(Averages for the year 1930 = 100)

Date	Total food	Total not food	All Articles	Basic materials (excluding fuel)	Intermediate Products	Manufactured articles	Building materials
March 1946	158·2	179·1	172·1	184·6	190·2	168·6	164·3
April „	158·5	179·9	172·7	185·1	191·1	169·6	165·7
May „	158·7	180·6	173·2	185·8	191·5	170·6	168·2
June „	158·5	181·2	173·4	186·9	192·2	171·0	169·0
July „	160·9	184·7	176·5	192·6	196·0	173·4	177·7
August „	159·1	186·2	176·8	192·9	197·8	175·7	178·8
September „	158·2	187·8	177·4	194·2	198·8	178·9	184·3
September 1945	*158·3*	*175·5*	*169·8*	*188·4*	*182·5*	*161·4*	*159·0*
August 1939	*90·4*	*102·2*	*98·1*	*94·5*	*104·0*	*108·7*	*104·1*
Percentage increase in September 1946 over—							
September 1945	—	7·1	4·5	3·1	8·9	10·8	15·9
August 1939	75·0	83·8	80·8	105·5	91·2	64·6	77·0

There was no change during the six months from April 1st to October 1st, 1946, in the index-numbers prepared by the Ministry of Labour and National Service showing the *cost of living* of working-class families. There were, however, slight counterbalancing increases and decreases in the prices of various groups of commodities. There was a slight rise in working-class rents in April, amounting to about 1 per cent., and almost entirely due to increase in rates. In May there was an increase in the price of bread due to the order made by the Minister of Food reducing the size of the loaf by one-eighth without a corresponding reduction in price. There was, on the other hand, a reduction of 2*d.* per lb. in the maximum price of butter which tended to counterbalance the increased cost of bread. There were seasonal variations in the price of potatoes, and the price per 7 lbs. at October 1st was 6½*d.*, compared with 8½*d.* at April 1st. There were also slight reductions in the retail prices of certain kinds of fish. Outside the London Area there was an increase of one-seventh in railway fares. The prices of men's suits and overcoats rose to some

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extent, but clothing materials were somewhat cheaper. The price of coal advanced to some extent in many towns in July, and in the same month there was a rise of ½*d.* per lb. in the price of soap.

The index-numbers for the various groups of commodities are given below for the six months period.

(Prices at July 1914 = 100)

Date	Food	Rent and rates	Clothing	Fuel and Light	Other items	Total
April 1st, 1946	169	166	345	275	293	203
May ,,	169	168	345	276	291	204
June ,,	169	168	345	276	290	203
July ,,	171	168	345–350	276	294	205
August ,,	171	168	345–350	278	296	205
September ,,	168	168	345–350	278	296	203
October ,,	168	168	345–350	278	297	203
October 1st, 1945	*169*	*166*	*345*	*276*	*292*	*203*
September 1st, 1939	*138*	*162*	*205–210*	*180–185*	*180*	*155*
Percentage increase in October 1946 over—						
October 1945	0·6 *	1·2	0·7	0·7	1·7	—
September 1939	22	4	67	53	66	31

* Decrease

Comparative figures for certain other British index-numbers of wholesale prices and also for that of the United States Bureau of Labour are given below.

Date	Board of Trade (1930 = 100)	*Economist* (1927 = 100)	*Statist* (1866–77 = 100)	*The Times* (1913 = 100)	U.S. Bureau of Labour (1926 = 100)*
March 1946	172·1	122·9	166·6	193·9	108·3
April ,,	172·7	124·5	168·4	195·1	109·4
May ,,	173·2	124·6	169·3	196·0	110·4
June ,,	173·4	124·8	169·7	196·8	111·9
July ,,	176·5	126·3	174·4	199·1	121·6
August ,,	176·8	126·7	175·7	204·8	127·4
September ,,	177·4	129·1	180·5	207·6	123·0
September 1945	*169·8*	*119·5*	*162·7*	*188·6*	*105·0*
August 1939	*98·1*	*70·3*	*90·4*	*114·5*	*74·8*
Percentage increase in September 1946 over—					
September 1945	4·5	8·0	10·9	10·1	17·1
August 1939	80·8	83·6	99·7	81·3	64·4

* Mean of weekly figures.

During the six months April to September 1946 there was no great change in the total number of unemployed workpeople registered at the employment offices of the Ministry of Labour and National Service. At the middle of September the number on the registers were 373,603 insured and uninsured workpeople, a fall of 7,499 since March 11th. Of these 359,206 were insured against unemployment. There was a fall of over 30 per cent. in the number of women and girls and an increase of nearly 12 per cent. in that of men and lads. No doubt the considerable decline in the number of females registered is due to the fact that many are not now seeking re-employment, and does not mean necessarily that they are finding work more readily than men.

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Number of unemployed persons on the registers of the Employment Offices of the Ministry of Labour and National Service at the undermentioned dates

Date	Men and boys	Women and girls	Total (insured unemployed)	Total (insured and uninsured)
March 11th, 1946	239,427	132,489	371,916	381,102
April 8th, „	247,128	124,282	371,410	380,581
May 13th, „	257,567	117,309	374,876	386,881
June 17th, „	270,403	105,764	376,167	382,898
July 15th, „	263,755	95,579	359,334	367,814
August 12th, „	270,979	92,742	363,721	382,441
September 16th, „	267,895	91,311	359,206	373,603
October 12th, 1945	*131,832*	*101,494*	*233,326*	*245,810*

The total for September 1946 includes 45,203 married women, some of whom are retiring from industrial employment, and 33,008 ex-Service men and women who have had no employment since leaving the Forces. The 14,397 uninsured persons on the registers at September 16th, 1946 include 6,675 boys and girls under 16 who have not yet entered employment. The totals do not include men and women on the register who are classified as unsuitable for ordinary employment. These included at September 16th 23,068 insured men, 633 insured women and 1,353 uninsured persons.

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(2) Other Statistics

Over the first eight months of the trade year (February to September 1946) the statistics prepared by the Bank of England from returns received from retail distributors' associations, co-operative societies, departmental stores, etc., show that there was an increase of 15·7 per cent. in the sales per selling day, compared with the sales in the corresponding period of 1945. Sales of food and perishables increased 9·8 per cent. and those of non-food merchandise 24·8 per cent. Sales were greatest in furniture departments (82·5 per cent.) and in hardware (65·7 per cent.). For women's wear the increase was 18·2 per cent., and for men's and boy's wear 15·3 per cent. Stocks of food and perishables had decreased 3·7 per cent. at the end of September 1946 compared with the end of September 1945, and stocks of non-food merchandise had increased 21·1 per cent.

The Board of Trade has commenced obtaining returns from independent retailers in three trades—chemists, furniture dealers and men's wear shops—where the returns received by the Bank of England apply chiefly to larger units and multiple shops. Figures are now available in the *Board of Trade Journal* for November 9th covering sales for the three months July, August and September 1946. Over the period there were increased sales of 11 per cent. per week in chemists, of 73 per cent. in furniture dealers and of 46 per cent. in men's wear shops (including bespoke tailors). It is hoped ultimately to obtain sufficient regular returns from retailers to enable area figures for sales to be published.

The Food and Agriculture Organization of the United Nations has published a Report, dated August 15th, 1946, on *Forestry and Forest Products* bringing together the available statistics relating to the major regions of the world and leading up to recommendations for national and international action. Generally the Report takes a poor view of the position. "Some of the indirect effects of war have conspired to aggravate situations already critical and now threatening to become worse. War requirements placed heavy demands on the forests and post-war requirements are placing an added burden. Paper deficits have been felt throughout the world, and the reconstruction of war-torn areas, the resumption of house building and other forms of civilian wood use are creating enormous pressures, and the need to replenish exhausted stocks will add substantially to these demands."

The Report draws particular attention to the inadequacy of statistics relating to forestry. The classical balance sheet in forestry is a statement showing annual growth on the one side and annual drain on the other, but such statistics exist only for the world's most advanced forestry regions, and even so figures for different regions have widely different significance. Even within the same country

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inadequate definitions of terms often lead to considerable confusion and produce widely fluctuating and contradictory figures. The most serious statistical gap occurs with regard to consumption and requirements. Few Governments keep any systematic records of wood consumption and little effort has been made to determine year by year the quantities used for building, railway sleepers, pit props, pulp, etc., or to establish any systematic relationship between production and consumption. This gap has always impeded the making of long-range forecasts and has made the economic exploitation of forests highly speculative,

A great statistical task, it is said, lies before the F.A.O., and the calling of a world conference is recommended to agree on definitions and measurement units and to establish clear relationships and conversion factors between national forestry statistics and the F.A.O.'s standard terms and units. It is suggested that the F.A.O. should take immediate steps to establish essential data and balance sheets dealing with the world's forest resources and major forest products, coupling this with preparation for a world forest census and leading to the preparation of a handbook containing standard information relating to the forests of all nations.

CONSEQUENT on the establishment of the *Food and Agriculture Organisation of the United Nations* arrangements have been made for the transfer to the Organisation as from August 1, 1946, of the whole of the functions of the International Institute of Agriculture in Rome, including the statistical work hitherto undertaken by the latter body. This work has been concerned largely with the preparation of annual and seasonal estimates of the production and supply of the principal crops and agricultural products entering the international market, and the publication of Reports and Bulletins dealing with their economic significance. The Institute has also issued for many years an *International Year-Book of Agricultural Statistics*. As a result of the transfer, the collection of current statistics will in future be concentrated in the newly-established Food Reporting Service of the F.A.O. at Washington, though as a temporary measure the work necessary for the publication of the current issue of the Institute's *Year Book* will be completed in Rome. In the meantime the F.A.O. has issued as an interim publication a *Handbook of Agricultural Statistics* containing for each country figures of crop areas and production, live stock numbers and output so far as they are readily available.

A question arising in connection with the transfer is the disposal of the Institute's valuable Library, which includes much statistical material. No useful purpose would be served by moving the library to the U.S.A., as such a move would result in considerable duplication of resources in America, while seriously depleting the limited research resources of Europe. It has been suggested that the library should be equipped to serve as a working reference collection for Europe, probably in connection with a regional European Office of the F.A.O., but the matter is under consideration.

A recent report of the F.A.O. on the transfer concluded with the observation that " it is hoped that continuity between the work of the Institute and that of the F.A.O. will be assured and that the good-will built up by the Institute over forty years of pioneer work will be maintained." (A previous note on the F.A.O. will be found in this *Journal*, Parts III–IV, 1944, p. 306.)

In his annual figures of Statistics of Failures in the United Kingdom and Ireland for the year 1946, Mr. Richard Seyd again shows an increase over the previous year in the total number of failures—420 (377 bankruptcies and 43 deeds of arrangement, including Scottish Trust Deeds) compared with a total of 329 in 1945. In the 1946 figures 45 (40 bankrupticies and 5 deeds of arrangement) were in the financial, wholesale and manufacturing trades, 107 (105 bankruptcies and 2 deeds of arrangement) in the professional classes, and 268 (232 bankruptcies and 36 deeds of arrangement in the retail trades and working classes. Of these, 5 were in Eire (1 wholesale, 1 professional, and 3 retail). During the year, 1,814 Limited Companies were wound up (1,624 English, 154 Scottish, 12 Northern Ireland, and 24 Eire). Of these, 102 were on account of liabilities, of which 69 were in financial, wholesale and manufacturing categories, 4 in the professional classes, and 29 in the retail trades. The number of failures of private persons was 61 as compared with 55 in 1945. In the retail trades, builders and clerks and commercial travellers showed the highest number of failures (32 each). Farmers came second with a total of 26.

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CURRENT NOTES

The Revue de l'Institut International de Statistique, 13e Année, Livraison 1/4, 1945 (issued in October, 1946) contains the announcement of which the following is a translation.

Notice to Members

25th Session of the International Institute, Washington, September 13th to 25th, 1947.

I have pleasure in informing the members of the International Statistical Institute that the 25th Session of the Institute will be held, by invitation of the Government of the United States, at Washington, from September 13th to 25th, 1947. This Session was to have taken place in 1940, but was indefinitely adjourned because of the international situation.

The Organizing Committee, under the direction of the President, Mr. Stuart A. Rice, has already been engaged for some time in the work of preparing for the success of this Session. It now remains for the members to do their part, in the first place by being present, and next, by contributing to the Proceedings. As it is important to have early information with respect to the topics put forward for discussion, I shall be glad to receive without delay the title of any proposed communication, with a brief summary of its content. Papers should as a rule not exceed 8 pages of printed text (of the Bulletin).

In addition to the scientific portion, the programme of the Session will include a discussion of the present position of the International Statistical Institute in the world of international statistics and the moment is opportune for considering whether, in view of the circumstances and the needs of the present time, the existing organization of the Institute is still fully adequate for the attainment of its objects.

There must be the closest possible co-operation with the other international organizations concerned with statistics. At the same time, the fundamental character of the Institute, as an autonomous international academy of statistics, must be wholly maintained. It is eminently desirable that as many members as possible should participate in the discussions on this matter, which are of the utmost importance for the future of the Institute.

Members who intend to take part, unless prevented by unforeseen causes, are requested to inform the Secretary-General. A form for a definite statement to this effect will be sent to them in due course.

In accordance with custom, a limited number of invitations to take part in the Session may be sent to persons of high competence in statistical matters who are not members of the Institute. Proposals for such invitations should be sent to the Secretary-General.

H. W. Methorst,
Secretary-General.

The Sixth International Conference of Labour Statisticians will meet in Montreal on August 4th, 1947. The decision to convene the Conference was taken in September last by the Governing Body of the International Labour Office which fixed the agenda of the Conference as follows:

I. Employment and Pay-roll Statistics.

Methods of obtaining an over-all picture of the volume of man-power and employment, as well as its fluctuations, in the economy as a whole and in the major branches of economic activity. The methods and uses of pay-roll statistics which are obtained in conjunction with employment data will also be considered.

II. Unemployment Statistics.

Methods of obtaining statistics of the numbers and proportions of workers unemployed, and the fluctuations in these, in the economy as a whole and in the different economic sectors.

III. Cost-of-Living Statistics.

War-time and post-war developments in methods and techniques.

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IV. Amendments to the Resolutions on Statistics of Industrial Accidents adopted by the First International Conference of Labour Statisticians in October 1923.

Proposed restatement of methods of measuring industrial accident rates.

The task of the Conference, like that of its predecessors, will be to formulate international standards to serve as a basis for improving the quality and international comparability of the statistics compiled in different countries. (For an account of the work of earlier International Conferences of Labour Statisticians, together with the texts of their Recommendations, reference may be made to International Labour Office: *The International Standardization of Labour Statistics* (Revised edition, Montreal, 1943).)

Invitations to be represented at the Conference will be sent to the Governments of all States Members of the International Labour Organization, and to the United Nations and other inter-governmental agencies concerned with the international standardization of statistics. The Governing Body of the I.L.O. will also be represented.

On each of the topics on its agenda the Conference will have before it a report of the International Labour Office describing and comparing the methods at present followed in different countries, and containing a draft of proposed international standards for the consideration of the Conference.

The *Economist*, in completing its 151st volume for the half-year ending December 1946, announces changes in the publication beginning with the first issue in 1947. The *Records and Statistics* section included in the *Economist* is being reduced to a minimum, and in its place a special *Records and Statistics Supplement* to the *Economist* will be separately published each week. This *Supplement* is to consist of at least 16 pages with an expanded monthly issue. In addition to a comprehensive service of British and overseas statistics, it will contain a variety of economic and industrial reports, records of political and public affairs, and digests of official documents. The *Supplement* is also to embody the various regular supplements on special subjects hitherto published in the *Economist*, such as those on banking and insurance, and the Annual Commercial History. The *Records and Statistics Supplement* will be supplied by postal subscription only. The annual subscription will be £1 for those who are already direct postal subscribers to the *Economist*, and £1 10*s*. for all others.

Students of Statistics will welcome the appearance of a second impression of the sixth edition of Professor Bowley's *Elements of Statistics*. The sixth edition itself, published in 1937 and reviewed in the *Journal* in 1938, pp. 458–9, was, in the main text, unchanged from the fifth edition, new matter being incorporated as notes or supplements. This second impression of it is a photo-type copy. It is published by Staples Press at 30*s*.

In the issue of *Psychometrika* for September 1946 a method is described for using simple diagrams to compute tetrachoric correlation coefficients. In conjunction with the tables for computing the standard error of this coefficient presented in the September 1943 issue of *Psychometrika*, the diagrams may be used to determine a tetrachoric correlation for many problems in which its complicated formula formerly made its calculation too tedious.

Reprints of these two articles, together with a set of twelve computing diagrams, are available on request from the Marketing and Research Service, Dun and Bradstreet, Inc., 290 Broadway, New York 8, N.Y., so long as the supply of copies lasts.

The following excerpt from Hansard, Tuesday, January 21st, 1947, is sent at the suggestion of a member of the Committee:

Research Work (Advisory Committee)

MR. PARKER asked the Chancellor of the Exchequer the terms of reference and Membership of the standing Inter-Departmental Committee on Social and Economic Research which has been set up following the Report of the Clapham Committee.

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MR. DALTON: Yes, Sir. The Committee will survey and advise upon research work in Government Departments, and in particular (*a*) will bring to the notice of Departments the potential value for research purposes of the material which they collect and will suggest new methods and areas of collection; and (*b*) will advise on how there could be made available to research workers information gathered for their own purposes by the Departments which has potential value as material for research.

The members are:—George North, M.C., LL.D., General Register Office (Chairman); Sir A. Carr-Saunders, LL.D., Director of the London School of Economics; D. N. Chester, Nuffield College; G. A. Clark, M.D., Ministry of Health; K. G. Fenelon, Ph.D., Ministry of Food; J. H. Kirk, Ministry of Agriculture; J. G. Kyd, C.B.E., Registrar-General for Scotland; G. H. Maddex, Government Actuary; J. E. Meade, Economic Section Cabinet Office; A. Patterson, Ministry of National Insurance; W. B. Reddaway, Board of Trade; A. Reeder, C.B.E., Ministry of Labour; Professor T. S. Simey, Professor of Social Science, Liverpool University; J. Stafford Central Statistical Office; J. R. N. Stone, C.B.E., Director of the Department of Applied Economics, Cambridge University; The Secretary of the Committee is L. M. Feery, North-East Wing, Somerset House, W.C. 2.

We note the following appointments of Fellows:

Mr. G. H. Maddex has been appointed Government Actuary in succession to the late Sir Percy Harvey.

Professor J. H. Jones has retired from the Professorship of Economics in the University of Leeds and has been made a Professor Emeritus.

Mr. M. G. Kendall has been appointed Joint Assistant General Manager of the Chamber of Shipping of the United Kingdom. At the same time he retains his title of Statistician to the Chamber.

Dr. B. L. Welch has been appointed Lecturer in Statistics in the University of Leeds. Dr. H. O. Hartley succeeds him as Lecturer in the Department of Statistics, University College, London.

Dr. H. E. Daniels has left the Wool Industries Research Association to take up an appointment as Lecturer in Mathematics (Statistics), University of Cambridge.

Dr. M. S. Bartlett was appointed in September 1946 Visiting Professor to the Institute of Statistics, University of North Carolina, for a period of five months.

Mr. O. Kempthorne has been appointed Associate Professor of Statistics at the State College of Agriculture and Mechanic Arts, Ames, Iowa.

The Department of Applied Economics, University of Cambridge, announces that on its recommendation the Nuffield Foundation has appointed Dr. R. C. Geary Senior Research Fellow in the Department. The Government of Eire has granted Dr. Geary leave of absence from the Statistics Branch, Department of Industry and Commerce, to work at Cambridge during the present academic year. He will work principally on the analysis of economic time series.

The Honorary Secretaries will be glad to hear of any future appointments or transfers of Fellows which are of general interest to the Society.

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OBITUARY

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Sir Percy Harvey, K.B.E., C.B., F.I.A.

The public service, the actuarial profession, and the general body of statistical practitioners have suffered a grievous loss by the premature death of Sir Percy Harvey, the Government Actuary since December 1944.

Harvey was born in North London on November 1st, 1887, and was educated at the Grocers' Company School. In 1905 he entered the actuarial profession in a junior capacity with the Atlas Assurance Company. In 1913 he obtained the Fellowship of the Institute of Actuaries and entered the Civil Service as a member of the brilliant team of young actuaries whom Watson was gathering around him. Watson had just been appointed Chief Actuary to the National Health Insurance Joint Committee and was the Government's financial and actuarial adviser on a great social experiment, the new scheme of National Health Insurance. The atmosphere was rarefied. There was, of course, the dominating personality of Watson himself. But there were also, among his immediate aides, such men as Henry, whose death in 1926 at the age of 38 was a major loss to the actuarial profession and the public service, Epps and Derrick—to mention only a few—all of whom early made their mark, and contributed in their various ways to the enhancement of the prestige of the group of actuaries who later formed the nucleus of the Government Actuary's Department, when it was constituted in 1920. It is fair to say that in those early days nobody foresaw in that pale young man, with, at that time, a pronounced impediment in his speech, the future Government Actuary.

It was my good fortune in 1919 to become one of Harvey's assistants, at a time when his growth to maturity was, in effect, beginning, and no actuarial novice could have had a better chief. His consideration and patience were alike infinite, and I take this opportunity of recording how much I, like others, have owed to him. And yet, in those days, Harvey's merits were by no means as obvious to the onlooker as they subsequently became. He was not endowed with that ease of personal approach and with that faculty for diplomacy by which many an otherwise mediocre person not infrequently achieves success. Watson himself was frankly unsympathetic to him in those early days. One had, in fact, to work closely with Harvey, day in and day out, and above all to gain his confidence, in order to get to know his quality, and to discover that here was somebody of fundamental capacity, conscious of a latent power for getting things done, who was suffering from a deep sense of frustration because he was not regarded with any great sympathy in the highest quarters. At no time, however, did his work suffer on this account; rather did he seem to be spurred on to yet higher levels of accomplishment.

It was in this difficult atmosphere, and with no very high rank in the Government Actuary's Department to support him, that it fell to him, in 1919, to pull together both the technique and the organization of the work of that major actuarial exercise, the First Valuation of Approved Societies, which was to be effected as at December 31st, 1918. At that time many large questions were engaging the attention of Watson and Henry, who became Deputy Government Actuary in the following year, and the successful conduct of the First Valuation can in large measure be attributed to Harvey. Soon, however, he was working in wider spheres, immediately under Henry, and gradually, it is not unfair to say, he assumed responsibility for the detailed work on most of the larger "political" references which came Henry's way. Henry was a brilliant man and expected quick responses. That Harvey satisfactorily met Henry's demands on a wide range of exacting actuarial and statistical problems is, to those who knew Henry, a sufficient testimony to Harvey's capacity in both respects.

It was only when Henry died suddenly at the height of his powers that the extent of Harvey's contribution to the work of the Department came to be fully appreciated by Watson. Watson was a just man and in his own way and in his own time acknowledged the position, handsomely as was his wont, by obtaining the special preferment of Harvey to the rank of Principal Actuary. Harvey

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had at last arrived, and when Watson died in harness in 1936 he was appointed Deputy Government Actuary. In 1939 he was seconded to the Ministry of War Transport, becoming Joint Statistical Adviser with Sir William Elderton, and Director of Statistics and Intelligence, until his appointment as Government Actuary at the end of 1944. For his services at the Ministry he had received the honour of being made C.B. in the same year. Back at the Government Actuary's Department as Government Actuary, he was soon immersed in the problems of National Insurance, and his work was recognized when, in 1946, he was made K.B.E.

At the Institute of Actuaries he also made his mark, serving on the Council and being elected a Vice-President for the years 1935 to 1938, during which period his quality of rising to the occasion was revealed by the success with which he deputized for the President during the latter's illness. He joined the Royal Statistical Society in 1937 and served on the Council from 1941 to the time of his death. He was a member of the Council's Committee on the Revision of the Bye-laws and spared much time and thought to that quite heavy piece of work.

In these days appointments to the headships of public departments are not lightly made; and, in the case of so small a profession as that of actuary, those who achieve the highest places are subject to very keen scrutiny by their professional contemporaries. This is particularly so when it is a matter of filling a post which, short of the Presidency of the Institute, may be regarded, from many points of view, as the most important actuarial post in the profession. This office was created in 1917, and there had been only two predecessors to Harvey as Government Actuary, namely Sir Alfred Watson and Sir George Epps; to the actuary, the mere fact that Harvey was deemed worthy to be entrusted with the succession would be a sufficient tribute to his capacity.

Enough has been said, however, to indicate that the path to advancement and honour was no easy one. Every step in Harvey's official career seemed to involve a struggle and had, so to speak, to be hewn out of the rock. When I first knew him, he felt that every man's hand was against him, and it was not unnatural, therefore, that his mode of self-expression tended to be battle. Like most of us, however, he mellowed as time went on, even though his love of a fight for its own sake never entirely deserted him and, from the point of view of his Department, perhaps it was as well that this was so. What then was his essential quality that brought him resounding success, despite such handicaps as those that have been indicated? It goes without saying that he was a thoroughly sound actuary, both in principle and in practice. In the sphere of statistics, in its less mathematical aspects, he was equally effective, as indeed his war-time work at the Ministry of War Transport amply demonstrated. If he lacked the intuition of a Watson, a Henry or an Elderton, he never failed to solve a problem in a thoroughly practical, if not always economical, manner. In military metaphor, his method was often not so much that of an attack on one front as the "all-in" offensive with all arms on all fronts, sometimes irrespective of the strength of the position to be captured. His justification was that the goods were always and infallibly delivered. He was never a procrastinator. He always got on with the job. The mass attacks on a problem, as the just appreciation in *The Times* stated, fructified frequently in a series of masterly short notes, which stand on record as a guide for the future. Nobody will ever take up a job where Harvey had left it and wonder, years afterwards, what in fact he had done. Those who worked with him on the Statistical Committee of the Medical Research Council will recall his probing thoroughness in all he did. In this quality of thoroughness and in another vital aspect he clearly modelled himself on Watson. The departmental tradition of clarity in exposition, particularly in White Papers for the edification of Parliament and the informed public, was worthily maintained by Harvey. No pains were too great, even in purely departmental memoranda, to ensure that the matter in hand was elucidated. His faculty of self-criticism gave a high quality to all his work. But over and above these indispensable qualities in those who hold responsible office, there was one personal quality which suffused and made distinguished much of what he did for his Department and his profession: that was the determination and the unflagging courage by which his ambition was sustained. I suppose he was what is sometimes called a "late developer." As time went on, however, with ever-growing opportunities of proving his capacity, his personality developed and became more balanced, and whilst acquiring in some measure the arts of diplomacy he never lost that directness and occasional bluntness of approach, from which perhaps he derived his greatest strength. I am sure that my old colleagues will acknowledge that Harvey's death has left a great gap in the ranks of the Department.

F. A. A. M.

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George Rae, D.Sc. 1884–1946

To a majority of the many new Fellows, who have joined the Society during the war and the subsequent years, the name of George Rae may be only a name, one that appeared periodically on the list of the Council. For, modest and unassuming, he invariably kept himself in the background. Older Fellows and, in particular, members of the Council, will recall a colleague of considerable personal charm, one whose quiet and rather diffident contributions to a discussion were always precisely to the point and of value, and one who had worked long and successfully to advance the development and the right use of statistics in his own field of work—the tea and rubber industries.

Born in 1884, the son of a master-joiner of Aberdeen, Rae was educated at Robert Gordons College and at Aberdeen University at which he maintained himself by coaching and other work. He graduated B.Sc. in 1906 and many years later (in 1929) took his D.Sc. For a few years, 1911–15, he was an assistant lecturer in mathematics in the department of Professor McDonald. In 1915 he enlisted as a gunner in the Royal Garrison Artillery. Later in life he would often say that his experience in the ranks had been a time of peculiar happiness to him, and it was with some reluctance that he was persuaded to take a commission late in 1916. He was wounded at Nieuport and lost the sight of an eye, a disability of which few of his colleagues were aware. Invalided from the Army he joined, in 1918, the staff of the Board of Trade, where for a short time he was an assistant to a former President of the Society, Mr. H. W. Macrosty. In the following year he took up the work which occupied the rest of his life, as statistician to Messrs. Harrisons and Crosfield. At the time of his appointment the raw statistical materials for the tea and rubber industries were few and far between, and it was entirely through his orderly and enquiring mind that a sound system was set up to produce the relevant figures in a digestible form. His abilities were, however, frequently applied to a considerably wider field than that. The trading interests of his firm are world-wide and impinge on many and diverse industries. Rae was soon involved in problems of a general economic character and in economic research.

His talents, too, were well utilized by the various associations with which Messrs. Harrison and Crosfield were connected. In particular, he did much pioneer work for the Rubber Growers Association, and the exhaustive statistical tables published in its *Bulletin* from 1922 to 1934 were largely the product of his industry.

It was on this subject—The Statistics of the Rubber Industry—that he read, in 1938, his one paper before the Society. He had become a Fellow in 1920, served on the Council from 1936 to 1941 and again, for a second term, from 1942 until the time of his death. In 1946, shortly before his death, he became a Vice-President, an appointment which in view of his illness he was doubtful of accepting but which, it is known, gave him much pleasure.

One who worked in close association with him writes: "What struck me most about George Rae was his extraordinary generosity. He was always anxious to encourage young statisticians and to share with them his wide knowledge, not only of the rubber industry but also of the play of economic forces in a wider sphere." This generosity, bestowed with his customary modesty, will not be lightly forgotten by those he benefited. Those of us who met him only outside his work knew that upon his death in November 1946 the Society had lost a shy but warm-hearted friend and counsellor.

A. B. H.

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STATISTICAL AND ECONOMIC ARTICLES IN RECENT PERIODICALS

UNITED KINGDOM—

Agricultural Economics Society, Journal of Proceedings—

June 1946—The teaching of agricultural economics: discussion opened by *J. P. Maxton*. The development of agricultural economics in Great Britain: discussion opened by *J. F. Duncan*. Some economic aspects of planning land uses: discussion opened by *A. W. Ashby*. The output of British agriculture during the war: *J. H. Kirk*. Milk in wartime: *J. L. Davies*. Wartime egg control and post-war egg marketing policy in Great Britain: *P. M. Reason*. Lessons of wartime control of meat and livestock: *C. H. Blagburn*. Land settlement policy: *A. C. Richmond*.

Annals of Eugenics—

July 1946—Familiar data on 144 cases of anencephaly, spina bifida and congenital hydrocephaly: *L. S. Penrose*. Blood-group frequencies in New Zealand and Maori soldiers: *E. J. Godley*. A serological and genetical study of multiple antibodies formed in response to blood transfusion by a patient with lupus erythematosus diffusus: *Sheila T. Callender* and *R. R. Race*. Scores for ranked data in school examination practice: *F. Sandon*. The cumulants of the distribution of Fisher's 'u_{11}' and 'u_{31}' scores used in the detection and estimation of linkage in man: *J. B. S. Haldane*. On the familial appearances of maternal and foetal incompatibility: *L. S. Penrose*. The fitting of gene frequencies to data on *rhesus* reactions: *R. A. Fisher*.

The Banker—

July 1946—Inflationary pressures in the U.S.: *F. E. Daniels*. Reconversion contrasts—U.S. and U.K.: *Walter Hill*. Debate on Bretton Woods: *R. Harrod* and *P. Einzig*.

August 1946—Economics of the "Indian Union": *Sir Theodore Gregory*. Financing the small business: *J. H. Lawrie*. Fugitive German funds: *H. G. Daniels*. Guarantees for aliens' debts: *L. le M. Minty*.

Economica—

May 1946—The relation of economic history to economic theory: *T. S. Ashton*. Rural population in England and Wales, 1911 and 1931: *A. L. Bowley*. The classical view of the economic problem: *Hla. Myint*. The economics of planting density in rubber growing: *P. T. Bauer*.

August 1946—Full employment and international trade: *Frederic Benham*. The marginal cost controversy: *R. H. Coase*. A further note on the British balance of payments: *C. F. Carter* and *T. C. Chang*. Welfare economics and economic policy: *A. Radomysler*.

The Eugenics Review—

July 1946—Population trends in Palestine: *D. V. Glass*.

Faculty of Actuaries, Transactions—

Vol. XVII, Part VIII—Superannuation funds: *D. A. Porteous*. The role of statistics in the furtherance of medical research: *F. A. E. Crew*. Mortality experience of government life annuitants: *E. S. Jones*. The determination of the constants in Stoodley's varying-interest curve: *A. W. Joseph* and *G. J. Lidstone*.

Vol. XVIII. Part I—Presidential Address: *James Gray Kyd*. Mortality options in life assurance: *Sir William P. Elderton*.

Institute of Bankers, Journal—

July 1946—The nationalisation of the Bank of England: *L. le M. Minty*.

Manchester School of Economic and Social Studies—

May 1946—Man-power distribution 1939–1945: some international comparisons: *C. T. Saunders*. Future competition between natural and synthetic rubber: *P. T. Bauer*. Second thoughts on the white paper on employment policy: *J. Jewkes*.

Oxford Institute of Statistics, Bulletin—

July 1946—The productivity of labour: its concept and measurement: *T. Barna*. The report of the pottery working party: *G. D. N. Worswick*.

Review of Economic Studies—

Vol. XII. No. 32—The economies and diseconomies of industrial concentration: the wartime experience of Coventry: *A. Shenfield* and *P. Sargant Florence*. Some consequences of the habit of judging quality by price: *T. de Scitovsky*. A note on selling costs and the equilibrium of the firm: *J. P. Hayes*. A note on the theory of tariffs: *Henri Denis*. A note on arc elasticity of demand: *J. Gallego Diaz*.

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Royal Meteorological Society, Quarterly Journal—
January 1946—Temperature trend in Lancashire, 1753–1945: *G. Manley.* Upper winds over the world: *C. E. P. Brooks, C. S. Durst* and *N. Carruthers.* Sequences of wet and dry months and the theory of probability: *A. Beer, A. J. Drummond* and *R. Fürth.*

AUSTRALIA—
Economic Record—
June 1946—John Maynard Keynes. (Some personal notes): *L. F. G.* Applications and extensions of the Karmel formula for reproductivity: *Colin Clark* and *R. E. Dyne.* Australian income tax, 1945: *H. S. Carslaw.* Insulationism and the problem of economic stability: *C. G. F. Simkin.* Production goals for primary products: *K. O. Campbell.* Prospects of full employment in Australia: *J. S. G. Wilson.* Social accounting: *Dudley Seers.*

INDIA—
Indian Journal of Economics—
April 1946—Cost and efficiency in Indian business—I: *Kasturchand Lalwani.* Reserve Bank and the management of public debt—I: *M. Sitaramayya.* Food rationing in India: *Rabindranath Chatterji.* The new industrial policy of the Government of India: *D. R. Samant.* Rationing of food grains in the U.P. during World War II: *S. K. Rudra.* Our food: a critical study in retrospect and prospect: *K. L. Govil.* The Indian food problem: *R. Bhan.*

UNITED STATES—
American Academy of Political and Social Science, Annals—
July 1946—Making the United Nations work. (Whole number.)

American Economic Review—
June 1946—The National War Labor Board: *D. M. Keezer.* Business cycles and the modern theory of employment: *L. A. Metzler.* Debt management and economic policy: *H. C. Wallich.* The railway labor act and labor disputes: *H. R. Northrup.* The economics of minimum wage legislation: *G. J. Stigler.* Wage differences in local labor markets: *L. G. Reynolds.*

American Statistical Association, Journal—
June 1946—The membership of the American Statistical Association—an analysis: *Abner Hurwitz* and *F. C. Mann.* Problems and methods of the sample survey of business: *M. H. Hansen, W. N. Hurwitz* and *Margaret Gurney.* Actuarial analysis of the operating life of B-29 aircraft engines: *O. L. Altman* and *C. G. Goor.* Systematic sampling and its relation to other sampling designs: *L. H. Madow.* Calculating the geometric mean from a large amount of data: *Z. Szatrowski.* The design and analysis of methods for sampling microclimatic factors: *H. G. Wilm.* Application of machines to differencing of tables: *J. Laderman* and *M. Abramowitz.*

American Statistical Association, Biometrics Bulletin—
June 1946—Missing-plot techniques: *R. L. Anderson.*

Annals of Mathematical Statistics—
June 1946—Contributions to the theory of sequential analysis—I: *M. A. Girshick.* An approach for quantifying paired comparisons and rank order: *L. Guttman.* Relative accuracy of systematic and stratified random samples for a certain class of populations: *W. G. Cochran.* Operating characteristics for the common statistical tests of significance: *C. D. Ferris, F. E. Grubbs* and *C. L. Weaver.* Minimal variance and its relation to efficient moment tests: *J. R. Vatnsdal.* Tolerance limits for a normal distribution: *A. Wald* and *J. Wolfowitz.* Approximate formulas for the percentage points and normalization of t and X^2: *Henry Goldberg* and *Harriet Levine.* The effect on a distribution function of small changes in the population function: *B. A. Camp.* An experimental design for slope-ratio assays: *C. I. Bliss.* Computation of factors for tolerance limits on a normal distribution when the sample is large: *A. H. Bowker.* The probability distribution of the measure of a random linear set: *D. F. Votaw, Jr.* Information given by odd moments: *E. Churchill.* Some order statistic distributions for samples of size four: *J. E. Walsh.*

Econometrica—
July 1946—Lord Keynes and the *General Theory*: *P. A. Samuelson.* The region as a concept in business-cycle analysis: *R. Vining.* Pricing and price levels: *G. R. Davies.* Raise profits by raising wages?: *T. C. Schelling.* Standard errors of the tilling coefficients used in confluence analysis: *E. Ruist.* The United Nations statistical commission: *S. A. Rice.* "Free money" of large manufacturing corporations and the rate of interest: *A. J. Duncan.* "Free money" of large manufacturing corporations and the rate of interest: a reply: *A. Kisselgoff.*

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Journal of Experimental Education—
June 1946—A study of educational facilities in California for children with cerebral palsy: *L. R. Schulz.* Factors affecting the legibility of handwriting: *L. Quant.*

Journal of Political Economy—
August 1946—A post-mortem on transition predictions of national product: *L. R. Klein.* The Italian lira, 1938–45: *W. D. Grampp.* Realism and relevance in the theory of demand: *J. M. Clark.* The graphic depiction of elasticity of demand: *C. C. Holt* and *P. A. Samuelson.*

Mathematical Tables and other Aids to Computation—
July 1946—The electronic numerical integrator and computer. Scientific computing in Great Britain.

Social Research—
June 1946—The economic problem of Germany: *A. Johnson* and *E. Hamburger.* Keynes as an economist: *H. Neisser.*

BELGIUM—

L'Institut de Recherches Économiques et Sociales, Bulletin—
April 1946—Les finances publiques belges depuis la libération: *A. E. Janssen.* Les finances privées: *E. Mallien.* Le mouvement social en 1945: *A. Woronoff.* Le nouvel aspect du problème charbonnier belge: *C. Demeure.* La sidérurgie belge en 1945: *M. Peeters.* Les métaux non-ferreux en 1945: *R. de Strycker.* L'industrie textile en 1945: *C. Haustrate.* La construction: *F. Baudhuin.* L'agriculture belge depuis la libération: *A. van Houtte.* Le commerce extérieur: *J. Colard.* Les transports intérieurs: *Y. Urbain.*

FRANCE—

Revue D'Économie Politique—
January–March 1946—Le rôle du temps dans l'explication de l'intérêt: *R. Bertrand.* Recherches nouvelles sur le problème des débouchés: *J. Dayre.* Une agence de commerce extérieur sous la Convention (2e partie): *R. Mayer.*
April–June 1946—Remarques sur la théorie générale de Lord Keynes: *P. T. Bauer.* Offre et demande et débit: *H. Guitton.* Théorie des encaisses et théorie de l'intérêt: *R. Florin.* La balance des comptes et l'équilibre économique international: *J. Sriber.* Le livre de Colin Clark: " Les conditions du progrès économique ": *J. Fourastie.*

Journal de la Société de Statistique de Paris—
May–June 1946—Le problème des stocks: *E. Baticle.* La loi des appels de courant dans une société de distribution d'électricité: *Henri Hemar.* Chronique des salaires: *J. Lehoulier.*
July–August 1946—Congrès de la Société suisse de Statistique et d'Économie politique: Le champ d'application de la méthode statistique: *C. Penglaou.* Sang, tempéraments, travail, races: *L. Bourdel.*

ITALY—

Giornale degli Economisti e Annali di Economia—
January–February 1943—*January–February* 1946—La riforma agraria in Italia: *G. Medici.* Uno studio sulla teoria dei dazi d'importazione: *L. Federici.* La raffigurazione della rendita del consumatore rispetto alle curve d'indifferenza e dei baratti: *A. Gambino.*
March-April 1946—Le basi storiche della struttura sindacale italiana: *G. Demaria.* A proposito di un suggerimento del pareto al ministro della produzione in uno stato collettivista: *C. Dami.* Obbiezioni economiche alle valutazioni della ricchezza nazionale: *C. Gini.* Nuove ricerche sui bilanci familiari: *P. Luzzatto-Fegiz.* Nuovi aspetti della politica monetaria internazionale: *A. Ganrino Canina.*
May–June 1946—Singolarità economiche della Palestina: *R. Bachi.* In merito a un' osservazione critica del Pareto: *A. de Toma.* Significato economico delle valuazioni della richezza nazionale: *C. Gini.* Nuove ricerche sui bilanci familiari: *P. Luzzatto-Fegiz.* La politica annonaria de Venezia: *L. Dal Pane.*

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LIST OF ADDITIONS TO THE LIBRARY

Since the issue of Part II, 1946, the Society has received the publications enumerated below:

I.—OFFICIAL PUBLICATIONS

(a) United Kingdom.

Agriculture and Fisheries, Ministry of. Fisheries in war time: report on the sea fisheries of England and Wales . . . for the years 1939–44 inclusive. London, H.M.S.O., 1946. 9½″. 108 pp. 1*s.* 6*d.*

Customs and Excise Department, Statistical Office. Annual statement of the trade of the United Kingdom with British countries and foreign countries 1944 compared with the years 1940–43. Vol. I, xi + 315 pp. 12*s.* 6*d.* Vol. II, vi + 681 pp. 22*s.* 6*d.* Vol. II, Supplement, 87 pp. 3*s.* Vol. III, vii + 464 pp. 15*s.* Vol. IV, vii + 351 pp. 12*s.* 6*d.* London, H.M.S.O., 1946. 5 vols. 13″.

Food, Ministry of.

How Britain was fed in war-time: food control 1939–45. London, H.M.S.O., 1946. 9½″. ii + 65 pp. 1*s.*

Second review of the world food shortage July 1946. London, H.M.S.O., 1946. Cmd. 6879. 9½″. 35 pp. 6*d.*

Health, Ministry of.

Hospital survey. The Hospital services of the North-Eastern area. London, H.M.S.O., 1946. 13″. 132 pp. 5*s.*

On the state of the public health during the six years of war: report of the Chief Medical Officer of the Ministry of Health 1939–45. London, H.M.S.O., 1946. 9½″. iv + 280 pp. 5*s.*

Home Office. Report of the Commissioners of Prisons and Directors of Convict Prisons for the years 1939–41. London, H.M.S.O., 1946 Cmd. 6820. 9½″. 180 pp. 3*s.*

Inland Revenue, Board of. Report of the Commissioners of His Majesty's Inland Revenue. 83. 1939–40, Cmd. 6769, 1946. 47 pp. 9*d.* 84. 1940–41, Cmd. 6770, 1946. 39 pp. 9*d.* 85. 1941–42, Cmd. 6771, 1946. 42 pp. 9*d.* 86. 1942–43. Cmd. 6772, 1946. 32 pp. 6*d.* 87. 1943–44, Cmd. 6773, 1946. 30 pp. 6*d.* 88. 1944–45, Cmd. 6774, 1946. 556 pp. 1*s.* London, H.M.S.O., 1946. 6 parts. 9½″.

National Insurance, Ministry of. Increases in old age, widows' and blind persons' pensions. London, H.M.S.O., 1946. Cmd. 6878. 9½″. 11 pp. 2*d.*

Overseas Trade, Department of. Review of commercial conditions. Southern Rhodesia, Oct. 1945. 26 pp. 6*d.* Spain, Oct. 1945. 59 pp. 1*s.* London, H.M.S.O., 1946. 8½″. 2 parts.

Royal Commission on Equal Pay. Appendices to minutes of evidence taken before the Royal Commission on Equal Pay. Appendices IX and X, pp. 86–138. 2*s.* 6*d.* XI–XIX, pp. 139–92. 2*s.* 6*d.* London, H.M.S.O., 1946.

Scientific and Industrial Research, Department of. Fuel Research Survey Paper No. 58. Rapid survey of coal reserves and production: a first appraisal of results. London, H.M.S.O., 1946. 2 pp. 9½″. viii + 23. 9*d.*

Scotland, Department of Agriculture for. Agricultural survey of Scotland. Edinburgh, H.M.S.O., 1946. 10¾″ × 15″. 76 pp. 8*s.* 6*d.*

Trade, Board of.

Accounts relating to trade and navigation of the United Kngdom for each month . . . Jan. 1940–Dec. 1945. London, 1940–45. 9½″. 72 parts.

Working Party reports. Boots and shoes. 9½″. viii + 190 pp. 3*s.* 6*d.* Hosiery. 9½″. 2 + 224 pp. 3*s.* 6*d.* London, H.M.S.O., 1946.

Works, Ministry of. Production in building and civil engineering. Supplement No. 1, dealing with the analysis of man-hours and machine-hours expended in the advance preparation of housing sites and temporary housing sites. London, H.M.S.O., 1946. 13″. 8 pp., 3 folded tables. 9*d.*

(b) International and Others.

Canada.

Department of Reconstruction and Supply. Economic Research Branch. Reconversion, modernization and expansion: progress and programs in selected Canadian manufacturing industries 1945–47: an analysis of reports provided by 643 major plants formerly engaged in war production. Ottawa, August 1946. 14″. 105 fols.

Cuba.

Ministerio de Hacienda. Direccion General de Estadistica. Comercio exterior 1935–39 a 1944. Havana, 1946. 9¼″. xxxii + 630 pp.

Eire.

Department of Industry and Commerce. Trade and shipping statistics 1939, 1940, 1941, 1942. Dublin, Stationery Office, 1946. 10″. vi + 328 pp. 3*s.* 6*d.*

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(b) International and Others—*Cont.*

Egypt.

Ministry of Finance, Statistical Department. Statistics of wages and working hours in Egypt, July 1945. Cairo, 1946. 12″. 95 pp.

Finland.

Statistiska centralbyrån.
Finlands officiella statistik. I. A., Utrikeshandel årspublikation. 59, 1939; 60, 1940; 62, 1942; 63, 1943; 64, 1944. 5 vols. 11½″. III, 35, Maatalous, 1938. 36, 1939. 37B, Lanthushållning. Lantbruksarbetarnas löner åren 1940 och 1941. 38, 1. Allmänna lantbruksräkningen år 1941. Del I, Översikt av räkningens resultat i landskommunerna. 39B, Lantbruksarbetarnas löner åren 1942 och 1943. 5 vols. 11½″. XVIII, A. Industristatistik 56, 1939, 1940; 57, 1941; 58, 1942; 59, 1943; 60, 1944. 5 vols. 11½″.
Statistisk årsbok för Finland. 37, 1939; 38, 1940; 39, 1941; 40, 1942; 41, 1943; 42, 1944–45. 6 vols. 11½″. Helsingfors, 1940–46.

France.

Statistique Générale de la France. Annuaire statistique. 56me volume, 1940–45. Paris, 1946. 10¼″. xxxii + 392 pp.

International Labour Office.

The war and women's employment: the experience of the United Kingdom and the United States. (Studies and Reports, New Series, No. 1.) Montreal, 1946 (London, King and Staples). 9¼″. 287 pp. 6*s.*

Italy.

Istituto Centrale di Statistica. Compendio statistico italiano. Anno 1946. Serie II, Vol. I. Rome, 1946. 7¼″. xi + 204 pp.

League of Nations.

Economic Financial and Transit Department.
Fiscal Committee. Report on the work of the tenth session of the Committee held in London from March 20th to 26th, 1946. (1946, II, A, 4.) Geneva, 1946 (London, Allen & Unwin). 9½″. 79 pp.
The population of the Soviet Union: history and prospects, by Frank Lorimer. Geneva, 1946 (London, Allen & Unwin). 10¾″. xiv + 289 pp.

New Zealand.

Census and Statistics Department. Population census, 1936. Vol. xiii. Dwellings and households. Wellington, 1946. 12¼″. iii + 56 pp. 6*s.*

Southern Rhodesia.

Report of the Building Costs Commission, Salisbury, Southern Rhodesia, November 1945. Salisbury, 1945. 13″. 131 pp.

Switzerland.

Bureau Fédéral de Statistique.
Recensement fédéral de la population 1er décembre 1941. 9, Kanton Zürich (S.Q. der S. Heft 157), 278 pp. 10, Canton de Neuchâtel (S.Q. der S. Heft 158), 181 pp. 11, Kanton Luzern (S.Q. der S. Heft 160), 175 pp. 12, Canton de Genève (S.Q. der S. Heft 161), 185 pp. 13, Kantone Appenzell Ausser-Rhoden und Appenzell Inner-Rhoden (S.Q. der S. Heft 162), 166 pp. 14, Canton de Fribourg (S.Q. der S. Heft 166), 146 pp. Bern, 1946. 11¾″. 6 vols.
Recensement fédéral des entreprises du 24 août 1939. Vol. 4. Les exploitations industrielles et commerciales en Suisse 1939. Commentaires analytiques (Statistiques de la Suisse 146), Bern, 1945. 11¾″. 100 + 83 pp., maps. Vol. 7. Exploitations horticoles, forestières et de pêche en Suisse, 1939 (Statistiques de la Suisse 159). Bern, 1946. 11¾″. 103 pp.

United States of America.

Office of Economic Stabilization. Report of the President's Committee on the Cost of Living. Washington, Supt. of Documents, 1945. 9″. v + 423 pp. 60*c.*

II.—AUTHORS AND MISCELLANEOUS.

Banco do Brasil. Relatório do Banco do Brasil S.A. Apresentado à Assembléia Geral dos Acienistas na Sessão Ordinária de 30 Abril de 1946. Rio de Janeiro, 1946. 9½″. 328 pp.
Bôcher (Maxime). Introduction to higher algebra . . . prepared for publication with the coöperation of E. P. R. Duval. New York and London, Macmillan, 1907. 8½″. xi + 321 pp. 7*s.* 6*d.*

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Authors and Miscellaneous—*Cont.*

Burns (*Arthur F.*). Economic research and the Keynesian thinking of our times. Twenty-sixth annual report of the National Bureau of Economic Research, Inc. New York, 1946. 9″. 69 pp.

Burns (*Arthur F.*) and *Mitchell* (*Wesley C.*). Measuring business cycles. (Studies in business cycles, No. 2.) New York, National Bureau of Economic Research, 1946. 11¼″. xxvii + 560 pp. $5.

Burnside (*William*). Theory of probability, by the late William Burnside. . . . Cambridge, University Press, 1936. 8½″. xxxi + 106 pp. 10*s*. 6*d*.

Cramér (*Harald*). Mathematical methods of statistics. Princeton University Press, 1946. 8¾″. xvi + 575 pp. 39*s*.

Crum (*W. L.*) and *Schumpeter* (*Joseph A.*). Rudimentary mathematics for economists and statisticians. New York and London, McGraw-Hill, 1946. 8¼″. xi + 183 pp. 12*s*. 6*d*.

Cummins (*S. Lyle*). Empire and colonial tuberculosis: a general survey. Nat. Assoc. for Prevention of Tuberculosis, 1946. 8½″. 4 + 56 + 3 pp. 5*s*.

Damalas (*B. V.*). La crise du capitalisme et le problème de l'économie dirigée. Paris, Presses Universitaires de France, 1946. 10″. 327 pp. 240f. (Presented by the author.)

Farnsworth (*Helen C.*). Wheat-growers and the tariff. Berkeley, Cal., University of California Press, 1946. 9″. 29 pp. (Presented by the Food Research Institute.)

Federal Products Corporation. Federal dimensional quality control primer: a simplified method for applying statistical quality control to dimensions. Second printing with revisions and additions. Providence, R.I., Federal Products Corpn., 1946. 8½″. 39 pp.

Fisher (*Allan G. B.*). International implications of full employment in Great Britain. London and New York, Roy. Inst. of Int. Affairs, 1946. 7¾″. vii + 202 pp. 15*s*.

Fletcher (*A.*), *Miller* (*J. C. P.*), and *Rosenhead* (*L.*). An index of mathematical tables. London, Scientific Computing Service, Ltd., 1946. 9¾″. viii + 451 pp.

Furlan (*L. F.*). Das Harmoniegesetz der Statistik: eine Untersuchung über die metrische Interdependenz der sozialen Erscheinungen. Basel, Verlag für Recht und Gesellschaft, 1946. 9″. xii + 504 pp. (Presented by Mr. G. Udny Yule.)

Goulden (*C. H.*). Methods of statistical analysis. New York, John Wiley, 1945 (London, Chapman & Hall). 9″. vii + 277 pp. 21*s*.

Hawtrey (*R. G.*). Economic rebirth. London, Longmans Green, 1946. 7½″. 161 pp. 5*s*.

Johns Hopkins University. Collected papers from the Department of Biology of the School of Hygiene and Public Health of the Johns Hopkins University. Vol. xvii, 1940. Vol. xviii, 1941. Baltimore, Md, 1940–41. 10″. 2 vols. (Presented by Prof. Lowell J. Reed.)

Kaldor (*Nicholas*). The German war economy. Manchester Statistical Society, 1946. 8¾″. 35 pp.

Kendall (*Maurice G.*). The advanced theory of statistics. . . . Vol. II. London, Griffin, 1946. 10½″. vii + 521 pp. 50*s*.

Loeb (*Harold*). Full production without war. Princeton University Press, 1946. (London, Oxford University Press.) 9″. xviii + 284 pp. 23*s*.

Macmahon (*P. A.*). An introduction to combinatory analysis. . . . Cambridge University Press, 1920. 8½″. viii + 71 pp. 9*s*.

Milnes (*A. H.*). A practical guide to spinning mill control and testing statistics. Manchester, Cook & Co., 1946. 8¼″. 64 pp. (Presented by the Author.)

National Institute of Economic and Social Research. Occasional papers, X. A system of national bookkeeping illustrated by the experience of the Netherlands economy, by J. B. D. Derksen. Cambridge University Press, 1946. 9½″. 34 pp. 5*s*.

Qureshi (*Anwar Iqbal*). Islam and the theory of interest. Lahore, Shaikh Muhammad Ashraf, 1946. 8½″. xxiv + 219 pp. *Rs*. 5.

Snedecor (*George W.*). Statistical methods applied to experiments in agriculture and biology. 4th ed. Ames, Iowa, Iowa State College Press, 1946. 9″. xvi + 485 pp. $4.50.

Southard (*Frank A.*). Some European currency and exchange experiences : 1943–46. (Essays in International Finance, No. 7). Princeton, N.J., International Finance Section, Princeton University, 1946. 9″. 23 pp.

Statesman's year-book : statistical and historical annual for the states of the world for the year 1946, edited by M. Epstein. . . . 83rd annual publication. London, Macmillan, 1946. 7″. xli + 1461 pp., maps. 30*s*. (Presented by the Editor.)

Sukhatme (*P. K.*) An interim report on the crop-cutting survey for estimating the out-turn of wheat in the Punjab (1943–44). (*Proceedings of the Indian Academy of Sciences*, 21, 1945, pp. 328–41.) 9¾″.

—— On bipartitional functions. (*Phil. Trans. of the Royal Society*, A, 780, 237, 1938, pp. 375–409.) 12″.

Sukhatme (*P. V.*), *Kadam* (*B. S.*), and *Bhat* (*N. R.*). Random sampling for estimating rice yield in Kolaba, Bombay. (*Proceedings of the Indian Academy of Sciences*, 21, 1945, pp. 328–41.) 9¾″.

Wadsworth (*John*). Counter defensive : being the story of a bank in battle. London, Hodder & Stoughton, 1946. 9¾″. 3 + 106 pp. 12*s*. 6*d*.

Wallace (*William*). Enterprise first: the relationship of the state to industry; with particular reference to private enterprise. London, Longmans Green, 1946. 8½″. 112 pp. 8*s*. 6*d*.

Yule (*G. Udny*) and *Kendall* (*M. G.*). An introduction to the theory of statistics. 13th ed. London, Griffin, 1946. 9″. xiii + 570 pp. 25*s*.

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REGISTRATION OF THE UNITED KINGDOM

No. I.—ENGLAND AND WALES

BIRTHS, DEATHS, *and* MARRIAGES *registered in the Calendar years* 1941–1945 *and in the Quarters of those years. Numbers, Annual and Quarterly Rates* * *per* 1,000 *persons living. (Deaths under* 1 *year of age; rate per* 1,000 *Live Births. Stillbirths per* 1,000 *Births.)*

Years	1941		1942		1943 †		1944 †		1945 †	
Estimated Mid-year Popln. in thousands ‡	41,748		41,897		42,143		42,449		42,636	
	Number	Rate	Number	Rate	Number	Rate	Number	Rate	Number	Rate
Live Births	587,228	14·1	654,039	15·6	683,212	16·2	744,843	17·5	685,273	16·1
Stillbirths	20,902	34	22,335	33	21,275	30	21,212	28	19,451	28
Deaths [1]	535,180	12·8	480,137	11·5	501,412	11·9	492,176	11·6	488,112	11·4
Marriages	388,921	9·3	369,744†	8·8	296,432	7·0	302,714	7·1	395,458	9·3
Infant Mortality ...	34,550 [2]	59	32,258 [2]	49	33,432 [2]	49	33,456 [2]	45	31,961 [2]	47
Effective reproductive rate [3]	0·761		0·853		0·900		0·996		0·916*	
Quarters	Live Births in the Quarters of each Calendar Year									
Jan.–Mar.	147,027	14·3	158,225	15·3	172,271	16·6	183,973	17·4	177,946	16·9
Apr.–June	147,414	14·2	167,611	16·0	180,709	17·2	199,039	18·9	175,221	16·5
July–Sept.	147,621	14·0	168,335	15·9	169,306	15·9	183,594	17·2	167,807	15·6
Oct.–Dec.	145,166	13·8	159,868	15·1	160,926	15·1	178,237	16·7	164,299	15·3
	Stillbirths									
Jan.–Mar.	5,632	37	5,652	34	5,783	32	5,587	29	5,190	28
Apr.–June	5,407	35	5,787	33	5,577	30	5,496	27	4,853	27
July–Sept.	4,970	33	5,437	31	4,980	29	5,018	27	4,676	27
Oct.–Dec.	4,893	33	5,459	33	4,935	30	5,111	28	4,732	28
	Deaths [2] (excluding Stillbirths)									
Jan.–Mar.	178,637	17·4	151,262	14·6	137,578	13·2	146,039	13·8	157,745	15·0
Apr.–June	140,186	13·5	116,937	11·2	112,912	10·7	115,548	10·9	110,969	10·4
July–Sept.	101,336	9·6	97,336	9·2	98,474	9·3	107,254	10·1	97,159	9·0
Oct.–Dec.	115,021	10·9	114,602	10·9	152,448	14·4	123,335	11·6	122,239	11·4
	Marriages									
Jan.–Mar.	81,034	7·8	88,280†	8·4	62,684	6·0	62,879	5·9	76,614	7·3
Apr.–June	105,576	10·1	100,992†	9·6	82,577	7·8	82,583	7·8	99,241	9·3
July–Sept.	104,702	10·0	96,445†	9·2	81,569	7·7	82,312	7·7	118,723	11·1
Oct.–Dec.	97,609	9·4	84,027†	8·0	69,602	6·6	74,940	7·0	100,880	9·4
	Infant Mortality									
Jan.–Mar.	10,985	75	9,697	61	9,914	60	10,104	58	10,656	60
Apr.–June	8,712	59	7,820	47	8,028	46	7,936	43	7,201	41
July–Sept.	6,411	43	6,766	40	6,771	40	7,325	40	6,412	38
Oct.–Dec.	8,184	56	7,713	48	8,370	51	7,862	44	7,588	46

* All rates are based on the total estimated population as at the middle of the corresponding year.
† Provisional figures.
‡ Inclusive of non-civilians of England and Wales temporarily abroad; and exclusive of non-civilians of Foreign Countries, Dominions, etc., temporarily in England and Wales.
[1] Including deaths of non-civilians registered in England and Wales.
[2] The annual figures have been corrected, so that slight differences appear between them and the aggregates of the quarterly figures.
[3] Based not upon current, but upon estimated future mortality.

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No. II.—SCOTLAND

BIRTHS, DEATHS, *and* MARRIAGES *registered in the Calendar years* 1941–1945 *and in the Quarters of those years. Numbers, Annual and Quarterly Rates* per* 1,000 *persons living. (Deaths under* 1 *year of age; rate per* 1,000 *Live Births. Stillbirths per* 1,000 *Births.)*

Years	1941		1942		1943		1944 †		1945 †	
Estimated Mid-year Popln. in thousands ‡	4,819		4,751		4,662		4,654		4,748	
	Number	Rate	Number	Rate	Number	Rate	Number	Rate	Number	Rate
Live Births	89,743	17·5	90,697	17·6	94,682	18·4	95,941	18·5	86,932	16·9
Stillbirths	3,698	40	3,602	38	3,494	36	3,221	32	2,948	33
Deaths [1]	72,558	14·7	64,963	13·3	66,733	14·0	64,603	13·6	62,655	13·2
Marriages	47,620	9·3	47,402	9·2	38,271	7·4	37,111	7·2	48,728	9·4
Infant Mortality ...	7,426	83	6,283	69	6,174	65	6,237	65	4,889	56
Quarters	Live Births in the Quarters of each Calendar Year									
Jan.–Mar.	22,254	17·6	21,885	17·3	23,277	18·3	23,231	18·0	21,644	16·9
Apr.–June	23,423	18·3	24,142	18·8	25,439	19·8	25,618	19·9	22,579	17·6
July–Sept.	22,656	17·5	22,586	17·4	23,320	17·9	23,405	17·9	21,344	16·4
Oct.–Dec.	21,410	16·6	22,084	17·0	22,646	17·4	23,687	18·2	21,365	16·5
	Stillbirths									
Jan.–Mar.	1,039	45	970	42	940	39	816	34	742	33
Apr.–June	961	39	931	37	886	34	874	33	772	33
July–Sept.	869	37	835	36	825	34	751	31	670	30
Oct.–Dec.	829	37	866	38	843	36	780	32	764	35
	Deaths (excluding Stillbirths)									
Jan.–Mar.	23,332	19·3	19,444	16·3	17,746	15·1	17,988	15·2	18,674	16·0
Apr.–June	18,999	15·5	16,298	13·4	15,874	13·3	15,288	12·9	15,364	13·0
July–Sept.	14,464	11·6	13,783	11·2	14,243	11·8	14,745	12·3	13,082	10·9
Oct.–Dec.	15,763	12·7	15,438	12·6	18,870	15·8	16,582	13·9	15,535	12·9
	Marriages									
Jan.–Mar.	10,732	8·5	12,710	10·0	9,290	7·3	9,043	7·0	9,764	7·6
Apr.–June	12,358	9·7	12,209	9·5	9,721	7·6	9,266	7·2	12,384	9·6
July–Sept.	13,052	10·1	11,888	9·2	10,228	7·9	9,872	7·6	13,646	10·5
Oct.–Dec.	11,478	8·9	10,595	8·2	9,032	6·9	8,930	6·8	12,934	10·0
	Infant Mortality									
Jan.–Mar.	2,435	109	1,960	90	1,756	75	1,756	76	1,465	68
Apr.–June	1,965	84	1,544	64	1,479	58	1,462	57	1,234	55
July–Sept.	1,424	63	1,322	59	1,283	55	1,510	65	979	46
Oct.–Dec.	1,602	75	1,457	66	1,656	73	1,509	64	1,211	57

* Death rates are calculated on civilian deaths, and civilian populations, but the birth and marriage rates are based on a total population which includes an allowance for men and women in the forces.
† Provisional figures.
‡ Civilians only.
[1] Registered deaths, civilians and non-civilians.

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No. III.—NORTHERN IRELAND

BIRTHS, DEATHS, *and* MARRIAGES *registered in the Calendar years* 1941–1945 *and in the Quarters of those years. Numbers, Annual and Quarterly Rates* * *per* 1,000 *persons living. (Deaths under* 1 *year of age; rate per* 1,000 *Live Births.)*

Years	1941		1942		1943		1944		1945 †	
Estimated Mid-year Popln. in thousands ‡	1,288		1,296		1,304		1,314		1,319	
	Number	Rate	Number	Rate	Number	Rate	Number	Rate	Number	Rate
Live Births	26,887	20·9	29,645	22·9	31,521	24·2	30,900	23·5	29,007	22·0
Stillbirths [1]					Not registered					
Deaths ‡	19,640	15·2 [2]	17,256	13·3	17,437	13·4	16,791	12·8	16,264	12·3
Marriages	11,966	9·3	11,673	9·0	10,155	7·8	9,508	7·2	10,417	7·9
Infant Mortality [3]	2,059	77	2,265	76	2,464	78	2,083	67	1,970	68
Quarters	Live Births in the Quarters of each Calendar Year									
Jan.–Mar.	6,200	19·3	6,731	20·8	7,907	24·3	7,612	23·2	7,335	22·2
Apr.–June	7,227	22·4	7,987	24·7	8,695	26·7	8,312	25·3	7,509	22·8
July–Sept.	7,083	22·0	7,834	24·2	7,738	23·7	7,503	22·8	7,189	21·8
Oct.–Dec.	6,377	19·8	7,093	21·9	7,181	22·0	7,473	22·7	6,978	21·1
	Deaths ‡ (excluding Stillbirths)									
Jan.–Mar.	6,183	19·2	5,168	16·0	4,890	15·0	5,108	15·5	5,249	15·9
Apr.–June	5,731	17·8	4,287	13·2	4,335	13·3	4,072	12·4	3,962	12·0
July–Sept.	3,741	11·6	3,639	11·2	3,642	11·2	3,483	10·6	3,339	10·1
Oct.–Dec.	3,985	12·4	4,162	12·8	4,570	14·0	4,128	12·6	3,714	11·2
	Marriages									
Jan.–Mar.	2,087	6·5	2,466	7·6	2,318	7·1	2,044	6·2	2,131	6·5
Apr.–June	3,051	9·5	3,190	9·8	2,628	8·1	2,532	7·7	2,786	8·5
July–Sept.	3,827	11·9	3,396	10·5	2,904	8·9	2,646	8·1	2,980	9·0
Oct.–Dec.	3,001	9·3	2,621	8·1	2,305	7·1	2,286	7·0	2,520	7·6
	Infant Mortality									
Jan.–Mar.	563	91	652	97	676	85	650	85	700	95
Apr.–June	518	72	569	71	647	74	501	60	504	67
July–Sept.	467	66	511	65	589	76	420	56	384	53
Oct.–Dec.	509	80	527	74	546	76	508	68	382	55

* Rates are based on civilian population only.
† Provisional figures.
‡ Civilians only.
[1] Stillbirths are not registered in Northern Ireland. The births of one living child and one still-born, or one living child and two stillborn, is counted as one birth.
[2] Civilian death rate (excluding air raid casualties) 14·6.
[3] The annual figures have been corrected, so that slight differences appear between them and the aggregates of the quarterly figures.

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No. IV.—EIRE

BIRTHS, DEATHS, *and* MARRIAGES *registered in the Calendar years* 1941–1945 *and in the Quarters of those years. Numbers, Annual and Quarterly Rates* per* 1,000 *persons living. (Deaths under* 1 *year of age; rate per* 1,000 *Live Births.)*

Years	1941		1942		1943		1944		1945 †	
Estimated Mid-year Popln. in thousands	2,993		2,963		2,946		2,944		2,983	
	Number	Rate	Number	Rate	Number	Rate	Number	Rate	Number	Rate
Live Births	56,780	19·0	66,117	22·3	64,375	21·9	65,425	22·2	66,521	22·3
Stillbirths ‡	Not registered									
Deaths	43,797	14·6	41,640	14·1	43,494	14·8	45,128	15·3	42,828	14·4
Marriages	15,021	5·0	17,470	5·9	17,328	5·9	16,772	5·7	17,285	5·8
Infant Mortality ...	4,175	74	4,591	69	5,319	83	5,198	79	4,572	69
Live Births in the Quarters of each Calendar Year										
Jan.–Mar.	13,673	18·3	14,427	19·5	16,400	22·3	17,066	23·2	16,086	21·6
Apr.–June	14,955	20·0	18,181	24·5	16,916	23·0	17,663	24·0	17,892	24·0
July–Sept.	15,067	20·2	17,749	24·0	16,242	22·1	15,749	21·4	17,095	22·9
Oct.–Dec.	13,085	17·5	15,760	21·3	14,817	20·1	14,947	20·3	15,448	20·7
Deaths (excluding Stillbirths)										
Jan.–Mar.	14,653	19·6	12,122	16·4	12,627	17·1	14,814	20·1	13,762	18·5
Apr.–June	11,312	15·1	10,822	14·6	10,974	14·9	11,074	15·0	10,751	14·4
July–Sept.	8,679	11·6	8,497	11·5	8,955	12·2	8,785	11·9	8,898	11·9
Oct.–Dec.	9,153	12·2	10,199	13·8	10,938	14·9	10,455	14·2	9,417	12·6
Marriages										
Jan.–Mar.	3,520	4·7	4,013	5·4	4,548	6·2	4,222	5·7	3,658	4·9
Apr.–June	3,643	4·9	4,603	6·2	4,006	5·4	4,069	5·5	4,628	6·2
July–Sept.	4,202	5·6	5,021	6·8	5,132	7·0	4,599	6·2	4,889	6·6
Oct.–Dec.	3,656	4·9	3,833	5·2	3,642	4·9	3,882	5·3	4,110	5·5
Infant Mortality										
Jan.–Mar.	1,151	84	1,163	81	1,583	97	1,719	101	1,434	89
Apr.–June	1,080	72	1,149	63	1,304	77	1,241	70	1,058	59
July–Sept.	976	65	1,031	58	1,186	73	1,021	65	996	58
Oct.–Dec.	968	74	1,248	79	1,246	84	1,217	81	1,084	70

* All rates are based on the total estimated population as at the middle of the corresponding year.
† Provisional figures.
‡ Stillbirths are not registered in Eire. The birth of one living child and one stillborn, or one living child and two stillborn, is counted as a single birth.

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Journal of the Royal Statistical Society

PART IV, 1946

PROCEEDINGS OF A MEETING OF THE ROYAL STATISTICAL SOCIETY HELD ON JULY 16TH, 1946, PROFESSOR M. GREENWOOD, F.R.S., IN THE CHAIR.

THE CHAIRMAN said that the meeting had been summoned to give the Society the pleasure of hearing an account of the notable progress achieved in India under the direction of Professor Mahalanobis. The scientific achievements of Professor Mahalanobis were known to a good many of them. It was the universal propensity of human nature to admire most highly the person who did successfully and easily what one had tried to do oneself without much success. In a casual way he himself had been a student of Combinatory Analysis for a good many years. He began with MacMahon's little primer, and then worked through part of his two volumes. It might be remembered that MacMahon said that "the theory of probabilities is to a large extent concerned with and dependent upon the enumeration of combinations subject to conditions, and almost every theory in combinatory analysis had its application to that theory." That seemed to the speaker perfectly true, and he would be ashamed to say how much time he had spent—he admitted that it was in leisure moments—trying to translate many of those theorems into statistical practice, but it was very much like an amateur chess-player who plays over the games of great masters. If he was reasonably intelligent he saw the beauty of their combinations, but if he tried to do the like in his own games he discovered that there was something lacking, not in the methods but in himself.

He could not honestly say that he had translated many of these theorems into a form which could not be obtained more simply, and his admiration for Professor R. A. Fisher and Dr. Isserlis, which would be great in any circumstances, was enhanced by the way in which they could translate theorems of combinatory analysis into statistically useful results. He had the same feeling in reading some of the work done or inspired by Professor Mahalanobis. The work had, of course, covered a much larger field than the application of theorems in combinatory analysis. Professor Mahalanobis had had the opportunity to apply statistical methods, the scientific technique of sampling, and the design of experiments on a scale so vast as to seem to someone like himself, inhabiting a small, over-crowded island, almost astronomical.

Some of the Fellows would have read a brief account that appeared in *Nature* last December which had whetted their appetite for the feast they were about to enjoy that evening. That account told them not only of fine educational and research work achieved, but also of the penalties, paid in human lives, of neglecting the extension of that work—not, of course, deliberate neglect, but in war-time all sorts of things happened. A good many of them in that room could give examples, though not on the same astronomical scale, of losses due to neglect or misapplication of statistical methods.

Professor Mahalanobis had been the guiding spirit of the Indian Statistical Institute. Not only had he personally done fine work, but he had had the capacity of selecting and encouraging other men of ability, of organizing team-work in the highest sense of that rather dubious expression. The result was that they now had in India one of the foremost statistical research institutions in the world, and they knew to whom that achievement was primarily due. It was a great pleasure to them that last year Professor Mahalanobis became one of a too small group of persons elected into the Royal Society mainly in recognition of their statistical work. It was in the nature of things that admission into a Society, however eminent and however anxious to be just, which in fact contained a very small number of persons competent to judge statistical merit, could not be a wholly satisfactory criterion of achievement in that particular branch of science, but they would be very priggish if they—and, he hoped, Professor Mahalanobis also—did not feel pleasure in his election into the oldest of the British scientific Societies. Nevertheless, it was possible that the welcome of his fellow-workers that afternoon would give him in a sense more pleasure, because they were capable of appreciating his work in its full merit. They knew what he had already done, and they looked forward with confident anticipation to what he would do in the future.

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PROFESSOR P. C. MAHALANOBIS then read his paper:

RECENT EXPERIMENTS IN STATISTICAL SAMPLING IN THE INDIAN STATISTICAL INSTITUTE

Introduction

I WAS naturally gratified to receive in March 1946 an invitation to present a paper before the Royal Statistical Society during my visit to London. I had collected some material for this purpose and had written certain portions of the paper when I was suddenly obliged to leave India at the end of April to attend a session of the United Nations Statistical Commission in New York. Owing to unforeseen difficulties and pressure of work in connection with the Royal Society and Empire Scientific Conferences in London in June and July, I had no time to finish the paper. It was not even possible to circulate the tables which had to be presented before the meeting in a mimeographed form. I greatly appreciate the kindness shown to me in relaxing the rule about submitting the written paper in advance, and secondly in arranging the present meeting during the closed session. The present paper contains the observations made at the meeting as well as certain portions of my original notes. I acknowledge with thanks the help I have received from my young colleague, Prasad Banerjee, in putting this paper in its final shape.

Restricted scope of the paper. This paper has been deliberately named as "Recent Experiments in Statistical Sampling in the Indian Statistical Institute" to make it clear that I have no desire to speak on sample surveys in general, or to make any attempt to give an account of the many interesting and significant developments in statistical sampling which have taken place in recent years outside India. A large number of papers have been published on the subject in statistical and scientific journals, and Dr. F. Yates, in the paper recently presented before the Royal Statistical Society, has given a comprehensive review. It is not necessary for me to touch the ground covered by others. The aim of the present paper is to draw attention to certain experiments in statistical sampling conducted during the period 1937–45 by the Indian Statistical Institute, of which I am the Honorary Secretary.

The Indian Statistical Institute. It will be convenient if I give at this stage a brief account of the Indian Statistical Institute itself, as this would explain the set-up in which the work was done. The Institute was inaugurated on December 17th, 1931, and was formally registered on April 28th, 1932, as a non-profit-making scientific society. Work began on a small scale, and the total expenditure in the first year (1932–33) was about £40. Special courses in statistics were started in the second year, which later developed into regular post-graduate training classes. Research work was also begun, at first on a part-time basis, but later with the help of a whole-time nuclear staff. A scheme for conducting examinations and awarding certificates of proficiency for computers and statisticians had been prepared in 1935–36 which was brought into effect from 1938. In 1941 the Institute helped in establishing a post-graduate department of statistics in the Calcutta University offering whole-time courses for the degrees in Statistics of M.A. and M.Sc.; and the actual teaching work was done in the Institute itself by a practically joint Institute and University staff. *Ad hoc* enquiries on a small scale were being undertaken from 1935, but project work began to develop on a large scale from 1937 with the initiation of a scheme for improving the forecast of the area under jute in Bengal. The project side developed rapidly during the war, as the Institute had to undertake various statistical enquiries and surveys on behalf of Government Departments. In 1945–46 the total volume of employment was about 750 man-years, divided about equally between the statistical and field branches, and the total expenditure was of the order of £100,000. Excepting for a comparatively small research grant of about £4,000, practically the whole of the income was derived in the form of contract grants from Government Departments for specific enquiries and projects. It is interesting to observe that the machinery was practically the same which had developed independently in the U.S.A., where large schemes of statistical investigations relating to the war effort were being done in different Universities and scientific institutions with the help of contract grants from the Government.

Conditions of work. Several points deserve to be emphasized. Each project or enquiry had definite practical objectives which were laid down by the client (usually a Government Department); the permissible margin of error of the results was broadly indicated; the enquiry had to be conducted in accordance with a definite time schedule so as to enable interim or final reports to be submitted by particular dates; the whole work had to be done within the limits of an all-inclusive

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contract grant, so that any excess of expenditure would mean a financial loss to the Institute. Finally, owing to difficulties created by the war situation in North-east India, the human agency and the organizational side of the project had to be very carefully planned, and usually the field survey had to be carried out under great difficulties. The research that had to be undertaken was naturally of an applied type. Nevertheless fascinating theoretical problems were continually arising, some of which were and are being tackled on fundamental lines. In preparing the design of each sample survey the three most important considerations thus were time, cost, and the human agency—all, of course, in relation to the most important factor of the permissible margin of error which was usually stipulated in advance.

Statistical engineering. In the present paper I have very much in mind those problems of organization which arise when a sample survey has to be carried out on a very large scale. The difference in quantity is so great that it brings about practically a change in quality. The manufacture on a commercial scale of a chemical gives rise to problems of an entirely different type from those which have to be solved in small-scale working in a laboratory, and the difference is so great that we give expression to it by calling large-scale production a matter of chemical engineering rather than of pure chemistry. In the same way large-scale sample surveys may be appropriately called statistical engineering. A good deal of the present paper is in fact concerned with statistical engineering rather than the pure theory of sampling.

General observations

The sample enquiries discussed in the present paper fall into two broad types. A great deal of work has been done on sample surveys on an extensive scale covering whole provinces (50,000 to 140,000 square miles in extent in the same season), in which the object has been to obtain reliable estimates of the acreage, rate of yield per acre, and total production of important food and fibre crops like rice, wheat, jute, etc.; or of economic or demographic factors relating to indebtedness, unemployment, destitution, paddy land, plough cattle, birth rate, death rate, etc., of rural families. For such work the technique used has always been the area or grid method, in which comparatively small sampling units (of area or size ranging from about two acres to one square mile) were located at random, and information was collected for each sample-unit by direct physical inspection or investigation. Estimates for districts (about 2,500 square miles or so in area on an average) or the province as a whole were then obtained directly by multiplying observed averages by appropriate multiplying factors (equal to the ratio of the total geographical area of the region to the total area covered in the sample surveys).

In a second type of work the enquiry was more localized, and related to cost and level of living, housing, consumption of food, clothes, etc.; preferences for particular types of commodities; reactions to radio programmes; public opinion on various subjects, etc.

Terminology and classification of sample surveys

Before proceeding further it will be convenient to explain the terminology and classification of sample surveys. In the area method the smallest physical element which can be separately surveyed is called a *quad*. Typical examples are a single plant for determining the yield of crop; a single family or household (having food from the same kitchen) for ascertaining the cost of living; or a single individual in anthropometric or blood-group surveys. The whole region or field to be surveyed may be then considered to consist of a very large number of such ultimate physical elements or *quads*.

Quad and configurational sampling. Sampling may then proceed in two different ways. The ultimate physical element or quad may be adopted as the sampling unit, or a group of such elements taken together may be used as the unit for sampling; these two types are called "quad" and "configurational" (or "grid" or "cluster") sampling. The grid, in the area method, consists of a suitable number of adjoining quads or ultimate physical units. For example, in crop surveys in Bengal it was found that a suitable sampling unit was a grid of square shape and size 2·25 acre. Each grid thus consists of a compact group of adjoining quads.

It must be noted, however, that the sample-unit may also consist of quads separated from one another, but in a fixed spatial configuration—for example, 4 quads at the corner of a square of a given size. It is convenient therefore to distinguish between the "grid" and the "cluster," and restrict the former to a compact group of adjoining individual units, and the latter to other types

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of configurational sampling where the quads are not adjoining but are arranged in some particular pattern in one or two dimensions. Every *n*th individual unit in a linear series would also in this sense be an example of cluster or configurational sampling. From the present point of view all sampling methods may then be characterized as being of either (1) of *quad*, (2) of *grid*, or (3) of *cluster* type.

Zonal and non-zonal. The important point to be noted is that all the sample-units have to be located at random. But this can be done in two ways—namely, distributing the sample units over the region under survey as a whole, or by first dividing the whole region into a suitable number of sub-divisions or zones, allocating a suitable number of sample units to each zone, and then locating such sample units, grids, or clusters at random within each zone separately. This gives rise to two types—namely, (1) without zoning or non-zonal, and (2) zonal sampling.

Multi-stage. So far methods have been considered in which the sample units are located at random in one single stage over each zone or over the whole area under survey. In many enquiries this method cannot be used because of its high cost. For example, in estimating the rate of yield per acre of crops it is usually necessary to proceed by stages. A number of zones are first selected at random; within each selected zone a number of villages are next selected at random; within each village so selected a number of fields are then selected at random; and within each field so selected one or more sample cuts are finally located at random. In this method the act of randomization is performed, not in one single stage, but in successive stages. The coverage of the process of randomization takes place repeatedly over smaller and smaller regions. This type of sampling is called multi-stage.

Replicated networks of sample units. Information for each zone (or for the entire field under survey) may be collected in one single network of sample units; or the information may be collected separately for two or more networks of sample units. Each such network would give an independent estimate, and differences between such estimates supply immediately measures of the over-all or effective margin of error. A characteristic feature of the Statistical Institute's work has been the use of such independent replicated networks of samples.

It would be noticed that quad or configurational, zonal or non-zonal, and replicated sampling can be used with either the uni-stage or the multi-stage methods. Thus quad sampling may be used at one or more stages, and grid or cluster at another stage. In the same way, the sampling method may be non-zonal at one or more stages and zonal at others. There may be one single network of samples at one or more stages or more than one network at other stages. The present system of classification of sampling methods is thus flexible and comprehensive, and makes it possible to describe any particular design of sample survey in a precise and non-ambiguous manner. It is suggested that the present terminology (with such modifications that may be considered desirable) should be adopted as a standard system for the classification of sample surveys.

Large-scale surveys

When the work is done on an extensive scale covering a large geographical area it is almost inevitable that the sample units must be located at a considerable distance from one another. The investigators have therefore to spend a good deal of time in travelling from one sample unit to another. The total time (or the total cost) of field operations thus consists of two broad portions—namely, the actual time or cost required for enumeration or investigation of the sample units, and the time required for the journeys between different sample units. In such surveys it would be obviously uneconomical to collect information for a single quad in each locality. It is therefore usually arranged to use a grid or a cluster—that is, a group of quads for investigation in each locality visited by the investigator.

Cost function. Large-scale sample surveys are necessarily expensive, and considerations of cost are therefore of great importance.

As already noted, in surveys covering large geographical areas the cost of field operations depends on the time required for actual enumeration or investigation and the time required for journeys from one sample unit to another. Collecting information for grids of a large size would naturally take more time. If grids of a large size are used, then the total number which can be surveyed must be necessarily small. Grids of a large size would be thus more widely scattered, so that the time required for journeys would be larger for each journey on an average, but there would be fewer journeys to perform. Working with grids of a small size it

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would be possible to use a large number so that the average distance apart would be less. The time for journeys on an average would be less, but a large number of journeys would have to be undertaken. The total cost of operations thus depends on the size of the grids and their total number. Working within a fixed budget this means that once the size of the grids is settled, their total number must also be determined by the budget limit.

In enquiries which are geographically more localized, but which are conducted on a sufficiently large scale, the cost of operation depends on the way the field survey is organized—that is, on the nature of the human agency employed for the work, the methods adopted for collecting the primary data, arrangements for checking and supervision, etc. In every large-scale enquiry the cost is, therefore, determined by the particular plan or design of survey proposed to be adopted.

Variance function. Cost, however, is not the only consideration. The precision of the final results is also important. The variance of grids or sample units decreases as the size (*i.e.*, area) of the grid or sample unit is increased. In crop-survey work it became clear quite early that the decrease in variance with an increase in the size of the grid was appreciably slower than the normal rate of decrease (namely, inversely as the size of the sample unit). What particular size of grid would be most economical would therefore depend on how the variance changes with the size of the grid. This necessitates a study of the variance function which gives the relation between the variance and the size of the grid. More generally, each way of distributing the sample units—that is, each particular design of sampling—involves its own appropriate sampling error, so that corresponding to each design there exists its specific variance function.

Optimum solutions. It is now possible in terms of the cost and variance functions to state the conditions for an optimum or most economical design for sampling. The aim may be stated in either of the following two alternative forms: (*a*) to determine the size and distribution or density of grids or sample units in such a way that the variance (or margin of error) of the final estimate is a minimum when the total cost of the sample survey is fixed; or, alternatively, (*b*) to determine the size and distribution of sample units so as to reduce the variance (or margin of error) of the final estimate to any desired level at a minimum cost. It has been shown elsewhere how concrete solutions (which are the same for either of the alternative forms) can be obtained with the help of an empirical knowledge of the variance and cost functions.

Two points are worth noting. The above approach is indispensable when the scale of operations is sufficiently large, and individual grids or sampling units are so widely separated that an appreciable amount of time and cost is incurred in journeys between grids of sample units, or when a large number of investigators are employed for the field survey so that the cost depends materially on the way in which the field staff is organized. Secondly, such large-scale sample surveys usually give rise to many problems involving the human agency which have engaged the special attention of the Indian Statistical Institute for a long time, and to which more detailed references have been made in later sections.

Exploratory (or sequential) development

The planning of a sample survey thus has several aspects—namely, (1) zoning and/or stratification; (2) size of the grid or sample unit; (3) arrangements for replication; (4) density or distribution of grids in different zones or strata; (5) preparation of forms and schedules; (6) structure and organization of the field staff; (7) arrangements for the statistical processing of the material, etc. It has been explained above how the efficiency of the design of the sample survey can be maximized from joint considerations of variance and cost functions. To use such methods it is necessary to have previous information about those two functions. Other relevant information relating to the region or universe to be surveyed is also of great value in preparing the design. In fact, the greater the amount and accuracy of such information the greater is the possibility of reducing the cost of operations.

When absolutely no information is available, and yet the survey has to be completed at one single operation, there is no other alternative but to use unrestricted random sampling without zoning, or with such zoning or stratification as may be considered convenient from a purely organizational point of view.

Fortunately, some information is usually available. Also, especially in large-scale work, the survey has often to be continued from year to year or at suitable intervals. It is therefore usually possible to adopt the exploratory method, in which a sample survey is first carried out on a very small scale with the primary object of collecting basic information required for preparing an efficient

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design for later surveys. Sometimes such preliminary surveys have to be carried out more than once, and the scale of operations is gradually increased until finally the whole area or universe is fully covered. For example, in the Bengal crop survey a pilot survey was first carried out in 1937 covering only 124 square miles at a total cost of about £1,100. In 1938, a second survey was organized, in the light of the experience gained in the previous season, on a larger scale covering about 400 square miles at a cost of £2,500; next year the total area covered was nearly 2,600 square miles at a cost of £6,000. In 1940 the area covered was nearly 20,600 square miles at a cost of £8,100; and finally, in 1941 the whole jute tract of Bengal, measuring about 60,000 square miles, was surveyed at a cost of £10,100.

In the exploratory stage the object is to gather as quickly as possible some rough idea of the variance function, cost of operations, type and nature of human agency likely to be suitable for the field work or fluctuations from one region to another (*i.e.*, the distribution over space) of the elements or statistical variates proposed to be estimated in the survey. An intensive study is often essential in the exploratory stage. The cost per square mile (or per unit) is therefore necessarily high, but as basic information begins to be gathered the design of the survey steadily improves, and the cost per unit rapidly decreases in subsequent surveys.

In the Bihar crop survey the exploratory work was done in only two districts covering about 8,000 square miles from February to April 1944. As conditions were found in many ways similar to those in Bengal, it was possible to organize a full-scale provincial crop survey in Bihar covering about 70,000 square miles in the next crop season extending from July to September 1944. This survey was, however, admittedly weak in certain respects. In the next survey in the main winter rice season from October to December 1944 the Bihar crop survey was, however, fully organized. This shows that where previous experience is available it is possible to expedite the exploratory phase.

In extensive surveys it is usually necessary to employ a large number of staff in both field and statistical branches. For example, in the Bengal crop survey the field staff consists of about 350 workers, who actually go round from village to village, with a number of inspectors and supervisors at the top. The total statistical staff in the Institute for computational and technical work consists at present of about 375 workers inclusive of all grades in 1945–46. Giving necessary training to the staff is an important part of the work, and takes a good deal of time. The exploratory approach is often convenient from this point of view.

In socio-economic or demographic enquiries also it has been found extremely helpful to conduct a small-scale preliminary enquiry, and then make final arrangements in the light of the experience gained thereby. A pilot survey enables forms and schedules, staff organization and other details being thoroughly worked out in advance, and often saves its cost many times over.

In large-scale surveys which are repeated from season to season or from year to year the exploratory development of technique, of course, continues all the time. As pointed out by A. Wald, the underlying logic is the same as in sequential tests. Advantage is taken of all available information in preparing the design of the survey. As information accumulates, the efficiency of the survey continues to improve from one survey to another.

Multiple surveys. A special form of the exploratory approach also deserves mention. Consider an actual example. In 1941 it was desired to develop a method for forecasting the yield of cinchona bark in blocks of standing plants each covering from 20 to 40 acres. In the first survey a number of physical measurements, such as girth, height, number of shoots, surface area of the plant, etc., were made on a number of standing plants picked up at random. These plants were then uprooted and the yield of bark was determined for each plant separately. Coefficients of correlation were then determined between the yield of bark and the different physical measurements. Three measurements—namely, height of the plant, girth at a height of 6 inches above ground, and the number of shoots—were selected for final use. A number of plants in the particular block under survey can be then picked up at random and these three physical characters measured for each plant, from which the average values of these three characters for the block as a whole can be determined. The plants are not, however, uprooted, and no direct measurements of bark yield are made during the second survey. (The number of plants in the block can be determined, if necessary, by a subsidiary survey on a grid basis.) Using regression equations based on the coefficients of correlation determined in the first survey, it is then possible to estimate the total yield of bark for the block as a whole.

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Such methods have been used, for example, in forestry work in the U.S.A., and have been sometimes called " double sampling." Various other possibilities are of course open; recourse may be had to triple or quadruple or *n*-ple samples, so that this type of work may be suitably called the method of multiple sampling (which is, of course, to be carefully distinguished from multi-stage sampling). In one sense it is really an extension of the method of concomitant variations. The statistical relation between two (or more) variates is first determined by a preliminary sample survey or a series of such surveys. Estimates are then prepared for one of the variates by a second or subsequent survey, and estimates for the other correlated variates are then calculated with the help of the previously determined regression relations. A simple example occurs in crop-cutting experiments. In the Indian Statistical Institute the weight of green plants of jute or the weight of paddy immediately after harvesting are recorded on an extensive scale. In only a small fraction of cases (of the order of 10 per cent.) the jute plant is steeped in water, retted and the dry fibre extracted and its weight determined directly, or the paddy is dried, husked and the weight of rice measured separately. These auxiliary measurements serve to supply the regression relation between the weight of green plants of jute or the weight of paddy immediately after harvesting and the yield of dry fibre of jute or of husked rice, respectively, which can then be used to estimate the corresponding final yields from the more extensive weights taken immediately after harvesting. Such a procedure simplifies the field work enormously without any appreciable loss of precision in the final results.

Such methods, in which the estimates made in later surveys are based on correlations determined in earlier surveys, may perhaps be called " covariate sampling."

Planning of sample surveys

Coming back to the planning of sample surveys, the ultimate aim from the statistical point of view is, of course, to reduce the over-all or effective margin of error to the desired extent. It has been explained that in large-scale work this involves considerations of time, cost, and the human agency. These three factors are intimately connected, and are merely different facets of the same thing—namely, the plan or design of the survey. Any change in human agency, in general, would involve changes in the time programme, the total cost, and also (a point to be remembered) in the effective margin of error. In fact we learnt quite early the overwhelming importance of the human factor in the organization of statistical sampling on a large scale.

The over-all or effective margin of error does not consist simply of sampling variations, but also of errors of ascertainment arising from the fallibility of the human agency. W. Edwards Deming has emphasized this point on many occasions, and has given a comprehensive review in his paper on " Errors in Surveys " (*American Sociological Review*, Vol. IX, No. 4, August 1944, pp. 359–369). He gives a list of thirteen factors affecting the ultimate usefulness of a survey. I shall adopt a simpler classification for present purposes and omit differences arising from variability in response or changes in the universe, or the population or the field under survey. These sources of error are of particular importance in socio-economic or public opinion surveys and offer fruitful subjects for research. A good deal of work has been already done and is still going on in this field in the U.S.A. and U.K. In the Indian Statistical Institute we have also made a few studies, but on a comparatively small scale, as the great bulk of the Institute work has been concerned primarily with crop surveys, in which these problems are not of great importance.

Classification of errors

Very early it became clear that mistakes committed in the compilation of primary data and, to a less extent, at different stages of processing the material, were equally or even more important than errors of sampling. Continuous attention has had to be given, therefore, to this aspect of the problem. Faults in planning (including defects in the design and technique of sampling, the preparation of questionnaires and field schedules), as also mistakes in the final analysis or interpretation, belong to the purely scientific and technical level of the work, and have been kept out of the present discussion. The remaining sources of inaccuracy have been usually considered in the Institute under three broad heads described below.

Sampling variations, which have to be considered by familiar methods of theoretical statistics. The large sample theory (based on the normal distribution) has been naturally used most extensively.

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Observed frequency distributions are, however, sometimes definitely non-normal in character, and special forms have been or are being investigated in certain cases. The logarithmic (for example, in crop yields) or square-root transformations (in certain economic and demographic studies) or truncated distributions (for example, in studying the correlation between weight of crop immediately after harvesting and weight of grain after drying) have been found useful. The subject, however, deserves and requires extensive investigations.

Recording mistakes have been usually considered under two sub-heads: mistakes which occur either through bias or personal equation of the observers or gross negligence or even deliberate dishonesty on their part; and mistakes which arise at the stage of compilation, classification, tabulation, computation, and final analysis and presentation of the material. Both types have been investigated in considerable detail in the Statistical Institute, and a great deal of attention is still being given to developing methods for controlling such recording mistakes.

Physical fluctuations. It has also been found that for every element (or quantity) sought to be estimated by the sampling method there exists a normal margin of physical uncertainty which is a characteristic feature of the element under survey. For example, it is believed that wheat increases in weight by as much as 4 or 5 per cent. by the absorption of moisture during the rainy season in a dry province like Sind. Traders, in fact, move the grain by railways at such times that the gain accruing from the increase of weight practically pays for the cost of storage and freight. In this case the exact meaning of the term "total production of wheat" has to be carefully specified. But even when this has been done, a residual margin of physical fluctuations of the order of probably 1 or 2 per cent. would still remain. It is clear that no useful purpose would be served by trying to reduce the sampling error to a value below the margin of such physical fluctuations.

Controls at the stage of statistical processing

Output and mistakes occurring at different stages of copying, compilation, or numerical computations have been studied in considerable detail. For this purpose, computational work has been broken down into a large number of jobs or items with standard specifications such as copying three-figure tables, adding four-figure quantities, squaring three-figure entries, preparing frequency tables with not more than ten classes, etc. For each item, standard rates of output have been worked out, as also permissible limits of rates of mistakes. For each item standard rates of payment have also been set up. A daily job account is kept of the item, time spent, and output for each computer separately, and these are punched on Hollerith cards and tabulated for the valuation of the total work done by each computer in each month.

Individual differences are also studied, which enables inefficient workers to be weeded out on an objective basis, and work to be distributed among different workers in a suitable way. (Some workers were found to have such high rates of mistakes that their net valuation was negative, showing that a premium would have to be paid to the Institute for employing them.) The advantage of the above system is two-fold. It supplies a scientific method for controlling mistakes at different stages of the processing of the primary material. It also supplies a sound basis for cost accounting, which is a characteristic feature of work in the Institute.

In the valuation of computational work it is necessary, of course, to take into consideration not merely the total output, but also the number of mistakes committed. The method of making deduction on account of mistakes is simple in principle. Each mistake involves a certain amount of expenditure in correcting it; and, in theory, the deduction made should be equated to this additional expenditure. In practice the matter is, however, complicated, but the subject is being studied, and attempts are being made to set up standard procedures.

Sub-samples in statistical analysis. In large-scale computational work it has been sometimes found convenient and useful to carry out the statistical analysis in the form of a number of sub-samples. The sample units are allotted consecutive serial numbers in the order in which these are located on maps or picked up for being included in the survey. It is possible therefore to divide the sample data into a number of independent random sets by choosing sample units with odd or even serial numbers, or with numbers ending in particular digits. The actual computational work and analysis are then done separately for each such randomized sub-set of the material. This has three distinct advantages. Firstly, it often enables dimensional results to be obtained very quickly, which is sometimes of great help to administrators. Secondly, a comparison of the results of different sub-sets often gives clues for the detection of gross mistakes in procedure or in calculations.

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Finally, results for different sub-sets supply a very good idea of the over-all or effective margin of error.

Controls in mechanical tabulation. It may be mentioned here that during the last two years a great deal of the Institute work has been mechanized. The Institute has at present a full equipment of Hollerith eighty-column sorter, tabulator, and multiplier units with a battery of about fifty hand-operated punching and verifying machines, and a summary reproducing punch. In the Hollerith system each card is punched, and then passed a second time through a verifying machine operated by a different worker. With about fifty punching and verifying workers, the human factor becomes quite important. In the beginning there was some difficulty in getting the verifying work done with proper care. A simple but effective statistical control has been now introduced which is probably worth mentioning. A concrete example will make the method clear. Consider the procedure for checking the accuracy of punching of say 10,000 cards. A suitable number (say 100 or 200 or 500) of punched cards are withdrawn from the set at random. A dummy set of an equal number of test cards are punched deliberately with mistakes in assigned columns; and this second set is substituted for the first set at the appropriate places. The whole set of 10,000 cards is then verified. It is clear that, if the work is done properly, then the set of (100, 200 or 500, as the case may be) test cards must be rejected at the stage of verification in the exact order of their serial number. Besides the test cards it is possible and likely that other cards also would be rejected; but the test cards must come out in any case. This supplies a good foolproof check at the stage of verification.

Efficiency at the stage of processing. Generally speaking, the basic problem may be stated in the following way. A given volume of computational work (in the most general sense, including scrutiny of primary material, sorting, tabulation, statistical analysis, checking, etc.) is required to be done. The problem is to organize the processing work in such a way that it can be completed with maximum accuracy in minimum time and at minimum cost. The subject is important and fascinating, and attempts are being made to study it as far as possible along with the regular work of the Indian Statistical Institute. Valuable information has been collected and indications for future investigations have been secured, but it has not yet been possible to tackle the problem on fundamental lines for lack of resources.

Illustrative examples

I shall now try to give some idea of the kind of information which it has been possible to collect by the method of statistical sampling in recent years. It is quite impossible, of course, to give a detailed account of the numerous enquiries undertaken by the Indian Statistical Institute during the last five or six years. I have therefore made a selection of typical examples which are likely to be of interest outside India. The use of replicated networks of samples (in which information for each network is collected independently by different sets of investigators) has been a characteristic feature of the work in the Institute. It includes as a special case the same sample being surveyed twice or more often by independent investigators. Such methods have been used very successfully in furnishing estimates of the over-all margin of error, and I have intentionally laid stress on the use of replication in choosing my examples.

Crop surveys

This is the field in which the largest volume of work has been done in the Indian Statistical Institute. An account has been given elsewhere (*Philosophical Transactions*, B 584) of the general development of work which took place from 1937 to 1941, in which year a sample survey of the area under jute was carried out throughout the province of Bengal. The survey of jute acreage was repeated in 1942. Although by that time, owing to Japan's entry into the war, the food situation in Bengal had already become difficult, I failed completely to persuade the Government to extend the sample survey to cover the paddy crop in Bengal. The Bengal famine occurred in 1943. Since that year we have had the opportunity of carrying out a sample survey of both jute and rice crops throughout the province.

Bihar Crop Survey. As already mentioned, a pilot survey had been started in only two districts, comprising about 8,000 square miles, in the *rabi* (winter wheat) season in February 1944. The scheme was rapidly expanded to cover the whole province, and a full-scale survey of *aghani* (winter rice) of 1944–45 season was carried out only a few months later.

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TABLE 1

Bihar Crop Survey. Progressive reports on winter rice, 1944–45

Estimated area (in hundred thousand acres) under different crops.

Serial no.	Field work up to	Grids surveyed		Sampling fraction one in	Area in hundred thousand acres under			
		Number	Percentage		Rice	Pulses	Sugar cane	Potatoes
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	October 17th	2,588	4·0	4175	133·35	29·55	—	—
2	November 2nd	14,361	22·3	752	135·37	30·65	3·72	—
3	,, 11th	19,813	30·8	545	140·70	31·28	3·74	—
4	,, 18th	25,934	40·4	417	141·71	30·31	3·94	—
5	,, 26th	32,312	50·3	334	140·89	29·77	3·87	—
6	December 4th	38,818	60·4	278	139·46	29·78	4·01	—
7	,, 12th	45,006	70·0	240	138·72	30·40	4·03	—
8	,, 21st	51,502	80·1	210	140·05	29·63	4·06	—
9	January 12th, 1945	64,260	100·0	168	136·50	29·03	3·97	1·12
	(Total area covered = 432·23) proportion =				31·58%	6·72%	0·92%	0·26%

The above table shows the actual progressive estimates which were submitted to the Government from time to time. It will be noticed that the first estimates were based on only 2,588 grids (or about 4 per cent. of the total number ultimately surveyed), which represented a sampling fraction of one in 4,175, and yet the estimates of the area sown with rice (133·35) and with pulses (29·55) were of right dimensional order and compare quite favourably with the final figures, 136·50 and 29·03 (in 100,000 acres), respectively. After receiving the first estimates of rice and pulses, the Government desired to have similar figures for the area under sugar-cane, which were submitted regularly from the second progressive report. At the last stage, at the request of the Government, an estimate of 1·12 (in 100,000 acres) was supplied for the area under potatoes.

The total geographical area covered in the survey was 432·23 (in 100,000 acres), so that the proportion of land was 31·58 per cent. under rice, 6·72 per cent. under pulses, 0·92 per cent. under sugar-cane, and only about 0·26 per cent., or one quarter of 1 per cent., under potatoes. This would supply a good idea of the wide range of the survey. It, however, scarcely needs mentioning that the percentage (but not the absolute) error would be naturally quite high for potatoes or sugar-cane which have comparatively small acreages.

Another point is worth explaining. Sowings of rice continue for a good length of time, possibly till the middle of November, depending on actual conditions of rainfall in different areas. In Bihar (as well as in Bengal) the rice crop is mainly rain-fed, and usually a certain proportion of marginal land is put under cultivation in the hope of a favourable distribution of rainfall. When the rainfall is not exceptionally favourable (as often happens) it is inevitable that some of the lands which in the earlier part or middle of the season are under a particular crop will go out of cultivation owing to lack of rain at the end of the season. The total area under rice therefore increases from the beginning to the middle of the season, and then usually decreases. For this reason the earlier estimates would often agree fairly well with estimates based on material collected at the end of the season.

Bengal Crop Survey. A brief description of the Bengal Crop Survey would be useful at this stage. The province of Bengal has been divided into about 1,100 "cells," each approximately 64 square miles in area. In doing this the boundaries of *mauzas* (broadly corresponding to villages, and roughly about one square mile in area) have been kept intact, so that each *mauza* (or village) lies entirely within a single cell. This has been done in order to facilitate the building up of estimates for higher administrative or revenue units such as *thana* (or police circle of 150 square miles on an average), sub-division (about 850 square miles) or districts (about 2,500 square miles), and finally the province as a whole (70,000 square miles).

Flexibility of the design. The design of the sample survey has naturally to take into consideration the intensity of cultivation or the importance of particular economic variates desired to be investigated in different regions. For example, in crop surveys it is legitimate to omit areas known to be

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under forests. In the case of particular crops like jute it is permissible and necessary to allot a comparatively small number of sample units to regions or cells where jute is known to be grown only in small quantities (when it is desired to obtain an accurate estimate of the total production of the crop for the province as a whole). The arrangement by which the whole province is divided into a large number of cells gives great flexibility in designing, as it permits unimportant cells to be left out of the survey altogether or to be allotted a comparatively small number of grids. The first stage in the design of the survey is thus to settle the size (area) and the number of grids to be allotted to the different regions or cells. For surveys covering a number of crops or a number of different elements it is often convenient to adopt a uniform density, or the same number of grids for each cell included within the survey. But this is a special case; in general, the number in each cell can be varied as desired.

In Bengal the total number of sample units allotted to each cell (of 64 square miles) is at present of the order of 100 for crop surveys. The position of each grid is located at random and marked on maps. In this way each *mauza* gets none or one or more random points. These random points are numbered serially as they are marked on the map, and a grid (of 2·25 acres for crop surveys) is located at each random point. All fields falling within the grid are subsequently surveyed by the field staff.

Two sub-samples. Within each cell (in which the act of randomization is carried out separately) the sample units, as they are located at random, are given consecutive serial numbers, so that the set of sample units or grids with even serial numbers and the set of sample units or grids with odd serial numbers form two independent, but inter-penetrating and random networks of sample units, each of which covers the same area. These two sets are allotted to different parties of field investigators, so that information for each set is collected independently. These two sets of data thus supply two independent estimates for each cell as a whole.

Duplicate grids. A further control is used in crop surveys as a routine measure at present. A certain number of grids or sample units are allotted to both networks so that information for such "duplicated" sample units is collected by both sets of field investigators. In the Bengal Crop Survey in 1946, for example, out of fifty-four grids allotted to each network of samples, fourteen are "duplicated" or common to both sets. A detailed comparison of the two sets of field records for these fourteen grids in each cell shows how far the field survey has been done with care. The field investigators know that certain grids are duplicated, but have no knowledge as to which particular grids have to be enumerated twice. The field programme is arranged in such a way that the two different parties of investigators are separated by at least two or three days' journey, so that there is no chance of the investigators copying the records from one another. This method has proved extremely useful in supplying a good control at the point of collection of the primary material.

It is worth mentioning briefly another control used in connection with duplicated (or sometimes replicated) grids or sample units which are enumerated twice (or more often) by different parties of investigators. In the crop survey the investigators are required to note the average height of the plant in each field. As the duplicated (or replicated) surveys are done at different times—sometimes with an interval of even two or three weeks—this supplies a good check. For example, cases have occurred in which the height of plants were recorded as being lower at the time of the later survey than the height recorded during an earlier survey. Such discrepancies can arise only through mistakes in identifying the fields, or through gross negligence or dishonesty on the part of the investigators. Such controls also serve the useful purpose of supplying an objective basis for the rejection of unreliable primary material.

Agreement between half-samples. Table 2 shows the two independent (replicated) estimates of crop acreages in Bengal during the three years 1943, 1944, and 1945. The design was that of two interpenetrating random networks of samples. The actual design of the survey was different in each year, but it is not possible to enter into these details. The two independent samples (*A* and *B*) are usually called half-samples as both together make up the complete sample. These might also have been called "double" samples; but this would have probably raised misapprehensions about cost in the minds of administrators.

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TABLE 2

Bengal Crop Survey. Half-sample comparisons, 1943–45

Year	Number of grids surveyed			Area under crops in thousand acres			Difference ($A - B$)		Standard error of difference	Fisher's t
	Sample (A)	Sample (B)	Combined	Sample (A)	Sample (B)	Combined	Actual	Percentage		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
					Jute crop					
1943	29,676	29,676	59,352	2,759	2,757	2,758	2	0·1	100	0·02
1944	30,487	30,037	60,524	2,150	2,056	2,106	94	4·45	50	1·87
1945	53,504	52,623	106,127	2,512	2,528	2,520	− 16	0·8	*	—
					Aus (monsoon) rice					
1943	29,676	29,676	59,352	6 807	6,923	6,865	−116	−1·7	446	0·26
1944	30,457	30,037	60,524	7,815	7,942	7,873	−127	−1·6	90	1·41
1945	53,504	52,623	106,127	6,966	6,805	6,864	161	2·3	*	—
					Aman (winter) rice					
1943	31,216	31,215	62,431	23,840	24,044	23,942	−204	−0·8	703	0·29
1944	50,501	50,107	100,608	22,491	21,903	22,201	588	2·7	550	1·07
1945	46,148	45,971	92,119	20,970	21,202	21,087	−232	−1·1	103	2·26†

* Error calculations for 1945 were not completed at the time of writing.
† Significant at 5 per cent. level.

The two crop estimates in thousand acres based on the two independent samples (A and B) are shown in cols. 5 and 6 respectively, and the pooled estimate in col. 7. The difference between the two estimates $A - B$ in thousand acres is shown in col. 8, and the same difference expressed as a percentage of the combined value given in col. 7 is shown in col. 9. The standard error of the difference calculated from the samples is given in col. 10, and Fisher's t in col. 11. This table shows the kind of results which are actually obtained in practice. I should like to note, however, that the field survey in 1943 had to be organized at very short notice and under great difficulties (owing to the incidence of the Bengal famine), and supervision was probably not quite adequate; the close agreement between the two half-samples may be, therefore, to some extent spurious.

Discrepancies in crop enumeration. Since 1937, when work was started by the sampling method on a very small scale, the relative accuracy of the results obtained by a so-called complete enumeration and by the sample survey respectively have been the subject of acute controversy in India. From the very beginning attempts were therefore made to investigate the reliability or otherwise of crop enumeration done by ordinary field investigators in actual practice. The method adopted has been to carry out independently twice (or more often) complete enumerations of crops on fields in the same village or region by entirely different sets of investigators. If crop enumeration can be carried out with complete accuracy, then the two sets of records should be in perfect agreement. Certain discrepancies are, however, introduced by the fact that the two surveys have necessarily to be carried out one after the other in order that there is no chance of one party of investigators copying from the records prepared by the other party. If the first survey is carried out before sowings are completed, it is possible that certain fields which were uncultivated at the time of the first survey might have been brought under cultivation by the time the second survey is made. Or if the second survey is made rather late in the season, it is possible that some of the fields having standing crops at the time of the first survey might go out of cultivation at the time of the second survey for lack of rainfall or other causes. Such discrepancies can, however, be usually distinguished from a knowledge of crop conditions in relation to the two dates of survey.

A great deal of material of this nature has been compiled, year by year, since 1937. Even a cursory examination of the material leaves no doubt that very large discrepancies occur at the stage of crop enumeration. Some of the mistakes are genuine, in the sense that the investigators actually fail to distinguish between different crops, or make wrong estimates of the proportion of the field under different crops. Mistakes due to gross negligence, however, also occur; and sometimes, of

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course, entries are made by pure guesswork by unreliable investigators who want to avoid the trouble of going round to the fields. It must be remembered that conditions in which the crop enumeration has to be done in Bengal are particularly difficult. In summer the temperature would often go up to 110° F. or more; during the monsoon many of the roads are submerged; throughout the crop survey a large number of investigators suffer from malaria and other diseases every year. This makes it all the more necessary to have adequate controls at the point of collection of the material.

I am giving below a few tables to indicate the kind of material collected and nature of comparisons made in the Indian Statistical Institute on an extensive scale from year to year.

TABLE 3

Illustrative summary comparisons of duplicate crop records prepared by different investigators (A *and* B)

Province and zone under survey	Years of survey	Name of crop	Estimated acreage		Discrepancies ($A - B$) between the two records				Percentage to A	
					Actual		Sum			
			(A)	(B)	Positive	Negative	Algebraic	Absolute	Algebraic	Absolute
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Bengal 2 *thanas* ...	1937	Jute	355	385	103	133	−30	236	−8·5	66·5
Bengal 8 *thanas* ...	1938	Jute	3,584	3,646	466	528	−62	994	−1·7	27·7
Bengal 6 *thanas* ...	1944	Wheat	286	298	100	112	−12	212	−4·3	74·6
Bihar 6 *thanas* ...	1944	Gram	730	636	244	150	94	394	12·9	53·9
Bihar 86 *thanas* ...	1944	*Bhadoi* paddy	6,715	6,481	1,658	1,424	234	3,082	3·5	45·9

The above table gives a comparison of duplicated crop records prepared independently by two different sets of investigators called *A* and *B*. Cols. 4 and 5 give the two independent figures of total acreage under the same crop in the same geographical area. A detailed comparison of the two sets of records was carried out for each field separately. For example, a particular field may be shown as being under jute by the first party of investigators (*A*), but not under jute by the second party of investigators (*B*). In such cases the discrepancies $A - B$ may be considered to be positive. When a field is shown under a particular crop by party *B*, but not by party *A*, the discrepancy $A - B$ would be then naturally considered to be negative. If such discrepancies are added up for each individual plot, one would get the total positive or negative discrepancies. These are shown in cols. 6 and 7, and the point to be emphasized is that such absolute discrepancies are actually very large, and in the above table range from 27·7 per cent. to 74·6 per cent. So far as detailed crop records are concerned, it is quite clear, therefore, that the so-called complete enumeration cannot supply reliable results under conditions actually existing at present in Bengal or Bihar.

Many of the mistakes are, however, unbiased and tend to cancel out. The total algebraic errors are obtained by simply adding the positive and negative discrepancies, and are shown in col. 8. For example, in the first line of Table 3 the figure −30 in col. 8 is obtained by adding the positive and negative discrepancies in cols. 6 and 7, and is of course also simply the difference between the two figures given in cols. 4 and 5 respectively. In comparison with the absolute discrepancies shown in col. 9 it will be seen that algebraic discrepancies are proportionately much smaller, owing to the cancelling out of positive and negative mistakes. In other words, results of complete enumeration of crops as carried out at present in Bengal or Bihar have exactly the same qualitative kind of statistical reliability as results based on sample surveys, with the important difference that in the so-called complete enumeration (as usually conducted without any superposed sampling controls) there is no possibility of estimating the margin of error. This is a conclusion of the greatest practical significance.

Two illustrative examples of a detailed comparison, plot by plot, of duplicated crop records are given below. Table 4 refers to material collected in *mauza* (village) Kazikandi in P.S. Goalundo

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(an administrative unit) in Bengal in 1943. In this village 332 fields were surveyed by investigator *A* on August 14th, 1943. He noted the name of the crop (or mixture of crops) growing on each individual field. A second complete enumeration was made by investigator *B* about a fortnight later, on September 2nd, 1943, who also noted independently the name of the crop or mixture of crops growing on each individual field. Two records were thus available for each field, and can be shown in the form of a two-way table as in Table 4.

TABLE 4

Bengal Crop Survey, 1943. *Comparison of duplicated complete enumeration in P. S. Goalundo*, mauza *Kazikandi*

(*B*-survey September 2nd, 1943)

A-survey August 14th, 1943	Jute	*Aman* rice	Jute-*aman*	No crop	Total
Jute	4	15	4	3	26
Aus rice	4	12	1	4	21
Jute-*aus*	17	66	2	9	94
Jute-*aus*-*amam*	—	2	—	—	2
Aus-*amam*	1	—	—	—	1
No crop	37	45	4	102	188
Total	63	140	11	118	332

If the crop enumerations made by the two investigators, (*A* and *B*) were in complete agreement, then all fields shown as being under jute by investigator *A* would also be shown under jute by investigator *B*, and vice versa, so that entries would occur only in the diagonal cells. A glance at Table 4 would show that the actual position was entirely different. For example, according to investigator *B* there were altogether 63 fields under jute on September 2nd. According to investigator *A*, who had surveyed the same identical fields a fortnight earlier, apparently only four were growing jute, four *aus* (monsoon) rice, seventeen had a mixture of jute and *aus* rice, one field had mixture of *aus* (monsoon) and *aman* (winter) rice while no fewer than thirty-seven fields were without any crops at all. Discrepancies are equally striking for other crops. In fact out of 332 fields surveyed, the two investigators were in agreement only in regard to four plots, which both recorded to have been under jute, and also in regard to 102 fields, which were shown by both as having no crops at all. In other words, in only 106 out of 332 entries was there agreement between the two independent crop enumerations. This is clearly unsatisfactory, and shows that either one or both the investigators had failed to do their work with reasonable care.

TABLE 5

Bengal Crop Survey, 1943. *Comparison of duplicated complete enumeration in P. S. Chuadanga*, mauza *Monirampur*

(*B*-survey August 10th, 1943)

A-survey June 28th, 1943	*Aus*	Jute-*aus*	None	Total
Jute	1	—	1	2
Aus	123	2	8	133
Jute-*aus*	—	3	—	3
None	7	—	189	196
Total	131	5	198	334

A more satisfactory example is shown in Table 5 for another pair of investigators working in Bengal in the same year. In this case out of 131 fields recorded as having *aus* (monsoon) rice by investigator *B* on August 10th, 1943, no fewer than 123 had been shown as having rice also by investigator *A* during the earlier survey on June 28th, 1943. Out of five fields shown as having both jute and *aus* (monsoon) rice by investigator *B*, three were shown so in the earlier survey, while two fields were shown as having only *aus* rice by investigator *A*. This is quite possible, as *aus* rice is sown earlier, and these two fields might have been sown with jute after the first survey was over.

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TABLE 6

Bihar Crop Survey, Bhadoi, 1944–45. *Comparison of two independent estimates of* p (*proportion of land under* bhadoi *rice in each individual plot*) *by two parties of investigators*

District, Hazaribagh; Thana, Giridih; No. of villages, 86

First survey / Second survey		Anna estimates of proportion of land under rice during first survey between August 3rd and September 29th, 1944																
		Zero	1-anna	2-anna	3-anna	4-anna	5-anna	6-anna	7-anna	8-anna	9-anna	10-anna	11-anna	12-anna	13-anna	14-anna	15-anna	16-anna	Total
Anna estimates of proportion of land under rice during 2nd survey between Aug. 6th and Oct. 12th	Zero	23,988	72	116	29	116	26	51	14	108	10	33	13	68	15	31	24	4,307	29,021
	1-anna	87	6	8	3	2	—	1	—	3	—	—	1	1	—	—	3	33	148
	2-anna	115	6	12	3	4	—	1	2	9	2	—	—	2	1	—	2	79	238
	3-anna	32	6	3	1	2	—	—	—	3	—	5	—	1	—	—	—	24	76
	4-anna	129	6	9	2	16	1	7	1	9	—	4	1	2	—	2	—	102	291
	5-anna	14	1	1	1	—	—	—	—	1	—	1	—	1	—	—	—	17	37
	6-anna	59	—	3	1	6	—	3	1	5	1	—	3	2	2	—	—	56	142
	7-anna	23	—	—	—	3	—	—	—	1	—	1	—	—	—	—	—	19	47
	8-anna	130	4	3	1	7	—	5	—	13	1	7	1	4	—	5	1	157	339
	9-anna	9	—	1	—	2	—	2	—	1	—	1	—	—	—	—	—	13	29
	10-anna	44	—	3	3	3	—	2	—	3	1	8	2	3	1	2	—	77	152
	11-anna	18	—	—	—	2	2	1	—	—	—	—	—	—	—	—	—	13	36
	12-anna	63	—	—	1	5	—	3	1	6	—	1	—	4	1	2	—	109	196
	13-anna	7	1	—	—	—	—	—	1	1	—	1	—	—	—	1	—	26	38
	14-anna	43	—	2	—	—	—	—	—	5	—	2	—	4	—	1	3	112	172
	15-anna	21	—	—	—	3	—	1	—	2	—	1	—	3	3	4	1	121	160
	16-anna	3,721	27	25	11	65	10	43	10	113	18	70	16	82	34	109	82	20,635	25,071
	Total	28,503	129	186	55	236	39	120	30	283	33	135	37	177	57	157	116	25,900	56,193

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As regards uncultivated fields, the two investigators had agreed in the case of 189 out of 196 fields. There was agreement in 315 (or 94·3 per cent.) out of 334 fields. This may be contrasted with the agreement in the case of only 106 (31·9 per cent.) out of 332 fields in the previous table. Such differences show the wide variation in the quality of field work done by different investigators. The importance of providing statistical controls like duplicated grids can be therefore easily appreciated.

I may now give a quantitative example. In the sample survey of crops the field staff is required to estimate the proportion of land in each field which is under a particular crop. These estimates are made in terms of the Indian coin anna, sixteen of which make up a rupee. In other words, crop estimates are made in units of 6¼ per cent.

Table 6 shows a comparison of results of such estimates of the proportion of land sown with rice on the same group of 56,193 fields which were surveyed independently by two sets of investigators. The comparison between the two sets of records has been made with increasing latitude of discrepancy. For example, if the two entries (estimates of the proportion of field under rice) relating to the same field (made by the two different parties of investigators) agree within 1 anna, (or 6¼ per cent.), then the latitude of comparison is 1 anna, or 6¼ per cent. In the same way, if the two entries agree within 4 annas, then the latitude of comparison is 25 per cent.; and if they agree within 8 annas, the latitude is 50 per cent. Finally for a 16-anna (or 100 per cent.) latitude of comparison two entries relating to the same field would be considered to be in agreement if both the parties record the field to be under rice irrespective of the quantitative proportion.

TABLE 7

Bihar Crop Survey, 1944, Bhadoi (*monsoon*) *rice. Comparison of complete enumeration by two sets of investigators*

Latitude of comparison	Number of plots surveyed			Total number of plots compared	Percentage of plots surveyed		
	In agreement		Not in agreement		In agreement		Not in agreement
	No crop	With rice			No crop	With rice	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1 anna (6¼%) ...	23,988	20,699	11,506	56,193	42·69	36·84	20·47
4 anna (25%) ...	23,988	21,269	10,936	,,	,,	37·85	19·46
8 anna (50%) ...	23,988	21,742	10,463	,,	,,	38·69	18·62
16 anna (100%) ...	23,988	22,657	9,548	,,	,,	40·32	16·99

A glance at the summary Table 7 would show that if a 6¼ per cent. latitude of comparison is permitted, then records relating to 11,506 (or 20·47 per cent.) of 56,193 fields were discrepant. Increasing the latitude of comparison to 25 per cent., the number of discrepant fields is only slightly reduced to 10,939, or 19·46 per cent. Allowing a much higher margin of comparison of 50 per cent. the number of discrepant fields is again only very slightly reduced to 10,463, or 18·62 per cent. Finally, allowing the maximum possible latitude of comparison—that is, considering merely whether the field is recorded to have rice or not—it is seen that no fewer than 9,548 or 16·99 per cent. of fields still show discrepant entries. The latitude of comparison thus makes very little difference, which shows that mistakes usually occur in the identification of the fields rather than in making quantitative estimates of "*p*" (the proportion of land under a crop).

Comparison of "duplicate" grids. It has been already mentioned that in the same survey of crops the present practice is to have a certain proportion of grids enumerated in duplicate by two independent sets of investigators. This supplies a valuable check on the quality of the field survey. The field investigators make an entry for each grid of the proportion (called "*p*") of the total land included within each grid (namely, 2·25 acres in the Bengal crop survey of 1945–46) which is estimated to be under *aman* (winter) rice. Such estimates were prepared twice by independent parties of field staff so that for each grid there are two values of "*p*," one estimated by party *A* and the other by party *B*. The two sets of records can therefore be shown in the form of a two-way table as in Table 8.

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TABLE 8

Bengal Crop Survey, 1945–46. *Comparison of two independent estimates of* p (*proportion of grid under* aman *rice*) *by two parties of investigators*

Half-sample (*B*)

Half-sample (*A*) — *p* (in %)	0	1–10	11–20	21–30	31–40	41–50	51–60	61–70	71–80	81–90	91–99	100	Total
0	1,159	84	50	33	30	17	18	15	27	24	28	62	1,547
1–10	74	150	39	22	13	9	11	11	4	14	3	13	363
11–20	31	35	88	14	14	12	10	11	9	3	5	16	248
21–30	26	20	22	65	18	18	16	6	5	7	6	8	217
31–40	15	10	16	25	97	42	15	15	13	16	11	11	286
41–50	17	11	7	14	31	85	32	14	17	12	14	9	263
51–60	17	6	9	8	18	29	71	30	24	11	16	30	269
61–70	22	6	7	6	13	10	27	78	37	31	22	25	284
71–80	24	7	7	8	11	23	17	45	91	45	32	45	355
81–90	28	7	9	11	9	9	25	15	45	129	52	57	396
91–99	29	4	7	6	13	11	12	17	29	48	264	143	583
100	68	13	11	14	13	23	23	31	54	64	152	927	1,393
Total	1,510	353	272	226	280	288	277	288	355	404	605	1,346	6,204

Frequency constants: mean $p(A) = 52{\cdot}0$ per cent., mean $p(B) = 51{\cdot}9$ per cent.
s.d. of $A = 40{\cdot}9$ per cent., s.d. of $B = 41{\cdot}2$ per cent., coefficient of correlation $= 0{\cdot}739$.

TABLE 8A

Summary table of proportion of agreement with different latitudes of comparison

	Number of grids	Accumulated totals	Accumulated percentage
Complete agreement	3,204	3,204	51·6
1–10% margin	1,069	4,273	68·9
11–20% ,, ...	511	4,784	77·1
21–30% ,, ...	367	5,151	83·0
31–40% ,, ...	225	5,376	86·7
41–50% ,, ...	185	5,561	89·6
51–60% ,, ...	142	5,703	91·9
61–70% ,, ...	96	5,799	93·5
71–80% ,, ...	106	5,905	95·2
81–90% ,, ...	86	5,991	96·6
91–99% ,, ...	83	6,074	97·9
100% ,, ...	130	6,204	100

If the two sets of crop enumeration made by the two different parties (*A* and *B*) were in complete agreement, then the entries would occur only in the diagonal cells and the coefficient of correlation between the two sets of records would be $+1$. This of course cannot happen in practice, as different investigators would have different "personal equations" of observation and estimation. In the table shown above it would be noticed that in 3,204 (or 51·6 per cent.) out of 6,204 grids, the two sets of records are in complete agreement. If agreement is defined to include a margin of variation up to 10 per cent. on either side then 4,273 (or 68·9 per cent.) of all grids are in agreement. In the same way about 83 per cent. of the grids would be in agreement if a latitude of comparison up to 30 per cent. is permitted. This is not unsatisfactory.

The agreement between two sets of records can also be expressed in the form of a coefficient of correlation of $+0{\cdot}739$. Owing to the cancellation of positive and negative errors the two mean values $p(A) = 52{\cdot}0$ per cent. and $p(B) = 51{\cdot}9$ per cent. are in entirely satisfactory agreement. Duplicated readings thus show two things—namely, (*a*) detailed agreement, plot by plot or grid by grid, can never be attained in practice even when the field work is done with reasonable care:

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and yet, (*b*) the agreement between mean values may be quite satisfactory so that reliable sample estimates can be made with confidence.

Bias in the estimation of crop yields

It may be mentioned that in crop-cutting work the principle of random sampling was explicitly recognized for the first time in the classical experiments of J. (now Sir John) Hubback of the I.C.S. in India between 1923 and 1925. He used sampling units of a very small size, 1/3,200 acre, in the form of a triangular cut obtained by a special implement devised by him. He was the first person to point out that owing to high correlation between yields in adjoining parts of the same field there was not much gain in precision by using sample units of a large size. He further emphasized the convenience and economy of using small cuts, of which a much larger number can be collected in the same time by investigators moving from one place to another. He gave a general account of his work in a paper on "Sampling for Rice Yield in Bihar and Orissa" published by the Government of India in 1927 as Bulletin No. 166 of the Imperial Agricultural Institute, Pusa. C. D. (now Sir Chintaman) Deshmukh, of the I.C.S., used Hubback's method a little later between 1928 and 1931 in his crop-cutting work on rice conducted together with settlement operations in two districts in the Central Provinces. About the same time, P. S. Rau, I.C.S., used the same method in two other districts of the same province. I succeeded recently in recovering the original material collected by C. D. Deshmukh, but all papers relating to P. S. Rau's work appear to have been destroyed. As Hubback's paper has been out of print for a long time, it has been reprinted in *Sankhyā* (the Indian Journal of Statistics), Vol. 7, Part 3, pp. 281–94, and in the same number I have given a brief account of Deshmukh's work. In 1938–39 H. P. V. Townend (also of the I.C.S.) used cuts of a comparatively small sampling unit (27·04 sq. ft.) in the form of a square, harvested with the help of a rigid frame, in his work on rice in certain districts in Bengal. It is interesting to observe that four administrators belonging to the I.C.S. used cuts of a small size evidently because they were guided by reasons of convenience and economy.

It is worth mentioning that R. A. Fisher's crop-cutting work on wheat at Rothamsted was influenced by that of Hubback, as stated by Fisher in a memorandum on crop estimating surveys submitted by him to the Imperial Council of Agricultural Research in India on March 2nd, 1945. Subsequently, Yates and others used sample cuts of a small size.

When we first started work on the estimation of crop yields in 1939, we naturally thought of using cuts of a small size. At the same time I thought it advisable to investigate whether the size of the sample unit had any influence on the results. It was necessary at this stage to distinguish between two different possible effects. If the size of the sample unit is appreciable in comparison with the size of the field, then there will be obviously a chance of over-sampling the central area of the field. I have discussed such over-sampling elsewhere (in the paper "On Large-Scale Sample Surveys," *Phil. Trans.* Vol. 231(B), No. 584, Appendix 6, p. 404). There is a second possibility of bias arising from bordering plants being included within the sample cut. It was this second effect which I had in mind in undertaking an experimental study of the problem.

In our very first series of crop-cutting experiments on jute in Bengal in 1939 we collected some material with sampling units of five sizes ranging between 5·5 sq. ft. and 66 sq. ft. There was some evidence of over-estimation with small cuts, but the material was meagre. In 1940 we used five different sizes of sample cut. A square of size 16 ft. by 16 ft. was located at random in each field (which also was selected at random) and the crop was harvested in the form of a number of separate sub-cuts supplying yield rates for sizes 1 ft. × 1 ft., 3 ft. × 3 ft., 12 ft. × 4 ft., 12 ft. × 12 ft., and 16 ft. × 16 ft. Cuts of different sizes were intentionally obtained from the same spot so as to secure effective local control for purposes of comparison. Information was collected for many items, such as number of plants per acre, the weight of green plants immediately after harvesting, and also the final yield of dry fibre per acre (after the plants were retted, dried, and the fibre extracted). I am giving in Table 9 figures for one particular item—namely, the yield of jute expressed as the weight of green plants (immediately after harvesting) in pounds per acre collected from four different districts in Bengal in 1940.

Yield rates based on 1 ft. × 1 ft. (1 sq. ft.) sample cuts were clearly over-estimates. Such extremely small sizes were therefore omitted in subsequent years. The next larger size 3 ft. × 3 ft. (9 sq. ft.) also appeared to give over-estimates, but this size was retained for purposes of comparison. Since 1940 similar material has been collected year by year, for a number of crops like jute, wheat,

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TABLE 9

Bengal Crop Survey, 1940–41. *Yield of jute (green plant) in lbs. per acre*

	Size of cut (sq. ft.)	Dacca (n = 80)	Mymensingh (n = 80)	Rangpore (n = 94)	Tipperah (n = 66)	Combined (n = 320)
	(1)	(2)	(3)	(4)	(5)	(6)
1	(1 × 1)	36,603 ± 2,288	24,893 ± 2,535	23,453 ± 1,860	24,292 ± 2,370	27,271 ± 1,160
9	(3 × 3)	20,466 ± 1,317	15,462 ± 872	15,800 ± 749	18,598 ± 1,259	17,462 ± 535
48	(12 × 4)	10,188 ± 576	20,145 ± 774	16,293 ± 650	19,997 ± 848	16,080 ± 411
144	(12 × 12)	9,784 ± 494	19,305 ± 905	18,038 ± 617	20,309 ± 856	16,763 ± 428
256	(16 × 16)	9,332 ± 593	18,836 ± 774	17,552 ± 617	20,433 ± 790	16,828 ± 395

aus (monsoon), and *aman* (winter) rice in different provinces of India. An account of the first series of experiments on jute in 1940 was given in a report submitted to the Indian Central Jute Committee, which had financed the work. The report was printed by this Committee for official use, but was not released to the public owing to war-time restrictions. Other reports on crop-cutting work were submitted by us in subsequent years, but the Government restricted these also for official use. Although a good deal of material has been accumulated, it has not yet been possible to publish a comprehensive account.* I am giving a few typical results in Table 10.

TABLE 10

Crop Survey. Percentage yield rates based on sample cuts of different sizes

Size of cut (sq. ft.)	Bengal jute, '40 320	Bengal jute, '41 185	U.P. wheat, '41 178	U.P. wheat, '42 346	Bengal rice, 1943–44 40	Average index (unweighted)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
9	103·8	116·1	121·4	118·1	113·3	114·7
18	—	—	111·6	109·4	—	110·5
25, 27	—	100·7	—	109·0	—	105·0
36	—	—	100·1	99·0	112·1	103·7
48, 49	95·5	95·3	—	—	—	95·4
54	—	—	—	96·0	—	96·0
64	—	105·9	—	—	—	—
81	—	—	—	93·6	—	—
135	—	—	99·9	—	—	—
144	99·6	96·4	—	—	101·2	—
225	—	*100·0*	97·4	—	—	—
256	*100·0*	—	—	—	—	—
324	—	—	—	*100·0*	—	—
576	—	—	*100·0*	—	*100·0*	—

In this table the results have been expressed as percentages of the yield rate based on sample cuts of the largest size used in different series of experiments. On the whole, the above table indicates that the bias decreases as the size of the cut is increased from 9 sq. ft. to probably something of the order of 40 or 50 sq. ft., and becomes negligible with cuts of larger size.

The results given in Table 10 all refer to sample cuts which were located in the field with pegs and ropes. Experiments were also made with rigid and semi-rigid frames made of wood or of wood and iron I have no time here to discuss these results, beyond stating that there was evidence of over-estimation with small cuts in such cases also.

I may mention at this stage that in the paper "On Large-Scale Sample Surveys" written at the end of 1942 I referred to the over-estimation arising from the use of sample cuts of small sizes, and explicitly stated (*Phil. Trans.*, Vol. 231(B), No. 584, p. 409):

> "It was found that there was persistent over-estimation in working with units of very small size. In the case of field survey the obvious explanation is that the investigator has a tendency to include rather than to exclude plants or land which stand near the boundary line or perimeter of the grid. This boundary effect naturally becomes less and less important as the size of the

* A few observations have been made in *Sankhyā*, Vol. 7, part 3, April 1946, pp. 269–80.

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grid is increased. In crop-cutting work on jute it was found, for example, that mean values for all the characters studied (such as number of plants per acre, weight of green plants, weight of dry fibre) were much higher for sample units of small size, so that it was not at all safe to work with cuts of a size less than say 25 sq. ft."

At that time the idea I had in mind was that the unconscious pulling in of bordering plants was probably effective so long as the linear dimensions of the sample cuts were smaller than the human span.* In 1944 Jitendra Mohan Sen Gupta of the Statistical Laboratory suggested the use of circular-shaped cuts to be harvested with the help of an arm rotating over a pivot with a light stylus attached at the end of the rotating arm to catch the plants. This proved very convenient in practice, and various improvements were made. The latest model is now a standard implement in our work. It is the usual practice at the present time to take three (or four) concentric circular cuts on the same field. This is done by drawing out the rotating (radius) arm to three (or four) fixed lengths one after another, and harvesting the appropriate cuts. In this way yield rates are obtained for three or four different sizes.

TABLE 11

Bengal Crop Survey, 1945–46. *Yield in lb. per acre and index-number of yields of* aman *rice (not in husk) for circular cuts of different sizes*

Serial no.	Name of district	No. of cuts	Yield in lb. per acre			Index-number of yield with yield based on largest size of cut as 100	
			12·57 sq. ft.	50·27 sq. ft.	100·88 sq. ft.	12·57 sq. ft.	50·27 sq. ft.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	Bakarganj	180	961·1	880·5	891·2	107·8	98·8
2	Bankura	109	750·4	686·3	723·3	103·8	94·9
3	Birbhum	87	947·1	864·0	883·8	107·2	97·8
4	Bogra	76	728·2	639·4	637·7	114·2	100·3
5	Burdwan	122	801·5	724·9	723·3	110·8	100·2
6	Chittagong	25	1,156·1	935·6	924·9	125·0	101·1
7	Dacca	113	922·4	734·0	758·7	121·6	96·7
8	Dinajpur	218	933·1	819·6	822·0	113·5	99·7
9	Faridpur	36	1,036·0	655·8	597·4	173·4	109·8
10	Hoogly	45	706·8	613·0	616·3	114·7	99·5
11	Howrah	38	604·8	420·5	401·6	150·6	104·7
12	Jalpaiguri	60	655·8	578·5	579·3	113·2	99·9
13	Jessore	86	723·3	613·8	621·3	116·4	98·8
14	Khulna	91	832·7	741·4	750·4	111·0	98·8
15	Malda	73	799·8	738·9	749·6	106·7	98·6
16	Midnapur	249	767·7	703·5	700·2	109·6	100·5
17	Murshidabad	55	870·6	785·8	777·6	112·0	101·1
18	Mymensingh	263	766·9	671·4	679·7	112·8	98·8
19	Nadia	38	680·5	598·2	613·8	110·9	97·4
20	Noakhali	58	789·9	701·1	669·0	118·1	104·8
21	Pabna	55	512·6	431·2	429·5	119·3	100·4
22	Rajshahi	83	805·6	687·1	665·7	121·0	103·2
23	Rangpur	204	794·1	715·9	715·9	110·9	100·0
24	Tipparah	92	704·4	598·2	610·6	115·4	98·0
25	24-Parganas	113	706·0	602·3	618·0	114·2	97·5
	Total	2,569	812·2	704·4	706·8	—	—
	Percentage	—	114·9	99·7	100	117·3	100·1
				weighted		unweighted	

Illustrative results of certain recent experiments are given in Table 11. Two things are clear from the above table. Circular cuts of 12·57 sq. ft. are still over-estimating when the work is done

* Dr. P. V. Sukhatme in a letter to *Nature* of May 11th, 1946, has published some material showing the over-estimation of crop yields with cuts of a small size. He writes: "The reason for over-estimation appears to be the human tendency to include border plants inside the plot. This factor becomes serious when the perimeter of the plot is large in proportion to its area." This is exactly what I had suggested in the paper written in 1942. Dr. Sukhatme has not referred to my paper, or is not aware of the observations made by me much earlier. I have recently sent a letter to *Nature* on this subject.

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on an extensive scale by a large field staff scattered over the whole province of Bengal, comprising about 60,000 or 70,000 square miles. Secondly, it is equally clear that there is practically no bias when the size of the cut is increased to 50·27 sq. ft., which is in good agreement with previous observations.

Along with extensive experiments carried out by the ordinary field staff, arrangements were also made to repeat the work at a few selected centres by experienced investigators working under the direct supervision of trained statisticians. Illustrative results are given below in Table 12, which shows the yield in lbs. per acre of *aman* (winter) rice at three centres in Bengal in 1945–46. The numerical values show that the systematic over-estimation with cuts of 2 ft. radius is no longer appreciable. This is confirmed by Student's paired "*t*" values of which are given at the bottom of the table.

TABLE 12

Bengal Crop Survey, Aman (*winter*) *rice*, 1945–46. *Yield in lb. per acre for different sizes of cuts*

Symbol	Size of sample cut in sq. feet	Mean yield in lb. per acre with s.e.		
		Katwa (n = 124)	Gouripur (n = 64)	Sainthia (n = 48)
x_1	$\pi(2')^2$ = 12·57	1,261 ± 33	726 ± 25	1,593 ± 46
x_2	$\pi(4')^2$ = 50·27	1,283 ± 26	747 ± 20	1,518 ± 31
x_3	$\pi(5\frac{5}{8}')^2$ = 100·88	1,304 ± 26	761 ± 19	1,523 ± 28
x_4	$\pi(8')^2$ = 201·06	1,340 ± 29	764 ± 18	1,495 ± 24
		Values of Student's paired "*t*"		
$x_1 - x_2$	—	1·53	1·33	2·18 *
$x_2 - x_3$	—	2·55 *	1·81	0·43
$x_3 - x_4$	—	3·07 †	0·63	2·47 *

* Significant at 5 per cent. level. † Significant at 1 per cent. level.

We thus find that when the work is done under adequate statistical supervision, the over-estimation with cuts of a small size becomes practically negligible. In view of this finding, I believe that unconscious pulling in of bordering plants may not be the only, or even the most important factor in this matter. In fact, I am now inclined to adopt a line of explanation offered by F. Yates in his paper on bias in sampling in the *Annals of Eugenics* in 1936. Discussing the observed bias in crop yields harvested from within hoops (of area 10 sq. ft.) supposed to have been thrown at random on fields in the United Provinces, Yates suggested:

> "The bulk of the bias, however, is probably due to the tendency, conscious or unconscious, to cast the hoop on the good parts of the crop."

It is likely that there are patches of greater fertility distributed either in a random manner or in a mildly patterned fashion over each field. In locating the sample cut, the investigator may unconsciously tend to favour such more fertile patches by shifting about the "random point" to some extent. From the fact that the bias appears to become negligible beyond 40 or 50 sq. ft., it would seem, without entering into detailed investigations, that the average size of such patches would be probably fairly small and of the order of only a few square feet. Unconscious pulling in of bordering plants may also be a contributing factor.

Another point is perhaps just worth mentioning. In case there is any bias in favour of locating the sample cuts on or near more fertile patches, then one would expect over-estimation with the smaller cuts to be followed by compensating under-estimation in cuts of intermediate size. Some of the figures of Table 10 may perhaps be indicative of this, but more precise and detailed experimentation is necessary to settle this point.

A certain amount of loss of grain in the actual process of harvesting, threshing, drying, weighing, etc., is inevitable which will necessarily introduce a component of "error." The proportional error arising from this factor may be independent of or may increase or decrease with the size of the cut. In case the loss of grain is proportionally smaller with small cuts (owing to the greater care with which the different processes can be carried out), then there might be an apparent over-estimation with small cuts, which, however, would really indicate an under-estimation with large cuts.

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It is pertinent to observe in the present connection the danger of under-estimation in using cuts of a comparatively large size which are demarcated by pegs and ropes on the field. It is doubtful whether ordinary investigators can make accurate measurements of distance on the field. Any sagging of the ropes is bound to reduce the real size of the cut, and hence lead to under-estimation. There are serious difficulties in deciding the allowance to be made for the boundary ridges or demarcations between adjoining fields, a point which is of particular importance in a province like Bengal, where the average size of the field is very small and less than half an acre, so that the perimeter is large in comparison with the area of the field. In fact, it is difficult to define the "whole field" in an unambiguous manner; and until this is done the concept of total production must also remain to some extent vague. This subject clearly requires much further study.

In the meantime I may perhaps mention that the over-estimation of cuts of a very small size can itself be used as an excellent control in the following way. As already mentioned, yield rates are being obtained by the Statistical Institute for three (or four) concentric circular cuts of different sizes on the same plot. If the work is done honestly by ordinary investigators, then yield rates based on sample cuts of the smallest size would show over-estimation in comparison with results based on sample cuts of larger size. Results based on the two (or three) larger cuts should, however, be in good agreement. At the same time, the variance of the yield per acre should decrease with increasing sizes of sample cuts. Besides being extremely convenient and economical, concentric circular cuts thus enable a two-fold control (one over mean rates of yield and the other over variances) to be exercised at the point of collection of the primary material.

Bengal Labour Enquiry : Jagaddal 1941, 1942, 1945

As already mentioned, in socio-economic surveys special efforts have been made to study the personal equation or bias of the investigating staff. Here also the use of replicated sub-samples proved to be of great value. I shall give illustrative examples from certain enquiries carried out in 1941, 1942, and 1945 in an industrial area at Jagaddal, about 20 miles to the north of Calcutta. The Government of Bengal had asked us to survey family budgets, housing, and other economic conditions of factory workers in this area.

Design of the survey. The general approach can be best studied from the design of the enquiry of 1941. In order to ensure proper randomization, the addresses of working-class employees were obtained from the factories located in this area, on the basis of which a preliminary survey was made of the geographical distribution of working-class families at Jagaddal, which is a municipal area. The number of families residing in different groups of buildings was noted on large-scale maps. A sample check was also carried out, but no attempt was made to attain high precision in the preliminary survey, as this was not considered to be of great importance.

On the basis of the preliminary survey the area was divided into five blocks (which were of unequal size and irregular in shape and), which were demarcated on certain general considerations. For example, blocks nos. 1, 2, and 3 were located in the immediate vicinity of factories; block no. 4 was within the town, but was situated at a greater distance from the factories themselves; finally, block no. 5 covered the area outside municipal limits. For present purposes the first three blocks were most important; block no. 4 of moderate importance, while block no. 5 was included more or less as a control and for comparative purposes. The buildings from which the families were to be surveyed were picked out purely at random on maps.

The sample units within each block were divided into 5 equal sub-samples, each of which was independently random and covered the whole of the block. These five sub-samples thus constituted five independent networks of sample units within each block. Each such sub-sample was assigned to a different investigator. In this way information for each block was collected by five investigators, all of whom in consequence worked in all the blocks The number of families surveyed in 1941 and 1942 by the different investigators in different blocks are shown in Table 13.

Possibilities of analysis of variance. The interesting point is that the design of the survey makes it possible to carry out a Fisherian analysis of variance. For example, the 1941 and 1942 designs resemble in appearance Fisherian Latin squares. These are, however, not true Latin squares, but only analogues, as the sub-samples are not geographically distinct. In fact there is no intrinsic difference between the Latin square and the randomized block, so that an analysis of variance can be carried out for any two or more or all the blocks at the same time.

The unequal numbers in the different block-investigator cells introduce a difficulty. There are

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TABLE 13

Bengal Labour Enquiry—Jagaddal, 1941–42. Number of families surveyed by blocks and investigators

Blocks	Investigators					Total
	1	2	3	4	5	
(1)	(2)	(3)	(4)	(5)	(6)	(7)
			1941 survey			
1	33	33	33	31	27	157
2	36	36	31	31	34	168
3	37	36	45	34	29	181
4	20	18	20	23	18	99
5	7	9	8	6	7	37
Total	133	132	137	125	115	642
			1942 survey			
1	41	37	40	39	38	195
2	41	36	46	39	35	197
3	38	42	39	41	37	197
4	19	24	19	24	21	107
5	9	9	8	8	10	44
Total	148	148	152	151	141	740

three possibilities. If the first three blocks are selected the numbers are not very different, so that a simple analysis of variance on orthodox lines would give approximate results. A second possibility (for the same three blocks) is to equalize the number in each cell by rejecting an appropriate number of schedules at random from each cell. Finally, the method of fitting constants can of course be used, but would involve a great deal of computational labour.

The results of an analysis of variance (for the first three blocks) for the family budget enquiry at Jagaddal in 1942 are given for a few selected items in Table 14. The total number of household schedules was 589 with 588 degrees of freedom, which can be split up into two degrees of freedom for comparisons between the three blocks, four degrees of freedom for comparisons between five investigators, and eight degrees of freedom for the interaction making up altogether fourteen degrees of freedom for comparisons between sub-samples.

In the standard Fisherian analysis of variance it is usual to use the interaction as error to test the significance of difference between blocks, etc. In the present case a direct estimate of the error can be made from the variations " within block-investigators cells " (that is, within each portion of the sample for which the material is collected by the same investigator in the same geographical area) based on 574 degrees of freedom.

It is also possible to carry out a similar analysis after equalizing the frequencies in each cell by rejecting an appropriate number of schedules at random. This was done, and the results are given in Table 14 for convenience of comparison. The total number of schedules used for this purpose was 525, so that the total variance was based on 524 degrees of freedom. Deducting fourteen degrees of freedom for comparisons among the sub-samples, this left 510 degrees of freedom for estimating the variance within sub-samples.

The ratios of variance with unequal as well as equalized frequencies in each cell are shown in the lower half of Table 14. The results are broadly similar, but equalization of frequencies makes the comparison more stringent, as a number of ratios are significant here, but not in the analysis with unequal frequencies.

Several points are worth noting. Firstly, it will be noticed that, except in the case of monthly expenditure on cereals, the interaction (blocks × investigators) variances are not significant, showing that the Fisherian assumption of the interaction variance being about the same as the " true " within-cell variance is broadly confirmed.

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TABLE 14

Bengal Labour Enquiry—Jagaddal, 1942. *Analysis of variance*

(3 blocks × 5 investigators)

Sources of variation	D.f.	Age in years	Expenditure in rupees per month per capita			Consumption of cereals in lbs. per head per month
			Total	Food	Cereals	
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Values of variance						
(*a*) Actual (unequal) number of families per cell ($n = 589$)						
Between blocks	2	76·1	606·5	23·4	0·08	82·5
Between investigators	4	211·6	220·4	19·5	1·3	61·8
Blocks × investigators ...	8	134·1	114·1	11·2	1·9	212·4
Between sub-samples	14	147·9	214·8	15·3	1·5	151·1
Within sub-samples	574	112·7	172·9	10·2	0·6	97·3
Total	588	113·5	173·9	10·3	0·6	98·6
Mean value	—	38·84	19·16	7·80	3·05	35·59
Standard deviation	—	9·71	11·41	3·15	0·73	8·64
Coefficient of variation ...	—	25·0	59·5	40·4	23·9	23·8
(*b*) Equalized number of families per cell ($n = 525$)						
Between blocks	2	62·13	805·52	36·5	0·07	79·6
Between investigators	4	304·84	275·78	22·3	1·25	114·7
Blocks × investigators ...	8	78·47	129·38	9·6	1·47	152·8
Between sub-samples	14	140·81	267·80	16·8	1·21	132·0
Within sub-samples	510	127·74	168·12	9·9	0·49	100·3
Total	524	128·09	170·78	10·1	0·51	100·8
Mean value	—	39·31	20·86	8·15	3·09	35·96
Standard deviation	—	11·32	13·07	3·18	0·71	10·04
Coefficient of variation ...	—	28·7	62·6	39·0	22·9	27·9
Ratios of variance						
(*a*) Actual (unequal) number of families per cell ($n = 589$)						
Between blocks	2	0·68	3·51 *	2·29	0·13	0·85
Between investigators	4	1·88	1·27	1·91	2·17	0·63
Blocks × investigators ...	8	1·19	0·66	1·10	3·17 †	2·18
Between sub-samples	14	1·31	1·24	1·50	2·50 †	1·55
Within sub-samples	574	—	—	—	—	—
Total	588	—	—	—	—	—
(*b*) Equalized number of families per cell ($n = 525$)						
Between blocks	2	0·49	4·79 †	3·69 *	0·14	0·80
Between investigators	4	2·39 *	1·64	2·26	2·55 *	1·15
Blocks × investigators ...	8	0·61	0·77	0·98	3·00 †	1·53
Between sub-samples	14	1·10	1·59	1·70	2·47 †	1·32
Within sub-samples	510	—	—	—	—	—
Total	524	—	—	—	—	—

* Significant at 5 per cent. level. † Significant at 1 per cent. level.

The significant value itself was further analysed, block by block and investigator by investigator, by Janardan Poti and K. C. Cheriyan; and it was found that the abnormally high values were due to one single investigator in one particular block. This shows the possibilities of deeper analysis offered by replicated inter-penetrating samples.

Investigator differences were insignificant in two cases, and significant at 5 per cent. level in two cases. Personal equations had not been completely eliminated but their influence was not large.

Nagpur labour enquiry 1942–43. I am giving a second example from a labour enquiry carried out in Nagpur in 1942–43 by M. P. Shrivastava, at that time a worker of the Institute, under the

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technical guidance of the Indian Statistical Institute. In this case the design was arranged in the form of a randomized block of five zones and four investigators with practically the same number of family schedules—namely, fifty in each block-investigator cell. The analysis of variance is shown in Table 15.

TABLE 15

Nagpur Family Budget Enquiry, 1943. *Analysis of variance*

(5 zones × 4 investigators)

Sources of variation	D.f.	Total income	Monthly expenditure		
			Total	Food	Cereals
(1)	(2)	(3)	(4)	(5)	(6)
		(*a*) Values of variance			
Between zones	4	4,439·58	3,707·91	708·41	206·83
Between investigators	3	85·43	597·08	77·09	3·70
Zones × investigators	12	382·54	397·28	177·75	49·80
Between sub-samples	19	1,189·74	1,127·07	237·58	75·61
Within sub-samples	977	401·57	384·71	84·73	24·99
Total	996	424·67	398·87	88·33	25·95
Mean values (in rupees)	—	36·09	34·96	20·09	11·41
Standard deviations	—	18·60	16·12	8·53	4·96
Coefficients of variation	—	51·5	46·1	42·5	43·5
		(*b*) Ratios of variance			
Between zones	4	11·06 †	9·64 †	8·36 †	8·28 †
Between investigators	3	0·21	1·55	0·91	0·15
Zones × investigators	12	0·95	1·03	2·10 *	2·00 *
Between sub-samples	19	2·96 †	2·93 †	2·80 †	3·02 †
Within sub-samples	977	—	—	—	—
Total	996	—	—	—	—

* Significant at 5 per cent. level. † Significant at 1 per cent. level.

It will be noticed that in this case the sub-samples are all significantly differentiated. Block differences are also highly significant in each case.

It may be mentioned here that in the Nagpur enquiry the blocks had been deliberately demarcated on lines which could almost certainly be expected to show large differences in income and conditions of living. For example, one block consisted almost exclusively of employees of the Bengal Nagpur Railway Workshop, who certainly earned higher wages and had generally a higher standard of living. The other blocks also were differentiated in many ways.

It is, however, most satisfactory to find that investigator's differences were negligible in every case, showing that personal equations had been completely eliminated. It is only by an analysis of the present kind that one can be certain on this point.

Margin of error of Cost of Living Index. Similar studies are also possible in regard to the margin of error of the index-number of cost of living. For example, the C.L. index-number can be calculated separately for each block-investigator cell. Adopting 1941 consumption pattern as base and 1942 and 1945 prices, the results are given in Table 16. In this table the number of 1941 family schedules on which each index-number is based is given within brackets. From the values of the index in different cells it is possible to calculate the standard error of the marginal index-numbers separately for each block or for each investigator; and finally, in the same way, the standard error of the overall index number. These are shown in Table 16.

It will be noticed that the 1942 mean index is 122 with a standard error of about 0·4 (which is only about 0·3 per cent. of the mean value). The standard errors of marginal index numbers are quite small and usually less than 1.

The 1945 C.L. index has a standard error of 1·6 or about 0·5 per cent. of the mean index-number 273. Standard errors in 1945 were, however, much higher in comparison with standard errors in

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TABLE 16

Bengal Labour Enquiry. Jagaddal cost-of-living index 1942 *and* 1945, *with* 1941 *as base by blocks and investigators (with size of sample within brackets)*

Blocks	Investigators					Total	
	1	2	3	4	5	Size of sample in '41	Mean index with s.e.
(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)
				1942 cost-of-living index			
1	120 (33)	122 (33)	122 (33)	124 (31)	124 (27)	157	122 ± 0·67
2	122 (36)	120 (36)	121 (31)	124 (31)	118 (34)	168	121 ± 1·88
3	121 (37)	123 (36)	121 (45)	121 (34)	121 (29)	181	121 ± 0·36
4	119 (20)	122 (18)	120 (20)	123 (23)	121 (18)	99	121 ± 0·65
5	121 (7)	124 (9)	124 (8)	124 (6)	124 (7)	37	123 ± 0·53
n (1941)	(133)	(132)	(137)	(125)	(115)	642	—
Mean index	121 ± 0·46	122 ± 0·57	121 ± 0·41	123 ± 0·41	121 ± 1·04	—	122 ± 0·33
				1945 cost-of-living index			
1	270 (33)	280 (33)	275 (33)	271 (31)	279 (27)	157	276 ± 2·0
2	263 (36)	265 (36)	284 (31)	269 (31)	267 (34)	168	269 ± 3·6
3	264 (37)	274 (36)	278 (45)	272 (34)	269 (29)	181	272 ± 2·4
4	263 (20)	274 (18)	271 (20)	297 (23)	263 (18)	99	275 ± 6·2
5	260 (7)	279 (9)	296 (8)	274 (6)	284 (7)	37	279 ± 6·2
n (1941)	(133)	(132)	(137)	(125)	(115)	642	
Mean index	265 ± 1·38	273 ± 2·56	279 ± 2·69	276 ± 4·57	271 ± 2·87	—	273 ± 1·64

1942. This is partly accounted for by the fact that whereas price relatives had varied very little among themselves (roughly about 10 per cent. of their mean values) in 1942, price relatives had varied among themselves about 80 per cent. of their mean values in 1945.

Analysis of variance has been also carried out both for five blocks × five investigators and for three blocks × five investigators (for the three blocks in which the number of schedules are nearly equal). The results are shown in Table 17.

TABLE 17

Bengal Labour Enquiry—Jagaddal, 1941, 1942, 1945.
Analysis of variance: cost-of-living index for 1942 *and* 1945, *with* 1941 *as base*

Source of variation	5 blocks × 5 investigators			3 blocks × 5 investigators			Expected ratio	
	D.f.	Variance	Ratio	D.f.	Variance	Ratio	5%	1%
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
				1942 with 1941 as base				
Between blocks	4	5·25	2·40	2	2·60	0·80	4·46	8·65
Between investigators ...	4	4·50	2·12	4	2·06	0·63	3·84	7·01
Residual	16	2·12	—	8	3·27	—	—	—
Total	24	3·04	—	14	2·83	—	—	—
				1945 with 1941 as base				
Between blocks	4	59·75	0·81	2	37·80	1·54	4·46	8·65
Between investigators ...	4	194·00	2·64	4	69·00	2·81	3·84	7·01
Residual	16	73·44	—	8	24·55	—	—	—
Total	24	91·25	—	14	39·14	—	—	—

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It is seen that block and investigator differences were both insignificant for both 1942 and 1945. This is satisfactory and gives confidence in using the present index-numbers.

It is of course also possible to study the cost-of-living index for different groups of items separately in each block-investigator cell. Illustrative results are given in Table 18.

TABLE 18

Bengal Labour Enquiry—Jagaddal, 1941, 1942, 1945. *Cost-of-living index by block × investigator*

Block no.	Investigator no.	Food		Clothing		Fuel and light		Miscellaneous		Total	
		1942	1945	1942	1945	1942	1945	1942	1945	1942	1945
I	1	120	276	159	310	126	258	114	275	120	270
	2	121	279	159	304	129	285	117	293	122	280
	3	122	277	160	310	127	277	119	285	122	275
	4	125	280	156	301	130	289	120	271	124	271
	5	120	280	160	314	125	275	125	287	124	279
	Total	122	280	159	308	127	276	119	283	122	275
II	1	123	273	159	306	127	280	114	253	122	263
	2	121	270	158	304	126	289	114	262	120	265
	3	122	279	155	297	125	267	117	303	121	284
	4	125	272	156	305	127	283	120	271	124	269
	5	121	270	156	304	126	281	113	273	118	267
	Total	122	273	157	304	126	280	116	273	121	269
III	1	122	275	156	297	129	293	114	249	121	264
	2	127	277	161	304	124	281	114	279	123	274
	3	121	280	160	306	125	288	117	289	121	278
	4	121	281	153	298	125	283	115	266	121	272
	5	122	269	159	305	125	280	115	278	121	269
	Total	123	277	158	302	126	286	115	272	121	272
IV	1	119	272	153	277	126	280	114	256	119	263
	2	124	274	158	289	126	285	115	292	122	274
	3	121	274	157	242	131	303	114	290	120	271
	4	122	306	161	313	124	273	120	311	123	297
	5	124	275	154	302	123	274	112	253	121	263
	Total	122	281	157	293	126	283	115	283	121	275
V	1	120	266	159	304	128	259	118	281	121	260
	2	124	288	158	303	127	266	119	292	124	279
	3	119	273	162	312	125	266	123	340	124	296
	4	123	283	158	315	125	240	120	280	124	274
	5	125	280	163	309	131	276	114	315	124	284
	Total	122	277	160	308	127	264	119	308	123	279
I–V	1	121	274	157	303	127	278	114	258	121	265
	2	123	276	159	304	126	284	115	280	122	273
	3	121	278	159	296	126	282	118	293	121	279
	4	124	283	157	305	127	282	119	277	123	276
	5	123	276	158	307	126	278	117	278	121	271
	Total	122	277	158	303	126	280	117	276	122	273

N.B.—No change in "rent and tax."

There are many other possibilities of breakdowns. For example, the cost-of-living index can be studied by groups of items for different over-all size of households. Relative figures are given in Table 19, from which it would be noticed that the change in the cost-of-living index-numbers did not depend appreciably on the size of the household.

Similar detailed figures for the change in the cost of living in 1942 and 1945 for different families in different expenditure levels are given in Table 20. Here also the cost of living appears to have changed more or less in the same way in all expenditure levels. It must be remembered, however, that the enquiry was restricted to working-class families in which the consumption pattern was fairly homogeneous.

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TABLE 19

Bengal Labour Enquiry—Jagaddal, 1941, 1942, *and* 1945. *Cost-of-living index by size of family*

Size of family in 1941	No. of families in 1941	Food		Clothing		Fuel and light		Miscellaneous		Total	
		1942	1945	1942	1945	1942	1945	1942	1945	1942	1945
1	266	122	274	160	310	124	274	117	272	121	270
2	94	124	275	156	298	127	286	114	285	123	273
3	69	123	280	157	299	127	287	114	275	122	272
4	70	122	286	156	297	129	288	117	281	122	277
5	56	121	285	154	296	127	284	118	287	122	279
6	34	122	275	157	297	129	282	118	285	122	271
7	19	124	280	155	295	131	284	120	286	125	275
8	13	122	280	157	302	132	294	126	284	124	274
9	12	123	285	154	294	131	273	120	292	124	278
10	1	125	294	160	324	119	228	119	272	123	270
11	3	124	288	159	300	127	304	124	266	127	279
13	3	125	279	157	296	138	316	128	286	128	278
14	1	124	284	161	306	139	323	119	346	126	236
Total	641	122	277	158	305	126	280	117	276	122	273

N.B.—No change in " rent and tax."

TABLE 20

Bengal Labour Enquiry—Jagaddal, 1941, 1942, *and* 1945. *Cost-of-living index by expenditure levels in rupees per month per family*

Expenditure levels in 1941	No. of families in 1941	Food		Clothing		Fuel and light		Miscellaneous		Total	
		1942	1945	1942	1945	1942	1945	1942	1945	1942	1945
0– 9 ...	22	123	276	156	308	126	285	121	301	123	282
10–19 ...	189	122	275	158	306	126	201	114	281	121	268
20–29 ...	199	122	275	158	306	125	278	115	273	121	270
30–39 ...	107	123	277	157	301	126	279	117	268	122	269
40–49 ...	56	122	291	157	301	128	285	118	279	122	281
50–59 ...	30	122	281	161	305	129	286	123	276	124	273
60 & above	38	124	276	161	304	129	284	123	277	125	272
Total ...	641	122	277	158	305	126	280	117	276	122	273

N.B.—No change in " rent and tax."

Comparison of consumption in 1941 *and* 1945. As already mentioned, family budget enquiries were repeated in the same area in 1941 and also in 1942 and again in 1945. This enables a direct comparison being made of levels of consumption in different years. Somewhat similar material also happens to be available for Calcutta middle-class families in 1939 and in 1945. As prices and consumption patterns had not changed very much in Bengal between 1939 and 1941, the above two series can also be used to compare (at least approximately) the changes in patterns of consumption in working-class and middle-class families.

Relevant data are given in Table 21. The name of the item is given in col. 1, and the unit of measurement in col. 2. The over-all *per capita* consumptions in 1945 and 1941 for working-class families are given in cols. 3 and 4 respectively. The consumption in 1945 expressed as a percentage of the corresponding consumption in 1941 is given in col. 5; and the price ratios in col. 6. Similar data for *per capita* consumptions in 1945 and 1939 in middle-class families are given in cols. 7 and 8. The ratio of consumptions is given in col. 9, and price ratios in col. 10.

I should mention here that food began to be rationed in 1943. The ration of rice was somewhat restricted owing to shorter supply, and the price control was arranged in such a way as to make " wheat " and " wheat products " comparatively cheaper. Sugar was strictly rationed, and there was a partial rationing of oil.

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TABLE 21

Comparison of consumption in different years for working and middle-class families in Bengal

Item	Unit per head per year	Jagaddal working-class				Calcutta middle-classes			
		Consumption *per capita* per year		Index-number, 1945 : 1941		Consumption *per capita* per year		Index number, 1945 : 1939	
		1945	1941	Consumption	Prices	1945	1939	Consumption	Prices
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Rice and products ...	lbs.	214·0	321·8	66·5	273	221·4	224·7	98·5	308
Wheat and products ...	,,	133·3	113·6	117·4	187	85·6	74·1	115·5	258
All cereals	,,	345·6	435·3	79·4	—	306·9	298·7	102·8	—
Pulses	,,	68·3	70·0	97·6	366	34·5	32·1	107·6	317
Potatoes	,,	50·2	64·8	77·5	533	54·9	83·1	66·0	377
Milk and products ...	,,	18·3	43·2	42·4	300	77·8	174·5	44·6	333
Butter and ghee ...	,,	0·66	3·9	16·8	210	1·6	16·5	10·0	333
Oil	,,	20·5	15·2	134·7	304	19·1	23·0	83·0	235
Salt	,,	12·0	15·9	75·6	190	11·8	17·3	68·3	300
Sugar and gur ...	,,	26·4	15·0	176·0	185	27·9	48·6	57·5	172
Fish	,,	12·1	13·1	92·2	290	24·5	50·2	48·8	589
Meat	,,	8·0	12·0	66·6	343	7·7	11·5	67·0	360
Eggs	no.	—	—	—	—	11·8	42·4	28·0	562
Coal	lbs.	297·1	279·0	106·5	377	—	—	—	—
Fire wood	,,	302·0	405·7	74·4	330	—	—	—	—
Cloth (dhuti)	yds.	6·9	9·9	69·7	283	—	—	—	—
Cloth (lungi)	,,	0·6	0·5	120·0	416	—	—	—	—
Number of families ...	—	755	641	—	—	610	1,151	—	—
,, persons ...	—	2,313	1,866	—	—	4,335	10,539	—	—
,, persons per family	—	3·06	2·91	—	—	7·11	9·16	—	—

The consumption of all cereals had dropped very appreciably to about 79 per cent. in working-class families, although there was an increase in the consumption of wheat, owing no doubt to more favourable prices. In middle-class families the cereal consumption (which had been originally much lower in comparison with working-class families) had remained more or less steady, but there was a higher consumption of wheat. The consumption of sugar had increased a great deal (to 176 per cent.) among working-class families, but had decreased appreciably in middle-class families, due almost certainly to rationing. The consumption of oil had increased somewhat in working-class families, which was probably due to partial rationing and as an offset against the practical discontinuance of the use of higher-quality fats like butter or ghee (clarified butter). In practically all other items the consumption had decreased very seriously in 1945 in both working-class and middle-class families at Jagaddal and in Calcutta.

Margin of error of index-number of earnings and consumption. As information had been collected about total earnings of different families in 1941 and 1945, it is possible to calculate the average increase in earnings. The geographical blocks had been kept the same in both years. It is possible therefore to calculate average earnings for each block separately in 1941 and 1945, and hence to calculate the ratio of earnings for each individual block. Similar material is also available for a number of other items, like monthly expenditure, consumption of cereals, pulses, vegetables, vegetable oil, meat and fish, dairy products, and clothing. Index-numbers for 1945 with 1941 as base can be therefore calculated for all these items for each of the five geographical blocks in Jagaddal. These index-numbers are shown in Table 22. Similar figures were available for index-numbers for 1942 with 1941 as base, but these have not been printed here.

Table 22 supplies some idea of variations in the index-numbers from block to block. It is possible to form an unweighted average index for the whole area, and also to calculate the unweighted standard error from the figures for the different blocks. These unweighted average index numbers for 1942 on 1941 are shown in col. 3, and for 1945 on 1941 in col. 4 of Table 24.

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TABLE 22

Bengal Labour Enquiry—Jagaddal, 1945 *and* 1941. *Index-numbers for* 1945 *with* 1941 *as base* (= 100)

Items	(*a*) Per family basis					(*b*) *Per capita* basis				
	Block 1	Block 2	Block 3	Block 4	Block 5	Block 1	Block 2	Block 3	Block 4	Block 5
n (1941)	157	168	181	99	37	516	484	434	268	164
n (1945)	195	212	192	104	51	580	601	467	374	284
Harmonic mean	174	188	186	101	43	546	536	450	312	208
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Cost of living ...	275	269	272	275	279	275	269	272	275	279
Earnings	173	195	207	228	221	191	199	204	172	176
Expenditure	165	179	194	212	206	182	183	191	160	164
Cereals	78	80	80	107	81	86	82	79	81	65
Pulses	89	115	93	126	98	98	118	91	95	78
Vegetables	90	98	103	113	92	99	100	101	85	73
Vegetable oil ...	94	149	176	166	136	104	153	173	111	108
Meat and fish ...	71	78	104	95	72	79	80	102	72	57
Spices and salt ...	75	74	71	88	109	83	76	70	67	87
Dairy products ...	30	48	46	47	32	34	50	46	36	26
Clothing	73	73	83	87	92	81	75	81	66	73

Information regarding the size of the samples in 1941 and in 1945 in the different blocks is given together with their harmonic means at the top of each column in Table 22. Using such harmonic means as weights, a series of weighted averages with standard errors were calculated, and are shown in col. 5 of Table 24.

TABLE 23

Bengal Labour Enquiry—Jagaddal, 1941, 1942, *and* 1945. *Average earning and expenditure and quantity consumed per month*

Serial no.	Item	Unit of measurement per month	1941	1942	1945
(1)	(2)	(3)	(4)	(5)	(6)
	average size of family		2·91	2·79	3·06
	(*a*) Per family basis				
1	Earning	rupees	26·93 ± 0·92	36·38 ± 1·22	55·40 ± 5·16
2	Expenditure	,,	30·11 ± 1·16	35·57 ± 1·15	57·61 ± 5·38
3	Cereals	pounds	103·41 ± 5·70	98·08 ± 5·53	87·49 ± 7·63
4	Pulses	,,	15·47 ± 0·70	12·61 ± 0·56	15·93 ± 0·82
5	Vegetables	,,	32·32 ± 2·20	31·41 ± 1·52	31·56 ± 2·55
6	Vegetable oil	,,	3·56 ± 0·19	3·48 ± 0·19	5·10 ± 0·47
7	Meat and fish	,,	6·09 ± 0·35	5·90 ± 0·33	5·00 ± 0·33
8	Spices and salt	,,	5·74 ± 0·33	5·74 ± 0·29	4·96 ± 0·97
9	Dairy products	,,	13·31 ± 1·69	12·08 ± 1·46	5·10 ± 0·88
10	Clothing	yds. per year	60·48 ± 2·11	55·42 ± 2·52	49·77 ± 5·78
	(*b*) *Per capita* basis				
1	Earning	rupees	9·01 ± 0·42	13·32 ± 0·53	16·54 ± 1·09
2	Expenditure	,,	9·93 ± 0·35	12·08 ± 0·55	17·20 ± 1·15
3	Cereals	pounds	33·37 ± 0·74	31·76 ± 0·53	26·08 ± 1·48
4	Pulses	,,	5·16 ± 0·25	4·20 ± 0·16	4·98 ± 0·64
5	Vegetables	,,	10·45 ± 0·45	10·55 ± 0·56	9·46 ± 0·62
6	Vegetable oil	,,	1·15 ± 0·04	1·13 ± 0·02	1·52 ± 0·21
7	Meat and fish	,,	1·97 ± 0·02	2·04 ± 0·10	1·52 ± 0·14
8	Spices and salt	,,	1·85 ± 0·06	1·89 ± 0·06	1·40 ± 0·06
9	Dairy products	,,	4·05 ± 0·23	3·79 ± 0·33	1·50 ± 0·06
10	Clothing	yds. per year	19·92 ± 0·64	18·33 ± 0·58	14·70 ± 0·87

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Mean values of monthly earnings, expenditure, and consumption of various commodities for working-class families in Jagaddal in 1941, 1942, and 1945 are shown in Table 23. In this table figures have been given both on a "per family" and "*per capita*" basis, and the standard errors have been calculated from the block figures in 1945 and 1942. From this material, index-numbers can be prepared for 1945 with 1941 as base by dividing the mean values in 1945 by the corresponding mean values in 1941. One measure of the standard error of the index-number can also be obtained by compounding the standard error of the two mean values. These index numbers are shown in col. 6 of Table 24, in which the C.L. index-number has been also given at the top for convenience of reference.

TABLE 24

Bengal Labour Enquiry—Jagaddal, 1941, 1942, *and* 1945. *Index-numbers of earnings, expenditure, and consumption*

Serial no.	Item	1942 : 1941	1945 : 1941		
		Unweighted average of ratios	Unweighted average of ratios	Weighted average of ratios	Ratios of average
(1)	(2)	(3)	(4)	(5)	(6)
	Cost-of-living index	122 ± 0·33	273 ± 1·64	272·8 ± 1·3	—
	(*a*) Index-number on "per family" basis				
1	Earnings	137 ± 4·3	205 ± 9·8	199·1 ± 8·3	205·8 ± 20·50
2	Expenditure	121 ± 5·1	191 ± 8·6	186·0 ± 7·3	191·3 ± 19·35
3	Cereals	98 ± 6·1	85 ± 5·5	83·5 ± 4·4	84·6 ± 8·73
4	Pulses	86 ± 5·4	104 ± 7·0	103·1 ± 6·2	103·0 ± 7·09
5	Vegetables	105 ± 6·9	99 ± 4·1	99·2 ± 3·4	97·6 ± 10·33
6	Vegetable oil	101 ± 5·5	144 ± 4·3	144·1 ± 14·0	143·3 ± 15·28
7	Meat and fish	106 ± 7·0	84 ± 6·6	85·3 ± 6·1	82·1 ± 7·20
8	Spices and salt	105 ± 7·0	83 ± 7·0	77·7 ± 4·3	86·4 ± 17·61
9	Dairy products	100 ± 9·3	41 ± 4·0	41·8 ± 3·5	38·3 ± 8·37
10	Clothing	92 ± 5·8	82 ± 3·8	78·9 ± 2·8	82·3 ± 9·98
	(*b*) Index-number on "*per capita*" basis				
1	Earnings	141 ± 7·2	188 ± 6·3	191·5 ± 5·1	183·6 ± 14·83
2	Expenditure	122 ± 5·7	176 ± 6·0	179·1 ± 4·8	173·2 ± 13·09
3	Cereals	97 ± 3·0	79 ± 3·6	80·5 ± 3·4	78·2 ± 4·75
4	Pulses	85 ± 4·3	96 ± 6·5	99·2 ± 5·6	96·5 ± 13·22
5	Vegetables	106 ± 7·5	92 ± 5·5	94·9 ± 4·1	90·5 ± 7·15
6	Vegetable oil	98 ± 3·6	130 ± 14·0	133·4 ± 5·8	132·2 ± 18·52
7	Meat and fish	109 ± 8·1	78 ± 7·3	80·5 ± 6·5	77·2 ± 7·31
8	Spices and salt	105 ± 5·0	77 ± 3·8	76·3 ± 3·0	75·7 ± 4·14
9	Dairy products	100 ± 9·0	38 ± 4·3	40·3 ± 4·2	37·0 ± 2·58
10	Clothing	93 ± 3·4	75 ± 2·8	76·4 ± 2·2	73·8 ± 4·95

Index-numbers for 1945, calculated in three different ways, are given in cols. 4, 5, and 6 of Table 24. It will be noticed that it does not matter very much which particular method of calculation is followed, as the three index-numbers are in broad agreement.

It is of course possible to make a deeper analysis by taking into consideration the sampling errors within each block, which, however, would entail a great deal of computational labour. My purpose here is to draw attention to the possibility of making rough but quick comparisons by using the mean values for replicated sub-samples.

I have no desire to discuss the economic implications, but may just note that, with a cost-of-living index of 273 and an index-number of earnings of 205, the index-number of real wages was 75 in 1945. The index-numbers of consumption shown in col. 5 of Table 21 or cols. 5 or 6 of Table 24 are in broad agreement with this value. Consumption of essential articles had naturally remained more steady, while consumption of other commodities had decreased more appreciably. I am not competent to discuss the economic aspects of the subject. My object is to point out that even approximate values of the margin of error are likely to be useful in obtaining a more critical appreciation of the economic situation.

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I may mention here that in working-class family enquiries at Jagaddal arrangements had been made to keep a certain number of families common in the enquiries in different years. This enables a direct comparison of the change in the consumption pattern being made on a family or household basis. In fact the form in which the material has been collected makes it possible to undertake a critical study of such enquiries. Owing to lack of resources, we have not been able to take up this work seriously, but Ambika Ghosh and H. K. Chaturvedi are doing what they can.

TABLE 25

Bengal Sample Surveys (1936–42). *Estimated consumption of food items in lb. per head per year by expenditure levels*

Food items	Levels of family expenditure in rupees per month								All levels
	0–10	10–20	20–30	30–50	50–100	100–200	200–300	300 +	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Rice	310·22	371·93	370·28	360·41	342·31	278·95	257·55	262·49	341·48
Rice products	4·11	5·76	10·70	9·87	10·70	10·70	9·87	5·76	8·23
Wheat products	1·64	7·41	13·16	17·28	26·33	44·43	59·24	75·70	19·75
All cereals	315·97	385·10	394·14	387·56	379·34	334·08	326·66	343·95	369·46
Pulses	15·63	27·98	37·03	36·20	32·91	31·27	34·56	36·20	32·91
Salt	9·87	13·27	13·99	14·07	15·63	15·63	17·28	18·10	15·30
Potatoes	12·34	26·33	32·09	37·03	48·55	63·36	79·81	108·61	48·55
Vegetables	22·22	18·45	39·50	44·43	65·00	94·21	109·44	139·88	64·26
Oil	5·76	9·87	11·52	12·34	15·63	19·75	21·39	25·51	14·81
Sugar and gur	3·29	9·05	12·34	14·42	23·86	54·31	36·20	41·96	24·31
Fish	6·58	8·23	11·52	14·81	23·86	41·96	49·37	57·60	25·51
Meat	1·64	4·11	5·76	5·76	8·23	9·05	11·52	17·28	9·05
Eggs	0·13	0·16	0·25	0·70	2·01	3·28	4·84	6·04	2·74
Milk and milk products ...	5·76	10·70	23·86	49·37	83·93	145·77	167·86	236·98	84·75
Butter and ghee	0·53	0·82	1·64	2·47	5·76	10·70	13·16	23·86	6·58
No. of persons per family ...	3·67	4·52	4·66	5·68	7·19	9·12	11·52	15·36	5·29
Total no. of families ...	3,212	4,142	2,885	2,510	1,519	649	268	224	15,409
Total no. of persons ...	11,788	18,712	13,443	14,247	10,919	5,916	3,088	3,441	81,554

TABLE 26

Estimated index-numbers of per capita *consumption of various food items by expenditure level*

Food items	Units of measure-ment	Monthly expenditure in rupees							
		0–10	10–20	20–30	30–50	50–100	100–200	200–300	300 and above
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1. Rice	maunds	90·8	108·9	108·4	105·5	100·2	81·7	75·4	76·9
2. Rice products	,,	50·0	70·0	130·0	120·0	130·0	130·0	120·0	70·0
3. Atta flour, etc.	,,	8·3	37·5	66·7	87·5	133·3	225·0	300·0	383·3
4. All cereals	,,	85·5	104·2	106·7	104·9	102·7	90·4	88·4	93·1
5. Pulses	seers	47·5	85·0	112·5	110·0	100·0	95·0	105·0	110·0
6. Salt	,,	64·5	86·7	91·4	91·9	102·1	102·1	112·9	118·3
7. Potatoes	,,	25·4	54·2	66·1	76·3	100·0	130·5	164·4	223·7
8. Vegetables	,,	34·6	28·7	61·5	69·1	101·1	146·0	170·3	217·7
9. Oil	,,	38·9	66·7	77·8	83·3	105·6	133·3	144·4	172·2
10. Sugar and gur	,,	13·5	37·2	50·8	59·3	98·1	223·3	148·9	172·6
11. Fish	,,	25·8	32·3	45·2	58·1	93·6	164·5	193·6	225·8
12. Meat	,,	18·2	45·4	63·6	63·6	90·9	100·0	127·3	190·9
13. Eggs	number	4·9	5·7	13·1	25·5	73·2	119·8	177·0	220·6
14. Milk and milk products	seers	6·8	12·6	28·2	58·2	99·0	172·0	198·1	279·6
15. Butter and ghee ...	,,	8·1	12·5	25·0	37·5	87·5	162·5	200·0	362·5
Index no. of persons per family	—	69·4	85·4	88·1	107·4	135·9	172·4	217·8	290·4

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Per capita *consumption of food items by expenditure levels.* I may give another illustration of the use of the sample survey in investigating consumption patterns. A number of enquiries were undertaken in both rural and urban areas in Bengal between 1936 and 1942. Although economic conditions and prices had changed during this period, the actual magnitude of such changes was small in comparison with that which occurred under war conditions. It has therefore been considered permissible to prepare a composite table showing the *per capita* consumption of various food items at different expenditure levels (in rupees per month per family) based on the above enquiries. Relevant data are given in Table 25. The same table has been expressed in the form of index-numbers in Table 26 in which the over-all average consumption has been adopted as 100.

The consumption of all cereals is naturally fairly steady, but shows a slight falling off above the expenditure level of Rs.100 per month. Rice products and wheat are increasingly preferred with rising expenditure level in Bengal. Index-numbers for pulses and salt are also, on the whole, fairly steady. The consumption of all other items increased steadily with rising expenditure level. The disparity in consumption is very high in sugar, fish and meat, and still higher in the consumption of eggs, milk and milk products, and butter and ghee (clarified butter). Another point is worth noting—namely, the steady increase in the size of the household with rising economic status.

Radio Programme Preference Survey

I shall now give an example of what is usually called "listener research" in the United Kingdom. This work was undertaken at the request of the Government of India. The object was to ascertain public reactions to the broadcast of war programmes from the All-India Radio in the earlier part of 1941. It was apprehended, however, that any direct enquiry would meet with considerable psychological resistance. In this situation the Indian Statistical Institute undertook to organize a comprehensive public preference survey with a broad coverage, including reactions to war broadcasts.

As regards the design of the survey, it may be briefly mentioned that the sample was picked out of households possessing radio licences, of which a list was supplied by the Government. The households were selected on the basis of pure space randomization, and arranged in the form of several inter-penetrating replicates. Information for each replicate or sub-sample was collected by an independent set of investigators. The field work was carried out in April–May 1941, and a comprehensive report was submitted to the Government in July 1941, but it was not published owing to wartime restrictions.

Four tables are reproduced below to give a general idea of the approach. Table 27 shows the frequency (in the form of percentages) of listening "often" to different items of the radio programme broken down by age, educational, and occupational groups. The size of the sample in each group is shown at the head of each column.

The nature of the fluctuations in different age-groups is of considerable interest, and is on the whole in keeping with what may be called common-sense expectations. For example, "war news" had practically the highest preference in almost every group, which is just what might have been expected. "News talk" and "foreign news" were also, on the whole, generally popular, as these were also mostly concerned with the war.

In the music and entertainment section it was found that "modern Bengali" and "(Rabindranath) Tagore songs," as well as "plays" and "instrumental music," were, on the whole, highly popular, with "devotional music" holding quite a good place; "classical music" was, however, distinctly less popular. There were also interesting age-variations in this group. Modern Bengali and Tagore songs, plays, and instrumental music were far more popular among the younger people, and showed a definite and considerable decrease in popularity with increasing age. Devotional music, on the other hand, showed a sharp contrast and increasing popularity with age.

There are many other interesting points which, however, need not detain us. It is the sampling errors to which I should like to draw attention. The values for three independent but inter-penetrating samples or replicates are shown in Table 28. The variance (calculated from three replicates—that is, on two degrees of freedom) for the different items is shown in col. 6. The "within binomial variance" for the whole material is given in col. 7; and the ratio of variances in col. 8. The population is not Gauss-Laplacian, and strictly speaking Fisher's F cannot be used to test the significance. The use of F may, however, give approximate results, and significance has been shown in the usual manner.

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TABLE 27

Calcutta radio programme preferences—April–May, 1941. *Frequency of listening "often"*

Item of radio programme	All groups	By age-groups (in years)					By educational groups			By occupational groups					
		14–18	19–25	26–35	36–55	55–	Non-matrics	Under-graduates	Graduates	Student	Petty trade	Service	Business	Profession	Landlords
	$n = 803$	$n = 24$	$n = 170$	$n = 278$	$n = 298$	$n = 33$	$n = 156$	$n = 354$	$n = 293$	$n = 149$	$n = 11$	$n = 300$	$n = 184$	$n = 95$	$n = 64$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
WAR															
1. War news	54·7	54·2	52·9	60·1	49·9	60·6	50·3	58·5	52·2	54·0	54·5	54·0	54·9	54·7	57·8
2. News talk	50·8	45·8	52·4	52·2	47·9	57·6	46·5	54·2	48·8	46·0	54·5	49·0	52·2	58·9	54·7
3. Foreign news ...	43·5	41·7	44·1	46·4	39·3	54·5	35·1	46·6	45·1	46·7	27·3	41·3	41·8	49·5	43·8
MUSIC															
4. Modern Bengali ...	46·0	70·8	59·9	47·1	37·9	18·2	45·9	49·4	42·0	58·7	54·5	41·0	44·6	40·0	51·6
5. Tagore	40·2	66·6	49·4	39·6	35·6	21·2	30·6	44·4	40·3	49·3	27·3	38·3	37·0	42·0	35·9
6. Plays	42·5	54·2	51·2	37·4	41·9	36·4	51·6	51·1	33·1	48·7	45·5	42·0	45·7	38·9	53·1
7. Instrumental ...	42·0	50·0	51·2	43·5	36·2	27·3	41·4	46·6	36·5	46·7	54·5	39·7	40·2	41·1	45·3
8. Devotional	35·1	25·0	30·0	32·4	39·3	51·5	43·3	36·4	29·0	30·0	27·3	35·3	35·3	36·8	43·8
9. Classical	23·3	16·7	29·9	24·8	19·8	12·1	27·4	23·2	21·2	26·6	27·3	22·0	23·4	18·9	26·6
TALKS															
10. Humorous	29·8	29·2	28·8	32·4	28·2	27·3	29·3	31·6	27·6	26·7	36·4	30·7	28·3	26·3	40·6
11. Scientific	26·2	37·5	27·1	28·1	22·5	30·3	20·4	24·9	30·4	28·0	9·1	27·9	21·7	27·4	26·6
12. Literary	18·7	20·8	21·2	15·8	18·5	30·3	8·9	25·7	21·5	24·7	9·1	22·7	14·7	21·1	23·4
13. Educational	24·9	33·3	26·5	25·2	22·5	30·3	17·8	30·2	24·6	37·3	27·3	26·3	16·8	23·2	25·0
14. Music lessons ...	20·7	33·3	37·6	15·1	15·8	15·2	21·7	24·9	15·0	34·7	36·4	18·3	17·4	11·6	18·8
MISCELLANEOUS															
15. Children's	16·1	37·5	21·8	13·3	13·4	18·2	16·6	18·9	12·3	26·0	9·1	11·7	15·8	13·7	18·8
16. Women's	14·7	20·8	17·6	12·2	13·8	24·2	19·1	16·7	9·9	22·7	9·1	10·0	14·1	12·6	23·4
17. Folk music	16·9	25·0	17·0	16·9	16·1	18·2	15·3	18·9	15·4	14·7	0	17·0	20·6	14·7	17·2
18. Rural	12·0	12·5	18·2	11·2	8·1	21·2	10·8	12·4	11·9	16·0	9·1	11·3	12·5	7·4	10·9

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TABLE 28

Calcutta radio programme preferences—April–May, 1941. Percentages of persons "listening often" to different items of programme

Calcutta radio programme item early 1941	Samples (replicates)			Pooled values with s.e. based on replicates $n = 803$	Variance		Ratios of variance
	1	2	3		Between samples d.f. = 2	Within samples d.f. = 800	
	$n = 287$	$n = 268$	$n = 248$				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
WAR							
War news	55·8	50·7	57·7	54·7 ± 2·52	3,340	2,479	1·35
News talk	54·0	42·9	55·6	50·8 ± 3·99	12,728	2,477	5·14 †
Foreign news	45·3	40·3	44·8	43·5 ± 1·59	2,032	2,461	0·83
MUSIC							
Modern Bengali	48·8	39·6	49·6	46·0 ± 3·38	8,284	2,472	3·35 *
Tagore	39·4	35·1	46·8	40·4 ± 3·42	8,978	2,391	3·75 *
Plays	43·6	41·4	42·3	42·4 ± 0·67	319	2,452	0·13
Instrumental	42·2	40·7	43·1	42·0 ± 0·70	402	2,444	0·16
Devotional	31·0	28·8	46·8	35·5 ± 5·67	24,734	2,225	11·12 †
Classical	26·1	20·1	23·4	23·2 ± 1·74	2,506	1,794	1·40
TALKS							
Humorous	28·9	28·0	32·7	29·9 ± 1·44	1,567	2,094	0·75
Scientific	27·9	21·3	29·4	26·2 ± 2·49	4,958	1,926	2·57
Literary	24·4	16·0	14·9	18·4 ± 3·00	7,364	1,506	4·89 †
Educational	23·7	20·9	30·6	25·1 ± 2·88	6,450	1,861	3·47 *
Music lessons	18·1	19·8	24·6	20·8 ± 1·95	2,953	1,639	1·80
MISCELLANEOUS							
Children's programme ...	14·3	14·9	19·4	16·2 ± 1·61	1,970	1,349	1·46
Women's	12·9	9·3	22·6	14·9 ± 3·97	12,036	1,228	9·80 †
Folk music	14·3	16·0	21·0	17·1 ± 2·01	3,130	1,404	2·23
Rural programme ...	13·2	10·1	12·5	11·9 ± 0·94	748	1,055	0·71

* Significant at 5 per cent. level. † Significant at 1 per cent. level.

The important point to note is that, except in three cases, errors calculated from the replicated samples are greater than the theoretical binomial errors. This shows the need for caution.

I am giving below a table to indicate the practical use of such investigations. In this table the index-numbers of frequency of listening are shown separately for men and women in cols. 2.1 and 2.2 respectively. Index-numbers of demand (as expressed by the desire of persons interviewed to have more time given to particular items) are shown in cols. 3.1 and 3.2. Pooled index-numbers of preference were also calculated by taking the average of the two previous index-numbers, and are shown in cols. 4.1 and 4.2. Index-numbers of broadcast time actually allotted to various items at the time of the survey are shown in col. 5. Dividing the pooled index of preference in cols. 4.1 and 4.2 by the index of broadcast time we get index-numbers of disparity given in cols. 6.1 and 6.2 for men and women separately.

These index-numbers of disparity indicate to what items more time should be given. The disparity index cannot, of course, be treated as a quantitative measure of the time likely to be required for satisfaction. For example, the relative demand for talks is very high, but it is possible that a comparatively small increase would enable the satisfaction point to be reached. This merely means that the index of disparity has to be re-estimated as conditions change.

Finally, in Table 30 I give the reactions to war broadcasts. It is quite clear that early in 1941 enemy broadcasts were found more interesting as well as more convincing as compared to All-India Radio propaganda. The margin of error of the results (calculated from replicated samples) was reasonably small. In any case, the possibility of estimating the margin of error (by using independent net-works of samples) is clearly a great gain.

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TABLE 29

Radio programme preference—Calcutta, April–May, 1941. *Index-numbers of preference, broadcast time and disparity by sexes*

Name of programme item	Index-numbers of								
	Listening		Demand		Preference (pooled)		Broadcast time	Disparity	
	Men	Women	Men	Women	Men	Women		Men	Women
(1)	(2.1)	(2.2)	(3.1)	(3.2)	(4.1)	(4.2)	(5)	(6.1)	(6.2)
WAR									
War news	299	186	148	47	224	116	154	145	75
News talk	155	92	116	27	136	60	24	567	250
MUSIC									
Modern Bengali	140	148	207	210	174	179	262	66	68
Tagore	123	128	184	236	154	182	47	328	387
Plays	130	188	172	236	151	212	125	121	170
Instrumental	128	114	175	176	151	144	134	113	107
Devotional	107	114	142	155	124	134	219	57	61
Classical	71	87	47	54	59	90	208	28	43
TALKS									
Humorous	91	94	104	108	98	101	8	1,225	1,262
Scientific	80	27	102	34	91	30	4	2,275	750
Literary	57	47	74	27	66	37	3	2,200	1,233
Educational	76	67	90	68	83	68	20	415	340
Music lesson	63	101	28	94	46	98	65	71	151
MISCELLANEOUS									
Children's	49	74	45	33	47	54	174	27	31
Women's	45	136	38	203	42	170	73	58	233
Folk music	57	40	20	7	38	24	49	78	49
Rural	37	54	9	13	23	20	131	18	15

TABLE 30

War broadcast reactions: Calcutta sample, April–May, 1941.

Source of broadcast	Percentage of persons finding the broadcast								
	Interesting			Convincing			Effective propaganda		
	n	mean	s.e.	*n*	mean	s.e.	*n*	mean	s.e.
(1)	(2)	(3)		(4)	(5)		(6)	(7)	
Enemy	139	36·1	2·02	133	34·5	2·18	156	40·5	2·60
Allied (mostly A.I.R.) ...	56	14·5	1·11	41	10·6	1·10	57	14·8	1·30
Neutral (incl. U.S.S.R.) ...	34	8·8	0·74	50	13·0	1·15	19	4·9	0·62
No opinion	156	40·5	1·74	161	41·8	2·77	153	39·7	2·72
Total	385	99·9%		385	99·9%		385	99·9%	

N.B.—Standard errors based on eighteen sub-samples.

Calcutta Public Preference Survey, 1941

I shall give another example of a different type, something like what is called public preference or "Gallup" polls in the west. The survey was carried out in certain sections of Calcutta middle-class families along with the broadcast reaction survey. The schedule covered a large number of items including preferences for classical and modern literature in both English and Bengali,

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different kinds of games, tobacco, *pan* (betel leaves), etc., and a number of social and political questions. A few tables are given here for purposes of illustration.

TABLE 31

Calcutta public preference survey, 1941. *Possession of horoscopes*

Groups	Number of persons			Difference between observed and expected	χ^2
	Total	Having horoscopes			
		Observed	Expected		
(1)	(2)	(3)	(4)	(5)	(6)
Total sample	1,470	919	$p = 0{\cdot}625$		
(*a*) By age group					
Below 18 years	38	23	23·8	− 0·8	0·07
19–25 ,, ...	259	145	161·9	−16·9	4·70
26–35 ,, ...	486	297	303·8	− 6·8	0·41
36–55 ,, ...	601	405	375·6	29·4	6·14
Above 55 ,, ...	86	49	53·8	− 4·8	1·14
$P(\chi^2) = 0{\cdot}015$, d.f. $= 4$, $\chi^2 = 12{\cdot}46$					
(*b*) By educational groups					
Non-matrics	380	195	237·5	−42·5	20·28
Undergraduates	656	437	410·0	27·0	4·74
Graduates	434	287	271·3	15·7	2·42
$P(\chi^2) < 0{\cdot}001$, d.f. $= 2$, $\chi^2 = 27{\cdot}44$					
(*c*) By occupational groups					
Students	189	107	118·1	−11·1	2·78
Petty trade	101	43	63·1	−20·1	17·07
Service	522	329	326·3	2·7	0·06
Business	390	245	243·8	1·2	0·16
Learned profession	177	125	110·6	14·4	5·00
Landlords	91	70	56·9	13·1	8·04
$P(\chi^2) < 0{\cdot}001$, d.f. $= 5$, $\chi^2 = 33{\cdot}11$					
(*d*) By expenditure levels (per month per family)					
Below Rs. 40	105	45	65·6	−20·6	17·25
Rs. 41–Rs. 100	370	210	231·2	−21·2	5·18
Rs. 101–Rs. 200	386	249	241·2	7·8	0·67
Rs. 201–Rs. 400	316	216	197·5	18·5	4·62
Above Rs. 400	241	173	150·6	22·4	8·88
$P(\chi^2) < 0{\cdot}001$, d.f. $= 4$, $\chi^2 = 36{\cdot}60$					
(*e*) By communities					
Hindus	1,389	907	868·1	38·9	4·64
Muslims	81	12	50·6	−38·6	78·98
$P(\chi^2) < 0{\cdot}001$, d.f. $= 1$, $\chi^2 = 83{\cdot}62$					

Table 31 gives the percentages of persons having horoscopes, broken down by age, educational and occupational groups, expenditure levels, and communities. The total number of persons covered in the survey is shown in col. 2, and the number possessing horoscopes in col. 3. The over-all proportion of persons having horoscopes is 62·5 per cent. The expected number of persons having horoscopes is shown in col. 4, and the difference between the observed and expected number in col. 5. The corresponding value of χ^2 is given in col. 6.

Variations by age-groups are not important, but non-matrics have comparatively fewer horoscopes (possibly because non-matrics on the whole belong to lower expenditure levels). The occupational distribution shows that the learned profession or landlords proportionately have more horoscopes. The distribution by expenditure levels clearly brings out that richer people

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on the whole have a proportionately larger number of horoscopes. Finally, from the distribution by communities it is clear that Hindus are much more interested in horoscopes than Muslims.

TABLE 32

Calcutta Public Preference Survey, 1941. *Remarriage of widows and widowers*

Groups	Total number of persons giving opinion about remarriage of		Percentage opinion in favour of							
			Unconditional support		Conditional support		Unconditional opposition		Indifferent	
	Widows	Widowers	Widows	Widowers	Widows	Widowers	Widows	Widowers	Widows	Widowers
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
(*a*) By educational groups (males only)										
Non-matrics	367	348	19·07	22·42	52·32	58·05	21·15	13·78	7·36	5·75
Undergraduates ...	658	634	17·02	16·56	62·31	67·67	14·44	9·78	6·23	5·99
Graduates	436	410	26·15	24·15	62·16	65·60	6·19	4·88	5·50	5·37
Total	1,461	1,392	20·26	20·26	59·75	64·65	13·69	9·34	6·30	5·75
(*b*) By sex (all cards)										
Men	1,461	1,392	20·26	20·26	59·75	64·66	13·69	9·34	6·30	5·74
Women	37	37	5·41	10·81	48·65	48·65	40·54	35·14	5·40	5·40
Total	1,498	1,429	19·89	20·01	59·48	64·24	14·35	10·01	6·28	5·74
(*c*) By community (all cards)										
Hindus	1,408	1,341	17·47	17·52	60·87	65·92	15·13	10·59	6·53	5·97
Muslims	90	88	57·78	57·95	37·78	38·64	2·22	1·14	2·22	2·27
Total	1,498	1,429	19·89	20·01	59·48	64·24	14·35	10·01	6·28	5·74

Table 32 shows the distribution of opinion about the re-marriage of widows and widowers by educational groups, communities, and sex. Figures for unconditional opposition to the re-marriage of widows and widowers shown respectively in cols. 8 and 9 are of considerable interest. It is clear that there is much greater opposition to the re-marriage of widows. Opposition definitely decreased with increasing educational status, and women are more opposed to re-marriages than men. Finally, unconditional opposition to re-marriage is practically restricted to Hindus.

Table 33 shows public opinion on inter-group marriages, broken down by educational groups. From col. 6 it is seen that unconditional opposition decreases in every case with increasing educational status. On the whole, there still exists considerable opposition to inter-marriage between sub-castes (28·2 per cent.) which is greater against inter-marriage between different castes (36·41 per cent.). It is interesting to observe that the opposition is greater (49·14 per cent.) against marriages within the same *gotra* (that is, within traditionally the same patrilineal family or clan) than against marriages between provinces (44·61 per cent.). Opposition against marriage between communities—*i.e.*, Hindus and Muslims (60·64 per cent.)—is as strong as opposition to marriages between different nationalities (58·66 per cent.). It is interesting to note, however, that on the whole 12 or 13 per cent. of the persons surveyed and about 20 or 22 per cent. of graduates are unconditionally in favour of inter-communal or international marriages.

Finally, Table 34 shows public opinion about religious instructions in colleges, broken down by educational groups. From col. 6 it is seen that 59·34 per cent. are in favour of such instruction and 18·83 per cent. not in favour, while nearly the same proportion appears to be indifferent. From col. 7 it is clear that opinion in favour of religious instruction in colleges definitely decreases with increasing educational status and that nearly one-third of graduates are not in favour of such instruction.

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TABLE 33

Calcutta Public Preference Survey, 1941. *Inter-group marriages by educational groups*

Type of inter-group marriage	Educational groups	Total number	Percentage of			
			Unconditional support	Conditional support	Unconditional opposition	Indifferent
(1)	(2)	(3)	(4)	(5)	(6)	(7)
(*a*) Between sub-castes	Non-matrics	367	19·35	27·79	38·42	14·44
	Undergraduates	658	31·00	27·66	30·55	10·79
	Graduates	436	44·50	30·96	16·06	8·48
	Total	1,461	32·10	28·68	28·20	11·02
(*b*) Between castes	Non-matrics	367	17·17	22·88	45·78	14·17
	Undergraduates	658	23·56	25·84	40·42	10·18
	Graduates	436	38·53	30·05	22·48	8·94
	Total	1,461	26·42	26·35	36·41	10·82
(*c*) Within same *gotra*	Non-matrics	348	12·64	15·23	54·89	17·24
	Undergraduates	635	16·70	20·94	51·18	11·18
	Graduates	409	26·40	21·03	41·08	11·49
	Total	1,392	18·53	19·54	49·14	12·79
(*d*) Between provinces	Non-matrics	348	10·92	22·99	53·16	12·93
	Undergraduates	635	15·91	27·87	47·87	8·35
	Graduates	409	26·65	30·32	32·27	10·76
	Total	1,392	17·82	27·37	44·61	10·20
(*e*) Between communities (Hindus and Muslims)	Non-matrics	367	8·17	10·90	67·03	13·90
	Undergraduates	658	9·57	14·13	66·42	9·88
	Graduates	436	22·25	19·50	46·55	11·70
	Total	1,461	13·00	14·92	60·64	11·44
(*f*) Between nationalities	Non-matrics	367	6·27	9·53	67·85	16·35
	Undergraduates	658	10·03	17·17	62·16	10·64
	Graduates	436	19·50	21·79	45·64	13·07
	Total	1,461	11·91	16·63	58·66	12·80

TABLE 34

Calcutta Public Preference Survey, 1941. *Religious instruction in colleges by educational groups*

Educational group	Number of persons				Percentage of persons			
	In favour	Not in favour	Indifferent	Total	In favour	Not in favour	Indifferent	Total
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Non-matrics	288	22	87	397	72·54	5·54	21·92	100·00
Undergraduates	422	116	125	663	63·65	17·50	18·85	100·00
Graduates	179	144	115	438	40·87	32·88	26·25	100·00
Total	889	282	327	1,498	59·34	18·83	21·83	100·00

Model Sampling Experiments

I must refer to another special feature of the Institute—namely, the extensive use of model sampling experiments in applied work. For example, in the Bengal crop survey, information

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was collected about the crop grown on each individual field for large tracts, and these were recorded on maps. It was then possible to carry out, in the Statistical Laboratory, a large variety of model sampling experiments using grids of different sizes and densities. Such studies proved most helpful in developing the actual sampling technique.

In the case of space distributions, it has been shown elsewhere that the patch number supplies a useful concept. Consider a map of, say, an agricultural area with fields which are assumed, for the sake of simplicity, to be of the same size. Suppose each field which is shown with paddy is coloured yellow, and fields without paddy are left blank or white. The map of any such region would be then broken up into a number of yellow and white patches, depending on the actual distribution over space of fields under paddy. If alternate fields in both directions are sown with paddy, then the pattern will be something like that of a chessboard; and in this case the total number of patches will be equal to the total number of fields. On the other hand, if half the whole area is sown with paddy in one compact bloc and the other half is left without paddy, then the patch number will be simply two. In any actual situation the patch number will lie between these two limits. If the fields to be sown with paddy (or to be marked yellow) are selected purely at random, then the patch number would show a characteristic distribution.

It is obvious that the design of the sample survey can be made more efficient if something is known about the distribution of the patches. For example, if the patch-pattern is of the chessboard type, then it will be quite sufficient to explore (in the ideal limit) only two adjoining cells to secure a complete picture. If the patch number is only two, then the best plan would be to try to settle the boundary between the two heaps. In any case, when there is any appreciable tendency towards patterning, it is often possible to have recourse to configurational (which is sometimes called "systematic") sampling.

For two colours (*i.e.*, for a binomial distribution) in one dimension the theoretical solution is known. For more than two colours in one dimension, or for two or more colours in two dimensions, complete theoretical solutions have not yet been given. Raj Chandra Bose is carrying on mathematical researches on the sampling distribution of the patch number in the case of a binomial field in two dimensions, and has obtained a number of useful results.

Investigations are also proceeding on purely experimental lines under the leadership of Jitendra Mohan Sen Gupta. A number of different sets of fields each of size 100 × 100—*i.e.*, consisting of 10,000 cells arranged in the form of a square—were constructed and filled with numbers from 0 to 9 at random (or, to be more strict, what are believed to be arranged at random) in two dimensions. Some were based on well-known tables of random numbers. A large number were produced mechanically with the Hollerith equipment by the British Tabulating Machine Co., Ltd. By assigning different digits to different colours, it is possible to prepare random distributions of patches on these fields and to study the patch number experimentally. Work is being done on a large scale, and has already yielded results which, it is believed, would supply useful information for the guidance of practical work, as well as valuable clues for mathematical researches.

I shall give one concrete example. Consider an $n \times n$ field: Let p be the probability for a cell to be black, and q to be white ($p + q = 1$). In counting black patches contact is recognized only along sides, but in counting white patches contact is permitted both along sides and at corners. Let B be the number of black patches and W the number of white patches (as defined above), but embedded within black patches. R. C. Bose has given the following results:

$$\text{Expectation } E(B - W) = p + 2(n-1)pq + (n-1)^2(pq^2 - p^3q)$$

$$\text{Variance } V(B - W) = n^2pq - 4(n-1)p^2q - (6n^2 - 10n + 4)p^2q^2 - (6n^2 - 40n + 50)p^3q^2 + (14n^2 - 56n + 54)p^3q^3 + (32n^2 - 104n + 84)p^4q^3 - (9n^2 - 30n + 25)p^4q^4$$

The results of one series of model sampling experiments is given in the following two tables, 35 and 36. The observed average number of (black minus embedded white) patches and the expected number calculated by the above formula are shown in cols. 2 and 3 of Table 35 for various values of p. The difference between the observed and expected number divided by the standard error of the difference (based on the theoretical value of the variance as calculated from the formula given above) is shown in col. 4. Although the distribution may not be strictly normal in the present case, it is likely that the figures in col. 4 may behave approximately as normal deviates. It will be noticed that out of thirty-six comparisons no fewer than ten are significant at 1 per cent. level and three at the 5 per cent. level.

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TABLE 35

Model sampling experiments. Comparison of expected and observed average number of (black minus embedded white) patches in a binomial field

p	Average number		Normal deviate	Average number		Normal deviate
	Observed	Expected		Observed	Expected	
(1)	(2)	(3)	(4)	(5)	(6)	(7)
	10 cells × 10 cells (n = 1000)			20 cells × 20 cells (n = 250)		
0·1	7·99	8·21	−3·07 †	31·54	32·44	−0·35
0·2	12·81	12·93	−1·56	49·78	50·18	−1·34
0·3	14·44	14·46	−0·32	54·83	54·52	0·95
0·4	13·30	13·21	0·32	48·34	47·67	1·76
0·5	10·28	10·06	2·17 *	33·62	32·56	2·51 *
0·6	5·29	5·70	−4·09 †	11·97	13·19	−2·93 †
0·7	0·96	1·25	−3·38 †	− 7·37	− 5·72	−4·70 †
0·8	−2·06	−2·02	−0·63	−18·34	−18·53	0·74
0·9	−2·89	−2·66	−4·85 †	−18·34	−18·75	2·03 *
	50 cells × 50 cells (n = 200)			100 cells × 100 cells (n = 50)		
0·1	199·93	201·24	−1·67	797·48	802·98	−1·76
0·2	305·87	307·84	−2·40 *	1,215·70	1,233·68	−4·46 †
0·3	328·44	328·45	—	1,294·34	1,297·93	0·98
0·4	278·42	277·47	0·87	1,090·06	1,082·91	1·64
0·5	177·40	175·06	1·93	681·74	662·56	3·93 †
0·6	41·03	47·17	−5·17 †	122·82	142·21	−4·05 †
0·7	− 77·50	− 74·52	−3·00 †	−352·26	−348·78	0·87
0·8	−151·82	−152·55	1·03	−654·86	−657·71	1·01
0·9	−143·17	−143·70	0·92	−605·84	−607·50	0·71

* Significant at 5 per cent. level. † Significant at 1 per cent. level.

A similar comparison of observed and expected values of the standard deviation is shown in Table 36. Using large sample theory, the differences between observed and expected values are found to be significant in three out of thirty-six cases at the 5 per cent., and three out of thirty-six cases at the 1 per cent. level of significance.

TABLE 36

Comparison of observed and expected values of standard deviations of the number of (black minus embedded white) patches in a binomial field

p	10 cells × 10 cells (n = 1,000)			20 cells × 20 cells (n = 250)			50 cells × 50 cells (n = 200)			100 cells × 100 cells (n = 50)		
	S.d.		Normal deviate	S.d.		Normal deviate	S.d.		Normal deviate	S.d.		Normal deviate
	Observed	Expected		Observed	Expected		Observed	Expected		Observed	Expected	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
0·1	2·24	2·27	−0·60	4·31	4·47	−0·80	11·17	11·08	0·16	21·85	22·09	−0·11
0·2	2·42	2·40	0·40	5·20	4·70	2·38*	12·90	11·63	2·19*	25·71	23·18	1·09
0·3	2·62	2·55	1·17	5·75	5·13	2·70	15·20	12·89	3·61†	30·26	25·84	
0·4	2·99	2·91	1·14	6·05	6·01	0·15	15·67	15·34	0·43	32·16	30·89	0·41
0·5	3·16	3·20	−0·57	6·35	6·68	−1·10	17·45	17·12	0·38	41·72	34·52	2·09*
0·6	3·19	3·17	0·29	6·38	6·58	−0·69	16·76	16·81	−0·06	29·71	33·85	−1·22
0·7	2·66	2·72	−1·00	5·55	5·55	0·00	13·57	14·03	−0·66	25·16	28·16	−1·06
0·8	1·82	1·99	−4·25†	3·92	4·01	−0·50	10·05	10·00	0·10	17·85	19·98	−1·06
0·9	1·70	1·50	6·67†	3·36	3·19	1·21	8·49	8·23	0·63	17·01	16·64	0·22

* Significant at 5 per cent. level. † Significant at 1 per cent. level.

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It is clear that expected values of the patch number or of the variance are not confirmed in a rigorous manner. The general agreement between observed and expected values indicates, however, that the theoretical results are probably not wrong. The failure is, therefore, most probably due to some or all the fields used for experimental sampling not being of a sufficiently random character. This often happens in practice. In fact, as pointed out in the paper on large-scale sample surveys (in the *Phil. Trans.*), the concept of degrees of randomness is particularly useful in situations similar to the present one. I may conclude this section by stating that we have a big programme of work for model sampling experiments, but progress is slow for lack of resources.

Sample survey of the economic background of the Bengal famine

A sample survey was undertaken in 1944–45 to collect information relating to the after-effects and the economic background of the Bengal famine of 1943. A first report by Ramkrishna Mukherjee, Ambika Ghosh, and myself has been published in *Sankhya* (the Indian Journal of Statistics), Vol. 7, Part 4, and it is not necessary to enter into details, but a few typical results may be of interest in the present connection. The enquiry covered 15,769 families selected at random from 386 villages, which themselves were selected at random from 41 (out of 86 rural) sub-divisions (administrative units) covering about 60,000 sq. miles in Bengal. The design was zonal, with stratification of sub-divisions by intensity of incidence of famine conditions; and randomization of villages was completed separately within each sub-division.

From the sample survey it appeared that the land position was precarious even before the famine, with one-third of all rural families having no paddy land, while two-fifths had less than 2 acres, so that about three-fourths of all rural families had either no paddy land or owned less than 2 acres. With an average production of 820 lbs. of rice per acre, an average consumption of about 320 lbs. per head per year and an average family size of 5·4 persons, the subsistence level would be about 2 acres of paddy land per family on an average. The actual over-all average for the province was, however, found by the sample survey to be about 1·8 acres of paddy land per rural family, which was below the subsistence level. It is not surprising therefore that, averaged over a number of years before the war, there was a small but net import of a little over 1 per cent. of total production of rice and other cereals into the province.

The sample survey showed that the classification of sub-divisions by amount of paddy land owned per family before the famine was roughly parallel to the degree of incidence of famine conditions, indicating that sub-divisions in which there were more families with paddy land below subsistence level were more vulnerable to the famine.

It was found that about 1·6 millions of families (about one-fourth of the number who had owned paddy land before the famine) had either sold in full or in part, or had mortgaged their paddy land during the famine period. About a quarter of a million of families were obliged to sell all their paddy land, and were thus reduced to the rank of landless labour.

The net loss of plough cattle was about a million, or 13 per cent., during the famine period. Sales of cattle largely exceeded purchases, showing that transfers had taken place not merely from one rural family to another, but that large purchases to the extent of about 600,000 head of cattle had been made by outsiders (possibly by contractors for the supply of meat for army consumption).

The economic deterioration was measured by the number of families transferred from occupations at a higher economic level to occupations at a lower level. Assessed by such methods, it was found that about 700,000 of rural families had suffered a lowering of economic status during the famine. It was also found that even at the beginning of 1943 (before the advent of famine conditions) there had been already an increase of 150,000 of destitute persons. There was a further increase of about 330,000 destitutes during the famine itself, so that nearly half a million persons were rendered destitute under war and famine conditions in Bengal.

Even in the pre-famine period (January 1939 to January 1943) about 6·84 per cent. of rural families had suffered economic deterioration against 3·32 per cent. who had improved their economic level, while the position of 1·07 per cent. was not clear. Economic deterioration had thus set in definitely in the pre-famine period. Rates of change became more rapid during the famine period. Improvement in economic status during the famine period was relatively twice as great as that in the pre-famine period; but this was offset by a three times greater rate

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of economic deterioration and twelve times greater rate of destitution. The famine period was thus one of greatly accelerated economic changes. Improvement of economic conditions was quicker, but was restricted to a comparatively small number of families. Deterioration and destitution were even more accelerated, and were shared by a large number of families. The poorer sections of the community, especially landless labour, fishermen, and village craftsmen, were most seriously affected, and many were rendered destitute; the middle group, who had land of their own and other assets, were naturally less vulnerable; and a comparatively small number of families in the upper stratum had remained immune, and sometimes even became prosperous.

The above summary gives an idea of the kind of information which it was possible to secure by the sample survey. The margin of error of the sample estimates in the cases investigated varied roughly between 3 and 5 per cent. Comparison with other available statistics also showed differences of the same order. For example, extrapolating from the cattle census figures of 1930 and 1940, the calculated number of plough-cattle in rural Bengal in the beginning of 1943 was about 8·3 millions. The sample estimate was about 7·9 millions, giving a difference of about 5 per cent. The sample estimate of 18·6 millions of acres of paddy land was also not inconsistent with other available estimates. On the whole, the above enquiry showed the possibility of using the sample survey in a quick and efficient manner to obtain information about economic conditions in rural areas with a margin of error sufficient for many practical purposes.

A multi-purpose survey in Bengal, 1946–47

I may briefly refer to an extensive survey (which is actually in progress in Bengal at the present time, in 1946–47) as an example of a multiple survey. The size of the grid (sample unit) in this case is just 1 sq. mile, and the information is being collected in the form of two independent but interpenetrating networks of sample units. In the first stage of the survey all households falling within each grid were surveyed with a rather short schedule covering a number of basic items, such as sex and age composition of the family, caste and community, occupation, agricultural land, number of cattle, total indebtedness, etc. Information has been already collected for 84,370 families from 475 grids scattered over about 60,000 sq. miles of rural Bengal. The information is being tabulated on Hollerith machines, and certain preliminary estimates have been made on the basis of about half the number of families surveyed.

The total rural population based on this portion of the material is found to be 50·16 millions, roughly as in March or April 1946, with a calculated standard error of 5·26 millions. The corresponding census population in rural Bengal in March 1941 was 47·185 millions. Owing to uncertainties in census counts it is difficult to calculate reliable rates of growth of population, especially in Bengal. It is also difficult to assess the exact effects of the Bengal famine of 1943 on the growth of population. We may, however, make some rough calculations. The total population (including rural and urban areas) of Bengal was about 46·7 millions in 1921, 50·1 millions in 1931, and 60·3 millions in 1941. Very rough estimates by linear extrapolation can be made on the above basis. Adopting the rate of growth during 1931–41, the extrapolated rural population as in March 1946 would be about 51·9 millions. There are, however, reasons to believe that the census count in 1931 was abnormally low on account of the non-co-operation movement. Adopting the average rate of growth during the twenty-year period 1921–41, the extrapolated figure for the rural population at the end of 1944 would be about 50·4 millions. The observed sample estimate of 50·1 millions is thus of the right dimensional order; and is fairly satisfactory with a sampling fraction of about 1 : 250.

The enquiry just completed is, however, only the first fold or layer of the survey. Arrangements are now being made to select about 20 per cent. of the families included in the first survey for a more detailed study of rural indebtedness and agricultural labour. If everything proceeds smoothly it should be possible to amplify the results for the second survey for making estimates for the sample covered in the first survey, and hence for the rural population of Bengal as a whole. After the second survey is over, if funds permit, it is proposed to reduce the number of families much further and carry out a detailed enquiry into family budget and other socio-economic conditions. At each successive survey the families would be selected at random, but with appropriate zoning and/or stratification, so that it should be possible to amplify the results of one survey to obtain estimates for the families covered in an earlier survey, and hence, finally, for all rural families in the province.

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Other work in statistical sampling

A good deal of other work on statistical sampling has been and is being done in the Indian Statistical Institute. In most cases these have definite practical ends in view. The important point which I should, however, like to emphasize is that such practical studies have continually given rise to theoretical problems of fundamental interest. The subject of crop surveys itself raised many questions of theoretical interest, to some of which I have already referred. I have also mentioned that probability problems relating to space distributions are being tackled on fruitful lines with the help of topological and combinatorial concepts by Raj Chandra Bose. The close integration of applied work and theoretical researches has been such a valuable feature of the work of the Institute that it seems worth while giving a few more examples.

Analysis of anthropometric measurements. The statistical analysis of anthropometric measurements—work which may be properly considered to be purely of an applied nature—had led, about 20 years ago, to interesting theoretical developments in the formulation of the generalized distance (D^2-statistic). It was the starting point of a good deal of mathematical research in the theory of sampling from multivariate correlated population by Raj Chandra Bose and Samarendra Nath Roy, whose work in this subject is already well known.

We had occasion recently to take up an anthropometric study of an extensive series of physical measurements covering 3,000 individuals belonging to twenty-two different castes and tribes in the United Provinces of India. All the measurements were taken by one individual observer, Dr. D. N. Majumdar of the Lucknow University, and were therefore free from differences of personal equation. This made the material particularly suitable for comparative purposes. The use of the D^2-statistic has, I believe, yielded information of great interest and significance. As the paper is in the press, and will be shortly available, I need not enter into details.

I may mention one result which is of some methodological interest. From general considerations I reached the conclusion that, when correlations between variates are taken properly into consideration (as in the D^2-statistic), the use of indices (such as the cephalic or the nasal index) do not supply any additional information. This has been fully confirmed by C. R. Rao by actual numerical calculations. In fact, the present study has given rise to a number of interesting theoretical developments on which Rao has done some very useful work.

More recently, in 1945–46, Dr. D. N. Majumdar, working under the auspices of the Institute, has collected a large volume of material relating to physical measurements and blood-group tests of about 4,000 individuals belonging to various communities, castes, and tribes of Bengal. The analysis of this material should yield valuable results. A survey of blood pressure which was concluded some time ago is perhaps worth mentioning in the present connection.

Design of experiments. At one time a great deal of work relating to the design of agricultural experiments had been done in the Institute. This had led to significant theoretical researches in which the concepts of the Galois field and finite geometry were used with great success by R. C. Bose and, under his leadership, by workers like K. Raghavan Nair, K. Kishen, C. R. Rao, Harikinkar Nandi and others.

Circulation of rupees and rupee notes. At the request of the Reserve Bank of India, experimental surveys were made and methods were devised for estimating the circulation of silver rupees of various dates by sample counts at a number of receiving centres, like banks or railway stations. A method was also devised, based on sample counts of rupee notes received back in the Reserve Bank, to estimate the average life of such notes. It may be mentioned, incidentally, that this project gave rise to an interesting problem of measuring the magnitude of the difference between samples drawn from multinomial populations which was tackled on fundamental lines on the theoretical side by Anil Kumar Bhattacharyya, with promising results. A brief note was issued in the Report of the Reserve Bank of India on Currency and Finance for the year 1940–41, pp. 49–54.

Sampling for yield of cinchona bark. I have already mentioned the project for estimating the yield of cinchona bark as an example of multiple sampling. It is interesting to note that the final choice of physical measurements was made in this case from considerations of cost. It was found, for example, that although the measurements of the surface area of the standing plant gave the highest correlation with the yield of bark (0·865 in one case against the next highest 0·690 for a different character), the use of three simple physical measurements like the "standing

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vertical height of the plant," the " girth at height of 6 inches above ground level," and the " number of stems of the plant at ground level," gave a multiple correlation of 0·848. Theoretically the use of surface area would be more efficient, inasmuch as observations on eighty-nine plants would give the same information as the measurement of the three other characters on 100 plants. The time and trouble to measure the surface area for eighty-nine plants would be, however, far greater than that for measuring the other three characters on 100 plants. This is a simple example of the use of cost considerations in settling the sampling programme.

This particular project also has given rise to many interesting problems of the estimation of appropriate errors in the case of multiple sampling of various kinds. Mrs. Chameli Bose has obtained a number of useful results and is working on the subject.

Population enquiries. After the Indian Census of 1941, the Government of India had decided, as a matter of economy, to cut down most of the standard tables; even age or occupational tables were not prepared. Fortunately, Mr. M. W. M. Yeatts, as Census Commissioner, had issued instructions for the preservation of a 2 per cent. sample consisting of every fiftieth individual census slip. In certain areas, mostly in Indian States, full tabulation had been carried out. A comparison of results based on the complete count and on the 2 per cent. Y-sample in such areas showed that it should be possible to reconstruct most of the tables with sufficient accuracy for practical purposes. About 7 millions of the 2 per cent. slips have been brought over to the Statistical Laboratory in Calcutta from all over India, and the work of transferring the information to Hollerith cards has already started. In only one province—namely, Bihar—the original census slips had also been preserved; and *ad hoc* sampling studies on the basis of this material are also proceeding.

The above projects were sanctioned on the recommendations of the Population Data Committee which was appointed by the Government of India in May 1944, and which submitted its report in June 1945. A comprehensive programme relating to population census and demographic statistics generally was prepared by this Committee, and it was definitely recommended that a continuing sample census of the Indian population should be carried out from year to year.

Statistics of road development. Mention may also be made of the use of sampling methods for traffic count on roads and connected socio-economic variates in the traffic catchment. The object in this project is to lay down a scientific foundation for preparing programmes of road development. K. B. Madhava and Satyabrata Sen are at present actively engaged in this work.

Postal traffic and revenue. An enquiry is proceeding at present to explore the possibilities of using the sampling method for estimating in advance the volume of traffic and revenue in different postal sectors. Exploratory surveys under the guidance of Satyabrata Sen have already yielded encouraging results.

Integrated programme of work

The need of handling computational work on a very large scale has been salutary in teaching us the importance of efficient organization of the human agency. J. M. Sen Gupta and N. T. Mathew and others have done valuable work in this direction. An account of this work, especially from the point of view of cost accounting, would, I think, be of considerable interest. Unfortunately, we have not yet had sufficient time or money to undertake this in a systematic manner.

Along with computational work may be mentioned the preparation of statistical tables of various kinds (which often require the calculation of numerical values of Bessel and other mathematical functions), which has been proceeding for many years, and offers scope for training in a very useful branch of statistical work. The names of J. M. Sen Gupta, Purnendu Kumar Bose, and Raja Rao may be mentioned.

The contact between statisticians and field workers has also been fruitful in every way. The credit for the successful organization of the field side is almost entirely due to the organizing leadership and the ability to appreciate statistical needs on the part of Nihar Chandra Chakravarti and workers in the Field Branch, like Dhirendra Mohan Ganguly, Pronay Kumar Chatterjee and others. Much valuable work on the field has also been done by statisticians like J. M. Sen Gupta, Birendranath Ghosh, N. T. Mathew, Purnendu Kumar Bose, Harikinkar Nandi, and others. I may perhaps also mention that during the last few years the usual practice was

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for all theoretical workers, including R. C. Bose and S. N. Roy, to live in camp on the field for a few weeks in connection with the crop-survey work.

Training in statistics has also been an important part of Institute activities from the very beginning. One-year post-M.Sc. courses, as also occasional special courses in particular subjects, are being arranged for a long time. The Institute workers have also been intimately associated with the two-year course leading to the M.A. or M.Sc. degree and the recently introduced two-year course leading to the Honours B.Sc. degree in Statistics in the Calcutta University.

In the light of the experience gained during the last fifteen years I am convinced that the close integration of applied work, theoretical research, and training offers the soundest line of advance in statistical work, especially on the professional side. It offers scope for developing the spirit of team work among persons with widely varying interests which alone can make it possible to tackle many problems on a scale large enough to ensure success. I have been fortunate in having secured a large group of able and enthusiastic workers, each of whom in his own line has much greater knowledge and ability than myself. Whatever success the Indian Statistical Institute has achieved is in fact essentially due to the spirit of co-operative effort.

References

[1] Bhattacharyya, A.: On a measure of divergence between two multinomial populations. *Sankhyā*, Vol. 7 (4), 1946, pp. 401–6.

[2] Deming, W. E.: Errors in Surveys. *American Sociological Review*, Vol. 9 (4), 1944, pp. 359–69.

[3] Hubback, J. A.: Sampling for rice yield in Bihar and Orissa. Imperial Agricultural Research Institute, Pusa, Bulletin No. 166, 1927, and reprinted in *Sankhyā*, Vol. 7 (3), 1946, pp. 281–94.

[4] King, A. J., and Jessen, F. R.: The master sample of agriculture. *Journal of the American Statistical Association*, Vol. 40, No. 229, 1945, pp. 38–56.

[5] Mahalanobis, P. C.: "A statistical report on the rupee census" published in the report on currency and finance, 1940–41, by Reserve Bank of India, June 1941, pp. 49–55.

[6] Mahalanobis, P. C.: Sample surveys. Presidential address, Section of Mathematics and Statistics. *Proceedings*, Indian Science Congress, 1942.

[7] Mahalanobis, P. C.: On large scale sample surveys. *Phil. Trans.*, Vol. 231 (B), No. 584, pp. 329–451.

[8] Mahalanobis, P. C., Mukkerjee, R. K., and Ghosh, A.: A sample survey of after effects of Bengal famine of 1943. *Sankhyā*, Vol. 7 (4), 1946, pp. 337–400.

[9] Sukhatme, P. V.: Bias in the use of small size plots in sample surveys for yield. *Nature*, Vol. 157, No. 3993, p. 630.

[10] Wald, A.: Sequential tests of statistical hypotheses. *Annals of Mathematical Statistics*, Vol. 16 (2), 1945, pp. 117–86.

[11] Yates, F.: Some examples of biassed sampling. *Annals of Eugenics*, Vol. 6 (2), 1934, pp. 202–13.

[12] Yates, F.: A review of recent statistical developments in sampling and sampling surveys. (Read before the Royal Statistical Society, January 23rd, 1945.)

Discussion on Professor Mahalanobis's Paper

Dr. F. Yates: I have much pleasure in proposing the vote of thanks. Professor Mahalanobis has given us a most interesting description of the work for which he has been responsible in India. His address has provided a vivid picture of the Institute which he directs, and at the same time has shown what enormous strides have been made there, and how extremely able his direction has been.

There is not time to discuss all the many interesting points that Professor Mahalanobis has raised, but I should first like to take the opportunity of expressing my admiration for the way in which he has handled the analysis of the social survey and broadcast survey data, the discussion of which formed the last past of his address. Those who have had experience of the analysis of data of this type will know that while it is comparatively easy to present tables giving overall percentages, etc., the critical analysis of such material does present very great difficulties. Professor Mahalanobis is certainly to be congratulated on the penetrating methods of analysis he has developed in connection with these surveys. There is no doubt that the statistical analysis is the most difficult part of social survey work of this kind, and it is much to be hoped that further efforts will be made to develop the methodology.

Professor Mahalanobis did not spend much time discussing his sampling methods. There are several points of mutual interest that I should like to have the opportunity of discussing with him personally, but which I will not trouble to bring up now. I fully agree with Professor Mahalanobis that the determination of the sampling procedure is often the easiest part of a survey, but I would stress that this ease is in fact based on experience, which has to be acquired by a thorough study of the variability of the material that is being sampled.

Professor Mahalanobis finds the sampling problems easy because his Institute has built up a body of experience in sampling of the particular types of material with which it is concerned.

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This contains a moral for us all. We cannot devise good statistical methods merely by sitting in our studies and theorizing. Good statistical methods almost invariably result from contact between the mathematical statistician and the workers who are responsible for collecting the data, and who are interested not in the statistical methods themselves, but in the conclusions that emerge from their work. Consequently, the statistician must be in intimate contact with the actual numerical material. He must not think solely in terms of algebraic symbols. Professor Fisher once made what I consider a most revealing remark: "Most of the statistics which I have learnt, I have learnt on the computing machine."

Professor Mahalanobis has been concerned with the problem of securing numerical accuracy in the computations. In this connection I might mention the experience I had in the geodetic survey of West Africa. In that part of the world it was necessary to carry out much of the main framework of the survey by means of traverses instead of triangulation. The difficulty with traverses is that they involve a vast amount of computation which is not self-checking. In triangulation, once the base line is determined, the remainder of the computations are self-checking—it is impossible to make an error of any magnitude in a computation which will not reveal itself later in glaring discrepancies.

We had a very great struggle to get the numerical work of our traverses correct. We eventually evolved a most rigorous system of computation. All computations were done in the field in duplicate, the duplicates being compared only at certain points. A third computation was then done at headquarters using the "setting-out" taping, which was carried out in metres instead of feet, and bearings 45° different from the true bearings, so as to give different sines and cosines. This last computation was carried out so as to eliminate errors of the type that are likely to be made independently by different computers—*e.g.*, writing an odd taping length of 449·321 ft. as 499·321 ft., when practically all the taping lengths are just under 500 ft.

I was interested, after this system had been evolved, in finding that the U.S. Coast and Geodetic Survey laid it down in their standard instructions that triangulation was to be used in preference to traverses, if the cost was not more than 100 per cent. greater, simply because of the danger of computing errors. (It is now recognized that with modern invar tapes traverses are quite as accurate, possibly more accurate, than triangulation.)

Professor Mahalanobis stressed the importance of providing the results of sampling surveys on the required dates. We had an interesting example of this—which illustrates the flexibility of sampling methods—during the war, when we were making an estimate of the amount of standing timber in the country. The first estimates were required within about five months of the decision to undertake the survey. Initially the survey had been planned to be undertaken in two 5 per cent. samples, the sampling units being 6-inch Ordnance Survey Quarter Sheets (3 miles × 2 miles). Owing to the almost inevitable delays in organization, collection of staff, etc., it was clear by the time we were ready to start work, that the first 5 per cent. sample would not be completed and analysed by the given date. I therefore insisted that the first 5 per cent. sample should be divided into two parts. This was done by the simple process of drawing a diagonal line across each sampled 6-inch Ordnance Survey Quarter Sheet. There was, in fact, considerable opposition to this, as it was thought that the results of a $2\frac{1}{2}$ per cent. sample would not be sufficiently accurate for the required purpose. Fortunately I knew, from previous experience, the variability of the material, and was able to promise that the results would be of the required accuracy, and that moreover when they were obtained, estimates of the actual standard errors could be furnished so as to provide concrete evidence of this accuracy. I am not sure whether these standard errors really carried conviction, but what certainly did convince those who were initially in doubt was the close agreement between the first $2\frac{1}{2}$ per cent. sample and the second $2\frac{1}{2}$ per cent. sample, and between these and the second 5 per cent. sample.

This illustrates an important point in survey work. In general, overall estimates for the whole area are first required. It is on these estimates that the broad administrative decisions are based. Subsequently more detailed estimates for different parts of the area may well be required, in order to implement the administrative decisions. By carrying out a sampling survey in stages, as was adopted in the Forestry Survey, and as is also, I think, Professor Mahalanobis's practice, the administrators can be given the necessary information on which to base their decisions, while additional data for more detailed and accurate estimates are being collected.

Professor Mahalanobis mentioned the large biases that he has encountered when taking small areas for estimating crop yields. To a certain extent, we have had the same trouble in this country, but only to a limited degree. I suspect that many of these troubles will gradually fade away, as a permanent organization of trained people is built up. In our survey work in Africa, for example, much of the field work was undertaken by native surveyors who were trained in a Survey School which had been specially set up for the purpose. We had very little trouble with these surveyors. The very rigorous methods of booking observations which are always followed in survey practice were a great help. All observations must be booked direct into properly printed field note-books which have numbered pages. The surveyor is completely at liberty to reject any observations with which he is not satisfied, but a permanent record of such observations is always available, so that it is possible to find out what the surveyor has really done. Sometimes they did very funny

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things. One surveyor—quite a conscientious man—had been taken to task because his field work was not reaching the required standard of accuracy. Subsequent to this he turned in a series of traverses in which the closing errors in his bearings (on astronomical azimuths) were far smaller than they should have been. At first it was thought that the results had been deliberately faked. Subsequent cross-questioning revealed, however, that having made a preliminary computation of his closing error and finding it larger than he thought it should be, he had gone back the next day and re-observed some of the angles, rejecting the old or the new observation of each angle according to which improved the closure. The methods of booking enabled him to prove that this was what had been done, and also enabled the matter to be set right without any re-observation.

DR. C. OSWALD GEORGE, in seconding the vote of thanks: Never have I so much enjoyed listening to an address of this character, not only because of its intrinsic interest, but also because of the particularly charming manner of its delivery.

The first point which strikes one is perhaps the difference between conditions in India and in this country. On the other hand, there are important similarities. Of the various headings Professor Mahalanobis has written on the blackboard, we in this country also have to consider problems of time and sampling errors, and—for there is a body here known as the Treasury—the cost of statistical enquiries. Calculating and recording errors are also not unknown here. Had I been speaking a fortnight ago, I might have spoken more confidently on the much higher standards of accuracy in this country, but I fear Professor Mahalanobis may have seen the unfortunate calculations of mean yields in a recent issue of what we had previously regarded as an almost infallible newspaper. So I will pass over that point.

What he calls errors from physical fluctuation, I assume, arise with crops such as jute, where one reporter may find two acres under the crop, while another reporter, visiting the same farm a fortnight later, may find no jute acreage whatever. Under the existing system in this country, where returns relate to a specified day, the same problem does not arise, but some of his sampling problems interest us. He tells of the various sampling methods he adopted, "unitary and zonal, configurational and unrestricted," but unfortunately he has not given us sufficient detail to permit any useful comment on their theoretical or practical merits. He frequently mentioned large-scale sampling theory, but if greater use had been made of stratified sampling, small sample theory would possibly have come more into the picture.

I was particularly interested by his detailed treatment of errors, for I think the question of errors (other than sampling errors) is of more importance than is sometimes believed. How many can lay their hands on their hearts and say that in their published work, errors—if not quite of the type most troublesome in India, nevertheless of serious import—have never reached the printers? But it is noteworthy that Professor Mahalanobis does not mention errors due to the machines: perhaps in the Indian climate machines are infallible!

Another point that struck me was the speaker's attitude towards a system of complete enumeration and his declaration, when comparing it, detrimentally, with sample surveys, that complete enumeration is entirely lacking in any kind of control and precludes any valid estimate of the margin of error of the final results. Without further explanation it is not easy to see why the underlying weaknesses are inherent in the one and entirely absent from the other, or why a system of duplicated surveys cannot be used in conjunction with complete enumeration. And whatever may be desirable in India, it is not likely that in this country complete enumeration will be entirely dispensed with, but rather that it will be used as a basis upon which sampling surveys may be developed.

Professor Mahalanobis has laid particular stress on the results of the Bihar survey shown in Table I, and it is to this that I should like to confine my final remarks. He says that his system of sampling was very successful, in that in the case of rice and pulses a very small sample of about 0·025 per cent. gave results within about 2 per cent. of the final result obtained from the total sample. We are not told what was the result of the small sample in the case of sugar-cane. But what strikes one most about Table I is perhaps the variations between the various estimates, particularly that between estimates 8 and 9, which suggest something more than sampling errors. Estimate 9 for rice gave 13,650,000 acres, a fall of 355,000 acres compared with estimate 8. It is noticeable that this is a larger difference than that between estimate 1 (based on a sample of one in 4,175) and estimate 9, although it resulted from a large increase in the sample (from 80·7 to 100·0 per cent., or 19·3 per cent. of the whole sample). If my arithmetic is not wrong, it seems that the total acreage in or about January, if it had been estimated from this 19·3 per cent. of the total sample, would be only about 12·2 million acres.

What is the explanation of such large discrepancies? We have not been given sufficient detail to decide this point. An apparent explanation would be that there are what Professor Mahalanobis referred to as physical fluctuations, which would presumably mean that the acreage under rice changes substantially from month to month, or even from day to day. But this can hardly be the explanation, for if it were, any similarity between the first estimate in October and the last in January could hardly be attributed to the merits of the sampling methods employed. The question would also arise of what was being estimated: was it the acreage under the crop in October, or

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in January, or the total cropped acreage for the season; and how are any of these to be estimated from a sample drawn over a lengthy period? As for the remaining crops, estimate 8 for pulses is 2,963,000 acres, as against 2,903,000 for estimate 9, and the sugar-cane estimates show similar intriguing differences.

I have no doubt that there is some very simple explanation of all these divergencies, and that this would have been made quite clear if circumstances had permitted Professor Mahalanobis to submit his paper in the customary form for printing and distribution to Fellows at this meeting.

Time will permit only one further very brief point. Professor Mahalanobis said that at the last stage, at the request of the Government, an estimate of 312,700 acres was also given for the area under potatoes, only about 0·26 per cent. of the total geographical area covered by the sample survey, and he added that this supplies a good idea of the wide range of the survey. It would therefore be interesting to have details of this estimate and to know the sampling errors involved, how they were calculated, and how they compare with those of the other estimates.

Dr. George concluded by expressing the deep pleasure it gave him to second the vote of thanks for such a valuable paper from so distinguished a visitor.

The vote of thanks was then put to the meeting, and carried unanimously.

DR. M. S. BARTLETT, after adding his personal thanks and his welcome to Professor Mahalanobis, said that there was one incidental point raised by the last speaker, Dr. George, on which he also was not very clear, and that was the precise meaning of "physical fluctuations" as distinct from "sampling variations."

When considering the paper generally, what he felt was perhaps not so much the novelty of the methods used, but the value of the integration or synthesis of methods used to meet a variety of demand. It was rare to find one department which used these methods for such varied activities as agricultural crops and the investigation of the cost of living and family budgets. Possibly in this country, especially on the social and economic side, those concerned with similar investigations would be studying further the methods which Professor Mahalanobis had described. In particular, there was a great deal to be said for this study of variability, a point which was well recognized in agricultural work, but which had been brought out that evening on the social and economic side. It not only enabled one to judge the accuracy of one's averages at the end, but it gave invaluable information on the material itself.

To take an example. Supposing they made an enquiry into the consumption of a particular food in connection with rationing. Such a study of variation, not only between the broadly classified groups of consumers, but within those classes, would not only give the accuracy of the mean level as shown by the sample, but it would give the variation between those classes. A knowledge of such variation was desirable in order to estimate the change in the mean level of consumption resulting from rationing and the amount of individual hardship which would be entailed if the people were rationed at definite mean levels.

There was one further theoretical point he wanted to mention. Although it was not covered by the paper that evening, it was mentioned in the paper in the *Philosophical Transactions* which had been referred to. He thought it had some relation too with the last Table which Professor Mahalanobis had produced, which he gathered had something to do with the question whether units were random and independent or not. In his paper in the *Philosophical Transactions* Professor Mahalanobis referred to what he called space correlation functions; this was theoretically an interesting concept related to the time correlation functions which occurred in time series. These had been encountered before; they had, for example, been used by Sir Geoffrey Taylor in the theory of turbulence. The property of stationarity in time series was the property of homogeneity in space correlation functions. Further questions that arose were: Were these functions isotropic?—that is, independent of the direction in which one was measuring the correlation—the answer in agriculture was probably "no"; and Did these functions ever show periodicity?

The sort of question which one asked in agricultural design work was usually answered on the basis of a uniformity trial, from which was worked out semi-empirically what was the optimum design. The same answer (for constant variability) must follow from a knowledge of the correlation function. Such an alternative approach did not of course affect the final analysis of any data which was accumulated on a basis of the design finally decided; that followed the standard analysis of variance along the lines that had been described.

The following contributions were received in writing after the meeting:

MAJOR M. D. W. ELPHINSTONE: Professor Greenwood mentioned his own admiration for Professor Mahalanobis's mathematical skill; I also worship from afar. But even more do I admire the courage which has enabled Professor Mahalanobis to build up his organization in the face of all the discouragements and difficulties which he has had to face.

I have during the war been responsible for certain administrative statistics in G.H.Q., India, and I can imagine some of the difficulties overcome.

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In the first place, there is apathy—an apathy difficult to imagine even in our own constant struggle against wilful ignorance at home. Even those who care for good work are apt to be worn down by the continuous output of energy required to get any new idea accepted. The strain is greater in a hot climate, and appears to affect white and brown-skinned races alike.

Then there is the temptation of intellectual dishonesty. Professor Mahalanobis gave examples throughout his paper: the temptation of the field worker to "fudge" his figures, the political pressure resulting in an inaccurate census, the hint of the virtual suppression of an unpalatable report. If Professor Mahalanobis is not absolutely honest in his own work, then he must have practised on us this evening one of the subtlest deceptions of his career. Heat engenders fixed opinions and unreasoned prejudices, and I know how easy it is to hunt for figures to prove a theory rather than to hunt in the figures for the truth.

Then there is the administrative skill and thoroughness with which his investigations are planned. His account of the size of his organization must have made some of us jealous, but he is right to emphasize the factors of time and cost even in his Bengal surveys. Bullock-carts are a slow way of moving about; his field workers must spend much of their time travelling. Then he let fall a mention of work done near Karachi—only some 1,500 miles away. Only by the most careful economy can so much work be done in such conditions by so small a staff.

To give more point to my praise, I may perhaps add that having discarded one of my staff for "negative efficiency" I felt that my section was shaping well. We carried out a small survey on the output of work by coolies, and I designed most carefully all the working sheets and rules for checking. When the figures came up to me, it appeared that the discarded one's successor (specially picked as likely to do well in such work) had not troubled to see that decimal points were in the right place, being of the opinions that (1) I should not notice and (2) that even if I did it would not matter, as the mistakes would average out. Not all people are like that in any part of the world, but it is clear that Professor Mahalanobis has had enough of them to have had to devise techniques for protecting himself against them, and, moreover, has used by-products of these techniques to yield useful statistical information.

Anywhere in the world his achievements would be noteworthy, but as Professor Greenwood admires his work in Combinatory Analysis, so I, who have had some very small experience of the difficulties he has had to face, admire the skill with which administrative troubles unconceived in this country have been overcome.

MR. D. M. SEN: The speaker has emphasized, and Dr. Yates has reiterated, the difficulties of dealing with recording mistakes in sample surveys in different countries. Dr. Yates has introduced into the discussion the much-abused concept of the "intellectual competence" of the field investigators. I have been associated, for some time now, with large-scale sample surveys in Great Britain. Recording mistakes are not a regional phenomenon, not even in their scale. They are common wherever large-scale surveys are undertaken. It would be totally wrong to try to correlate recording mistakes with "the state of intellectual competence," even if "intellectual competence" could be measured and calibrated, or averaged and graded, for countries as a whole.

To counter recording mistakes, various checks are necessary. I may perhaps be allowed to give some instances from my own firm, which is thought to be one of the largest sample survey organizations in the world. Out of the ten departments which share, between them, the whole work of these sample surveys, beginning from the collection of the data to their presentation to clients, two of the largest departments are charged with the functions of "checking field data" and "inspecting processed data." The Tabulating Department, moreover, applies several checks on its own work (punching, etc.), and on Checking Department's work, which has supposedly already checked the data recorded by field investigators. I remember a somewhat irritated fieldman asking the question, "who checks Checking Department's work?". As a matter of fact, the Tabulating Department itself checks Checking Department's work to an extent, as I have mentioned above, but there are other departments, too, which are doing exactly the same thing at various stages of processing the data. The Tabulating Department, for instance, uses the "unit" check on the Hollerith to ensure that purchases plus previous stocks equal sales plus present stocks on each card. The cards may have been checked rightly from the original information sheets which were vetted by Checking Department previously; yet it is felt necessary to employ the "unit test." Similarly for "unit costing." The fieldman records purchases in units and the total value. The cost per unit is computed, which serves as a check on the accuracy of fieldmen's records, since this unit purchase price is compared with the unit purchase price of other shops for the same goods or similar goods, and also compared against the "unit cost" price as recorded a month or so previously in the previous period's survey. This cost per unit is used by Tabulating Department on the Hollerith. The units are multiplied by the "unit cost" to arrive at the total purchase price. These totals for different shops are added up on the Hollerith, and their grand total for each item or brand from the Hollerith serves as a check against the sterling total, which the Checking Department has got on their comptometers for the purchases of the same item or brand.

It is therefore clear that within departments themselves there are various checks. There are

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also many checks between departments. Data are also checked against previous averages, which serves as a rough check, if not a precise one.

In the field itself, besides fieldmen, there are field supervisors, who in their turn are controlled by the field manager. The field supervisors' work consists of no actual work except constant supervision over the fieldmen's thoroughness of collecting data.

Over and above these checks in the field, and during the time when the data are passed through the Tabulator and processed for presentation in the form in which they are wanted by our clients, there is another department—" Inspection Department "—whose job it is to check the purchase, sales, stocks, price and other tables for internal correspondence and plausibility, having regard to known data about market conditions, etc. We often discover at the very last stages mistakes which should have been picked up weeks before. That reflects neither on the field investigators' " intellectual competence " nor on the " intellectual competence " of the checking and supervisory staff within the office itself. These mistakes, we find, are inevitable, and hence they are allowed for. Extra staff are employed to see that they do not go through. There are, of course, cases when deliberate dishonesty is noticeable. I recall an instance where the field investigator, who was supposed to have gone to certain shops and recorded their stocks of certain branded goods, never in fact went there. He filled in the field information sheets at his seaside hotel. In certain cases, where the fieldman detects inaccuracy in purchase data or his stock count, he is supposed to inform the office about it. We in the office know that a percentage of shop data will, over a period, be marked as inaccurate by the fieldmen, and if any one fieldman's audits are extraordinarily accurate, from that very fact alone, we sometimes suspect that the audits are not being conducted as thoroughly as they might have been. This particular fieldman went so far as to advise us from time to time that such and such shops should be thrown out of the sample for that period owing to inaccurate records, etc., and thus our confidence in his work was kept up. He was at last found out by certain discrepancies in his stock position which did not tally with our knowledge of the then existing zoning scheme for the distribution of certain products. There are thus instances of deliberate dishonesty which one can quote.

It is also very easy to question the " intellectual competence " of field investigators. In our work, for instance, present stocks are subtracted from past stocks by fieldmen themselves, so that if present stocks are higher than past stocks, the difference is shown up as a negative quantity. Obviously, this increase in stock, or this negative quantity, must be covered by an equivalent, or higher, purchase figure. It is physically impossible for stocks to increase without equivalent purchases, unless the previous stock-taking is erroneous. Now, one might say that all fieldmen should be intelligent enough to see that these minus quantities are covered by equivalent purchases. In fact, in a very large number of cases honest and very competent fieldmen fail from time to time to account for increases in stocks. Perhaps they would be called " physical fluctuations," but anyhow physical fluctuations are very difficult to explain in this context.

One word about the size of samples. The speaker has indicated that he often takes samples with a sampling fraction of 1 : 4,175, and his largest samples have been of the order of 1 : 168. The samples with which we deal in our particular field of Food and Drug trades, and which we know, from experience over a period of time, yield satisfactory results, are sometimes as large as 1 : 50 and even 1 : 25, but sometimes as small as 1 : 250. I should have thought the size of samples would be related to the variability of the data, among other factors, and consequently the sampling fraction is perhaps not in itself as significant as might be thought.

PROFESSOR MAHALANOBIS made a brief reply at the Meeting, and amplified it later, as follows:—

I am extremely gratified by the kind way in which my talk was received. Unfortunately, the paper could not be circulated nor presented before the meeting in a written form. This naturally left many gaps, to which references were made in the course of the discussion. Relevant material was in certain cases actually with me, but I had no time to discuss it at the meeting.

I greatly appreciate Dr. Yates's emphasis on the importance of close contact between mathematical statisticians and field workers and the need for intimate contact with numerical material. This is a point which I have always striven to emphasize in our Institute. My own view has always been that no one should be considered to have qualified as a statistician without having gone through an apprenticeship as a computer.

I am particularly interested in Dr. Yates's remarks about bias in estimating crop yields. I entirely agree with him in thinking that such troubles can probably be eliminated if a permanent organization of trained workers can be built up. Our problem has been to find out what can be done until such trained workers become available. I may also mention in passing that the size of the cut has been the subject of sharp controversy between the Indian Statistical Institute and the Imperial Council of Agricultural Research of the Government of India. I have been pressing for several years for a scheme to study and compare the differences in technique under the joint supervision of the I.S.I. and I.C.A.R., as this is the only way in which agreed conclusions can be reached. The proposal for joint investigations was strongly supported by Professor R. A. Fisher in his memorandum submitted to the I.C.A.R. in March 1945. Unfortunately, we have not yet succeeded in persuading the I.C.A.R. to agree to this proposal. I

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spoke to a number of statisticians in the United States a few weeks ago and I hope to take the matter up there again shortly.

Dr. Oswald George as well as Dr. Bartlett have asked what I meant by physical fluctuations. I have already referred to this point in the text. Conceptually, sampling variations may be considered to have arisen from the fact that only a small portion or a fraction (and not the whole) of the universe is being surveyed. Recording mistakes arise from the unconscious bias, or sometimes conscious and deliberate negligence or dishonesty, of the human agency. Even when these two sources of error are eliminated there still remains a certain margin of fluctuations which I have called physical. For example, it would not serve any useful purpose to try to determine the weight of a living human being correct to, say, a milligram. When the administrator demands an estimate for a certain entity which he calls "the acreage under rice in Bengal in a particular year" or "the per capita consumption of cereals, etc.," there is a certain margin of fluctuation which is residual in the dual sense of being something independent of sampling variations and recording mistakes and also of being of no importance for the practical purposes in view. Operationally, the different types of error are mixed up in practice; but it is possible to some extent to separate them by appropriate analysis. I may add that what I have in mind is something like the permissible margin of fluctuations in the quality control sense, where production may be considered to be under control even when the measurements do not strictly conform to statistical randomness. Determination of the limits of physical fluctuations (in relation, of course, to the purpose in view) is difficult and requires a purely experimental approach. We realize its importance, but cannot claim to have made much progress in the actual determination.

Dr. George referred to my use of the phrase "unitary and zonal, configurational," etc. I had given a preliminary account in the paper "On large-scale sample surveys" in *Philosophical Transactions*, and I had included a more up-to-date description of the terminology and notation of sample surveys in my original draft, which I had with me at the meeting, but to which I could not refer for lack of time. This has been included in the present paper.

Dr. George has pointed out the need for small sample theory. I agree, of course, that it might sometimes become necessary to use it. We found, however, that too fine a stratification was often uneconomical in the sense that the additional cost was not commensurate with the gain in precision. In fact, in large-scale surveys we usually found large sample theory to be adequate for practical purposes. But I agree with Dr. George that in appropriate cases small sample theory would no doubt prove to be of great value. Incidentally, I may mention that one of the important lines of work in our Institute has been the development of the exact or small sample theory of distribution for multivariate analysis by my colleagues R. C. Bose and S. N. Roy.

I am glad to have Dr. George's support about the importance of errors other than sampling error. I also agree that errors arising through faulty working of machines cannot be neglected. I did not refer to this, simply because there was nothing peculiar in the Indian experience to make it worth mentioning.

Dr. George has referred to my remarks on the defects of complete enumeration. Certain observations which I had no time to make at the meeting will be found in the text of the paper. I may add one or two more remarks. Complete statistics are often obtained almost as a by-product of routine administration—for example, the number of criminal cases sent up for trial. Apart from mistakes in compilation or similar clerical errors I have no doubt that such statistics would be substantially accurate. Consider another example, which is nearer to the subject of the present paper. In temporarily settled provinces of India, crop records are compiled by a permanent, and often hereditary, staff of village recorders (called *patwaries*) primarily for revenue purposes, as the assessment is fixed on the basis of crop production. In such provinces, crop-acreage figures are claimed to be sufficiently accurate for all practical purposes. I have no personal experience of the quality of the work done by *patwaries*, but it may very well be true that crop records prepared by them are substantially accurate. This is quite different from what happens in a permanently settled province such as Bengal or Bihar. I had in mind the so-called complete enumeration made by an *ad hoc* staff whose number is large (of the order of fifteen or twenty thousand) and who are scattered widely over a province like Bengal. All available evidence points to the fact that work done under such conditions is unreliable. What I had wanted to emphasize was that the fact that complete paper records had been made ready was no guarantee that such records would be reliable. I may perhaps quote one concrete example. In 1939 the Government of Bengal decided to prepare a complete record, plot by plot, of the land sown with jute. After these records were prepared the Government arranged to have certain portions checked by permanent Government officers. The primary records, when checked, were found so unreliable that the Bengal Govenrment ordered all the records to be destroyed.

I agree with Dr. George, who thinks that suitable systems of sample checks and controls can be used with complete enumeration with great advantage. Some of the checks described in this paper may be quite suitably taken over.

Dr. George has referred to the results of the Bihar crop estimates. He is quite right in pointing out that the area under any given crop is not determinate unless the date is specified. I have

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discussed this point elsewhere.* In a province like Bengal no machinery exists for obtaining the crop acreage on a specified date. The estimate obtained by a sample survey spread over a lengthy period must necessarily be somewhat indeterminate. In Bengal, by the alternative method of obtaining an estimate of complete enumeration, the result would be equally indeterminate, for the simple reason that the complete enumeration would have to be made by an *ad hoc* staff whose work would be spread over more or less the same period as that of a sample survey.

A little consideration would, however, show that although a specified date makes the crop acreage determinate it renders the estimate of total production necessarily somewhat vague. Whatever may be the crop acreage on a specified date it is likely that the harvested area would be somewhat different. As already mentioned, such differences are likely to be quite appreciable in India, especially in the case of rain-fed crops. In this situation we thought it advisable to develop a method for ascertaining the total production by a direct estimation of production per grid. This seems to be as far as one can go in existing conditions in Bengal.

I may mention here one particular procedure with which we have been experimenting for some time. At the time of harvesting, the yield per acre is determined by crop-cutting experiments in a number of grids. The total cropped area in each grid is estimated at the same time, and in this way the total yield for each such grid is ascertained. This enables a direct estimate to be made of the total production in each cell, zone or district, or in the province as a whole. All lands which go out of cultivation would not in this method contribute anything to the estimate.

In showing the progressive estimates for the *ad hoc* crop survey my object was to show the kind of data supplied to administrators. The sampling at each stage was not individually or independently representative of the whole "universe" as spread over space and time. Suitable modifications in the theory of estimation to cover such cases have not been worked out. Different progressive estimates necessarily referred to different regions of the province at different times, and were not strictly comparable. Differences were due partly to sampling variations and partly to actual physical differences in crop acreage arising from the different geographical coverages at different periods of the survey. My real point was that, in spite of such intrinsic differences, the figures supplied to administrators were probably quite useful for administrative purposes.

Dr. George has enquired about the proportional sampling errors. These, naturally, were large when the total acreage or the proportion of land under a particular crop was small. I have discussed this point in greater detail for the Bengal crop surveys, for which material was available for a number of years. The Bihar crop survey was, unfortunately, terminated after one year and sampling errors had not been fully investigated. I hope, however, to give an account of the effective margin of error in crop surveys in a subsequent paper.

I am glad Dr. Bartlett has referred to what I called space correlation functions, which I mentioned in the paper in *Phil. Trans.*, although I had no time to speak on this topic at the meeting. Since that paper was written certain other concepts have been developed; and, as already mentioned, some theoretical work has been done by R. C. Bose. The subject appears to be of considerable interest; but, unfortunately, owing to heavy pressure of other work, it has not yet been possible to tackle it on fundamental lines.

It may be useful, however, if I add a few brief remarks. Consider a field of, say, square or rectangular shape consisting of a very large number of rows and columns of elementary cells or quads. Along any single row (or column), the problem of space correlation would be one-dimensional, and identical with that of serial correlation in time series. The fact that space distributions are intrinsically two-dimensional, however, makes it necessary to have more generalized concepts. The approach at present adopted in the Statistical Laboratory is as follows.

Consider a single row or a single column consisting of a very large number of cells. (The boundary difficulty can be avoided by making the row or column endless by joining the two terminal cells.) Imagine any particular row or column divided into a number of segments, each segment itself still consisting of a very large number of individual cells or quads. The correlogram or serial correlation for different gaps can be then determined for each segment. The form of the correlogram or serial correlation for different gaps can then be determined for each segment. The form of the correlogram or the serial correlation function may then differ from segment to segment or may happen to be statistically the same for different segments. The serial correlation is usually called stationary if it happens to be statistically the same for different segments in whatever way the segments are formed (subject, of course, to the provision that each segment continues to have a sufficiently large number of quads to enable the serial correlation to be determined with the required precision).

Now consider the adjoining (or some other) row. The correlation function or serial correlation in this row may again be stationary, but may differ from the stationary serial correlation in the preceding row. Thus, although the serial correlation in each row is stationary, they may be all different. In a simplified case, the serial correlation along each row may be stationary and may be the same for all rows. Such a field may be called "uniform along rows."

* Presidential Address to the Section of Mathematics and Statistics, *Indian Science Congress*, Baroda 1942, p. 25.

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Now consider the columns. The serial correlation along each or some of the rows may differ from segment to segment. If they happen to be the same over all segments in any particular column, then the serial correlation is stationary over that column. In the same way, the serial correlations may be stationary for each column and finally may be the same for all columns, in which case the field would be "uniform along columns."

It is possible, however, to consider the serial correlation not merely along rows or columns but in different directions, making different angles with the direction of, say, rows. If the field is divided up into two-dimensional segments (each segment still consisting of a sufficiently large number of elements or quads) it is possible to determine the serial correlation in different directions. In this way it is possible to construct what may be called the surface of the serial or space correlation in such a way that a section of the surface in any particular direction supplies the corresponding curve of the serial correlation. If such surfaces of the correlation function happen to be the same for all segments, then the field may be called homogeneous. It might not, however, be isotropic. In fact, in each segment the field would be isotropic only if the correlation function is identical in all directions or, in other words, the surface of the correlation function is a surface of revolution.

The interesting fact then emerges that the surface may be isotropic for each segment, but the actual surface may be different in different segments, in which case the field would not be homogeneous, but would be isotropic. It is obvious that the field may be homogeneous, but not isotropic. Finally, when the field is both homogeneous and isotropic it may be called uniform.

Instead of working with the correlation function it is also possible to work with the variance function in the same way. As the two functions are mathematically connected the results based on them would be equivalent. For practical purposes the variance function would often be more convenient, as it usually enables the sampling errors to be calculated more directly.

I may add that the study of the surface of the correlation or variance function is of interest in connection with what I have called mapping problems, in which it is desired to ascertain the actual distribution of the variate under study over different elements or regions of the field under survey.

In the paper in *Phil. Trans.* I explained that fields may be of non-periodic, quasi-periodic or periodic types. I also explained the use of the patch number in characterizing the different types of fields. From the point of view of patch numbers, fields may be broadly considered as belonging to three types, namely, (*a*) fields in which the patch number is very small, corresponding to what I called "patterned" fields; (*b*) fields with patch numbers lying within an assigned range on either side of the model or expectation value, which I called fields of the random type; and finally, (*c*) fields in which the patch number was larger than the limit assigned for random fields, which I called the crystalline type and which would correspond to quasi-periodic and periodic types from the point of view of correlation function.

It is easy to see that previous knowledge of the type of the field can help in improving the efficiency of the sampling design, especially in the case of mapping surveys. For example, if the field is of a random type then a purely random sampling would be appropriate. If the field is however of quasi-periodic, or periodic, or what I have called of the crystalline type, then cluster or configurational sampling must prove quite useful. On the other hand, if the field is known to be of the patterned type then it should be possible to improve the efficiency by suitable zoning and stratification.

I am particularly grateful to Major Elphinstone for his kind remarks. I entirely agree with him in thinking that the apathy of administrators and the peculiar difficulties in which statistical work has to be carried out in India has to be experienced in order to be properly appreciated. The average administrator in India expects the scientific or statistical technician to supply evidence or proof in favour of what the administrator thinks to be right, rather than to give independent advice on objective grounds. Intellectual dishonesty, to which Major Elphinstone has referred, would in such circumstances be an actual advantage in securing promotion in official posts. This is why I have never favoured the idea of the Statistical Institute being run as a Government department or under predominating Government control. I think it is of the utmost importance that scientific organizations like the Statistical Institute should maintain an independent position, so as to be able to offer impartial technical service and advice on an objective basis. Mr. D. M. Sen has referred to the size of samples. I am afraid I had not made the position quite clear in the paper. It is not correct to say that we have used a sampling fraction of 1 : 4175. This was the fraction of the sample (in Table 1) on which we supplied a purely provisional first estimate. In the sample survey of crop acreage we are at present using a sampling fraction of the order of about 1 : 150 or 1 : 200. In estimating the yield of crops per acre the sampling fraction is necessarily far smaller and of the order of 1 : 6,000,000. On the other hand, in small-scale work one may go up to such large fractions as 1 : 20. Mr. Sen is entirely right in thinking that the size of samples would depend on the variability of the material. Naturally, it also depends on the precision it is desired to attain in the final results, and on permissible budget limits.

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Wholesale Prices in 1945

By the Editor of "The Statist"

(*The Statist's* Index Numbers in continuation of Mr. A. Sauerbeck's figures)

In Table I below are set out the Sauerbeck–*Statist* annual index-numbers for each year from 1846 to 1945, Jevons's figures for 1810 and 1818, adjusted to Sauerbeck's standard being also included. These index-numbers, based on averages for the period 1867–77, are calculated (with few exceptions) from the average of weekly quotations throughout each year for the forty-five commodities which they cover. Up to the end of 1912 they were calculated by the late Mr. Augustus Sauerbeck. Since that date they have been compiled by *The Statist*.

The figures of recent years have been dominated by, and are indeed largely a reflection of, official price policy. After the Budget of 1944 there was a distinct tendency for the element of subsidisation in the official price level to be reduced or eliminated in anticipation of an early end to hostilities and the restoration of more normal international trading conditions. This tendency continued in strong evidence in the first three months of 1945, each of which showed an increase of about 4 per cent. in the index for all commodities as compared with the corresponding month of 1944. The rate of increase slowed down, however, in the following months, being 3 per cent. for April and May, 4 per cent. for June, 3 per cent. for July, 1 per cent. for August, and less than 2 per cent. for each month thereafter. (During this time the difficulty of compiling the index was aggravated by the substitution of standard qualities for accepted commercial grades and by physical shortages.) Not until the second half of 1946 did the index figures begin to turn strongly upwards.

This pattern is similar to that set after the 1914–18 war, for while there were relatively moderate advances in 1918 and 1919 it was only in 1920 that the increase, which then took prices to record levels, became really marked. The relative experience *during* the two wars was, however, quite different. Taking the annual index for all commodities there was little change, in the earlier conflict, until the second full year of war. Then came advances of 27 per cent. in 1915, 26 per cent. in 1916 and 29 per cent. in 1917. In the later conflict, however, there was an early jump of 36 per cent. between 1939 and 1940, followed by moderate advances of 11 per cent. in 1941, 6 per cent in 1942, 2½ per cent. in 1943 and 3 per cent. in 1944, while in 1945 the rise was still only of the order of 2½ per cent.

Taking the quarterly figures, 1945 started off with an increase of 4 per cent., but this was brought down to 3 per cent. for the second and to less than 2 per cent. for the third and fourth quarters. An interesting feature is that, contrary to the experience in 1944, the index for all commodities was influenced appreciably by increases in the food group. Here, as compared with the 1944 showing of declines of 2, 2 and 1 per cent. and a fractional increase, respectively, there were advances of 1, 3, 1, and 1 per cent. This behaviour was opposed to that experienced with industrial materials, wherein 1945 the quarterly increases, as compared with those of 1944, were respectively: 1 per cent., against 3 per cent.; 4 per cent., against 5 per cent.; 2 per cent., against 7 per cent., and 2 per cent., against 7 per cent.

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TABLE I

THE STATIST'S *Annual Index Numbers (in continuation of Sauerbeck's figures)* (1867–77 = 100)

Year	Average No.	Year	Average No.	Year	Average No.	Year	Average No.	Year	Average No.
1945	164	1924	139	1904	70	1884	76	1864	105
'44	160	'23	128	'03	69	'83	82	'63	103
'43	155	'22	131	'02	69	'82	84	'62	101
'42	151	'21	155	'01	70	'81	85	'61	98
'41	142	'20	251	'00	75	'80	88	'60	99
'40	128	'19	206	1899	68	'79	83	'59	94
'39	94	'18	192	'98	64	'78	87	'58	91
'38	90	'17	175	'97	62	'77	94	'57	105
'37	102	'16	136	'96	61	'76	95	'56	101
'36	88	'15	108	'95	62	'75	96	'55	101
'35	83	'14	85	'94	63	'74	102	'54	102
'34	81	'13	85	'93	68	'73	111	'53	95
'33	78	'12	85	'92	68	'72	109	'52	78
'32	79	'11	80	'91	72	'71	100	'51	75
'31	82	'10	78	'90	72	'70	96	'50	77
'30	96	'09	74	'89	72	'69	98	'49	74
'29	114	'08	73	'88	70	'68	99	48	78
28	119	'07	80	'87	68	'67	100	'47	95
'27	122	'06	77	'86	69	'66	102	'18	159 *
'26	125	'05	72	'85	72	'65	101	'10	171 *
'25	136								

* Jevon's numbers adjusted.

TABLE II

THE STATIST'S *Annual Index Numbers—ten-year averages* (1867–77)

1848–1857 = 89	1898–1907 = 71	1911–1920 = 146	1924–1933 = 110
'58– '67 = 99	'99– '08 = 72	'12– '21 = 148	'25– '34 = 104
'68– '77 = 100	1900– '09 = 73	'13– '22 = 153	'26– '35 = 99
'78– '87 = 79	'01– '10 = 73	'14– '23 = 157	'27– '36 = 95
'88– '97 = 67	'02– '11 = 74	'15– '24 = 162	'28– '37 = 93
'90– '99 = 66	'03– '12 = 76	'16– '25 = 165	'29– '38 = 90
'91–1900 = 66	'04– '13 = 77	'17– '26 = 164	'30– '39 = 88
'92– '01 = 66	'05– '14 = 79	'18– '27 = 159	'31– '40 = 91
'93– '02 = 66	'06– '15 = 82	'19– '28 = 152	'32– '41 = 97
'94– '03 = 66	'07– '16 = 88	'20– '29 = 142	'33– '42 = 104
'95– '04 = 67	'08– '17 = 98	'21– '30 = 127	'34– '43 = 111
'96– '05 = 68	'09– '18 = 110	'22– '31 = 120	'35– '44 = 119
'97– '06 = 70	'10– '19 = 123	'23– '32 = 115	'36– '45 = 127

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*Monthly Fluctuations of the Index Numbers * of 45 Commodities, 1867–77 = 100*

	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year
1898	62·8	63·4	63·0	65·5	66·4	64·7	64·3	64·0	63·9	63·6	63·9	63·8	64
1900	74·0	75·1	75·7	75·6	75·5	75·7	76·2	76·0	75·5	74·7	73·9	73·4	75
'01	72·2	71·7	71·0	70·6	70·5	69·8	69·5	69·8	69·6	69·6	69·0	68·4	70
'02	68·8	68·9	69·2	69·7	70·9	70·4	70·0	69·5	69·3	68·8	68·6	69·1	69
'03	69·5	70·2	70·4	69·4	69·6	69·5	69·5	70·0	69·1	69·0	69·0	70·0	69
1904	70·4	70·8	70·8	70·5	69·9	69·4	69·9	70·4	70·7	71·0	71·2	70·9	70
'05	71·2	71·4	71·8	72·0	71·7	72·0	72·5	72·3	72·4	73·2	74·2	74·9	72
'06	75·2	75·0	75·7	76·5	77·0	76·9	76·4	76·7	77·5	78·5	78·6	79·7	77
'07	80·0	80·7	80·0	80·7	82·4	82·0	81·1	79·4	79·1	78·8	76·7	76·2	80
'08	76·0	74·5	74·1	73·8	73·6	72·9	73·1	72·2	72·5	72·2	72·2	72·3	73
1909	72·0	71·9	72·4	74·3	75·4	75·1	75·2	74·9	74·7	75·2	75·5	76·3	74
'10	77·1	78·1	79·1	78·5	78·2	76·9	78·1	78·2	77·6	77·2	77·8	77·9	78
'11	78·5	78·6	78·9	80·0	80·3	80·0	78·9	79·5	80·3	80·7	80·6	80·9	80
'12	81·8	82·9	84·4	85·0	85·3	85·5	86·5	85·9	86·7	85·8	85·3	86·4	85
'13	86·4	86·1	86·7	86·2	85·7	84·1	84·2	85·0	85·7	84·5	83·3	83·8	85
1914	83·5	83·8	82·8	82·3	82·3	81·2	82·4	87·9	89·3	89·8	88·8	91·6	85
'15	96·4	100·9	103·7	105·9	107·2	106·4	106·4	107·0	107·8	110·0	113·1	118·4	108
'16	123·6	127·0	130·4	134·2	135·4	131·0	130·5	134·5	134·4	141·5	150·8	154·3	136
'17	159·3	164·0	169·0	173·0	175·0	180·4	176·9	175·7	176·4	180·6	182·9	185·1	175
'18	186·2	187·3	188·0	189·8	191·1	192·3	192·9	195·9	197·1	197·8	195·3	196·0	192
1919	192·1	187·5	184·7	184·6	194·6	199·4	206·4	212·7	214·8	224·3	231·0	235·2	206
'20	245·3	260·4	261·8	266·1	260·0	255·7	254·6	253·5	248·7	239·9	223·8	207·2	251
'21	197·2	183·0	177·2	169·8	162·2	155·8	158·2	154·3	149·4	138·4	136·7	133·6	155
'22	132·5	132·2	133·3	134·8	135·5	135·6	134·0	129·6	127·9	130·1	130·6	129·1	131
'23	130·2	131·9	132·7	134·0	132·2	127·9	124·8	125·0	127·8	127·7	132·4	133·2	128
1924	137·2	138·8	137·0	136·8	136·4	136·3	138·4	138·0	141·3	146·1	145·5	147·7	139
'25	144·8	143·1	140·1	137·5	135·7	131·2	134·3	134·3	132·7	130·2	132·9	130·4	136
'26	129·3	127·9	126·1	125·5	125·7	124·9	126·0	127·0	128·0	131·0	130·8	123·9	125
'27	123·1	124·1	123·6	123·3	123·8	123·1	122·0	122·8	121·5	120·6	121·5	121·4	122
'28	120·9	121·1	123·6	125·6	126·2	122·6	120·3	118·0	116·8	116·8	117·9	117·9	119
1929	117·0	120·1	120·5	116·5	113·0	113·1	115·2	113·9	112·6	111·1	108·3	108·8	114
'30	106·6	104·8	103·0	101·5	98·8	95·8	94·4	92·2	90·8	90·4	88·6	86·9	96
'31	85·7	85·5	85·5	84·4	82·2	82·6	80·2	79·1	80·7	82·3	83·0	85·4	82
'32	84·7	86·7	84·1	82·5	80·2	77·0	78·9	80·7	80·4	77·8	77·9	77·7	79
'33	77·8	77·0	77·0	78·5	80·9	81·3	81·7	81·2	80·7	80·5	79·3	80·0	78
1934	82·5	82·5	82·2	81·0	81·1	80·7	82·4	83·4	82·1	81·1	81·0	82·8	81
'35	83·6	83·4	82·9	84·1	85·2	83·7	84·3	84·1	85·1	85·8	86·3	86·7	83
'36	87·1	87·1	86·7	86·2	85·6	84·8	87·1	89·0	90·4	91·7	94·5	98·9	88
'37	99·6	102·1	107·3	104·7	106·2	104·7	105·9	104·4	103·3	100·8	96·7	97·3	102
'38	96·5	96·4	94·2	93·5	91·4	91·4	91·1	88·6	88·6	88·8	87·4	89·1	90
1939	88·7	88·6	89·0	90·5	90·6	90·6	88·7	90·4	99·7	105·8	112·2	120·1	94
'40	124·1	124·5	123·4	126·0	128·0	130·0	129·5	131·5	131·6	131·8	132·7	134·5	128
'41	134·9	136·3	138·0	141·1	143·5	144·4	145·3	145·1	145·5	143·7	145·5	146·5	142
'42	148·6	153·4	153·5	154·5	156·6	154·4	150·0	149·4	149·7	150·4	151·3	152·2	151
'43	153·3	153·2	154·0	154·9	155·6	155·4	156·0	154·4	154·6	153·7	153·9	153·9	155
1944	154·9	155·1	155·7	157·9	159·5	160·0	161·9	161·5	160·1	159·6	160·3	160·7	160
'45	161·4	161·5	162·1	162·4	164·0	166·7	165·2	163·3	162·7	162·7	162·7	163·2	164
'46	165·6	166·2	166·6	168·4	169·3	169·7	174·4	175·7	180·5				

* The average of the twelve monthly figures of each year does not necessarily coincide with the annual figures, as the latter are calculated mostly from the average of 52 weekly quotations, while the former are based on end-of-the-month prices.

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Summary of Index Numbers. Groups of Articles, 1867–77 = 100

	Vegetable Food (Corn, etc.)	Animal Food (Meat, etc.)	Sugar, Coffee, and Tea	Total Food	Minerals	Textiles	Sundry Materials	Total Materials	Grand Total	Silver *	Wheat Harvest †	Average Price of Consols ‡	Average Bank of England Rate ‡
												£	Per cent.
1873 ...	106	109	106	107	141	103	106	114	**111**	97·4	80	92½	4·750
1896 ...	53	73	59	62	63	54	63	60	**61**	50·5	112	110¾	2·483
1911 ...	70	90	61	75	93	76	81	83	**80**	40·4	110	79 5/16	3·467
1918 ...	168	207	130	174	192	222	202	206	**192**	76·4	111	56⅞	5·0
'19 ...	179	213	147	185	220	228	219	222	**206**	85·3	98	54 1/16	5·166
'20 ...	227	263	198	234	295	262	244	264	**251**	76·1	96	47 1/16	6·71
'21 ...	143	218	83	158	181	140	145	153	**155**	48·1	118	47 2/32	6·092
1922 ...	107	184	82	130	142	134	122	131	**131**	51·6	105	56 15/32	3·692
'23 ...	98	162	101	122	155	140	115	133	**128**	49·4	105	57 23/64	3·496
'24 ...	119	158	105	130	158	170	119	145	**139**	50·7	107	56 63/64	4·0
'25 ...	118	162	89	128	154	165	117	142	**136**	52·5	114	56⅜	4·575
'26 ...	108	150	88	119	154	133	112	130	**125**	47·1	99	54 2/32	5·0
1927 ...	108	138	83	114	141	131	116	127	**122**	42·8	109	54 3/32	4·650
'28 ...	107	142	78	114	123	136	114	123	**119**	44·0	109	55 1/32	4·5
'29 ...	99	146	72	110	126	122	108	117	**114**	40·2	114	54 7/16	5·508
'30 ...	77	142	54	96	112	84	94	96	**96**	29·0	99	56	3·4
'31 ...	68	119	50	83	100	63	81	80	**82**	20·4	99	55 3/16	3·975
1932 ...	72	105	50	79	99	64	78	80	**79**	19·5	105	66 25/32	3·017
'33 ...	60	106	47	74	107	67	76	82	**78**	18·7	114	73 13/32	2·0
'34 ...	63	108	50	77	109	72	76	84	**81**	20·0	120	80 21/32	2·0
'35 ...	66	107	42	76	112	80	79	88	**83**	26·4	112	86½	2·0
'36 ...	76	109	41	81	118	83	84	93	**88**	18·5	100	85 11/64	2·0
1937 ...	93	117	49	93	142	93	98	109	**102**	18·4	99	76 9/32	2·0
'38 ...	81	111	43	84	136	75	83	95	**90**	17·6	122	74 5/32	2·0
'39 ...	74	115	47	83	137	93	88	102	**94**	17·1	112	66 11/64	2·5
'40 ...	112	141	58	111	167	149	117	140	**128**	17·1	108	72 9/16	2·0
'41 ...	140	142	65	125	181	162	131	154	**142**	18·0	109	79 13/16	2·0
1942 ...	170	148	66	140	184	163	142	160	**151**	18·1	123	82 5/16	2·0
'43 ...	156	156	72	138	187	166	156	167	**155**	18·1	123	80¼	2·0
'44 ...	152	156	73	137	197	182	161	178	**160**	18·1	121	80 15/32	2·0
'45 ...	155	156	78	139	209	189	159	182	**164**	23·1	—	87 9/16	2·0
Average													
1904–13	68	91	53	73	95	74	76	81	**77**	44·1	106	82 25/32	3·733
1890–99	61	80	63	68	71	56	66	64	**66**	55·8	103	103 9/16	2·958
'78–87	79	95	76	84	73	71	81	76	**79**	82·1	97	99 9/16	3·264
1818–27	109	90	151	111	128	105	106	112	**111**	98·0	—	—	3·692

* Silver (see note on p. 386), parity of 1 gold to 15½ silver = 100.

† Wheat harvest in U.K. to 1895: 29 bushels = 100; from 1896: 30 bushels = 100.

‡ Average price of Consols and the average Bank of England rate of discount are actual figures, not index-numbers; Consols 3% to 1888, 2¾% from 1889, 2½% from April, 1903.

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THE STATIST'S *Index Numbers—monthly average by groups*, 1867–77 = 100.

	Vegetable Food	Animal Food	Sugar, Tea, and Coffee	Food-stuffs	Minerals	Textiles	Sundry Materials	Total Materials	All Commodities
1943									
Jan. ...	158·5	155·7	73·3	139·5	187·8	165·5	147·1	163·8	153·3
Feb. ...	158·8	155·7	73·3	139·7	187·6	163·0	148·3	163·4	153·2
Mar. ...	160·0	155·7	71·9	139·9	188·5	163·3	149·8	164·3	154·0
April ...	160·3	155·7	72·9	140·2	187·9	164·3	152·4	165·6	154·9
May ...	159·9	155·7	73·0	140·1	188·7	165·3	154·5	167·0	155·6
June ...	159·4	155·7	73·5	140·0	188·4	165·6	153·5	166·6	155·4
July ...	163·3	155·7	73·8	141·7	187·6	163·9	154·9	166·5	156·0
Aug. ...	152·3	155·7	74·2	137·1	188·7	163·5	155·9	167·1	154·4
Sept. ...	152·7	155·7	74·3	137·3	188·2	162·8	157·1	167·2	154·6
Oct. ...	150·4	155·7	73·9	136·3	187·8	161·3	156·4	166·4	153·7
Nov. ...	149·6	155·7	74·7	136·1	187·5	160·6	158·2	166·8	153·9
Dec. ...	150·2	155·7	74·0	136·2	188·0	160·6	157·5	166·7	153·9
1944									
Jan. ...	151·2	155·7	74·1	136·6	190·1	160·8	159·4	168·1	154·9
Feb. ...	151·8	155·7	72·9	136·6	191·4	160·9	159·2	168·4	155·1
Mar. ...	151·9	155·7	74·2	137·0	192·8	161·6	160·1	169·3	155·7
April ...	151·9	155·7	74·5	137·0	192·3	178·3	156·9	173·2	157·9
May ...	151·9	155·7	74·3	137·0	194·3	180·2	161·2	175·9	159·5
June ...	151·5	155·7	74·7	136·9	194·2	181·5	162·6	176·9	160·0
July ...	156·5	155·7	75·6	139·2	194·8	182·2	165·5	178·5	161·9
Aug. ...	149·9	155·7	77·4	136·8	197·8	184·1	164·5	179·5	161·5
Sept. ...	150·3	155·7	74·4	136·3	205·3	184·9	154·3	177·5	160·1
Oct. ...	150·3	155·7	73·7	136·2	203·9	184·1	154·3	176·8	159·6
Nov. ...	151·4	155·7	74·0	136·7	205·0	184·1	155·4	177·6	160·3
Dec. ...	152·6	155·7	73·5	137·1	202·9	185·5	156·7	178·0	160·7
1945									
Jan. ...	153·8	155·7	73·8	137·6	203·4	185·5	158·2	178·8	161·4
Feb. ...	154·2	155·7	73·2	137·7	204·4	185·5	157·9	178·9	161·5
Mar. ...	156·3	155·7	74·2	138·8	206·6	187·0	155·9	179·1	162·1
April ...	153·0	155·7	74·5	137·5	205·5	189·7	158·3	180·7	162·4
May ...	158·3	155·7	74·8	139·8	209·1	189·8	158·3	181·6	164·0
June ...	170·1	155·7	74·4	144·6	213·5	189·8	158·4	182·9	166·7
July ...	162·3	155·7	72·2	140·9	213·5	189·8	158·4	182·9	165·2
Aug. ...	155·5	155·7	74·6	138·5	211·9	189·6	155·8	181·3	163·3
Sept. ...	154·4	155·7	74·6	138·1	212·0	189·6	154·3	180·7	162·7
Oct. ...	154·1	155·7	73·5	137·8	212·3	190·1	154·2	180·9	162·7
Nov. ...	155·0	155·7	74·5	138·3	212·6	190·0	153·0	180·4	162·7
Dec. ...	155·0	155·7	75·3	138·5	211·5	190·0	155·6	181·2	163·2
1946									
Jan. ...	155·4	155·7	77·8	139·2	225·8	190·4	155·0	184·6	165·6
Feb. ...	155·6	155·7	79·6	139·6	225·5	192·0	155·4	185·5	166·2
Mar. ...	156·1	155·7	78·5	139·6	225·3	192·0	157·2	186·3	166·6
April ...	157·9	155·7	78·5	140·4	230·6	192·0	160·1	188·8	168·4
May ...	156·7	153·6	82·3	139·9	230·6	196·1	161·4	190·7	169·3
June ...	157·0	153·6	82·1	140·0	231·6	197·2	161·4	191·4	169·7
July ...	160·7	153·6	84·2	141·9	241·3	207·7	163·7	198·1	174·4
Aug. ...	153·1	154·7	85·2	139·4	247·5	209·1	167·7	201·9	175·7
Sept. ...	154·2	154·7	84·0	139·6	256·7	218·1	175·3	210·3	180·5

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Quarterly Movements of Prices *

Summary of Index Numbers, 1867–77 = 100

Years	Quarters	Vegetable Food (Corn, etc.)	Animal Food (Meat, etc.)	Sugar, Coffee, and Tea	Total Food	Minerals	Textiles	Sundry Materials	Total Materials	Grand Total	Silver †
'34	I	59·4	110·1	53·0	76·7	111·7	73·6	79·9	86·5	**82·4**	19·1
	II	58·7	110·3	52·2	76·1	108·4	70·8	79·1	84·4	**80·9**	18·6
	III	71·0	109·5	48·0	80·4	108·5	70·7	78·7	84·3	**82·6**	19·7
	IV	66·7	107·3	44·8	77·0	109·7	70·9	79·6	85·0	**81·6**	22·1
'35	I	64·4	111·2	41·3	76·8	108·9	77·4	82·6	88·0	**83·3**	23·0
	II	67·1	107·9	42·3	76·9	112·3	79·4	82·8	89·8	**84·3**	29·3
	III	68·8	106·6	40·6	76·8	113·8	80·1	82·3	90·1	**84·5**	27·3
	IV	70·7	104·1	41·8	76·9	116·7	83·6	84·9	93·0	**86·3**	25·7
'36	I	72·1	104·6	41·2	77·6	116·1	84·6	86·4	93·8	**87·0**	18·2
	II	71·2	107·1	39·7	77·8	114·7	79·4	84·8	91·2	**85·5**	18·6
	III	75·3	112·6	39·1	81·5	116·7	80·5	90·0	94·2	**88·8**	18·6
	IV	85·8	110·7	42·6	85·9	129·6	88·2	93·8	101·7	**95·0**	18·9
'37	I	93·1	112·7	47·1	90·6	144·3	97·4	102·3	112·0	**103·0**	18·6
	II	94·3	121·0	49·6	94·7	143·5	99·9	102·9	112·8	**105·2**	18·7
	III	93·3	121·4	50·2	94·6	147·7	94·6	101·4	111·8	**104·5**	18·4
	IV	95·5	116·3	47·2	93·0	136·3	81·2	95·6	102·1	**98·3**	18·0
'38	I	92·3	116·4	43·1	90·8	134·8	77·8	92·4	99·3	**95·7**	18·5
	II	89·2	114·1	42·2	88·5	132·0	73·5	86·5	94·8	**92·1**	17·4
	III	78·0	107·4	42·9	81·4	135·9	73·9	85·0	95·3	**89·4**	17·3
	IV	68·8	105·8	43·3	77·1	140·6	74·3	85·1	96·7	**88·4**	17·4
'39	I	68·0	110·9	43·5	78·7	134·2	79·6	84·0	96·2	**88·8**	17·7
	II	66·7	112·9	46·9	79·6	134·4	85·7	85·3	98·7	**90·6**	17·3
	III	65·2	117·0	48·1	80·7	135·8	91·1	88·0	101·8	**92·9**	15·7
	IV	93·2	130·3	57·4	99·3	146·7	123·5	106·4	122·5	**112·7**	17·8
'40	I	104·2	141·2	57·4	108·0	160·1	141·2	116·1	135·6	**124·0**	16·3
	II	108·0	140·5	56·7	109·2	165·2	148·4	122·2	141·8	**128·0**	16·7
	III	117·3	140·5	58·6	113·5	171·1	148·6	122·2	143·4	**130·9**	17·6
	IV	127·5	140·5	58·8	117·8	173·5	151·9	119·8	144·1	**133·0**	17·9
'41	I	129·8	141·3	61·9	119·8	179·0	154·8	124·7	148·6	**136·4**	18·0
	II	137·9	142·8	65·3	124·4	181·2	161·6	137·2	156·6	**143·0**	18·0
	III	145·0	142·8	65·4	127·4	181·0	165·6	138·5	158·3	**145·3**	18·0
	IV	151·0	142·8	68·9	130·7	181·6	163·1	134·2	155·8	**145·2**	18·1
'42	I	179·6	144·3	69·3	143·4	181·6	161·1	140·6	157·9	**151·8**	18·1
	II	188·6	147·3	69·5	148·3	182·2	160·4	146·1	160·2	**155·2**	18·1
	III	158·4	147·3	71·4	136·0	185·8	158·3	144·1	159·7	**149·7**	18·1
	IV	156·6	151·1	71·4	136·7	186·8	164·9	145·0	162·0	**151·3**	18·1
'43	I	159·1	155·7	72·8	139·7	188·0	163·9	148·4	163·8	**153·5**	18·1
	II	159·9	155·7	73·1	140·1	188·3	165·1	153·5	166·4	**155·3**	18·1
	III	156·1	155·7	74·1	138·7	188·2	163·4	156·0	166·9	**155·0**	18·1
	IV	150·1	155·7	74·2	136·2	187·8	160·8	157·4	166·6	**153·8**	18·1
'44	I	151·6	155·7	73·7	136·7	191·4	161·1	159·6	168·6	**155·2**	18·1
	II	151·8	155·7	74·5	137·0	193·9	180·0	160·2	175·3	**159·1**	18·1
	III	152·2	155·7	75·8	137·4	199·3	183·7	161·4	178·5	**161·2**	18·1
	IV	151·4	155·7	73·7	136·7	203·9	184·6	155·4	177·5	**160·2**	19·6
45	I	154·8	155·7	73·7	138·0	204·8	186·0	157·3	178·9	**161·7**	19·5
	II	160·5	155·7	74·6	140·6	209·4	189·8	158·3	181·7	**164·4**	20·2
	III	157·7	155·7	73·8	139·2	212·5	189·7	156·2	181·6	**163·7**	31·1
	IV	154·7	155·7	74·4	138·2	212·1	190·0	154·3	180·8	**162·9**	

* The averages of the four quarterly figures to each year do not necessarily coincide with the annual averages, as the latter are based as far as possible on average weekly prices. See also the *Journal*, 1893, p. 221; 1895, p. 244; 1901, p. 90; and 1909, p. 70.

† Silver, parity of 1 gold to 15½ silver = 100.

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Construction of the Tabular Statements

The following table illustrates the method of construction of the index numbers. The index numbers here given are based on the average prices for the eleven years 1867–77. Take for instance, the *Gazette* price of English wheat:—

		s.	d.			
Average, 1867–77	... ...	54	6 = 100, average point.			
,, 1914	... ...	35	0 = 64, or 36 per cent.	*below*	the average point.	
,, 1930	... ...	80	7 = 148, ,, 48	,,	*above*	,, ,,
,, 1936	... ...	53	3 = 98, ,, 2	,,	*below*	,, ,,

The individual index numbers, therefore, represent simple percentages of the average point. The articles are grouped in six categories:—

	Index Nos.	1867–77 Total Numbers	Example for 1945	
			Total Numbers	Average
1. Vegetable food, corn, etc. (wheat flour, barley, oats, maize, potatoes, and rice)	8	800	1,241	155
2. Animal food (beef, mutton, pork, bacon, and butter)	7	700	1,090	156
3. Sugar, coffee, and tea	4	400	314	78
1–3. *Food*	19	1,900	2,645	139
4. Minerals (iron, copper, tin, lead, and coal)	7	700	1,462	209
5. Textiles (cotton, flax, hemp, jute, wool, and silk) ...	8	800	1,515	189
6. Sundry materials (hides, leather, tallow, oils, soda, nitrate, indigo, and timber)	11	1,100	1,753	159
4–6. *Materials*	26	2,600	4,730	182
General Average	45	4,500	7,375	164

The general average is drawn from all forty-five descriptions which are treated as of equal value, and is the simple arithmetic mean as shown above.

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Index of Silver Prices

The base of the index numbers given below is 60·84*d.* per standard oz. = 100 this being a parity of 1 fine oz. of gold to 15½ standard ozs. of silver.*

	Price per oz. standard	Index number		Price per oz. standard	Index number
	d.			*d.*	
Average 1873	59¼	=97·4	*Lowest* Nov., 1902	21 11/16	=35·6
,, '90–99	34	=55·8	End Dec., 1906	32 5/16	=53·1
,, 1917–26	40½	=66·6	,, Dec., '08	23 3/16	=38·1
,, 1893	35 5/8	=58·6	,, Dec., '12	29	=47·7
,, '96	30¾	=50·5	,, Dec., '13	26 9/16	=43·7
,, 1914	25 5/16	=41·6	,, Dec., '14	22 11/16	=37·3
,, '15	23 11/16	=38·9	,, Dec., '15	26¼	=43·1
,, '16	31 5/16	=50·4	,, Dec., '16	36½	=58·7
,, '17	40 7/8	=65·8	,, Dec., '17	43½	=70·0
,, '18	47½	=76·4	,, Dec., '18	48 7/16	=77·9
,, '19	57	=85·3	,, Dec., '19	77¼	=98·3
,, '20	61 9/16	=76·1	,, Dec., '20	40 7/8	=49·2
,, '21	36 7/8	=48·1	,, Dec., '21	34 5/8	=49·3
,, '22	34 7/16	=51·6	,, Dec., '22	31 9/16	=49·6
,, '23	31 15/16	=49·4	,, Dec., '23	33 7/16	=49·0
,, '24	34	=50·7	,, Dec., '24	31 13/16	=50·4
,, '25	32 1/8	=52·5	,, Dec., '25	31 11/16	=52·1
,, '26	28 11/16	=47·1	,, Dec., '26	25	=41·1
,, '27	26 1/32	=42·8	,, Dec., '27	26½	=43·6
,, '28	26¾	=44·0	,, Dec., '28	26⅜	=43·3
,, '29	24 7/16	=40·2	,, Dec., '29	21 7/16	=35·2
,, '30	17 11/16	=29·0	,, Dec., '30	14 7/16	=23·7
,, '31	14⅘	=20·4	,, Dec., '31	20 7/16	=21·6
,, '32	17½	=19·5	,, Dec., '32	16½	=17·2
,, '33	18⅛	=18·7	,, Dec., '33	19 1/16	=19·5
,, '34	21 7/32	=20·0	,, Dec., '34	24 5/8	=22·6
,, '35	29	=26·4	,, Dec., '35	22½	=20·6
,, '36	20 1/16	=18·5	,, Dec., '36	21 5/16	=19·4
,, '37	20 1/16	=18·4	,, Dec., '37	19 1/16	=17·7
,, '38	19 17/32	=17·6	,, Dec., '38	20 1/16	=17·3
,, '39	20 3/32	=17·1	,, Dec., '39	22½	=17·3
,, '40	22 11/64	=17·1	,, Dec., '40	23 3/16	=17·9
,, '41	23 7/16	=18·0	,, Dec., '41	23½	=18·1
,, '42	23½	=18·1	,, Dec., '42	23½	=18·1
,, '43	23½	=18·1	,, Dec., '43	23½	=18·1
,, '44	23½	=18·1	,, Dec., '44	23½	=18·1
,, '45	30½	=23·1	,, Dec., '45	44	=31·1

* All the index numbers in the table from 1916 to 1925 inclusive and from 1931 to date are calculated on the basis of the gold prices of silver instead of the sterling prices, though the latter are the price quotations given in the table. In arriving at the index numbers for these dates the prices of gold are taken as follows. For 1916, 1917 and 1918 the price is taken as 86*s.* 9½*d.* per fine oz., derived from the " pegged " New York rate of $4·76½ to the £. For 1919 the average price of gold is taken as 93*s.* 4¼*d.*, this being the parity price with the U.S. dollar, the average New York exchange in that year being $4·429. For the other dates the index numbers are based on the quotations in the London market for exportable gold. The quotation at the end of 1919 was 109*s.* 8¼*d.* per fine oz. The prices at the end of the years 1931–38 were 121*s.* 11*d.*, 123*s.* 9*d.*, 126*s.* 6*d.*, 141*s.* 0*d.*, 141*s.* 2*d.*, 141*s.* 7*d.*, 139*s.* 6*d.*, 149*s.* 7½*d.*, for the years 1939–44 168*s.* respectively, and 1945 172*s.* 3*d.* and the average prices in these years were 92*s.* 6¼*d.*, 118*s.* 0¾*d.*, 124*s.* 10½*d.*, 137*s.* 7¾*d.*, 124*s.* 1¼*d.*, 140*s.* 3½*d.*, 140*s.* 8¾*d.*, 142*s.* 6½*d.*, 154*s.* 4*d.*, 168*s.*, 168*s.*, 168*s.*, 168*s.* and 170*s.* 4*d.* respectively.

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World's Production of Silver (in millions of ounces)

	United States	Mexico	Canada	Australia	Other Countries	Total
1904	57·7	60·8	3·7	14·5	27·5	164·2
'05	56·1	65·0	5·9	15·0	30·3	172·3
'06	56·5	55·2	8·5	14·2	30·6	165·0
'07	56·5	61·0	12·8	19·0	34·8	184·2
'08	52·4	73·6	22·1	17·2	37·8	203·1
'09	54·7	73·9	27·5	16·3	39·7	212·1
'10	57·1	71·4	32·9	21·5	38·8	221·7
'11	60·4	79·0	32·7	16·6	37·5	226·2
'12	63·8	74·6	31·6	18·1	36·2	224·3
'13	66·8	70·7	31·5	3·5	51·4	223·9
'14	72·4	27·5	28·4	3·6	36·5	168·4
'15	74·9	39·5	28·4	4·1	37·3	184·2
'16	74·4	38·2	25·4	4·2	26·6	168·8
'17	71·7	35·0	22·2	10·0	35·3	174·2
'18	67·8	62·5	21·2	10·0	35·9	197·4
'19	56·7	62·7	15·7	7·4	32·0	174·5
'20	55·5	66·8	12·6	7·5	33·0	175·4
'21	53·1	64·5	13·1	4·9	35·7	171·3
'22	56·2	81·1	18·6	11·3	46·3	213·5
'23	73·3	90·9	17·8	13·3	50·7	246·0
'24	65·3	91·5	19·7	10·8	52·2	239·5
'25	66·1	92·9	20·2	11·1	54·8	245·1
'26	62·7	98·3	22·4	11·2	59·0	253·6
'27	60·4	104·6	22·7	9·0	57·3	254·0
'28	58·4	108·5	21·9	9·0	59·5	257·3
'29	61·2	108·7	23·1	9·0	59·7	261·7
'30	51·0	105·0	26·0	8·9	57·1	248·0
'31	31·0	86·0	21·0	7·6	50·4	196·0
'32	24·0	69·0	18·0	6·5	47·5	165·0
'33	22·8	68·1	15·2	11·0	52·0	169·1
'34	32·5	74·1	16·4	10·8	56·6	190·4
'35	45·6	75·6	16·6	11·4	71·5	220·7
'36	63·4	77·5	18·3	12·7	81·8	253·7
'37	71·3	84·7	22·7	14·3	80·9	273·9
'38	61·7	81·0	23·8	101·3		267·8
'39	63·9	75·9	24·5	101·6		265·9
'40	68·3	82·6	25·3	96·3		272·5
'41	71·1	78·4	23·4	90·1		263·0
'42	55·9	84·9	21·8	—		—
'43	44·8	71·2	18·5	—		—
'44	37·3	63·0	14·7	—		—
'45 *	28·3	60·0	14·0	—		—

* Provisional. (Estimate by Messrs. Samuel Montagu & Co.)

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(000's omitted)

Year	Value of output £	Year	Value of output £	Year	Value of output £
1851	17,200	1883	20,640	1915	97,114
'52	26,550	'84	20,830	'16	92,597
'53	31,090	'85	21,250	'17	87,236
'54	25,490	'86	21,430	'18	78,605
'55	27,015	'87	21,735	'19	73,078
'56	29,520	'88	22,644	'20	68,522
'57	26,655	'89	25,375	'21	67,848
'58	24,930	'90	24,421	'22	66,723
'59	24,970	'91	26,846	'23	77,888
'60	23,850	'92	30,134	'24	81,807
'61	22,760	'93	32,363	'25	82,267
'62	21,550	'94	37,229	'26	82,211
'63	21,390	'95	40,843	'27	82,582
'64	22,600	'96	41,559	'28	82,400
'65	24,040	'97	48,509	'29	84,500
'66	24,220	'98	58,949	'30	88,500
'67	22,805	'99	63,027	'31	95,100
'68	21,945	1900	52,312	'32	103,400
'69	21,245	'01	53,630	'33	107,700
'70	21,370	'02	60,975	'34	116,000
'71	25,400	'03	67,337	'35	125,700
'72	24,200	'04	71,380	'36	140,900
'73	23,600	'05	78,143	'37	148,700
'74	22,950	'06	82,707	'38	159,000
'75	22,700	'07	84,857	'39	165,900
'76	22,540	'08	90,995	'40	173,400
'77	23,830	'09	93,302	'41	170,400
'78	22,020	'10	93,544	'42	148,800
'79	21,400	'11	94,930	'43	123,300
'80	22,130	'12	95,783	'44	114,800
'81	21,150	'13	97,481	'45 (provisional)	112,600
'82	20,500	'14	92,709		

Gold—The table shows the world's annual gold production since 1851. Prior to 1911 the estimates are those of the Bureau of the U.S. Mint and other authorities. The estimates since 1926 are those of the Union Corporation, Limited. The value is taken throughout at £4·25 per fine oz.

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Average Prices of Commodities *

No. of Article / Year	0	1	2	3	4	5	6	7	8	1-8	9	10
		Wheat		Flour	Barley	Oats	Maize §	Potatoes *	Rice	Vegetable Food	Beef ‡	
	Silver †	English *Gazette*	American	Town Made white (now "G.R.")	English *Gazette*	English *Gazette*	American Mixed	Good English	Rangoon Cargoes to Arrive	Total	Prime	Middling
	d. per oz.	*s.* and *d.* per qr.	*s.* and *d.* per qr.	*s.* per sack (280 lbs.)	*s.* and *d.* per qr.	*s.* and *d.* per qr.	*s.* per qr.	*s.* per ton	*s.* and *d.* per cwt.		*d.* per 8 lbs.	*d.* per 8 lbs.
1922 ...	34 7/16	47·10	52·11	45 3/4	40·1	29·1	31 1/4	130	14·10	—	88 6/8	82
'24 ...	34	49·3	53·9	43 5/6	46·9	27·2	39 2/3	186	16·9	—	82 1/3	76 1/3
'25 ...	32 1/8	52·2	62·4	50 1/8	42·0	27·2	38 9/16	154	16·0	—	80	73 1/2
'26 ...	28 11/16	53·3	58·9	49 1/4	36·11	25·1	29 13/16	127	16·3	—	74	67
'27 ...	26 1/32	49·3	58·3	44 9/16	42·0	25·4	30 1/16	136	15·11	—	70	62
'28 ...	26 3/4	44·8	50·10	40 1/6	39·0	29·0	38 15/32	133	15·0	—	74	66 1/2
'29 ...	24 7/16	42·2	51·3	38 1/2	35·5	24·7	36 1/4	111	14·3	—	71	66
'30 ...	17 11/16	34·3	36·10	33 1/12	28·3	17·2	23	93	13·0	—	73	68
'31 ...	14 5/6	24·0	25·1	22 7/8	28·0	17 8	15 9/16	146	9·8	—	67	61
'32 ...	17 7/32	25·0	27·5	24 7/32	27·1	19·3	18 13/16	152	9·8	—	65	59
'33 ...	18 1/8	22·10	25·7	23 9/32	28·7	15·10	17 1/16	86	7·9	—	61	52
'34 ...	21 7/32	20·2	28·0	23 1/8	30·11	17·5	19 1/32	97	7·8	—	58	52
'35 ...	29	22·2	31·1	25 3/4	28·7	18·9	17 1/4	107	8·10	—	54	49
'36 ...	20 1/16	30·9	35·1	31 17/32	29·5	17·8	19 1/2	146	9·0	—	54	50
'37 ...	20 1/16	40·0	49·7	40 1/3	39·0	23·11	26 9/16	136	10·5	—	61	57
'38 ...	19 17/32	28·11	39·3	30 21/32	36·4	21·2	28 1/16	111	10·7	—	62	58
'39 ...	20 3/32	21·5	30·1	22 9/32	31·7	19·3	26 1/4	117	11·1	—	61	58
'40 ...	22 19/64	42·10	33·6	24 5/16	64·10	37·2	39 13/16	143	15·5	—	72	68
'41 ...	23 7/16	62·10	32·2	27 1/2	85·8	40·10	43 ‖	164	23·8	—	72	68
'42 ...	23 1/8	68·6	36·9	35	165·5	42·0	43	134	26·0	—	76	68
'43 ...	23 1/2	69·8	45·5	38 1/4	112·5	43·8	43 ‖	111	27·2	—	79	71
'44 ...	23 1/2	63·11	56·0	38 3/4	94·6	45·3	43 ‖	115	27·2	—	79	71
'45 ...	30 1/2	61·10	63·7	40	89·2	45·9	43 ‖	139	27·2	—	79	71
Average 1904–13	26 3/4	31 1/2	36	30	25 1/2	18 1/2	24 1/4	78	7 1/4	—	51	44 1/2
1890–99	34	28 1/2	31 1/2	27 1/2	25 1/2	17 1/2	19 1/2	72	6 1/4	—	47	37 1/2
'78–87	50	40	43 1/2	34 1/2	31 1/2	21	25	102	8	—	55 1/2	46
'67–77	58 1/2	54 1/2	56	46	39	26	32 1/4	117	10	—	59	50
	Index Numbers (or Percentages) of Prices, the Average of 1867–77 being 100											
1922 ...	51·6	88	95	100	103	112	96	111	148	853	150	164
'24 ...	50·7	90	96	95	120	105	122	159	167	954	139	152
'25 ...	52·5	96	111	109	108	105	119	132	160	940	136	147
'26 ...	47·1	98	105	107	95	96	92	109	163	865	125	134
'27 ...	42·8	90	104	98	108	97	95	116	159	867	119	124
'28 ...	44·0	82	91	87	100	112	118	114	150	854	125	133
'29 ...	40·2	77	91	84	91	95	112	95	143	788	120	132
'30 ...	29·0	63	66	72	72	66	71	79	130	619	124	136
'31 ...	20·4	44	45	50	71	68	48	125	93	544	114	122
'32 ...	19·5	46	49	53	69	74	58	130	93	572	110	118
'33 ...	18·7	42	46	52	73	61	53	74	78	479	103	104
'34 ...	20·0	37	50	50	79	67	60	83	77	503	98	104
'35 ...	26·4	41	56	56	73	72	53	91	88	530	92	98
'36 ...	18·5	56	63	69	75	68	60	125	90	606	92	100
'37 ...	18·4	73	89	88	100	92	82	116	105	745	103	114
'38 ...	17·6	53	70	67	93	81	86	95	106	651	105	116
'39 ...	17·1	39	54	48	81	74	82	100	111	589	103	116
'40 ...	17·1	78	60	53	166	143	122	122	154	898	122	136
'41 ...	18·0	115	57	60	220	157	132	140	236	1,117	122	136
'42 ...	18·1	126	66	76	424	162	132	115	260	1,361	129	136
'43 ...	18·1	128	81	83	288	168	132	95	272	1,247	134	142
'44 ...	18·1	117	100	84	242	174	132	98	272	1,219	134	142
'45 ...	23·1	113	114	87	229	176	132	118	272	1,241	134	142

* The annual prices are the average monthly or weekly quotations, except potatoes, which are the average weekly quotations during the eight months January to April and September to December.
† Not included in the general average.
‡ Meat (9—13), by the carcase, in the London Central Meat Market.
§ La Plata from 1924.
‖ Argentine maize (Feeding Stuffs): £10 per ton fixed by Ministry of Food.

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Average Prices of Commodities—Contd.

No. of Article	11	12	13	14	15	9–15	16A	16B	17	18A *	18B *	18
	Mutton		Pork	Bacon	Butter		Sugar			Coffee		
Year	Prime	Middling	Large and Small, average	Waterford	Friesland, Fine to Finest	Animal Food Total	British West Indian Refining‖	Beet, German, 88 p. c., f.o.b.	Java, Floating Cargoes §	Ceylon Plantation, Low Middling†	Rio, Good	Mean of 18A and 18B
	d. per 8 lbs.	*d.* per 8 lbs.	*d.* per 8 lbs.	*s.* per cwt.	*s.* per cwt.		*s.* per cwt.	*s.* per cwt.	*s.* per cwt.	*s.* per cwt.	*s.* per cwt.	
1922 ...	125	121 3/8	101	145 1/3	202 1/2	—	15	14 1/8	15 5/16	120 2/8	74 1/3	—
'24 ...	111 1/2	103 1/2	70	106	211	—	23 3/8	20 1/4	21 1/2	152 5/8	85 1/3	—
'25 ...	106 3/8	98 5/8	84 1/2	128 1/2	206 7/8	—	16 1/4	11 5/8	12 1/3	153 13/16	98 5/16	—
'26 ...	89	80 1/2	98 5/8	130	173	—	16 17/32	11 2/3	12 5/8	154 2/3	89 1/16	—
'27 ...	86	79 5/8	85	102 1/4	178	—	16 7/16	12 1/2	13 9/16	143 1/2	71 1/2	—
'28 ...	92 1/4	87	77	101 3/4	185 7/8	—	13 1/2	10 11/16	11 1/2	143 0/32	81 1/32	—
'29 ...	89 1/2	83	91	116 1/32	180 1/8	—	11 7/32	8 5/16	8 7/8	141 1/8	74 27/32	—
'30 ...	92	86	89	105 5/8	146 13/16	—	8 5/8	5 27/32	6 13/32	106 23/32	42 11/32	—
'31 ...	79	73	65	83 3/8	130	—	7 13/16	5 11/16	6 7/16	101 1/4	33 15/16	—
'32 ...	63	55	54	77	126 3/8	—	7 1/2	5 5/16	5 10/32	105 3/32	54 5/16	—
'33 ...	69	63	60	81 1/3	105 9/16	—	7 5/32	4 23/32	5 1/4	86 25/32	42 1/8	—
'34 ...	74	70	65	90 9/32	79 5/8	—	6 5/32	4 5/32	4 11/16	87 7/32	42 13/16	—
'35 ...	75	70	62	89	92 5/8	—	6 3/16	3 21/32	4 3/4	67 9/16	29 21/32	—
'36 ...	73	68	65	93 27/32	98 21/32	—	6 11/16	3 1/32	4 3/4	58 1/2	30 21/32	—
'37 ...	78	74	68	94	108 3/32	—	7 31/32	5 1/4	6 1/2	75 3/4	36 3/4	—
'38 ...	62	56	69	97 3/16	114 9/16	—	7 1/32	4 5/16	5 15/32	75	19 3/4	—
'39 ...	68	64	70	97 3/4	122	—	9 3/32	6 1/32	7 7/32	73 3/4	22 1/2	—
'40 ...	85	76	96	114 7/8	143	—	9 3/4	—	8 1/8	86 5/16	28 3/32	—
'41 ...	85	76	96	123 11/16	142 11/16	—	9 3/4	—	8 1/8	137 1/2	30 1/4	—
'42 ...	90	78	101	130	143 1/2	—	9 3/4 ‡	—	8 1/8 ‡	130 ‡	28 3/4 ‡	—
'43 ...	96	84	102	142	151 1/3	—	9 3/4 ‡	—	8 1/8 ‡	140 ‡	37 ‡	—
'44 ...	96	84	102	142	151 1/3	—	9 3/4 ‡	—	8 1/8 ‡	140 ‡	37 ‡	—
'45 ...	96	84	102	142	151 1/3	—	9 3/4 ‡	—	8 1/8 ‡	140 ‡	37 ‡	—
Average												
1904–13	58 1/4	51 3/4	47 5/8	67	113	—	10 1/8	10 7/8	12	75 1/2	43 3/4	—
1890–99	54 1/2	41 1/2	42 3/4	59	100	—	11 1/2	11 1/2	13 3/4	98	62	—
'78–87	64 1/2	53	49	71	116	—	17	18	21 1/2	78	52	—
'67–77	63	55	52	74	125	—	23	24	28 1/2	87	64	—
Index Numbers (or Percentages) of Prices, the Average of 1867–77 being 100												
1922 ...	199	221	194	196	162	1,286	62		54	140	116	128
'24 ...	177	188	135	143	169	1,103	93		75	175	133	154
'25 ...	169	180	162	174	165	1,133	60		43	176	154	165
'26 ...	141	146	190	176	138	1,050	60		44	178	139	159
'27 ...	136	145	163	138	142	967	62		47	165	112	139
'28 ...	146	158	148	137	149	996	51		40	165	127	146
'29 ...	142	151	175	157	144	1,021	42		31	162	117	140
'30 ...	146	155	171	143	117	992	31		22	123	66	95
'31 ...	125	133	125	113	104	836	29		23	120	53	87
'32 ...	100	100	104	104	101	737	27		20	121	85	103
'33 ...	110	114	115	110	84	740	25		18	100	66	83
'34 ...	117	127	125	122	64	757	22		16	100	67	84
'35 ...	119	127	119	120	74	749	21		17	78	46	62
'36 ...	116	124	131	127	79	763	21		17	67	48	58
'37 ...	124	135	125	127	86	820	28		23	87	57	72
'38 ...	98	102	133	131	92	777	24		19	86	31	59
'39 ...	108	116	135	132	98	808	32		26	85	35	60
'40 ...	135	138	185	155	114	985	42		29	99	44	72
'41 ...	135	138	185	167	114	997	42		29	158	47	102
'42 ...	143	142	194	176	115	1,035	42		29	150	45	98
'43 ...	152	153	196	192	121	1,090	42		29	161	58	109
'44 ...	152	153	196	192	121	1,090	42		29	161	58	109
'45 ...	152	153	196	192	121	1,090	42		29	161	93	127

‡Nominal.

* Index numbers not included in general average.
† E. India good middling from 1908. § Raw Centrifugals, 96% Pol., from 1924.
‖ White Javas, C.I.F., from 1924.

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Average Prices of Commodities—Contd.

No. of Article	19A *	19B *	19	16–19	1–19	20A	20B	21	22	23	24	25
	Tea			Sugar, Coffee, and Tea	Food	Iron			Copper	Tin	Lead	Coal
Year	Congou, Common	Average Import Price	Mean of 19A and 19B	Total	Total	Scottish Pig	Cleveland (Middlesbrough) Pig	Bars, Common	Standard	Straits	English Pig	Wallsend Hetton in London §
	d. per lb	*d.* per lb.				*s.* and *d.* per ton	*s.* and *d.* per ton	per ton	£ per ton	£ per ton	£ per ton	*s.* per ton
1922 ...	8 5/8	14·9	—	—	—	99·10	90·7	11 1/2	63 1/2	162	25 1/2	34 2/6
'24 ...	9 1/4	19·0	—	—	—	96·8	88·2	12 1/2	63 3/16	251	35 13/16	27 1/2
'25 ...	7 7/8	18·34	—	—	—	83·4	72·8	11 7/8	61 13/16	267	37 7/16	29 5/6
'26 ...	7 13/16	18·82	—	—	—	87·2	87·6	11 1/2	58 1/32	297 9/16	32 1/8	†30 7/12
'27 ...	6 3/4	18·58	—	—	—	80·5	73·0	11 1/2	55 3/8	303 7/8	25 9/16	23 1/10
'28 ...	6 5/16	16·84	—	—	—	69·9	65·9	9 15/16	63 3/4	229 7/16	22 5/16	21 7/10
'29 ...	6 1/16	16·11	—	—	—	74·0	70·3	9 3/4	75 9/16	207 7/16	24 1/32	23 3/4
'30 ...	5 7/32	15·12	—	—	—	76·0	67·0	9 31/32	54 3/4	144 23/32	19 1/2	24 23/32
'31 ...	4 13/16	13·29	—	—	—	71·0	58·6	10 3/32	38 1/8	121 1/8	14 9/32	24 5/8
'32 ...	4 9/32	10·75	—	—	—	68·2	58·6	10	31 21/32	140	13 5/16	23 11/32
'33 ...	6 31/32	11·87	—	—	—	66	62·3	9 11/16	32 21/32	202 1/2	13 1/4	22 5/8
'34 ...	8 21/32	13·20	—	—	—	69 6	66·11	9 17/32	30 9/32	232 1/4	12 3/4	20 1/4
'35 ...	6 23/32	13·06	—	—	—	70·6	67·10	9 5/8	32 15/32	230 15/32	16	20 9/32
'36 ...	6 1/4	13·19	—	—	—	78·6	73·2	10 3/16	37 13/32	207 21/32	19 15/32	23 3/32
'37 ...	6 9/16	14·58	—	—	—	104·6	94·4	12 5/16	54 21/32	246 1/4	24 13/32	24 1/2
'38 ...	6 15/32	14·04	—	—	—	118·0	109	13 1/4	41 21/32	193 31/32	17 1/16	25 11/16
'39 ...	6 1/4 ‖	14·18	—	—	—	104·3	100·7	12 3/8	44 1/4	232 17/32	17 3/8	25 1/2
'40 ...	—	15·33	—	—	—	114·10	116·4	14 5/16	62	273 5/16	26 1/2	28 1/16
'41 ...	—	15·13	—	—	—	123	128	15 5/8	62	284 5/16	26 1/2	30 2/12
'42 ...	—	16·25	—	—	—	123	128	15 5/8	62	275	26 1/2	32 1/2
'43 ...	—	18·44	—	—	—	123	128	15 5/8	62	275	26 1/2	34 16/16
'44 ...	—	19·48	—	—	—	123	128	16 1/2	62	300	26 1/2	38 5/6
'45 ...	—	20·06	—	—	—	139 15/16	140 1/2	18	62	300	29 1/6	42 2/6
Average 1904–13	7 7/8	8 1/8	—	—	—	57 3/4	51 1/2	6 7/8	67 3/4	164 7/8	15 5/8	18 1/6
1890–99	4 5/8	9 3/4	—	—	—	47	41 1/2	5 1/2	50	81	12	17 1/2
'78–87	6 3/4	12 3/4	—	—	—	46	38	5 1/2	55	89	14	16 3/4
'67–77	11 1/4	17 1/4	—	—	—	69	60	8 1/4	75	105	20 1/2	22

Index Numbers (or Percentages) of Prices, the Average of 1867–77 being 100

1922 ...	77 *	86 *	82	326	2,465	148	136	84	154	123	156
'24 ...	82	110	96	418	2,475	143	152	84	239	175	125
'25 ...	70	106	88	356	2,429	121	144	82	254	183	135
'26 ...	69	109	89	352	2,267	135	139	77	283	157	138
'27 ...	60	108	84	332	2,166	119	136	74	289	125	105
'28 ...	56	98	77	314	2,164	105	120	85	219	109	97
'29 ...	54	93	74	287	2,096	112	118	101	198	117	106
'30 ...	46	88	67	215	1,826	111	121	73	138	95	113
'31 ...	42	78	60	199	1,579	100	123	52	115	71	112
'32 ...	38	62	50	200	1,509	98	121	43	131	65	106
'33 ...	58	68	63	189	1,408	99	117	44	193	65	103
'34 ...	77	77	77	199	1,459	106	116	40	221	61	92
'35 ...	60	76	68	168	1,447	107	117	43	219	78	92
'36 ...	56	76	66	162	1,531	118	123	50	198	95	105
'37 ...	58	85	72	195	1,760	154	149	73	235	121	111
'38 ...	58	81	70	172	1,600	176	161	56	185	83	117
'39 ...	56	82	69	187	1,584	159	150	59	221	85	115
'40 ...	—	89	89	232	2,115	179	173	83	260	129	127
'41 ...	—	88	88	261	2,375	195	189	83	271	129	141
'42 ...	—	94	94	263	2,659	195	189	83	262	129	148
'43 ...	—	107	107	287	2,624	195	189	83	262	129	159
'44 ...	—	113	113	293	2,602	195	200	83	286	129	176
'45 ...	—	116	116	314	2,645	217	218	83	286	142	194

* Index numbers not included in the general average.
§ Best Yorkshire house after 1916.
† Average price January–April, 1926.
‖ First 9 months only.

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Average Prices of Commodities—Contd.

No. of Article	26	20–26	27	28	29A	29B	30A	30B	31	32A	32B	33
	Coal	Minerals	Cotton		Flax		Hemp		Jute	Wool		
Year	Average Export Price	Total	Middling American	Fair Dhollerah ‖	Petrograd ¶	Russian Average Import Price	Manila Fair Roping	Petrograd Clean (*a*)	Good Medium††	Merino, Port Phillip, Average Fleece	Merino, Adelaide, Average Greasy	English Lincoln Half Hogs
	s. per ton		*d.* per lb.	*d.* per lb.	£ per ton	£ per ton	£ per ton	£ per ton	£ per ton	*d.** per lb.	*d.* per lb.	*d.* per lb.
1922 ...	24·16	—	12·10	8	95	84 7/8	33 1/2	57 ‡	30 3/4	39	17 1/8	9 3/4
'24 ...	23·28	—	16·26	11·03	120	104 1/2	44	81	31 3/4	53 7/16	25 1/2	18 7/8
'25 ...	20·08	—	12·64	11·01	92 1/8	120 3/4	46 1/8	89 1/4	49 9/16	41 1/16	17 3/8	17 1/8
'26 ...	18·59	—	9·40	7·75	65	72 15/16	43	74 5/8	43 7/8	36 1/2	16 7/16	15
'27 ...	17·80	—	9·54	8·27	95 13/16	74 1/2	43 3/8	66 5/16	32 3/4	38 1/16	17 7/16	15 1/32
'28 ...	15·67	—	10·92	8·66	98 1/2	91 15/16	37 11/16	63 1/32	33 13/16	37	17 5/16	17 3/32
'29 ...	16·13	—	10·26	7·73	76 7/16	71 7/32	37 7/32	61	32	35 3/8	13 3/32	16 1/16
'30 ...	16·64	—	7·49	5·12	53 1/8	60 7/32	26 1/2	48 7/32	20	18 9/32	8 1/32	10 3/4
'31 ...	15·98	—	5·90	4·60	36	35 9/16	18 3/4	27 1/2	15 15/16	14·7	7·1	8 1/2
'32 ...	16·27	—	5·24	4·85	45 7/32	42 5/32	18 23/32	36	16 3/32	15·0	7·2	5 25/32
'33 ...	16·08	—	5·54	4·53	51 1/32	48 15/32	15 7/8	37	14 7/8	19·9	9·3	5 9/32
'34 ...	16·08	—	6·70	4·80	60 11/16	50 9/32	14 9/32	42 5/16	14 3/32	21 1/4	10·4	7
'35 ...	16·30	—	6·71	5·42	79 3/8	72 5/16	19 1/32	43 1/4	16 15/16	20·1	9·5	7 1/32
'36 ...	16·98	—	6·71	5·12	63 25/32	60 15/16	28 1/32	42 27/32	17 9/32	24·7	12·2	10 5/32
'37 ...	19·05	—	6·21	4·80	78 1/32	70 3/32	34 5/16	38 3/8	19 11/16	26·9	12·7	16·9
'38 ...	21·32	—	4·93	3·67	66 9/32	63 1/2	21 3/32	38 5/16	17 21/32	18·6	8·9	11·9
'39 ...	21·12	—	5·95	4·41	90 21/32	72 9/16	22 5/16	48 5/16	26 1/32	17·9	9·0	12·2
'40 ...	27·23	—	8·10	6·26	177 1/32	183 1/32	26 3/8	100 1/32	27 3/4	29·6	14·9	19·4
'41 ...	32·22	—	9·14	7·65	200 ‡	—	31 13/16	126 ‡	26 1/32	32	16 1/8	21
'42 ...	34·87	—	8·83	7·37	200 ‡	201·6	33 ‡	130 ‡	24 3/4	32	16 1/8	21
'43 ...	36·91	—	7·83	6·38	200 ‡	205 ‡	33 ‡	130 ‡	33 1/8	32	16 1/8	21
'44 ...	39·19	—	11·32	9·50	200 ‡	205 ‡	40 1/2 ‡	130 ‡	40 1/4	32	16 1/4	21
'45 ...	40·27	—	12·75	10·80	200 ‡	230 (*b*)	40 1/2 ‡	130 ‡	39 1/4	31·6	15·9	21·1
Average												
1904–13	11 7/8	—	6 1/2	5	32 1/2	36 7/8	30 3/8	31 1/8	18 7/8	17 3/8	9	10 7/8
1890–99	10 3/8	—	4 1/4	3	27	27	26 1/8	25	12 1/2	13 1/2	6 1/2	10
'78–87	9	—	6	4 1/4	33	34	35 1/2	26 1/2	15	18 1/2	8 3/8	11 3/4
'67–77	12 1/2	—	9	6 3/4	46	48	43	35	19	21 1/4	9 7/8	19 3/4
	Index Numbers (or Percentages) of Prices, the Average of 1867–77 being 100											
1922 ...	193	994	134	118	191		116		162	180		49
'24 ...	187	1,105	181	163	239		160		167	254		96
'25 ...	161	1,080	140	163	227		174		261	188		87
'26 ...	149	1,078	104	115	147		151		231	170		76
'27 ...	142	990	106	123	181		141		172	177		78
'28 ...	125	860	121	128	203		130		178	174		91
'29 ...	129	881	114	114	157		126		168	156		81
'30 ...	133	784	83	76	121		96		105	86		54
'31 ...	127	700	66	68	76		58		84	70		43
'32 ...	130	694	58	72	93		70		85	71		29
'33 ...	129	750	62	67	106		68		78	94		30
'34 ...	129	765	74	71	119		73		74	102		35
'35 ...	130	786	74	80	161		80		89	96		37
'36 ...	136	825	74	76	133		91		93	119		53
'37 ...	152	995	69	71	158		93		104	127		86
'38 ...	171	949	55	54	139		77		93	88		60
'39 ...	169	952	66	65	174		91		140	86		62
'40 ...	218	1,169	90	93	383		163		146	142		98
'41 ...	258	1,266	101	113	416		202		139	155		106
'42 ...	279	1,285	98	109	427		209		130	155		106
'43 ...	295	1,312	87	95	431		209		174	155		106
'44 ...	314	1,383	126	141	431		219		212	155		106
'45 ...	322	1,462	142	160	457		219		207	153		107

* Port Philip fleece washed nominal sinec 1895, exactly in proportion with the value of clean wool.
‡ Nominal. ‖ Now No. 1 Oomra, Fine.
¶ Livonian Z.K. from 1921. †† Lightnings from 1931 to 1942. Daisee ⅜ from 1943.
(*a*) Russian Siretz Group 1, Sort 1 from 1931–33; Jugo-Slav Peasant from 1934.
(*b*) Average of 4 months only.

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Average Prices of Commodities—Contd.

No. of Article / Year	34 Silk	27–34 Textiles	35A Hides	35B Hides	35C Hides	36A Leather	36B Leather	37 Tallow	38 Oil	39 Oil	40A Oil	40B Seeds
	Tastlee *	Total	River Plate, Dry	River Plate, Salted	Average Import Price	Dressing Hides	Average Import Price	Town	Palm	Olive	Linseed	Linseed
	s. per lb.		*d.* per lb.	*d.* per lb.	*d.* per lb.	*d.* per lb.	*d.* per lb.	*s.* per cwt.	£ per ton	£ per ton	£ per ton	*s.* per qr.
1922 ...	28¾	—	9⅛	8⅞	8·06	24⅛	36	34⅝	34⅗	75½ †	39⅕	75½
'24 ...	23 7/16	—	10 3/64	8⅝	8·63	22 13/16	33⅞	42⅜	40 3/16	79 15/16	42⅝	81 3/16
'25 ...	18⅛	—	11 7/16	8 13/16	9·87	23	33	42¼	40⅝	73⅜	43¼	80 9/16
'26 ...	15 15/16	—	10 5/16	8	9·32	21⅛	35 11/16	38⅛	37⅝	79 9/16	32⅛	63 7/16
'27 ...	15 7/16	—	12 5/16	10 7/16	9·85	22 15/16	36 15/16	33⅝	34¼	102⅝	31¾	64 5/16
'28 ...	14	—	15 7/16	11 23/32	12·09	23 13/16	37 7/16	36 15/16	35¾	80 7/16	29⅞	66 15/16
'29 ...	13⅞	—	10 9/16	8¼	10·80	19¼	38⅛	36 15/16	34 21/32	72	35 15/32	74⅛
'30 ...	10 15/16	—	6 5/32	6½	7·80	18½	33½	28 23/32	25¼	52⅛	36⅝	61 15/16
'31 ...	8 15/16	—	5½	5 9/16	6·12	17¾	32½	19⅓	19 7/16	53 9/16	18 3/16	38⅝
'32 ...	8 5/32	—	4 9/16	4¾	5·47	17 3/32	28⅞	21 1/16	17⅛	57 3/16	17	38 5/16
'33 ...	6⅜	—	5¼	4 11/16	5·65	17 1/32	26 9/32	19 7/32	15 11/16	53 15/16	20¾	39¾
'34 ...	5 7/16	—	4 13/16	4⅞	5·71	17½	25 5/16	17¾	13 17/32	62 3/16	21 13/32	42⅜
'35 ...	5 3/32	—	5 5/32	5⅛	5·51	17½	25 13/16	24⅝	19⅝	61 1/32	24 3/32	43¼
'36 ...	5 5/32	—	6¼	6	6·47	17½	27 1/32	23 9/32	19 31/32	70 3/16	28½	48 5/32
'37 ...	8 7/16	—	8 21/32	7 9/16	8·62	18 9/32	28 5/16	23 15/32	22 9/16	95 23/32	31⅛	54 1/32
'38 ...	7 31/32	—	6 21/32	5 9/16	6·35	14 5/16	24 1/32	17 3/32	14⅞	68 1/32	26 7/16	46 7/32
'39 ...	13 3/32	—	6¾	6⅛	6·39	18 5/32	23 7/16	16 9/32	14 5/16	73 15/16	29⅛	49 5/32
'40 ...	17½	—	8	8½	8·50	25 7/32	23¾	22 3/32	19	114¼	44 3/32	66⅛
'41 ...	15 21/32	—	7½	8½	8·45	24⅜	24¼	22½	20¾	118	41½	70¼
'42 ...	16 †	—	8⅝	9	9·25	26	26⅛	24 3/16	23	118	44 13/16	78 9/16
'43 ...	16 †	—	8⅞	9 1/16	9·49	27	24 15/16	34⅜	36	118 †	48½	109½
'44 ...	16 †	—	10 11/16	9⅞	10·14	27	28⅞	43½	42¼	118 †	60¾	107¾
'45 ...	16 †	—	10 11/16	9⅜	9·42	25	31⅛	43½	42¼	150 †	62	109½
Average 1904–13	11¾	—	9⅝	7½	6⅞	16	17	31½	31¼	43¾	26⅝	49½
1890–99	11½	—	6¼	5½	5	13¼	13¾	25	24½	35	19¼	38
'78–87	15	—	8⅝	6¾	6½	15	17	35½	32½	40	23	46
'67–77	23	—	9	7	6⅞	16	18¾	45	39	50	30	60

Index Numbers (or Percentages) of Prices, the Average of 1867–77 being 100

1922 ...	125	1,075	114	174	77	89	151	127
'24 ...	102	1,362	119	163	94	103	160	138
'25 ...	79	1,319	132	161	85	104	147	137
'26 ...	69	1,063	121	164	75	96	159	106
'27 ...	67	1,045	142	172	82	88	205	107
'28 ...	61	1,086	172	176	81	92	161	108
'29 ...	60	976	129	166	64	89	144	122
'30 ...	48	669	92	150	43	65	104	110
'31 ...	39	504	77	146	47	51	108	63
'32 ...	35	513	66	132	44	45	114	61
'33 ...	29	534	68	127	39	40	108	67
'34 ...	24	572	67	123	55	35	124	71
'35 ...	24	641	69	125	52	50	123	75
'36 ...	25	646	82	129	52	51	140	85
'37 ...	37	745	109	134	39	58	192	95
'38 ...	35	601	81	110	36	38	137	81
'39 ...	57	741	84	119	49	37	148	88
'40 ...	76	1,191	109	141	50	49	228	122
'41 ...	68	1,300	107	135	54	53	236	124
'42 ...	70	1,304	116	150	76	59	236	137
'43 ...	70	1,327	120	149	81	92	236	176
'44 ...	70	1,460	132	161	97	108	236	187
'45 ...	70	1,515	129	161	97	108	300	190

* Common New Style from 1921 to 1936. China, Extra "A" from 1937. † Nominal.

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Average Prices of Commodities—Contd.

No. of Article	41A ‡	41B ‡	41C ‡	41	42	43	44	45A	45B	35–45	20–45	1–45
	Petroleum *				Soda		Indigo	Timber				
Year	Motor Spirit c.i.f.	Kerosene (Burning Oil) c.i.f.	Gas Oil c.i.f.	Mean of 41A 41B and 41C	Crystals	Nitrate of Soda	Bengal, Good Consuming	Hewn, Average Import Price	Sawn or Split, Average Import Price	Sundry Materials Total	Materials Total	Grand Total
	d. per imp. gall.	*d.* per imp. gall.	*d.* per imp. gall.		*s.* per ton	*s.* per cwt.	*s.* per lb.	*s.* per load	*s.* per load			
1922 ...	15·89	6·32	3·98	—	123	14 1/3	9 5/6	46 5/8	117 1/2	—	—	—
'24 ...	10·14	5·69	4·28	—	101 3/4	13 5/8	6 1/10	49 4/5	122	—	—	—
'25 ...	9·98	4·96	4·22	—	100	13 3/8	5 3/4	47 15/16	122 15/16	—	—	—
'26 ...	9·21	5·66	4·25	—	100	13 1/4	5 5/8	48 29/32	107	—	—	—
'27 ...	7·25	5·15	4·23	—	100	12 9/16	5 1/2	45 7/16	107 9/16	—	—	—
'28 ...	5·96	4·48	3·32	—	100	10 31/32	5 1/2	45 7/8	111 5/32	—	—	—
'29 ...	6·75	4·87	3·21	—	100	10 1/4	5 1/2	44 3/4	107 21/32	—	—	—
'30 ...	6·52	4·54	3·21	—	100	9 27/32	5 1/2	44 1/2	102 3/16	—	—	—
'31 ...	3·63	3·10	2·33	—	100	9 1/8	5 1/2	37 19/32	83 3/8	—	—	—
'32 ...	3·97	3·34	2·39	—	100	8 5/8	5 1/2	35 1/2	75 1/16	—	—	—
'33 ...	3·43	2·94	2·48	—	100	8 11/32	5 1/2	31 7/8	75 19/32	—	—	—
'34 ...	3·28	2·65	2·45	—	100	7 13/16	5 1/2	31 1/2	79 13/32	—	—	—
'35 ...	3·51	2·88	2·32	—	100	7 5/8	5 1/2	32 1/4	73 5/32	—	—	—
'36 ...	3·77	2·76	2·34	—	100	7 5/8	5 3/4	37 9/16	78 27/32	—	—	—
'37 ...	4·52	3·71	3·14	—	100	7 25/32	5 3/4	58 3/8	103 7/32	—	—	—
'38 ...	4·08	3·49	3·22	—	100	8	5 3/4	61 3/32	94 9/16	—	—	—
'39 ...	5·00	3·71	3·61	—	100	8 1/8	5 3/4	57 9/16	107 9/32	—	—	—
'40 ...	5·79	5·68	5·31	—	100	9 7/8	5 3/4 †	106 29/32	169 15/16	—	—	—
'41 ...	6·97	5·95	5·93	—	100	13 1/32	5 3/4 †	191 1/16	214 7/16	—	—	—
'42 ...	7·77	7·00	7·07	—	102 1/2	13 1/2	5 3/4 †	235 1/8	236 1/4	—	—	—
'43 ...	8·93	7·51	7·61	—	107 1/2	13 1/2	5 3/4 †	269	246 1/4	—	—	—
'44 ...	8·31	6·78	6·83	—	107 1/2	15 1/4	6 †	222 15/16	273 15/16	—	—	—
'45 ...	6·23	5·33	5·38	—	107 1/2	15 1/4	6 †	190 1/2	235 1/4	—	—	—
Average 1904–13				—	60	10 5/8	3	38	56	—	—	—
1890–99	BASE, 1922			—	53	8 1/4	4 3/8	40	45	—	—	—
'78–87				—	62	12 1/2	6	47	47	—	—	—
'67–77				—	92	14	7 1/4	60	54	—	—	—
	Index Numbers (or Percentages) of Prices, the Average of 1867–77 being 100											
1922 ...	‡ 100	‡ 100	‡ 100	100	134	102	128	143		1,339	3,408	5,873
'24 ...	64	90	108	87	111	97	84	151		1,307	3,774	6,249
'25 ...	63	79	106	83	109	96	79	150		1,292	3,691	6,120
'26 ...	58	90	107	85	109	95	78	137		1,235	3,376	5,643
'27 ...	46	82	106	78	109	90	76	134		1,276	3,311	5,477
'28 ...	37	71	83	64	109	78	76	138		1,256	3,202	5,366
'29 ...	42	77	81	67	109	73	76	134		1,190	3,047	5,143
'30 ...	41	72	81	65	109	70	76	129		1,034	2,487	4,313
'31 ...	23	50	60	44	109	65	76	106		890	2,094	3,673
'32 ...	25	53	60	46	109	62	76	97		855	2,062	3,571
'33 ...	22	47	62	44	109	60	76	94		837	2,121	3,529
'34 ...	21	42	62	42	109	56	76	97		839	2,176	3,635
'35 ...	22	46	60	43	109	54	76	92		871	2,298	3,745
'36 ...	24	42	60	42	109	54	79	102		925	2,414	3,945
'37 ...	28	59	79	55	109	56	79	142		1,081	2,821	4,581
'38 ...	26	55	81	54	109	57	79	136		921	2,471	4,071
'39 ...	31	60	91	61	109	58	79	145		964	2,663	4,247
'40 ...	36	90	133	86	109	70	79	243		1,285	3,645	5,760
'41 ...	44	94	150	96	109	93	79	356		1,438	4,004	6,379
'42 ...	49	111	178	113	111	96	79	413		1,564	4,153	6,812
'43 ...	56	119	199	122	117	96	79	452		1,715	4,354	6,978
'44 ...	52	107	172	110	117	109	83	436		1,776	4,619	7,221
'45 ...	39	84	135	86	117	109	83	373		1,753	4,730	7,375

* Prior to 1922, Kerosene Burning Oil (in barrels), base period 1873–77; from 1922 c.i.f. values per imperial gallon of Motor Spirit, Kerosene (Burning Oil) and Gas Oil, base period 1922.

† Nominal.

‡ Index-numbers not included in the general average.

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MISCELLANEA

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Recent Advances in Mathematical Statistics

Bibliography of Mathematical Statistics (1940–42)

By F. A. Rigg, B.Sc.

(Communication from the National Physical Laboratory)

The series of bibliographies of papers on mathematical statistics which the Society intended to publish at approximately annual intervals was interrupted by the war. Dr. Hartley, who compiled the last bibliography, for 1939, recently made a start to cover the years 1940–42, but owing to pressure of other commitments found it impossible to complete this undertaking. Meantime, a card index of similar scope was being compiled by members of the statistics section of the Mathematics Division at the National Physical Laboratory, and they undertook, on the basis of Dr. Hartley's work, to prepare the bibliography for publication. Though it is designed to cover papers published in 1940, 1941 and 1942, a few papers published in 1939, which were not included in the previous list, are given. It is believed to be fairly comprehensive, but it is too much to hope either for completeness or for an absence of typographical errors, and the compiler would be grateful if readers would inform her of any errors or omissions.

Owing to the considerable increase in the number of papers to be listed, it was thought desirable to sub-divide the "Theory" group into twelve smaller sections. This makes it easier to pick out the majority of the papers on any particular subject, but it introduces the difficulty that a number of papers could appropriately have been placed in more than one section. For reasons of economy of space a system of cross-classification is impracticable here, and such papers have been listed only once under the heading which seemed most relevant. It will be appreciated that many of the papers have had to be classified without first being read, and that since titles are often misleading, some will probably be wrongly classified. It is felt, however, that the advantages of the system outweigh the disadvantages.

The following notes are included, to explain the method of classification adopted:—

A. BIOGRAPHICAL AND GENERAL

B. THEORY

I. *Axiomatics.*

II. *Probability Functions.* Including limit theorems on probability functions. Special probability functions appear under Section IV. See also Sections VII and VIII.

III. *Stochastic Processes.*

IV. *Special Distributions.* See Sections VII and VIII.

V. *Sampling Distributions.* Both for the general case (*e.g.*, a paper on the cumulants of k-statistics in terms of the cumulants of the parent population), and for special distributions (*e.g.*, the distribution of non-central t in samples from a normal distribution). See also Sections VII and VIII.

VI. *Problems of Estimation and Tests of Statistical Hypotheses.* Covering all problems of estimation and tests for univariate distributions. See also Sections VII and VIII.

VII. *Bivariate Problems.* Including ordinary correlation and regression, and all bivariate problems which, in the corresponding univariate case, would be divided among Sections II, III, IV, V and X.

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VIII. *Multivariate Problems.* Including multiple and partial correlation and regression, and all multivariate problems which, in the corresponding univariate case, would be divided among Sections II, III, IV, V and X. Also discriminant functions and canonical correlation.

IX. *Design of Experiment and Sampling Techniques.* Including papers on methods of construction and analysis of special designs, and on statistical methods used in sampling surveys. Some papers concerned more with applications of these subjects than with their theory will be found in Sections XIII and XV.

X. *Analysis of Variance.*

XI. *Special Problems.* Mainly combinatorial problems (not those connected with Design of Experiment) such as contingency, rank correlation and fundamental problems of the calculus of probability. Also renewal theory and random walk problems.

XII. *Grouping and Curve Fitting.* Treatment of observations, corrections for grouping, the graduation of frequency curves and the fitting of curves to experimental data.

C. APPLICATIONS

XIII. *Agriculture and Forestry.*

XIV. *Genetics.*

XV. *Actuarial and Population Studies.*

XVI. *Psychology.* Including the theory of factor analysis. Matching problems are in Section XI.

XVII. *Other Biological Applications.* Including probit methods and problems on insect populations.

XVIII. *Economic and Industrial Problems.* It seemed advisable that papers on economic time series should not be separated from papers on harmonic analysis which have, therefore, been included in this section (with the exception of some in Section VI).

XIX. *Computation.* Including computational methods on all statistical subjects, and all statistical tables.

D. MISCELLANEOUS

XX. Containing both papers on miscellaneous subjects which could not be classified easily under the above system, and some papers whose subject-matter is, for various reasons, unknown to the compiler.

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The compiler desires to acknowledge the comments and suggestions she has received from her colleagues in the Mathematics Division, National Physical Laboratory.

A. BIOGRAPHICAL AND GENERAL

(1) BARTLETT, M. S. The present position of mathematical statistics.
J. Roy. Stat. Soc., Vol. CIII, pp. 1–19, 1940.

(2) CAMP, B. H. Some recent advances in mathematical statistics. I.
Annals Math. Statistics, Vol. XIII, pp. 62–73, 1942.

(3) CRAIG, C. C. Recent advances in mathematical statistics. II.
Annals Math. Statistics, Vol. XIII, pp. 74–85, 1942.

(4) CRAMÉR, HARALD. Deux conférences sur la théorie des probabilités.
Skandinavisk Actuarietidskrift, 1941, pp. 34–69, 1941.

(5) CROXTON, F. E. Toward standardized symbols for basic statistical concepts.
J. Amer. Stat. Ass., Vol. 36, pp. 426–8, 1941.

(6) DEMING, W. E. Some thoughts on statistical inference.
J. Washington Acad. Sci., Vol. 31, pp. 85–93, 1941.

(7) DUNLAP, J. W. Recent advances in statistical theory and applications.
Amer. J. Psychol., Vol. 54, pp. 583–601, 1941.

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Digitized by Arya Samaj Foundation Chennai and eGangotri

(8) GONTCHAROFF, V., and KOLMOGOROFF, A. Le soixantenaire de S. Bernstein.
Bull Acad. Sci. U.R.S.S., Sér. Math., Vol. 4, pp. 249–60, 1940.
(9) HAYFORD, F. L. An inquiry into the nature and causes of statisticians.
J. Amer. Stat. Ass., Vol. 36, pp. 1–10, 1941.
(10) KENDALL, M. G. On the future of statistics.
J. Roy. Stat. Soc., Vol. CV, pp. 69–80, 1942.
(11) KUZMIN, R. O. The mathematical works of S. N. Bernstein.
Uspekhi Matem. Nauk 8, pp. 3–7, 1941.
(12) RIEFLER, W. W. Government and the statistician.
J. Amer. Stat. Ass., Vol. 37, pp. 1–11, 1942.
(13) VIRGILII, F. Le cause di errore nell'osservazione dei fatti. (The causes of errors in the observation of facts.)
Scientia, Milano, Vol. 67, pp. 188–92, 1940.
(14) WILKS, S. S. Karl Pearson: founder of the science of statistics.
Sci. Mon., N.Y., Vol. 53, pp. 249–53, 1941.

B. THEORY

I. AXIOMATICS

(15) BERNSTEIN, S. N. The Petersburg school of the theory of probability.
Leningrad State Univ. Annals (*Uchenye Zapiski*), Math. Ser., Vol. 10, pp. 3–11, 1940.
(16) BOREL, E. Une objection à la définition empirique de la probabilité.
C. R. Acad. Sci., Paris, t. 211, pp. 312–13, 1940.
(17) BRODERICK, T. S., and SCHRÖDINGER, E. Boolean algebra and probability theory.
Proc. Roy. Irish Acad. Sci., A, Vol. 46, pp. 103–12, 1940.
(18) CHURCH, A. On the concept of a random sequence.
Bull. Amer. Math. Soc., Vol. 46, pp. 130–35, 1940.
(19) COPELAND, A. H. Fundamental concepts of the theory of probability.
American Math. Monthly, Vol. XLVIII, pp. 522–30, 1941.
(20) COPELAND, A. H. Postulates for the theory of probability.
Amer. J. Math., Vol. LXIII, pp. 741–62, 1941.
(21) DANTZIG, D. van. Mathematical and empirical foundations of the calculus of probability.
Nederl. Tijedschr. Natuurkunde, Vol. 8, pp. 70–91, 1941. (Dutch. English summary.)
(22) DOOB, J. L. Probability as measure.
Annals Math. Statistics, Vol. XII, pp. 206–14, 1941.
(23) DUCASSE, C. J. Some observations concerning the nature of probability.
J. Phil., Vol. 38, pp. 393–403, 1941.
(24) HALMOS, P. R. Statistics, set functions, and spectra.
Rec. Math. N.S., Vol. 9 (51), pp. 241–8, 1941. (English. Russian summary.)
(25) JEFFREYS, H. Frequency interpretations in probability.
Nature, Vol. 143, p. 242, 1939.
(26) KAMKE, E. Zu meinem Aufsatz "Kritische Bemerkungen zu K. Marbe, Grundfragen der angewnadten Wahrscheinlichkeitsrechnung und theoretischen Statistik."
Iber. Deutsch. Math. Verein., Vol. 49, pp. 255–6, 1940.
(27) KEMBLE, E. C. Is the frequency theory of probability adequate for all scientific purposes?
Amer. J. Phys., Vol. 10, pp. 6–16, 1942.
(28) KEMBLE, E. C. The probability concept.
Philos. Sci., Vol. 8, pp. 204–32, 1941.
(29) KENDALL, M. G. A theory of randomness.
Biometrika, Vol. XXXII, pp. 1–15, 1940–41.
(30) KOOPMAN, B. O. The axioms and algebra of intuitive probability.
Annals of Mathematics, Vol. 41, pp. 269–92, 1940.
(31) KOOPMAN, B. O. The bases of probability.
Bull. Amer. Math. Soc., Vol. 46, pp. 763–74, 1940.
(32) KOOPMAN, B. O. Intuitive probabilities and sequences.
Annals of Mathematics, Vol. 42, pp. 169–87, 1941.

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Digitized by Arya Samaj Foundation Chennai and eGangotri

(33) LARGUIER, E. H. A matrix theory of n-dimensional measurement.
Duke Math. J., Vol. 5, pp. 729–39, 1939.
(34) MEIER, J. Zur theorie der unabhängigen Wahrscheinlichkeiten.
Mittgilungen d. Vereinigung schweiz. Versicherungs Math. H. 39, pp. 53–74, 1940.
(35) VON MISES, R. On the foundations of probability and statistics.
Annals Math. Statistics, Vol. XII, pp. 191–205, 1941.
(36) VON MISES, R. and DOOB, J. L. Discussion of papers on probability theory.
Annals Math. Statistics, Vol. XII, pp. 215–17, 1941.
(37) PEARSON, K. The laws of chance, in relation to thought and conduct.
Biometrika, Vol. XXXII, pp. 89–100, 1941.
(38) PÓLYA, G. Heuristic reasoning and the theory of probability.
American Math. Monthly, Vol. XLVIII, pp. 450–65, 1941.
(39) REICHENBACH, H. Note on probability implication.
Bull. Amer. Math. Soc., Vol. 47, pp. 265–67, 1941.
(40) SALTER, L. A. A comment on Deming's classification of problems of inference.
J. Amer. Stat. Ass., Vol. 37, pp. 540–2, 1942.
(41) WEIL, A. Calcul des probabilités, méthode axiomatique, intégration.
Revue Sci., Vol. 78, pp. 201–8, 1940.
(42) WRIGHT, G. H. V. On probability.
Mind, Vol. 49, pp. 265–83, 1940.

II. PROBABILITY FUNCTIONS

(43) AGNEW, R. P., and KAC, M. Translated functions and statistical independence.
Bull. Amer. Math. Soc., Vol. 47, pp. 148–54, 1941.
(44) BERNSTEIN, S. N. Some remarks concerning the limit theorem of Liapunov.
Doklady Akad. Nauk. S.S.S.R., Vol. 24, pp. 3–7, 1939.
(45) BERNSTEIN, S. Nouvelles applications des grandeurs aléatoires presqu'indépendantes.
Bull. Acad. Sci. U.R.S.S. Sér. Math., Vol. 4, pp. 137–50, 1940. (Russian. French summary.)
(46) BERNSTEIN, S. Sur les sommes de grandeurs aléatoires liées de classes (A, N) et (B, N).
C. R. Acad. Sci. U.R.S.S. (*N.S.*), Vol. 32, pp. 303–7, 1941.
(47) BERRY, A. C. The accuracy of the Gaussian approximation to the sum of independent variates.
Trans. Amer. Math. Soc., Vol. XLIX, pp. 122–36, 1941.
(48) BIRNBAUM, Z. W., and ZUCKERMAN, H. S. On the properties of a collective.
Amer. J. Math., Vol. LXII, pp. 787–91, 1940.
(49) BOAS, R. P., JR. A general moment problem.
Amer. J. Math., Vol. LXIII, pp. 361–70, 1941.
(50) BROWN, B. H. Simple examples of limiting processes in probability.
American Math. Monthly, Vol. XLVIII, pp. 98–102, 1941.
(51) BROWN, G. W. Reduction of a certain class of composite statistical hypotheses.
Annals Math. Statistics, Vol. XI, pp. 254–70, 1940.
(52) BURR, I. W. Cumulative frequency functions.
Annals Math. Statistics, Vol. XIII, pp. 215–32, 1942.
(53) CONSOLI, T. Généralisation d'un théorème sur la probabilité de la somme d'un nombre infini de variables aléatoires.
Rev. Fac. Sci. Univ. Istanbul, A, Vol. 5, pp. 1–17, 1940.
(54) CRAIG, G. C. On frequency distributions of the quotient and of the product of two statistical variables.
Amer. Math. Monthly, Vol. XLIX, pp. 24–32, 1942.
(55) CURTISS, J. H. On the distribution of the quotient of two chance variables.
Annals Math. Statistics, Vol. XII, pp. 409–21,1941.
(56) CURTISS, J. H. Generating functions in the theory of statistics.
Amer. Math. Monthly, Vol. XLVIII, pp. 374–86, 1941.
(57) CURTISS, J. H. A note on the theory of moment generating functions.
Annals Math. Statistics, Vol. XIII, pp. 430–3, 1942.

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Digitized by Arya Samaj Foundation Chennai and eGangotri

(58) DIEULEFAIT, C. On incomplete moments.
Rev. Ci. Lima, Vol. 41, pp. 543–7, 1939.

(59) DIEULEFAIT, C. E. On Thiele's semi-invariants and the conjugate Fourier function.
Ann. Soc. Ci. Argentina, Vol. 129, pp. 208–11, 1940.

(60) DIEULEFAIT, C. E. On Slutsky's sinusoidal limit law, derived from a new sequence of random variables.
An. Soc. Ci. Argentina, Vol. 134, pp. 257–85, 1942.

(61) DODD, E. L. The substitutive mean and certain subclasses of this general mean.
Annals Math. Statistics, Vol. XI, pp. 163–76, 1940.

(62) DODD, EDWARD L. Some generalizations of the logarithmic means of two variates which become indeterminate when the two variates are equal.
Annals Math. Statistics, Vol. XII, pp. 422–8, 1941.

(63) DOEBLIN, W. Sur un problème de calcul des probabilités.
Comptes Rendus, t. 209, pp. 742–3, 1939.

(64) DOEBLIN, W. Sur l'ensemble de puissances d'une loi de probabilité.
Studia Math., t. IX, pp. 71–96, 1940. (French. Ukrainian summary.)

(65) DUGUÉ, D. Sur quelques exemples de factorisation de variable aléatoires.
Comptes Rendus, t. 212, pp. 838–40, 1941.

(66) ERDÖS, P. On the law of the iterated logarithm.
Annals of Mathematics, Vol. 43, pp. 419–36, 1942.

(67) ERDÖS, P., and KAC, M. The Gaussian law of errors in the theory of additive number theoretic functions.
Amer. J. Math., Vol. LXII, pp. 738–42, 1940.

(68) ESSEEN, C.-G. On the Liapounoff limit of error in the theory of probability.
Ark. Mat. Astr. Fys., Vol. 28A, No. 9, 19 pp., 1942.

(69) FAESI, M. Über die Glättung statistischer Verteilungsreihen. Mitteilungen.
Vereinigung schweiz Versicherungs-Math., H.40, pp. 61–84, 1940.

(70) FELDHEIM, E. Sur quelques propriétés des lois de probabilité stables.
Rev. Mat. Union Interbalkan, Vol. 2, pp. 9–30, 1939.

(71) FELLER, W. Neuer Beweis für die Kolmogoroff-P. Levysche Charakterisierung der unbeschränkt teilbaren Verteilungsfunktionen.
Bull. Intern. Acad. Yougoslave. Cl. Sci. Math. Nat., Vol. 32, pp. 1–8, 1939.

(72) FORTET, R. Sur une suite également répartie.
Rev. Sci., Vol. 78, pp. 298–9, 1940.

(73) FORTET, R. Sur une suite également répartie.
Studia Math., t. IX, pp. 54–70, 1940. (French. Ukrainian summary.)

(74) GARTI, Y. Les lois de probabilité pour les fonctions statistiques (cas de collectifs à plusieurs dimensions).
Rev. math Union Interbalkan, Vol. 3, pp. 21–39, 1940.

(75) GEIRINGER, H. A generalization of the law of large numbers.
Annals Math. Statistics, Vol. XI, pp. 393–401, 1940.

(76) GNEDENKO, B. On the domains of attraction of stable laws.
C. R. Acad. Sci. U.R.S.S., Vol. 24, pp. 640–2, 1939.

(77) GNEDENKO, B. V. To the theory of limiting theorems of sums of independent random variables.
Bull. Acad. Sci. U.R.S.S., Sér. Math., 1939, pp. 181–232, 1939. (Russian. English summary.)

(78) GNEDENKO, B. V. On the theory of limit theorems for sums of independent variables.
Bull. Acad. Sci. U.R.S.S., Sér. Math., 1939, pp. 643–7, 1939. (Russian.)

(79) GNEDENKO, B. V. On limiting laws of the theory of probability.
Doklady Akad. Nauk. S.S.S.R., Vol. 23, pp. 868–71, 1939. (Russian.)

(80) GNEDENKO, B. To the theory of the domains of attraction of stable laws.
Uchenye Zapiski Moskov Gos. Univ. Matematika, Vol. 30, pp. 61–81, 1939. (Russian. English summary.)

(81) GNEDENKO, B. Quelques théorèmes sur l'ensemble des puissances d'une loi de probabilité.
Uchenye Zapiski Moskov Gos. Univ. Matematika, Vol. 45, pp. 61–71, 1940. (Russian. French summary.)

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(82) GNEDENKO, B. V. Limit theorems for the maximal term of a variational series.
C. R. Acad. Sci. U.R.S.S., N.S., Vol. 32, pp. 7–9, 1941.
(83) GNEDNEKO, B. On locally stable probability distributions.
C. R. Acad. Sci. U.R.S.S. (N.S.), Vol. 35, pp. 263–6, 1942.
(84) GNEDENKO, B. V. Locally stable distributions.
Bull. Acad. Sci. U.R.S.S., Sér. Math. Vol. 6, pp. 291–308, 1942. (Russian. English summary.)
(85) GNEDENKO, B. V., and GROSHEV, A. V. On the convergence of distribution laws of normalized sums of independent random variables.
Rec. Math. Moscow, N.S. 6 (48), pp. 521–41, 1939. (Russian. English summary.)
(86) HALDANE, J. B. S. The cumulants of the distribution of the square of a variate.
Biometrika, Vol. XXXII, pp. 199–200, 1941.
(87) HALDANE, J. B. S. The mode and median of a nearly normal distribution with given cumulants.
Biometrika, Vol. XXXII, pp. 294–9, 1942.
(88) HARTMAN, P., and WINTNER A. On the spherical approach to the normal distribution law.
Amer. J. Math., Vol. LXII, pp. 759–79, 1940.
(89) HARTMAN, P., and WINTNER, A. On the law of the iterated logarithm.
Amer. J. Math., Vol. LXIII, pp. 169–76, 1941.
(90) HARTMAN, P., and WINTNER, A. On the infinitesimal generators of integral convolutions.
Amer. J. Math., Vol. LXIV, pp. 273–98, 1942.
(91) HEDGE, L. B. Moment problem for a bounded region.
Bull. Amer. Math., Soc., Vol. 47, pp. 282–5, 1941.
(92) HOFFDING, WASSILIJ. Maztabinvariate Korrelationstheorie.
Schrinften Maths. Inst. u. Inst. Angew. Math. Univ. Berlin 5, pp. 181–233, 1940.
(93) DE JONGH, B. H. General minimum-probability theorem.
Nederl. Akad. Wetensch., Proc., Vol. 44, pp. 738–43, 1941.
(94) KAWATA, T. On the strong law of large numbers.
Proc. Imp. Acad. Tokyo, Vol. 16, pp. 109–12, 1940.
(95) KAWATA, T. On the division of a probability law.
Proc. Imp. Acad. Tokyo, Vol. 16, pp. 249–54, 1940.
(96) KAWATA, T. The Fourier series of the characteristic function of a probability distribution.
Tôhoku Math. J., Vol. 47, pp. 121–5, 1940.
(97) KAWATA, T. The function of mean concentration of a chance variable.
Duke Math. J., Vol. 8, pp. 666–7, 1941.
(98) KENDALL, M. G. Conditions for uniqueness in the problem of moments.
Annals Math. Statistics, Vol. XI, pp. 402–9, 1940.
(99) KENDALL, M. G. Corrections to a paper on the uniqueness problem of moments.
Annals Math. Statistics, Vol. XII, pp. 464–5, 1941.
(100) KENNEY, J. F. Characteristic functions in statistics.
Nat. Math. Mag., Vol. 17, pp. 51–67, 99–114, 1942.
(101) KOZAKIEWICZ, W. Sur la convergence presque certaine.
Bull. Sci. Math. II, Vol. 64, pp. 121–8, 1940.
(102) LOÈVE, M. Nouvelles classes de lois limites.
Comptes Rendus, t. 210, pp. 202–4, 1940.
(103) LOÈVE, M. La loi des grands nombres pour des variables aléatoires liées et des événements liés.
Comptes Rendus, t. 212, pp. 840–43, 1941.
(104) LOÈVE, M. La loi forte des grands nombres pour des variables aléatoires liées.
C. R. Acad. Sci. Paris, Vol. 212, pp. 1121–23, 1941.
(105) LOÈVE, M. La tendance centrale des sommes de variables aléatoires liées.
C. R. Acad. Sci. Paris, Vol. 213, pp. 9–11, 1941.
(106) MADOW, W. G. Limiting distributions of quadratic and bilinear forms.
Annals Math. Statistics, Vol. XI, pp. 125–46, 1940.
(107) MENGER, K. Statistical metrics.
Proc. Nat. Acad. Sci. U.S.A., Vol. 28, pp. 535–7, 1942.

CC-0. In Public Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(108) MOYAL, J. E. Approximate probability distribution functions for the sum of two independent variates.
J. Roy. Stat. Soc., Vol. CV, pp. 42–3, 1942.

(109) OTTAVIANI, G. Sulla probabilità che una prova su due variabili casuali X e Y verifichi la disuguaglianza $X < Y$ e sul corrispondente scarto quadratico medio.
Giorn. Ist Ital. Attuari, Vol. X, pp. 185–92, 1939.

(110) RAIFORD, T. E. Skewness of combined distributions.
J. Amer. Stat. Ass., Vol. 37, pp. 391–3, 1942.

(111) RIEBESELL, P. Neue deutsche Forschungen über das Gesetz der grossen Zahl. (New German research into the law of large numbers.)
Bl. Versich.-Math., Vol. 5, pp. 68–75, 1940.

(112) ROSENBLATT, A. Sur les théorèmes des grands nombres dans la théorie de la probabilité.
Actas Acad. Ci. Lima, Vol. 3, pp. 152–9, 1940.

(113) ROSENBLATT, A. On the law of large numbers in the theory of probability.
Publ. Inst. Mat. Univ. Nac. Litoral., Vol. 2, pp. 141–6, 1940.

(114) SELBERG, H. L. Ueber eine Verschärfung der Tchebycheffschen Ungleichung.
Arch. Math. Naturvid., Vol. 43, pp. 30–2, 1940.

(115) SELBERG, H. L. Zwei Ungleichungen zur Ergänzung des Tchebycheffschen Lemmas.
Skandinavisk Acturietidskrift, 1940, pp. 121–5, 1940.

(116) SELBERG, H. L. Ueber eine Ungleichung der mathematischen Statistik.
Skandinavisk Akturietidskrift, 1940, pp. 114–20, 1940.

(117) STEINHAUS, H. Sur les fonctions indépendantes. VI.
Studia Math., t. 9, pp. 121–32, 1940.

(118) STERNBERG, W. The general limit theorem in the theory of probability.
Bull. Amer. Math. Soc., Vol. 46, pp. 292–8, 1940.

(119) WINTNER, A. On the iteration of distribution functions in the calculus of probability.
Union Mat. Argentina, Publ. No. 18, 12 pp., 1941.

(120) WINTNER, A. On a family of Fourier transforms.
Bull. Amer. Math. Soc., Vol. 48, pp. 304–8, 1942.

III. STOCHASTIC PROCESSES

(121) AMBARZUMIAN, G. A. Study of a special case of a continuous stochastic process.
Leningrad State Univ. Annals (*Uchenye Zapiski*), *Math. Sér.* 10, pp. 120–38, 1940. (Russian.)

(122) AMBROSE, W. On measurable stochastic processes.
Trans. Amer. Math. Soc., Vol. XLVII, pp. 66–79, 1940.

(123) BACHELIER, L. Probabilités des oscillations maxima.
Comptes Rendus, t. 212, pp. 836–8, 1941.

(124) BEBOUTOFF, M. Markoff chains with a compact state space.
C. R. Acad. Sci. U.R.S.S. (*N.S.*), Vol. 30, pp. 482–3, 1941.

(125) BERNSTEIN, S. Correction d'une démonstration.
C. R. Acad. Sci. U.R.S.S. (*N.S.*), Vol. 25, pp. 707–9, 1939.

(126) BLACKWELL, D. Idempotent Markoff chains.
Annals of Mathematics, Vol. 43, pp. 560–7, 1942.

(127) CRAMÉR, H. On the theory of stationary random processes.
Annals of Mathematics, Vol. 41, pp. 215–30, 1940.

(128) DEDEBANT, G., MOYAL, J., and WEHRLÉ, P. Sur les équations aux dérivées partielles que vérifient les fonctions de distribution d'un champ aléatoire.
Comptes Rendus, t. 210, pp. 243–5, 1940.

(129) DOEBLIN, W. Sur certains mouvements aléatoires discontinus.
Skandinavisk Aktuarietidskrift, Vol. 1939, pp. 211–22, 1939.

(130) DOEBLIN, W. Sur l'équation de Kolmogoroff.
Comptes Rendus, t. 210, pp. 365–7, 1940.

(131) DOEBLIN, W. Sur l'équation matricielle $A^{(\alpha+\beta)} = [A^{(\alpha)}A^{(\beta)}]$ et ses applications au calcul des probabilités.
Bull. Sci. Math., Vol. 64, pp. 35–7, 1940.

CC-0. In Public Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(132) DOEBLIN, W. Sur des mouvements mixtes.
Comptes Rendus, t. 210, pp. 690–2, 1940.

(133) DOEBLIN, W. Eléments d'une theorie générale des chaines simples constantes de Markoff.
Annals de l'École Normale Sup., t. 57, pp. 61–111, 1940.

(134) DOOB, J. L. The law of large numbers for continuous stochastic processes.
Duke Math. J., Vol. 6, pp. 290–306, 1940.

(135) DOOB, J. L. Regularity properties of certain families of chance variables.
Trans. Amer. Math. Soc., Vol. XLVII, pp. 455–86, 1940.

(136) DOOB, J. L. Topics in the theory of Markoff chains.
Trans. Amer. Math. Soc., Vol. LII, pp. 36–64, 1942.

(137) DOOB, J. L. What is a stochastic process?
American Math. Monthly, Vol. XLIX, pp. 648–53, 1942.

(138) DOOB, J. L., and AMBROSE, W. On two formulations of the theory of stochastic processes depending upon a continuous parameter.
Annals of Mathematics, Vol. 41, pp. 737–45, 1940.

(139) DOUBROWSKY, V. Sur un problème limite de la théorie des probabilités.
Bull. Acad. Sci. U.R.S.S., Sér. Math., Vol. 4, pp. 411–16, 1940. (Russian. French summary.)

(140) FELLER, W. On the integro-differential equations of purely discontinuous Markoff processes.
Trans. Amer. Math. Soc., Vol. XLVIII, pp. 488–515, 1940.

(141) FORTET, R. Sur des fonctions aléatoires définies par leurs équations aux dérivées partielles.
Comptes Rendus, t. 212, pp. 325–6, 1941.

(142) FORTET, R. Sur le calcul de certaines probabilités d'absorption.
C. R. Acad. Sci. Paris, Vol. 212, pp. 1118–20, 1941.

(143) FORTET, R. Sur la notion de fonction aléatoire.
Rev. Sci., Vol. 79, pp. 135–9, 1941.

(144) FRANCKX, E. La théorie des chaînes de Markoff. Étude du cas régulier.
Skandinavisk Actuarietidskrift, 1939, pp. 200–10, 1939.

(145) FUKAMIYA, M. The Lipschitz condition of random function.
Tôhoku Math. J., Vol. 46, pp. 145–9, 1939.

(146) GINSBOURG, G. Sur les lois limites des distributions dans les procédés stochastiques.
Comm. Inst. Sci. Math. Méc. Univ. Kharkoff, Vol. 17, pp. 65–73, 1940. (Russian. French summary.)

(147) GINSBOURG, G. Sur les conditions suffisantes pour l'unicité des distributions limites.
C. R. Acad. Sci. U.R.S.S., N., Vol. 30, pp. 295–7, 1941.

(148) GNEDENKO, B. Investigation of the growth of homogeneous random processes with independent increments.
C. R. Acad. Sci. U.R.S.S., Vol. 36, 3–4, 1942.

(149) HÖFFDING, W. Stochastische Abhängigkeit und funktionaler Zusammenhang.
Skandinavisk Aktuarietidskrift, Vol. XXV, pp. 200–27, 1942.

(150) HOSTINSKÝ, B. Sur le coefficient de corrélation.
Memorial volume dedicated to D. A. Grave, Moscow, 1940, pp. 48–51.

(151) KAKUTANI, S. Some results in the operator-theoretical treatment of the Markoff process.
Proc. Imp. Acad. Tokyo, Vol. 15, 260–4, 1939.

(152) KAKUTANI, S. Ergodic theorems and the Markoff process with a stable distribution.
Proc. Imp. Acad. Tokyo, Vol. 16, pp. 49–54, 1940.

(153) KAWADA, YUKIYOSI. On the measurable stochastic process.
Proc. Phys.-Maths. Soc., Japan, III, s. 23, pp. 512–27, 1941.

(154) KHINTCHINE, A. Sur la croissance locale de processus stochastiques homogènes à accroissements indépendants.
Bull. Acad. Sci. U.R.S.S., Sér. Math., Vol. 1939, pp. 487–508. (Russian. French summary.)

(155) KITAGAWA, T. The limit theorems of the stochastic contagious processes.
Mem. Fac. Sci. Kyūsyū Imp. Univ. A., Vol. 1, pp. 167–94, 1941.

CC-0. In Public Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(156) KITAGAWA, T. The weakly contagious stochastic process which depends upon the Gaussian distribution.
Mem. Fac. Sci. Kyūsyū Imp. Univ. A., Vol. 2, pp. 37–65, 1941.

(157) KITAGAWA, T., and HURUYA, S. The application of the limit theorem of the contagious stochastic process to the contagious diseases.
Mem. Fac. Sci. Kyusyu Imp. Univ. A., Vol. 1, pp. 195–207, 1941.

(158) KOLMOGOROFF, A. N. Interpolation und Extrapolation von stationären zufälligen Folgen.
Bull. Acad. Sci. U.R.S.S., Sér. Math., Vol. 5, pp. 3–14, 1941.

(159) KOLMOGOROFF, A. N. Stationary sequences in Hilbert's space.
Bolletin Moskovskogo Gosudarstvenogo Universiteta. Matematika, Vol. 2, 1941.

(160) KOSULAJEFF, P. A. Sur les problèmes d'interpolation et d'extrapolation des suites stationnaires.
C. R. Acad. Sci. U.R.S.S., (*N.S.*), Vol. 30, pp. 13–17, 1941.

(161) LABUTIN, D. N. On a generating function.
Leningrad State Univ. Annals (*Uchenye Zapiski*), *Math. Sér.* 10, pp. 139–47, 1940. (Russian.)

(162) LÉVY, P. Sur certains processus stochastiques homogènes.
Compositio Math., Vol. 7, pp. 283–339, 1939.

(163) LÉVY, P. Intégrales stochastiques.
C. R. Acad. Sci. Paris, Vol. 212, pp. 1066–8, 1941.

(164) ONICESCU, O., and MIHOC, G. Sur l'application des équations fonctionnelles de Chapman et Smoluchowsky dans la théorie des chaînes de Markoff.
Acad. Roum. Bull. Sect. Sci., Vol. 21, pp. 110–12, 1939.

(165) ONICESCU, O., and MIHOC, G. Propriétés asymptotiques des chaînes de Markoff étudié à l'aide de la fonction caractéristique.
Mathematica, Vol. 16, pp. 13–43, 1940.

(166) ONICESCU, O., and MIHOC, G. Sur les sommes de variables enchaînées dans le cas d'un ensemble numérable de valeurs.
Acad. Roum. Bull. Sect. Sci., Vol. 22, pp. 231–6, 1940.

(167) OPATOWSKI, I. An inverse problem concerning a chain process.
Proc. Nat. Acad. Sci., Vol. 28, pp. 83–8, 1942.

(168) PITT, H. R. A special class of homogeneous random processes.
J. Lond. Math. Soc., Vol. 15, pp. 247–57, 1940.

(169) YOSIDA, K. The Markoff process with a stable distribution.
Proc. Imp. Acad., Tokyo, Vol. 16, pp. 43–8, 1940.

(170) YOSIDA, K., and KAKUTANI, S. Operator-theoretical treatment of Markoff's process and mean ergodic theorem.
Annals of Mathematics, Vol. 42, pp. 188–228, 1941.

(171) ZASUHIN, V. On the theory of multidimensional stationary random processes.
C. R. Acad. Sci. U.R.S.S., Vol. 33, pp. 435–7, 1941.

IV. SPECIAL DISTRIBUTIONS

(172) ARLEY, N. On the distribution of relative errors from a normal population of errors. A discussion of some problems in the theory of errors.
Danske Vid. Selsk. Math. Fys. Medd., Vol. 18, No. 3, 62 pp., 1940.

(173) BHATTACHARYYA, B. C. A note on the tractrix and the cycloid as statistical distribution curves.
Bull. Calcutta Math. Soc., Vol. 34, pp. 105–8, 1942.

(174) BIRNBAUM, Z. W. An inequality for Mill's ratio.
Annals Math. Statistics, Vol. XIII, pp. 245–6, 1942.

(175) DAVIES, G. R., and SMITH, R. H. Probabilities in logarithmic skewed distributions.
J. Amer. Stat. Ass., Vol. 36, pp. 493–9, 1941.

(176) DEHALU, M. Sur la démonstration de la formule de K. Pearson dans le cas du schéma simple des urnes.
Bull. Soc. Roy. Sci. Liége 11, pp. 146–51, 1942.

CC-0. In Public Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(177) DELGLEIZE, A. Sur la schéma simple des urnes.
Bull. Soc. Roy. Sci. Liége 11, 398–403, 1942.
(178) DIENLEFAIT, C. E. Sui momenti delle distribuzioni ipergeometriche.
Giorn. Ist Ital. Attuari, Vol. X, pp. 221–4, 1939.
(179) DUGUÉ, D. Sur un nouveau type de courbe de fréquence.
C. R. Acad. Sci. Paris, Vol. 213, pp. 634–5, 1941.
(180) DUGUÉ, D. Sur certaines composantes des lois de Cauchy.
C. R. Acad. Sci. Paris, Vol. 213, pp. 718–19, 1941.
(181) EVANS, W. D. Note on the moments of a binomially distributed variate.
Annals Math. Statistics, Vol. XI, pp. 106–7, 1940.
(182) FELDHEIM, E. Nuova dimostrazione e generalizzazione di un teorema di calcolo delle probabilità.
Giorn. Ist Ital. Attuari, Vol. X, pp. 229–43, 1939.
(183) FELDHEIM, E. Nouvelle démonstration et généralisation d'un théorème du calcul des probabilités dû à Simmons.
J. Math. Pure Appl., t. 20, pp. 1–16, 1941.
(184) FINNEY, D. J. On the distribution of a variate whose logarithm is normally distributed.
J. Roy. Stat. Soc., Suppl., Vol. 7, pp. 155–61, 1941.
(185) FRÉCHET, M. Sur une loi de probabilité considérée par J. F. Steffensen.
Skandinavisk Aktuarietidskrift, Vol. XXIV, pp. 214–20, 1941.
(186) FRÉCHET, M. Sur la correspondance entre certaines lois d'erreurs et certaines definitions de la distance.
Rev. Sci., Vol. 79, pp. 3–14, 1941.
(187) GEIRINGER, HILDA. A new explanation of non-normal dispersion in the Lexis theory.
Econometrika, Vol. X, pp. 53–60, 1942.
(188) GORDON, R. D. Values of Mills' ratio of area to bounding ordinate and of the normal probability integral for large values of the argument.
Annals Math. Statistics, Vol. XII, pp. 364–6, 1941.
(189) GREENWOOD, J. A. The significance of a critical ratio.
J. Elisha Mitchell Sci. Soc., Vol. 57, p. 209, 1941.
(190) GROSHEV, A. Sur le domaine d'attraction de la loi de Poisson.
Bull. Acad. Sci. U.R.S.S., Sér. Math., Vol. 5, pp. 165–72, 1941. (Russian. French summary.)
(191) GUMBEL, E. J. La dissection d'une répartition.
Ann. Univ. Lyon, A., Vol. 11, pp. 39–51, 1939.
(192) HALDANE, J. B. S. The cumulants and moments of the binomial distribution and the cumulants of χ^2 for a $(n \times 2)$ fold table.
Biometrika, Vol. XXXI, pp. 392–6, 1940.
(193) HALDANE, J. B. S. Moments of the distributions of powers and products of normal variates.
Biometrika, XXXII, pp. 226–42, 1942.
(194) HARTMAN, P. Normal distributions and the law of the iterated logarithm.
Amer. J. Math., Vol. LXIII, pp. 584–88, 1941.
(195) KAC, M. Note on the partial sums of the exponential series.
Univ. Nac. Tucumán., Vol. 3, pp. 151–3, 1942.
(196) KANELLOS, S. Étude des répétitions dans une suite infinie de tirages. Loi des répétitions.
Bull. Soc. Math. Grèce, Vol. 20, pp. 79–84, 1940.
(197) KÄRSNA, A. Über das System der einmodigen Häufigkeitskurven.
Acta Comment. Univ. Tartuensis. A., Vol. 35, No. 1, 65 pp., 1940.
(198) KENDALL, M. G. A recurrence relation for the semi-invariants of Pearson curves.
Biometrika, Vol. XXXII, pp. 81–2, 1940–1.
(199) KRISHNASWAMI AYYANGAR, A. A. The triangular distribution.
Math. Student, Vol. 9, pp. 85–7, 1941.
(200) LÉVY, P. Sur une loi de probabilité analogue à celle de Poisson et sur un sous-groupe important du groupe des lois indéfiniment divisibles.
Bull. Sci. Math., Vol. 63, pp. 247–68, 1939.

CC-0. In Public Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(201) LIDSTONE, G. J. Notes on the Poisson frequency distribution.
J. Inst. Actuaries, Vol. LXXI, pp. 284–91, 1942.
(202) LUKACS, E. A characterization of the normal distribution.
Annals Math. Statistics, Vol. XIII, pp. 91–3, 1942.
(203) QUENSEL, C. E. Truncated normal curves and correlation distributions.
Lunds Univ. Arsskr. N.F. 36, 17 pp., 1940.
(204) RODGERS, E. Probable errors for Poisson distributions.
Phys. Rev., Vol. 57, pp. 735–7, 1940.
(205) SATTERTHWAITE, F. E. Generalized Poisson distribution.
Annals Math. Statistics, Vol. XIII, pp. 410–17, 1942.
(206) VON SCHELLING, H. Zur Statistiik seltener Ereignisse.
Astron. Nachr., Vol. 270, pp. 189–92, 1940.
(207) SCHELLING, H. VON. Eine Formel für die Teilsummen gewisser hypergeometrischer Reihen und deren Bedeutung fur die Wahrscheinlichkeitstheorie.
Naturwissenschaften, Vol. 30, pp. 757–8, 1942.
(208) SCHREK, R., and LIPSON, H. I. Logarithmic frequency distributions.
Human Biology, Vol. 13, pp. 1–22, 1941.
(209) SHANNON, S. Comparative aspects of the point binomial polygon and its associated normal curve of error.
Rec. Amer. Inst. Actuar., Vol. 31, pp. 208–26, 1942.
(210) WINTNER, A. The singularities of Cauchy's distributions.
Duke Math. J., Vol. 8, pp. 678–81, 1941.

V. Sampling Distributions

(211) ANDERSON, R. L. Distribution of the serial correlation coefficient.
Annals Math. Statistics, Vol. XIII, 1–13, 1942.
(212) AROIAN, LEO A. A study of R. A. Fisher's z distribution and the related F distribution.
Annals Math. Statistics, Vol. XII, pp. 429–48, 1941.
(213) BAKER, G. A. Comparison of Pearsonian approximations with exact sampling distributions of means and variances in samples from populations composed of the sums of normal populations.
Annals Math. Statistics, Vol. XI, pp. 219–24, 1940.
(214) BATICLE, E. Sur la composition des probabilités de densités constantes.
Comptes Rendus, t. 211, pp. 420–2, 1940.
(215) CRAIG, C. C. The product semi-invariants of the mean and a central moment in samples.
Annals Math. Statistics, Vol. XI, pp. 177–85, 1940.
(216) CRAIG, C. C. Note on the distribution of non-central t with an application.
Annals Math. Statistics, Vol. XII, pp. 224–8, 1941.
(217) DANIELS, H. E. A property of the distribution of extremes.
Biometrika, Vol. XXXII, pp. 194–5, 1941.
(218) DRESSEL, P. Statistical semivariants and their estimates with particular emphasis on their relation to algebraic invariants.
Annals Math. Statistics, Vol. XI, pp. 33–57, 1940.
(219) DWYER, P. S. Combinatorial formulas for the rth standard moment of the sample sum, of the sample mean, and of the normal curve.
Annals Math. Statistics, Vol. XI, pp. 353–5, 1940.
(220) EVANS, W. D. The standard error of percentiles.
J. Amer. Stat. Ass., Vol. 37, pp. 367–76, 1942.
(221) FELDHEIM, E. Sul rapporto fra la media dei quadrati di più errori e il quadrato della media dei loro valori assoluti.
Atti Accad. Sci. Torino. Cl. Sci. Fis. Mat. Nat., Vol. 75, pp. 296–305, 1940.
(222) FINNEY, D. J. The joint distribution of variance ratios based on a common error mean square.
Annals of Eugenics, Vol. XI, pp. 136–40, 1941.
(223) HARTLEY, H. O. The range in random samples.
Biometrika, Vol. XXXII, pp. 334–48, 1942.

CC-0. In Public Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(224) HSU, C. T., and LAWLEY, D. N. The derivation of the fifth and sixth moments of the distribution of b_2 in samples from a normal population.
Biometrika, Vol. XXXI, pp. 238–48, 1939–40.

(225) JOHNSON, N. L., and WELCH, B. L. Applications of the non-central t-distribution.
Biometrika, Vol. XXXI, pp. 362–89, 1940.

(226) KENDALL, M. G. Some properties of k-statistics.
Annals of Eugenics, Vol. X, pp. 106–11, 1940.

(227) KENDALL, M. G. Proof of Fisher's rules of ascertaining the sampling semi-invariants of k-statistics.
Annals of Eugenics, Vol. X, pp. 215–22, 1940.

(228) KENDALL, M. G. Note on the distribution of quantiles for large samples.
J. Roy. Stat. Soc., Suppl., Vol. VII, pp. 83–5, 1940.

(229) KENDALL, M. G. On semi-invariant statistics.
Annals of Eugenics, Vol. XI, pp. 300–5, 1942.

(230) MARTINOTTI, P. Di alcune recenti medie.
Pont. Acad. Sci. Acta, Vol. 5, pp. 113–21, 1941.

(231) NAIR, A. N. K. Distribution of Student's "t" and the correlation coefficient in samples from non-normal populations.
Sankhyā, Vol. 5, pp. 383–400, 1941.

(232) VON NEUMANN, J. Distribution of the ratio of the mean square successive difference to the variance.
Annals Math. Statistics, Vol. XII, pp. 367–95, 1941.

(233) VON NEUMANN, J. A further remark concerning the distribution of the ratio of the mean square successive difference to the variance.
Annals Math. Statistics, Vol. XIII, pp. 86–8, 1942.

(234) VON NEUMANN, J., KENT, R. H., BELLISON, H. R., and HART, B. I. The mean square successive difference.
Annals Math. Statistics, Vol. XII, pp. 153–62, 1941.

(235) NORRIS, N. The standard errors of the geometric and harmonic means and their application to index numbers.
Annals Math. Statistics, Vol. XI, pp. 445–8, 1940.

(236) PAULSON, E. An approximate normalization of the analysis of variance distribution.
Annals Math. Statistics, Vol. XIII, pp. 233–5, 1942.

(237) PIZZETTI, E. Osservazione sulle medie esponenziali e baso-esponenziale. (Exponential and baso-exponential means.)
Metron, Vol. XIII, pp. 1–15, 1939.

(238) RADHAKRISHNA RAO, C. On the sum of n observations from different gamma type populations.
Science and Culture, Vol. 7, pp. 614–15, 1942.

(239) RAJALAKSHMAN, D. V. On the extreme values of samples taken from a rectangular population.
Math. Student, Vol. 9, pp. 103–11, 1941.

(240) ROJAS, G. C. Formula para el error standard de una razón.
Bol. Soc. Venezolana Cienc. Nat., Vol. 7, pp. 229–31, 1942.

(241) SATTERTHWAITE, F. E. Linear restrictions on chi-square.
Annals Math. Statistics, Vol. XIII, pp. 326–31, 1942.

(242) SHRIVASTAVA, M. P. The distribution of the mean for certain Bessel function populations.
Science and Culture, Vol. 6, pp. 244–5, 1940.

(243) SMIRNOFF, N. Sur les écarts de la courbe de distribution empirique.
Rec. Math. Moscou, N.S., Vol. 6, pp. 3–26, 1939.

(244) SMIRNOFF, N. On the estimation of the discrepancy between empirical curves of distribution for two independent samples.
Bull. Math. Univ. Moscou, Vol. 2, 16 pp., 1939.

(245) SMIRNOFF, N. On the estimation of the maximum term in a series of observations.
C. R. Acad. Sci. U.R.S.S., Vol. 33, pp. 346–50, 1941.

CC-0. In Public Domain. Gurukul Kangri Collection, Hafidwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(246) TRUSKA, L. The simultaneous distribution in samples of mean and standard deviation and of mean and variance.
Biometrika, Vol. XXXI, pp. 256–71, 1939–40.

(247) VALI, M. A. On the sampling distribution of harmonic means.
Bull. Calcutta Math. Soc., Vol. 34, pp. 87–91, 1942.

(248) WILLIAMS, J. D. Moments of the ratio of the mean square successive difference in samples from a normal universe.
Annals Math. Statistics, Vol. XII, pp. 239–41, 1941.

(249) YOUNG, L. C. On randomness in ordered sequences.
Annals Math. Statistics, Vol. XII, 293–300, 1941.

VI. Problems of Estimation and Tests of Statistical Hypotheses

(250) ABBOT, C. G. On periodicities in the Smithsonian solar-constant values.
Bull. Amer. Meteorol. Soc., Vol. 21, pp. 156–7, 1940.

(251) AITKEN, A. C., and SILVERSTONE, H. On the estimation of statistical parameters.
Proc. Roy. Soc. Edinb., Sect. A., Vol. 61, pp. 186–94, 1942.

(252) BAKER, G. A. Maximum likelihood estimation of the ratio of the components of non-homogeneous populations.
Tohôku Math. J., Vol. 47, pp. 304–8, 1940.

(253) BAKER, G. A. Test of homogeneity for normal populations.
Annals Math. Statistics, Vol. XII, pp. 233–6, 1941.

(254) BARTLETT, M. S. A note on the interpretation of quasi-sufficiency.
Biometrika, Vol. XXXI, pp. 391–2, 1940.

(255) BERKSON, J. A. A note on the chi-square test, the Poisson and the binomial.
J. Amer. Stat. Ass., Vol. 35, pp. 362–7, 1940.

(256) BERKSON, J. Tests of significance considered as evidence.
J. Amer. Stat. Ass., Vol. 37, pp. 325–35, 1942.

(257) BERNSTEIN, S. On " fiducial " probabilities of Fisher.
Bull. Acad. Sci. U.R.S.S., Sér. Math., Vol. 5, pp. 85–94, 1941.

(258) BONNIER, G. The four-fold table and the heterogeneity test.
Science, Vol. 96, pp. 13–14, 1942.

(259) BRIEGER, F. G. The χ^2 test.
Jor. Agron. (*São Paulo*), Vol. 3, pp. 103–10, 1940.

(260) BROOKNER, R. J. A note on the mean as a poor estimate of central tendency.
J. Amer. Stat. Ass., Vol. 36, pp. 410–12, 1941.

(261) CAMP, B. H. Further comments on Berkson's problem.
J. Amer. Stat. Ass., Vol. 35, pp. 368–76, 1940.

(262) CHANDRA SEKAR, C., and FRANCIS, M. G. A method to get the significance limit of a type of test criteria.
Sankhyā, Vol. 5, 165–8, 1941.

(263) COCHRAN, W. G. The distribution of the largest of a set of estimated variances as a fraction of their total.
Annals of Eugenics, Vol. XI, pp. 47–52, 1941.

(264) COCHRAN, W. G. The χ^2 correction for continuity.
Iowa State Coll. J. Sci., Vol. 16, pp. 421–36, 1942.

(265) DALY, JOSEPH F. A problem in estimation.
Annals Math. Statistics, Vol. XII, pp. 459–61, 1941.

(266) DANTZIG, G. B. On the non-existence of tests of "Student's" hypothesis having power functions independent of σ.
Annals Math. Statistics, Vol. XI, pp. 186–92, 1940.

(267) DIXON, W. J. A criterion for testing the hypothesis that two samples are from the same population.
Annals Math. Statistics, Vol. XI, pp. 199–204, 1940.

(268) DODD, E. L. Certain tests for randomness applied to data grouped into small sets.
Econometrica, Vol. 10, pp. 249–57, 1942.

CC-0. In Public Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(269) DRESSEL, P. L. A symmetric method of obtaining unbiased estimates and expected values.
Annals Math. Statistics, Vol. XII, pp. 84–90, 1941.

(270) FISHER, R. A. On the similarity of the distributions found for the test of significance in harmonic analysis, and in Stevens' problem in geometrical probability.
Annals of Eugenics, Vol. X, pp. 14–17, 1940.

(271) FISHER, R. A. The asymptotic approach to Behrens's integral, with further tables for the *d* test of significance.
Annals of Eugenics, Vol. XI, pp. 141–72, 1941.

(272) FISHER, R. A. The interpretation of experimental four-fold tables.
Science, Vol. 94, pp. 210–11, 1941.

(273) GEARY, R. C. The estimation of many parameters.
J. Roy. Stat. Soc., Vol. CV, pp. 213–17, 1942.

(274) GEBELEIN, H. Verfahren zur Beurteilung einer sehr geringen Korrelation zwischen zwei statistischen Merkmalsreihen.
Z. Angew. Math. Mech., Vol. 22, pp. 286–98, 1942.

(275) GEORGE, A. On the problem of interval estimation.
Sankhyā, Vol. 6, pp. 111–20, 1942.

(276) GORDON, ROBERT D. The estimation of a quotient when the denominator is normally distributed.
Annals Math. Statistics, Vol. XII, pp. 115–18, 1941.

(277) GUMBEL, E. J. La probabilité des hypothèses.
Comptes Rendus, t. 209, pp. 645–7, 1939.

(278) GUMBEL, E. J. Simple tests for given hypotheses.
Biometrika, Vol. XXXII, pp. 317–33, 1942.

(279) HALDANE, J. B. S. The mean and variance of χ^2 when used as a test of homogeneity, when expectations are small.
Biometrika, Vol. XXXI, pp. 346–55, 1940.

(280) HARTLEY, H. O. Testing the homogeneity of a set of variances.
Biometrika, Vol. XXXI, pp. 249–55, 1939–40.

(281) HOTELLING, H. Experimental determination of the maximum of a function.
Annals Math. Statistics, Vol. XII, pp. 20–45, 1941.

(282) JEFFREYS, H. Note on the Behrens–Fisher formula.
Annals of Eugenics, Vol. X, pp. 48–51, 1940.

(283) JEFFREYS, HAROLD. Some applications of the method of minimum χ^2 test.
Annals of Eugenics, Vol. XI, pp. 108–14, 1941.

(284) JEFFREYS, H. On the significance tests for the introduction of new functions to represent measures.
Proc. Roy. Soc., A., Vol. 180, pp. 256–68, 1942.

(285) JOHNSON, N. L. Parabolic test for linkage.
Annals Math. Statistics, Vol. XI, pp. 227–53, 1940.

(286) KENDALL, M. G. On the method of maximum likelihood.
J. Roy. Stat. Soc., Vol. CIII, pp. 388–99, 1940.

(287) KOLMOGOROFF, A. Confidence limits for an unknown distribution function.
Annals Math. Statistics, Vol. XII, pp. 461–3, 1941.

(288) KOLMOGOROFF, A. Sur l'estimation statistique des paramètres de la loi de Gauss.
Bull. Acad. Sci. U.R.S.S., Sér. Math., Vol. 6, pp. 3–32, 1942.

(289) KOOPMANS, T. Serial correlation and quadratic forms in normal variables.
Annals Math. Statistics, Vol. XIII, pp. 14–33, 1942.

(290) KULLBACK, S., and FRANKEL, A. A simple sampling experiment on confidence intervals.
Annals Math. Statistics, Vol. XI, pp. 209–13, 1940.

(291) LÉVY, P. Sur la détermination expérimentale de la loi des erreurs.
Enseignement Math., Vol. 38, pp. 227–31, 1942.

(292) MACSTEWART, W. A note on the power of the sign test.
Annals Math. Statistics, Vol. XII, pp. 236–9, 1941.

(293) MADOW, W. G. Note on tests of departure from normality.
J. Amer. Stat. Ass., Vol. 35, pp. 515–17, 1940.

CC-0. In Public Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(294) MANN, H. B., and WALD, A. On the choice of the number of class intervals in the application of the chi-square test.
Annals Math. Statistics, Vol. XIII, pp. 306–17, 1942.

(295) MOORE, G. H., and WALLIS, W. A. A significance test for time series.
Nat. Bur. of Econ. Res., Tech. Pap. No. 1, 1941.

(296) MULLER, H. J. On judging the significance of a difference obtained by averaging essentially different series.
Amer. Nat., Vol. 75, pp. 264–71, 1941.

(297) NAIR, K. R. The median in tests by randomization.
Sankhyā, Vol. 4, pp. 543–50, 1940.

(298) NAIR, K. R. Table of confidence interval for the median in samples from any continuous population.
Sankhyā, Vol. 4, pp. 551–8, 1940.

(299) NAIR, U. S. A comparison of tests for the significance of the difference between two variances.
Sankhyā, Vol. 5, pp. 157–64, 1941.

(300) NEYMAN, J. On a statistical problem arising in routine analyses and in sampling inspections of mass production.
Annals Math. Statistics, Vol. XII, pp. 46–76, 1941.

(301) NEYMAN, J. Fiducial argument and the theory of confidence intervals.
Biometrika, Vol. XXXII, pp. 128–50, 1941.

(302) NEYMAN, J. Basic ideas and some recent results of the theory of testing statistical hypotheses.
J. Roy. Stat. Soc., Vol. CV, pp. 292–327, 1942.

(303) OLSHEVSKY, L. Two properties of sufficient statistics.
Annals Math. Statistics, Vol. XI, pp. 104–6, 1940.

(304) PAULSON, EDWARD. On certain likelihood-ratio tests associated with the exponential distribution.
Annals Math. Statistics, Vol. XII, pp. 301–6, 1941.

(305) PEARSON, E. S. A note on further properties of statistical tests.
Biometrika, Vol. XXXII, pp. 59–61, 1940–1.

(306) PEARSON, E. S. Notes on testing statistical hypotheses.
Biometrika, Vol. XXXII, pp. 311–16, 1942.

(307) PITMAN, E. J. G. The estimation of the location and scale parameters of a continuous population of any given form.
Biometrika, Vol. XXX, pp. 391–421, 1939.

(308) PRZYBOROWSKI, J., and WILENSKI, H. Homogeneity of results in testing samples from Poisson series, with an application to testing clover seed for fodder.
Biometrika, Vol. XXXI, pp. 313–23, 1940.

(309) ROMANOVSKI, V. I. On inductive conclusions in statistics.
C. R. Acad. Sci. U.R.S.S., N.S., Vol. 27, pp. 419–21, 1940.

(310) ROSANDER, A. C. The use of inversions as a test of random order.
J. Amer. Stat. Ass., Vol. 37, pp. 352–8, 1942.

(311) SAVUR, S. R. A test of significance in approximate periodogram analysis.
Sankhyā, Vol. 6, pp. 77–84, 1942.

(312) SAWKINS, D. T. Remarks on goodness of fit of hypotheses and on Pearson's χ^2 test.
J. Proc. Roy. Soc. New South Wales, Vol. 75, pp. 85–95, 1941.

(313) SCHEFFÉ, H. On the theory of testing composite hypotheses with one constraint.
Annals Math. Statistics, Vol. XIII, pp. 280–93, 1942.

(314) SCHEFFÉ, H. On the ratio of the variances of two normal populations.
Annals Math. Statistics, Vol. XIII, pp. 371–88, 1942.

(315) VON SCHELLING, H. Kennzeichen für eine zufällige Folge der Werte in einer zeitlich geordneten Beobachtungsreihe.
Astron. Nachr., Vol. 269, pp. 155–9, 1939.

(316) VON SCHELLING, H. Zur Beurteilung einer alternativen Stichprobe von n Beobachtungen.
Deutsche Math., Vol. 5, pp. 107–15, 1940.

CC-0. In Public Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(317) VON SCHELLING, H. Statistische Schätzungen auf kombinatorischer Grundlage.
Z. Angew Math. Mech., Vol. 21, pp. 52–8, 1941.
(318) VON SCHELLING, H. Mutungsgrenzen für den Ausfall einer geplanten Versuchsreihe beschrankten Umfangs.
Naturwiss., Vol. 30, pp. 64–5, 1942.
(319) SIMAIKA, J. B. On an optimum property of two important statistical tests.
Biometrika, Vol. XXXII, pp. 70–80, 1940–1.
(320) TILDESLEY, M. L. Sources and extent of errors in estimating standard deviations of normally distributed populations.
Man, Vol. 40, pp. 146–55, 1940.
(321) VAN DER WAERDEN, B. L. Vertrauensgrenzen für unbekannte Wahrscheinlichkeiten.
Ber. Verh. Sächs. Akad. Wiss. Leipzig, Vol. 91, pp. 213–28, 1939.
(322) WALD, ABRAHAM. Asymptotically most powerful tests of statistical hypotheses.
Annals Math. Statistics, Vol. XII, pp. 1–19, 1941.
(323) WALD, A. Some examples of asymptotically most powerful tests.
Annals Math. Statistics, Vol. XII, pp. 396–408, 1941.
(324) WALD, A. On the principles of statistical inference.
Notre Dame Mathematical Lectures, No. 1, Univ. of Notre Dame, Notre Dame, Ind., 1942.
(325) WALD, A. Asymptotically shortest confidence intervals.
Annals Math. Statistics, Vol. XIII, pp. 127–37, 1942.
(326) WALD, A. Setting of tolerance limits when the sample is large.
Annals Math. Statistics, Vol. XIII, pp. 389–99, 1942.
(327) WALD, A., and WOLFOWITZ, J. On a test whether two samples are from the same population.
Annals Math. Statistics, Vol. XI, pp. 147–62, 1940.
(328) WALD, A., and WOLFOWITZ, J. Note on confidence limits for continuous distribution functions.
Annals Math. Statistics, Vol. XII, pp. 118–19, 1941.
(329) WALLIS, W. A. Compounding probabilities from independent significance tests.
Econometrica, Vol. X, pp. 229–48, 1942.
(330) WALLIS, W. A., and MOORE, G. H. A significance test for time series analysis.
J. Amer. Stat. Ass., Vol. 36, pp. 401–9, 1941.
(331) WILKS, S. S. Confidence limits and critical differences between percentages.
Publ. Opin. Quart., Vol. 4, pp. 332–8, 1940.
(332) WILKS, S. S. Determination of sample sizes for setting tolerance limits.
Annals Math. Statistics, Vol. XII, pp. 91–6, 1941.
(333) WILKS, S. S. Statistical prediction with special reference to the problem of tolerance limits.
Annals Math. Statistics, Vol. XIII, pp. 400–9, 1942.
(334) WILSON, E. B. The controlled experiment and the four-fold table.
Science, Vol. 93, pp. 557–60, 1941.
(335) WILSON, E. B. On confidence intervals.
Proc. Nat. Acad. Sci. U.S.A., Vol. 28, pp. 88–93, 1942.
(336) WILSON, E. B. On contingency tables.
Proc. Nat. Acad. Sci. U.S.A., Vol. 28, pp. 94–100, 1942.
(337) WILSON, E. B., and WORCESTER, J. Note on the *t*-test.
Proc. Nat. Acad. Sci. U.S.A., Vol. 28, pp. 297–301, 1942.
(338) WILSON, E. B., and WORCESTER, J. Contingency tables.
Proc. Nat. Acad. Sci. U.S.A., Vol. 28, pp. 378–84, 1942.
(339) WILSON, E. B., and WORCESTER, J. The association of three attributes.
Proc. Nat. Acad. Sci. U.S.A., Vol. 28, pp. 384–90, 1942.
(340) WOLFOWITZ, J. Additive partition functions and a class of statistical hypotheses.
Annals Math. Statistics, Vol. XIII, pp. 247–79, 1942.
(341) YATES, F. An apparent inconsistency arising from tests of significance based on fiducial distributions of unknown parameters.
Proc. Camb. Phil. Soc., Vol. 35, pp. 579–609, 1939.

CC-0. In Public Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

VII. Bivariate Problems

(342) Bachmann, W. K. L'ellipsoïde d'erreur.
Schweiz. Z. Vermessgswes., Vol. 28, pp. 181–97, 201–8, 213–16, 1940.

(343) Baker, G. A. Linear regression when the standard deviation of arrays are not all equal.
J. Amer. Stat. Ass., Vol. 36, pp. 500–6, 1941.

(344) Baker, G. A. Correlations between functions of variables.
J. Amer. Stat. Ass., Vol. 37, pp. 537–9, 1942.

(345) Banerjee, D. P. Note on the limit of correlation and regression coefficients in mingled records.
Math. Student, Vol. 9, pp. 155–7, 1941.

(346) Bojarski, A. Sur la corrélation géométrique.
Bull. Acad. Sci. U.R.S.S., Sér. Math., Vol. 5, pp. 159–64, 1941.

(347) Bonferroni, C. E. Di una estensione del coefficiente di correlazione.
Giorn. degli Economista (*N.S.*), Vol. 1, pp. 797–826, 1939.

(348) Bose, P. On the exact distribution of the ratio of two means belonging to samples drawn from a given correlated bivariate normal population.
Bull. Calcutta Math. Soc., Vol. 34, pp. 139–41, 1942.

(349) Casanova, T. Corrections to correlation coefficients on account of homogeneity in one variable.
J. exp. Educ., Vol. 8, pp. 341–5, 1940.

(350) Charnley, F. Some properties of a composite bivariate distribution in which the means of the component normal distributions are linearly related.
Canadian Jour. Res. A., Vol. 19, pp. 139–51, 1941.

(351) Charnley, F. The variances of the means and the variance of the slope of the line of relation of a linear, composite, bivariate distribution.
Canadian Jour. Res. A., Vol. 20, pp. 6–9, 1942.

(352) Cherian, K. C. A bi-variate correlated gamma-type distribution function.
J. Indian Math. Soc., N.S., 5, pp. 133–44, 1941.

(353) Clough, H. W. A note on methods of correlation.
Bull. Amer. Meteorol. Soc., Vol. 23, p. 410, 1942.

(354) Dieulefait, C. On a result of Prof. Beppo Levi and its relation to the problem of frequency surfaces.
Ann. Soc. Ci. Argentina, Vol. 129, pp. 249–53, 1940.

(355) Dwyer, P. S. The skewness of the residuals in linear regression theory.
Annals Math. Statistics, Vol. XII, pp. 104–10, 1941.

(356) Dwyer, P. S. Recent developments in correlation technique.
J. Amer. Stat. Ass., Vol. 37, pp. 441–60, 1942.

(357) Gebelein, H. Das statistische Problem der Korrelation als Variations- und Eigenwert-problem und sein Zusammenhang mit der Ausgleichsrechnung.
Z. Angew. Math. Mech., Vol. 21, pp. 364–79, 1941.

(358) Gebelein, H. Bemerkung über ein von W. Höffding vorgeschlagenes, massstabsinvariantes Korrelationsmass.
Z. Angew. Math. Mech., Vol. 22, pp. 171–3, 1942.

(359) Geppert, M. P. Su una classe di distributioni in due variabili casuali.
Giorn. Ist Ital. Attuari, Vol. X, pp. 225–8, 1939.

(360) Geppert, M. Über eine Klasse von zweidimensionalen Verteilungen.
Z. Angew. Math. Mech., Vol. 20, pp. 45–9, 1940.

(361) Hsu, C. T. On samples from a normal bivariate population.
Annals Math. Statistics, Vol. XI, pp. 410–26, 1940.

(362) Hsu, C. T. Samples from two bivariate normal populations.
Annals Math. Statistics, Vol. XII, pp. 279–92, 1941.

(363) Humm, D. G. Note on the product-moment correlation of non-rectilinear data.
Amer. J. Psychol., Vol. 55, pp. 127–30, 1942.

(364) Kendall, M. G. Proof of relations connected with the tetrachoric series and its generalization.
Biometrika, Vol. XXXII, pp. 196–8, 1941.

CC-0. In Public Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(365) KIBBLE, W. F. A two-variate gamma-type distribution.
Sankhyā, Vol. 5, pp. 137–50, 1941.
(366) KOSAMBI, D. D. A bivariate extension of Fisher's *z* test.
Current Sci., Vol. 10, pp. 191–3, 1941.
(367) KRAMER, M. Frequency surfaces in two variables each of which is uniformly distributed
Amer. Jour. Hyg., Vol. 32, pp. 45–65, 1940.
(368) LIENAU, C. C. Discrete bivariate distribution in certain problems of statistical order.
Amer. J. Hyg. A., Vol. 33, pp. 65–85, 1941.
(369) DE LURY, D. B. A test for significant difference between corresponding coefficients of two regression lines. Appendix to "Digestibility studies with ruminants. V."
Scientific Agriculture, Vol. 20, 175–204, 1939.
(370) NAIR, U. S. Probability statements regarding the ratio of standard deviations and correlation coefficient in a bivariate normal population.
Sankhyā, Vol. 5, pp. 151–6, 1941.
(371) NICHOLSON, C. A geometrical analysis of the frequency distribution of the ratio between two variables.
Biometrika, Vol. XXXII, pp. 16–28, 1940–1.
(372) O'NEIL, J. B. The analysis of covariance by the method of individual comparisons.
Scientific Agriculture, Vol. 22, pp. 721–4, 1942.
(373) ORTS, J. M. The stability of the normal probability distribution for two random variables.
Revista Mat. Hisp.-Amer., Vol. 1, pp. 34–6, 1941.
(374) PAULSON, E. A note on the estimation of some mean values for a bivariate distribution.
Annals Math. Statistics, Vol. XIII, pp. 440–5, 1942.
(375) RADHAKRISHNA RAO, C. On bivariate correlation surfaces.
Science and Culture, Vol. 8, pp. 236–7, 1942.
(376) SAMUELSON, PAUL A. A note on alternative regressions.
Econometrika, Vol. X, pp. 80–3, 1942.
(377) SARMANOFF, O. W. Sur la corrélation isogène.
C. R. Acad. Sci. U.R.S.S., N., Vol. 32, pp. 28–30, 1941.
(378) SCHNEIDER, O. On a parameter used to characterize bivariate statistical distributions. (Spanish.)
Ann. Soc. Ci. Argentina, Vol. 133, pp. 397–401, 1942.
(379) SCHREK, R. Logarithmic correlation coefficients and regression equations.
Human Biology, Vol. 14, pp. 95–103, 1942.
(380) SHRIVASTAVA, M. P. Bi-variate correlation surfaces.
Science and Culture, Vol. 6, pp. 615–16, 1941.
(381) SINGLETON, R. R. A method of minimizing the sum of absolute values of deviations.
Annals Math. Statistics, Vol. XI, pp. 301–10, 1940.
(382) SITARAMAN, B. On correlation constants in mingled records.
Math. Student, Vol. 8, pp. 73–5, 1940.
(383) STEFFENSEN, J. F. On the coefficient of correlation for continuous distributions.
Skandinavisk Actuarietidskrift, 1941, pp. 1–12, 1941.
(384) STEFFENSEN, J. F. On the ω test of dependence between statistical variables.
Skandinavisk Actuarietidskrift, 1941, pp. 13–33, 1941.
(385) WALD, A. The fitting of straight lines if both variables are subject to error.
Annals Math. Statistics, Vol. XI, pp. 285–300, 1940.
(386) WOOLLEY, E. B. The method of minimized areas as a basis for correlation analysis.
Econometrica, Vol. IX, pp. 38–62, 1941.

VIII. MULTIVARIATE PROBLEMS

(387) AITKEN, A. C. On the independence of linear and quadratic forms in samples of normally distributed variates.
Proc. Roy. Soc. Edinb., Vol. 60, pp. 40–6, 1940.
(388) BARTLETT, M. S. A note on tests of significance in multivariate analysis.
Proc. Camb. Phil. Soc., Vol. 35, pp. 180–185, 1939.

CC-0. In Public Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(389) BARTLETT, M. S. The standard errors of discriminant function coefficients.
J. Roy. Stat. Soc. Suppl., Vol. VI, pp. 169–73, 1939

(390) BARTLETT, M. S. The statistical significance of canonical correlations.
Biometrika, Vol. XXXII, pp. 29–37, 1940.

(391) BHATTACHARYYA, B. C. An alternative method of the distribution of Mahalanobis's D^2 statistic.
Bull. Calcutta Math. Soc., Vol. 33, pp. 87–92, 1941.

(392) BHATTACHARYYA, D. P., and NARAYAN, R. D. Moments of the D^2-statistic for populations with unequal dispersions.
Sankhyā, Vol. 5, pp. 401–12, 1941.

(393) BOSE, P. On the reduction formulae for the incomplete probability integral of the multiple correlation coefficient of the second kind.
Science and Culture, Vol. 7, pp. 171–2, 1941.

(394) BOSE, P. Certain moment calculations connected with multivariate normal populations.
Science and Culture, Vol. 7, pp. 411–12, 1942.

(395) BROSTER, E. J. Correlation analysis by margins.
J. Amer. Stat. Ass., Vol. 37, pp. 359–66, 1942.

(396) DALY, J. F. On the unbiased character of likelihood-ratio tests for independence in normal systems.
Annals Math. Statistics, Vol. XI, pp. 1–32, 1940.

(397) DAY, B. B., and SANDOMIRE, M. M. Use of the discriminant function for more than two groups.
J. Amer. Stat. Ass., Vol. 37, pp. 461–72, 1942.

(398) FISCHER, C. H. A sequence of discrete variables exhibiting correlation due to common elements.
Annals Math. Statistics, Vol. XIII, pp. 97–101, 1942.

(399) FISHER, R. A. The precision of discriminant functions.
Annals of Eugenics, Vol. X, pp. 422–9, 1940.

(400) GÅRDING, L. The distributions of the first and second order moments, the partial correlation coefficients, and the multiple correlation coefficient in samples from a normal multivariate population.
Skandinavisk Actuarietidskrift, Vol. XXIV, pp. 185–202, 1941.

(401) GEARY, R. C. Inherent relations between random variables.
Proc. Roy. Irish Acad., A., Vol. 47, pp. 63–76, 1942.

(402) HOEL, P. G. The errors involved in evaluating correlation determinants.
Annals Math. Statistics, Vol. XI, pp. 58–65, 1940.

(403) HOTELLING, H. The selection of variates for use in prediction with some comments on the general problem of nuisance parameters.
Annals Math. Statistics, Vol. XI, pp. 271–93, 1940.

(404) HSU, P. L. On generalized analysis of variance. I.
Biometrika, Vol. XXXI, pp. 221–37, 1939–40.

(405) HSU, P. L. An algebraic derivation of the distribution of rectangular co-ordinates.
Proc. Edinburgh Math. Soc., II, s. 6, pp. 185–9, 1940.

(406) HSU, P. L. On the limiting distribution of the canonical correlations.
Biometrika, Vol. XXXII, pp. 38–45, 1940–1.

(407) HSU, P. L. On the problem of rank and the limiting distribution of Fisher's test function.
Annals of Eugenics, Vol. XI, pp. 39–41, 1941.

(408) HSU, P. L. Canonical reduction of the general regression problem.
Annals of Eugenics, Vol. XI, pp. 42–6, 1941.

(409) HSU, P. L. On the limiting distribution of roots of a determinantal equation.
J. Lond. Math. Soc., Vol. 16, pp. 183–94, 1941.

(410) HSU, P. L. The limiting distribution of a general class of statistics.
Acad. Sinica Science Record, Vol. 1, pp. 37–41, 1942.

(411) KENDALL, M. G. The derivation of multivariate sampling formulae from univariate formulae by symbolic operation.
Annals of Eugenics, Vol. X, pp. 392–402, 1940.

CC-0. In Public Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(412) KENDALL, M. G. The relationship between correlation formulæ and elliptic functions.
J. Roy. Stat. Soc., Suppl., Vol. CIV, pp. 281–3, 1941.
(413) KOSAMBI, D. D. A test of significance for multiple observations.
Current Sci., Vol. 11, pp. 271–4, 1942.
(414) LESER, C. E. V. Inequalities for multivariate frequency distributions.
Biometrika, Vol. XXXII, pp. 284–93, 1942.
(415) MASUYAMA, M. Correlation between tensor quantities.
Proc. Phys.-Maths. Soc., Japan, III, s. 21, pp. 638–47, 1939.
(416) MASUYAMA, M. On the meaning of symmetric correlation coefficient between vector sets.
Proc. Phys.-Maths. Soc. Japan, III, s. 22, pp. 579–85, 1940.
(417) MASUYAMA, M. On the subdependency.
Proc. Phys.-Math. Soc., Japan, III, s. 22, 855–8, 1940.
(418) MASUYAMA, M. The variance tensor of vector set and a nature of the symmetric correlation coefficient.
Proc. Phys.-Math. Soc., Japan, III, s. 22, pp. 858–61, 1940.
(419) MASUYAMA, M. The standard error of the mean vector.
Proc. Phys.-Maths. Soc., Japan (3), Vol. 23, pp. 194–5, 1941.
(420) MASUYAMA, M. The normal law of frequency for vector quantities.
Proc. Phys.-Math. Soc., Japan, III, s. 23, pp. 196–9, 1941.
(421) MASUYAMA, M. On the characteristic values of the correlation tensor and a new measure of correlation between vector quantities.
Proc. Phys.-Maths. Soc., Japan (3), Vol. 23, pp. 199–204, 1941.
(422) MASUYAMA, M. The totally orthonormalized vector set and the normal form of correlation tensor.
Proc. Phys.-Maths. Soc., Japan, III, s. 23, pp. 346–51, 1941.
(423) MASUYAMA, M. The mean angle between two vector sets.
Proc. Phys.-Maths. Soc., Japan (3), Vol. 23, pp. 351–5, 1941.
(424) MAUCHLY, J. W. Significance tests for sphericity of a normal n-variate distribution.
Annals Math. Statistics, Vol. XI, pp. 204–9, 1940.
(425) MITROPOLSKY, A. On the multiple non-linear correlation equations.
Bull. Acad. Sci. U.R.S.S., Sér. Math., Vol. 1939, pp. 399–406, 1939. (Russian. English summary.)
(426) MOOD, A. M. On the joint distribution of the medians in samples from a multivariate population.
Annals Math. Statistics, Vol. XII, pp. 268–78, 1941.
(427) MOWBRAY, A. H. Observation on correlation analysis.
J. Amer. Stat. Ass., Vol. 36, pp. 248–52, 1941.
(428) OBUCHOFF, A. M. Eine Korrelationstheorie der Vektoren.
Uchenye Zapizki Moskov. Gos. Univ. Matematika, Vol. 45, pp. 73–92, 1940. (Russian. German summary.)
(429) O'NEIL, J. B. A note on an easy check on means adjusted by partial regressions.
Scientific Agriculture, Vol. 20, pp. 234–5, 1939.
(430) ROFF, M. Linear dependence in multiple correlation work.
Psychometrika, Vol. 5, pp. 295–8, 1940.
(431) ROY, S. N. The sampling distribution of p-statistics and certain allied statistics of the non-null hypothesis.
Sankhyā, Vol. 6, pp. 15–34, 1942.
(432) ROY, S. N. Analysis of variance for multivariate normal populations: the sampling distribution of the requisite p-statistics on the null and non-null hypotheses.
Sankhyā, Vol. 6, pp. 35–50, 1942.
(433) ROY, S. N., and BOSE, R. C. The use and distribution of the Studentized D^2-statistic when the variances and co-variances are based on k samples.
Sankhyā, Vol. 4, pp. 535–42, 1940.
(434) ROY, S. N., and BOSE, P. The distribution of the root-mean-square of the second type of the multiple correlation coefficient.
Science and Culture, Vol. 6, p. 59, 1940.

CC-0. In Public Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(435) SCHEFFÉ, H. An inverse problem in correlation theory.
American Math. Monthly, Vol. XLIX, pp. 99–104, 1942.
(436) SHRIVASTAVA, M. P. On the D^2-statistic.
Bull Calcutta Math. Soc., Vol. 33, pp. 71–86, 1941.
(437) WALD, A., and BROOKNER, R. J. On the distribution of Wilks' statistic for testing the independence of several groups of variates.
Annals Math. Statistics, Vol. XII, pp. 137–52, 1941.
(438) WAUGH, F. V. Regressions between sets of variables.
Econometrica, Vol. X, pp. 290–310, 1942.

IX. Design of Experiment and Sampling Techniques

(439) ANDERSON, P. H. Distributions in stratified sampling.
Annals Math. Statistics, Vol. XIII, pp. 42–52, 1942.
(440) BATEN, W. D. Formulas for finding estimates for two and three missing plots in randomized block layouts.
Tech. Bull. Michigan Agric. Exp. Sta., Vol. 165, pp. 1–16, 1939.
(441) BHATTACHARYYA, A. A note on Ramamurti's problem of maximal sets.
Sankhyā, Vol. 6, pp. 189–92, 1942.
(442) BOSE, C. The variance of the forecasted mean value subjecting to two-way fluctuations.
Science and Culture, Vol. 7, p. 514, 1942.
(443) BOSE, R. C. Discussion of the mathematical theory of the design of experiments.
Sankhyā, Vol. 5, pp. 169–74, 1941.
(444) BOSE, R. C. On some new series of balanced incomplete block designs.
Bull. Calcutta Math. Soc., Vol. 34, pp. 17–31, 1942.
(445) BOSE, R. C. A note on two series of balanced incomplete block designs.
Bull. Calcutta Math. Soc., Vol. 34, pp. 129–30, 1942.
(446) BOSE, R. C. A note on the resolvability of balanced incomplete designs.
Sankhyā, Vol. 6, pp. 105–10, 1942.
(447) BOSE, R. C. A note on two combinatorial problems having applications in the theory of design of experiments.
Science and Culture, Vol. 8, pp. 192–3, 1942.
(448) BOSE, R. C., and KISHEN, K. On the problem of confounding in the general symmetrical factorial design.
Sankhyā, Vol. 5, pp. 21–36, 1940.
(449) BOSE, R. C., and NAIR, K. R. On complete sets of Latin squares.
Sankhyā, Vol. 5, pp. 361–82, 1941.
(450) BRANDT, A. E. The relation between the design of an experiment and the analysis of variance.
J. Amer. Stat. Ass., Vol. 36, pp. 283–92, 1941.
(451) COCHRAN, W. G. Sampling theory when the sampling-units are of unequal sizes.
J. Amer. Stat. Ass., Vol. 37, pp. 199–212, 1942.
(452) CORNISH, E. A. The estimation of missing values in quasi-factorial designs.
Annals of Eugenics, Vol. X, 137–43, 1940.
(453) CORNISH, E. A. The analysis of quasi-factorial designs with incomplete data.
J. Australian Inst. Agric. Sci., Vol. 7, pp. 19–26, 1941.
(454) COX, G. M. Enumeration and construction of balanced incomplete block configurations.
Annals Math. Statistics, Vol. XI, pp. 72–85, 1940.
(455) FISHER, R. A. An examination of the different possible solutions of a problem in incomplete blocks.
Annals of Eugenics, Vol. X, pp. 52–75, 1940.
(456) FISHER, R. A. New cyclic solutions to problems in incomplete blocks.
Annals of Eugenics, Vol. XI, pp. 290–9, 1942.
(457) FISHER, R. A. The theory of confounding in factorial experiments in relation to the theory of groups.
Annals of Eugenics, Vol. XI, pp. 341–53, 1942.

CC-0. In Public Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(458) FISHER, R. A. Some combinatorial theorems and enumerations connected with the numbers of diagonal types of a Latin square.
Annals of Eugenics, Vol. XI, pp. 395–401, 1942.

(459) FISHER, R. A. Completely orthogonal 9 × 9 squares. A correction.
Annals of Eugenics, Vol. XI, pp. 402–3, 1942.

(460) GANGULI, M. A note on nested sampling.
Sankhyā, Vol. 5, pp. 449–52, 1941.

(461) GANGULI, M. A method of estimating variance of sample grand-mean and zone variances in unequal nested sampling.
Science and Culture, Vol. 6, p. 724, 1941.

(462) GARRETT, H. E. The representativeness of a sample.
Amer. J. Psychol., Vol. 55, pp. 580–1, 1942.

(463) IYER, P. V. K. The analysis of asymmetrical experiments with special reference to the partition of treatment sum of squares.
Proc. Indian Acad. Sci. A., Vol. XI, pp. 369–75, 1940.

(464) IYER, P. V. K. Standard error of the difference between two estimates for incomplete block experiments.
Current Sci., Vol. 10, p. 165, 1941.

(465) KERMACK, W. O., and MCKENDRICK, A. G. The design and interpretation of experiments based on a four fold table: the statistical assessment of the effect of treatment.
Proc. Roy. Soc. Edinb., Vol. 60, pp. 362–75, 1940.

(466) KISHEN, K. On a simplified method of expressing the components of the second order interaction in a 3^3 factorial design.
Sankhyā, Vol. 4, 577–80, 1940.

(467) KISHEN K. Split-plot technique in field experimentation.
Proc. 26th Indian Sc. Congress, Lahore, 1939, III Section IX, No. 47. Abstract in *Sankhyā*, Vol. 4, pp. 590, 1940.

(468) KISHEN, K. Symmetrical unequal block arrangements.
Sankhyā, Vol. 5, pp. 329–44, 1941.

(469) KISHEN, K. On expressing any single degree of freedom for treatments in an *s* factorial arrangement in terms of its sets for main effects and interactions.
Sankhyā, Vol. 6, pp. 133–40, 1942.

(470) MADHAVA, K. B. Technique of random sampling.
Proc. of the 2nd Session of the Indian Statistical Conference, Lahore, 1939, pp. 94–6, 1940.

(471) MAHALANOBIS, P. C. Discussion on planning of experiments.
Sankhyā, Vol. 4, pp. 530–1, 1940.

(472) MAHALANOBIS, P. C. Mathematics and statistics. Sample surveys.
Science and Culture, Vol. 7, Suppl., pp. 1–2, 1942.

(473) MANN, H. B. The construction of orthogonal Latin squares.
Annals Math. Statistics, Vol. XIII, pp. 418–23, 1942.

(474) NAIR, K. R. Balanced confounded arrangements for the 5^n type of experiment.
Sankhyā, Vol. 5, pp. 57–70, 1940.

(475) NAIR, K. R. The application of the technique of analysis of covariance to field experiments with several missing or mixed-up plots.
Sankhyā, Vol. 4, pp. 581–8, 1940.

(476) NAIR, K. R. A note on the method of " fitting of constants " for analysis of non-orthogonal data arranged in a double classification.
Sankhyā, Vol. 5, pp. 317–28, 1941.

(477) NAIR, K. R., and RADHAKRISHNA RAO, C. Confounded designs for asymmetrical factorial experiments.
Science and Culture, Vol. 6, pp. 313–14, 1941.

(478) NAIR, K. R., and RADHAKRISHNA RAO, C. A note on partially balanced incomplete block designs.
Science and Culture, Vol. 7, pp. 568–9, 1942.

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Digitized by Arya Samaj Foundation Chennai and eGangotri

(479) NAIR, K. R., and RADHAKRISHNA RAO, C. Incomplete block designs for experiments involving several groups of varieties.
Science and Culture, Vol. 7, pp. 615–16, 1942.

(480) NAIR, K. R., and RADHAKRISHNA RAO, C. A general class of quasi-factorial designs leading to confounded designs for factorial experiments.
Science and Culture, Vol. 7, pp. 457–8, 1942.

(481) O'NEIL, J. B., and GUTTERIDGE, H. S. A note on the calculation of standard errors for treatment means after adjustment for regression.
Scientific Agriculture, Vol. 21, pp. 358–9, 1941.

(482) OSBORNE, J. G. On the precision of estimates from systematic versus random samples.
Science, Vol. 94, pp. 584–5, 1941.

(483) OSBORNE, J. G. Sampling errors of systematic and random surveys of cover-type areas.
J. Amer. Stat. Ass., Vol. 37, pp. 256–64, 1942.

(484) RAMAMURTI, B., and SITARAMAN, B. On maximal sets of confounded interactions in a $(2^n, 2^k)$ confounded design.
Sankhyā, Vol. 6, pp. 183–8, 1942.

(485) ROY, S. N., and BANERJEE, K. On hierarchical sampling, hierarchical variances and their connexion with other aspects of statistical theory.
Science and Culture, Vol. 6, p. 189, 1940.

(486) STEVENS, W. L. The completely orthogonalized Latin square.
Annals of Eugenics, Vol. IX, pp. 82–93, 1939.

(487) TOLMAN, B. Multiple versus single-factor experiments.
Proc. Amer. Soc. Sugar Beet Technol., 1942, pp. 170–80, 1942.

(488) YATES, F. The comparative advantages of systematic and randomized arrangements in the design of agricultural and biological experiments.
Biometrika, Vol. 30, pp. 440–66, 1939.

(489) YATES, F. The recovery of inter-block information in balanced incomplete block designs.
Annals of Eugenics, Vol. X, pp. 317–25, 1940.

(490) YATES, F. Lattice squares.
J. Agric. Sci., Vol. XXX, pp. 672–87, 1940.

X. ANALYSIS OF VARIANCE

(491) BATEN, W. D., and HENDERSON, E. W. A method of handling ratios by the analysis of variance.
Poultry Sci., Vol. 20, pp. 227–31, 1941.

(492) BEALL, G. The transformation of data from entomological field experiments so that the analysis of variance becomes applicable.
Biometrika, Vol. XXXII, pp. 243–62, 1942.

(493) CHANDRA SEKAR, C. A note on the inverse sine transformation.
Sankhyā, Vol. 6, pp. 195–8, 1942.

(494) COCHRAN, W. G. The analysis of variance when experimental errors follow the Poisson or binomial law.
Annals Math. Statistics, Vol. XI, pp. 335–47, 1940.

(495) ENGELHART, M. D. The analysis of variance and covariance techniques in relation to the conventional formulas for the standard error of a difference.
Psychometrika, Vol. 6, pp. 221–33, 1941.

(496) GEIRINGER, H. Observations of analysis of variance theory.
Annals Math. Statistics, Vol. XIII, pp. 350–69, 1942.

(497) GODARD, R. H., and LINDQUIST, E. F. An empirical study of the effect of heterogeneous within-groups variance upon certain *F*-tests of significance in analysis of variance.
Psychometrika, Vol. 5, pp. 263–74, 1940.

(498) HSU, P. L. Analysis of variance from the power function standpoint.
Biometrika, Vol. XXXII, pp. 62–9, 1940–1.

(499) IRWIN, J. O. On the distribution of a weighted estimate of variance and on analysis of variance in certain cases of unequal weighting.
J. Roy. Stat. Soc., Vol. CV, pp. 115–18, 1942.

CC-0. In Public Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(500) MADOW, W. G. The distribution of quadratic forms in non-central normal random variables.
Annals Math. Statistics, Vol. XI, pp. 100–3, 1940.

(501) SATTERTHWAITE, F. E. Synthesis of variance.
Psychometrika, Vol. 6, pp. 309–16, 1941.

(502) SATTERTHWAITE, F. E. A generalized analysis of variance.
Annals Math. Statistics, Vol. XIII, pp. 34–41, 1942.

(503) TSAO, F. Tests of statistical hypotheses in the case of unequal or disproportionate numbers of observations in the subclasses.
Psychometrika, Vol. 7, pp. 195–212, 1942.

(504) DE VARENNES E MENDONÇA, P. Orthogonality and analysis of variance.
Portugaliae Math., Vol. 3, pp. 234–52, 1942.

(505) WALD, A. A note on the analysis of variance with unequal class frequencies.
Annals Math. Statistics, Vol. XI, pp. 96–100, 1940.

(506) WALD, A. On the analysis of variance in case of multiple classifications with unequal class frequencies.
Annals Math. Statistics, Vol. XII, pp. 346–50, 1941.

(507) WALD, A. On the power function of the analysis of variance test.
Annals Math. Statistics, Vol. XIII, pp. 434–9, 1942.

XI. SPECIAL PROBLEMS

(508) BATTIN, I. L. On the problem of multiple matching.
Annals Math. Statistics, Vol. XIII, pp. 294–305, 1942.

(509) BERNSTEIN, S. Sur un problème du schéma des urnes à composition variable.
C. R. Acad. Sci. U.R.S.S. (*N.S.*), Vol. 28, pp. 5–7, 1940.

(510) BOREL, E. Applications du calcul des probabilités aux problèmes concernant les nombres premiers. Théorème de Goldbach.
Comptes Rendus, t. 212, pp. 317–20, 1941.

(511) BOREL, É. Théorie de l'hérédité: définitions et problèmes.
C. R. Acad. Sci. Paris, Vol. 212, pp. 777–80, 1941.

(512) BOREL, É. Sur certains problèmes d'hérédité connexes au problème de la ruine des jouenurs.
C. R. Acad. Sci. Paris, Vol. 212, pp. 821–5, 1941.

(513) BOREL, É. Sur l'emploi du théorème de Bernoulli pour faciliter le calcul d'une infinité de coefficients. Application au problème de l'attente à un guichet.
C. R. Acad. Sci. Paris, Vol. 214, pp. 452–6, 1942.

(514) BROWN, A. W. A note on the use of a Pearson type III function in renewal theory.
Annals Math. Statistics, Vol. XI, pp. 448–53, 1940.

(515) CAMPBELL, N. R. The replacement of perishable members of a continually operating system.
J. Roy. Stat. Soc., Suppl., Vol. VII, pp. 110–30, 1941.

(516) CHUNG, KAI-LAI. Sur un théorème de M. Gumbel.
Comptes Rendus, t. 210, pp. 620–1, 1940.

(517) CHUNG, KAI-LAI. On the probability of the occurrence of at least m events among n arbitrary events.
Annals Math. Statistics, Vol. XII, pp. 328–38, 1941.

(518) CHUNG, KAI-LAI. On mutually favourable events.
Annals Math. Statistics, Vol. XIII, pp. 338–49, 1942.

(519) DANIELS, H. E. The probability distribution of the extent of a random chain.
Proc. Camb. Phil. Soc., Vol. 37, pp. 244–51, 1941.

(520) DEMING, W. E. and STEPHAN, F. F. On a least squares adjustment of a sampled frequency table when the expected marginal totals are known.
Annals Math. Statistics, Vol. XI, pp. 427–44, 1940.

(521) DIEULEFAIT, C. E. Some new derivations of limiting probability functions.
Univ. Nac. Tucumán. Rev. A., 2, pp. 25–30, 1941.

(522) DIEULEFAIT, C. E. Note on a method of sampling.
Annals Math. Statistics, Vol. XIII, pp. 94–7, 1942.

CC-0. In Public Domain. Gurukul Kangri Collection, Hafidwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(523) DOEBLIN, W. Remarques sur la théorie métrique des fractions continues.
Compositio Math., Vol. 7, pp. 353–71, 1940.

(524) FELLER, W. On the time distribution of so-called random events.
Phys. Rev., Vol. 57, 906–8, 1940.

(525) FELLER, W. On the integral equation of renewal theory.
Annals Math. Statistics, Vol. XII, pp. 243–67, 1941.

(526) FERRARI, E. On a paradox of Bertrand.
Revista Union Mat. Argentina, Vol. 7, pp. 1–6, 1940.

(527) FERRARI, E. On Bertrand's paradox. II.
Union Mat. Argentina, Vol. 7, pp. 74–80, 1941.

(528) FISHER, R. A. The likelihood solution of a problem in compounded probabilities.
Annals of Eugenics, Vol. XI, pp. 306–7, 1942.

(529) FRÉCHET, M. Compléments à un théorème de T. S. Broderick concernant les événements dépendants.
Proc. Edinburgh Math. Soc., Vol. 6, pp. 109–13, 1939.

(530) FRÉCHET, M. Conditions d'existence de système d'événements associées à certaines probabilités.
J. Math. Pure Appl., t. 19, pp. 51–62, 1940.

(531) FRIEDMAN, M. A comparison of alternative tests of significance for the problem of *m* rankings.
Annals Math. Statistics, Vol. XI, pp. 86–92, 1940.

(532) GARWOOD, F. An application of the theory of probability to the operation of vehicular-controlled traffic signals.
J. Roy. Stat. Soc., Suppl., Vol. VII, pp. 65–77, 1940–1.

(533) GEARY, R. C. The mathematical expectation of the mean square contingency when the attributes are mutually independent.
J. Roy. Stat. Soc., Vol. CIII, pp. 90–1, 1940.

(534) GEIRINGER, H. A note on the probability of arbitrary events.
Annals Math. Statistics, Vol. XIII, pp. 238–45, 1942.

(535) GIRSHICK, M. A. Note on the distribution of roots of a polynomial with random complex coefficients.
Annals Math. Statistics, Vol. XIII, pp. 235–8, 1942.

(536) GIRSHICK, M. A. A correction.
Annals Math. Statistics, Vol. XIII, p. 447, 1942.

(537) GONTCHAROFF, W. Sur la distribution des cycles dans les permutations.
C. R. Acad. Sci. U.R.S.S. (*N.S.*), Vol. 35, pp. 267–9, 1942.

(538) GREENWOOD, J. A. The first four moments of a general matching problem.
Annals of Eugenics, Vol. X, pp. 290–2, 1940.

(539) GREVILLE, T. N. E. The frequency distribution of a general matching problem.
Annals Math. Statistics, Vol. XII, pp. 350–4, 1941.

(540) HADWIGER, H. Bemerkung zum Problem des Ruins beim Spiele.
Mitteilungen d. Vereinigung Schweiz. Versicherungs-Math., *H* 40, pp. 41–4, 1940.

(541) HARTMAN, PHILIP, and WINTNER, AUREL. On the needle problem of Laplace and its generalisations.
Bol. Mat., Vol. 14, pp. 260–3, 1941.

(542) HORN, D. A correction for the effect of tied ranks on the value of the rank difference correlation coefficient.
J. Educ. Psych., Vol. XXXIII, pp. 686–90, 1942.

(543) JORDAN, C. Problèmes de la probabilité des épreuves répétées dans le cas général.
Bull. Soc. Math. France, t. LXVII, pp. 223–42, 1939.

(544) KAC, M. On a problem concerning probability and its connection with the theory of diffusion.
Bull. Amer. Math. Soc., Vol. 46, pp. 534–7, 1940.

(545) KENDALL, M. G. Note on the estimation of a ranking.
J. Roy. Stat. Soc., Vol. CV, pp. 119–21, 1942.

(546) KENDALL, M. G. Partial rank correlation.
Biometrika, Vol. XXXII, pp. 277–83, 1942.

CC-0. In Public Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(547) KENDALL, M. G., and BABINGTON SMITH, B. On the method of paired comparisons.
Biometrika, Vol. XXX, pp. 324–45, 1940.

(548) KERAWALA, S. M. The enumeration of the Latin rectangle of depth three by means of a difference equation.
Bull. Calcutta Math. Soc., Vol. 33, pp. 119–27, 1941.

(549) KOLMOGOROFF, A. N. Ueber das logarithmisch normale Verteilungsgesetz der Dimensionen der Teilchen bei Zerstückelung.
C. R. Acad. Sci. U.R.S.S. (*N.S.*), Vol. 31, pp. 99–101, 1941.

(550) KRISHNASWAMI AYANGAR, A. A. Random lines—a statistical study.
Sankhyā, Vol. 5, pp. 307–12, 1940.

(551) KULLBACK, S. Note on a matching problem.
Annals Math. Statistics, Vol. 10, pp. 77–80, 1939.

(552) LEGRAS, P. Über das asymptotische Verhalten der Erneuerungsfunktion.
Mitteilungend. Vereinigung Schweiz. Versicherungs-Math., Vol. 33, pp. 183–204, 1942.

(553) LEVI, F. W. Remarks on Mr. V. Narasimha Murthi's paper: On a problem of arrangements. I.
J. Indian Math. Soc. (*N.S.*), 4, pp. 45–6, 1940.

(554) LOÈVE, M. Sur les systèmes d'événements; application à deux théorèmes classiques.
Comptes Rendus, t. 212, pp. 261–3, 1941.

(555) LOÈVE, M. Systèmes d'événements en nombre fini.
Ann. Univ. Lyon. A., Vol. 5, pp. 55–74, 1942.

(556) MCCREA, W. H., and WHIPPLE, F. J. W. Random paths in two and three dimensions.
Proc. Roy. Soc. Edinb., Vol. 60, pp. 281–98, 1940.

(557) MCMILLAN, B. On two problems of sampling.
Annals of Mathematics, Vol. 42, pp. 437–45, 1941.

(558) MOHR, ERNST. Bemerkungen zu Mises' Behandlung des Nadelproblems von Buffon.
Deutsche Math., Vol. 6, pp. 108–13, 1941.

(559) MOHR, E. Bemerkung zum Buffonschen Nadelproblem.
Z. Angew. Math. Mech., Vol. 22, pp. 170–1, 1942.

(560) MOOD, A. M. The distribution theory of runs.
Annals Math. Statistics, Vol. XI, pp. 367–92, 1940.

(561) MUNZNER, H., and SCHWARZ, H. Ein Zusammenhang zwischen Erneuerungszahlen und dem Moivre'schen Problem.
Arch. Math. Wirtsch.-Sozialforschg., Vol. 6, pp. 46–9, 1940.

(562) NAIR, A. N. K. On the probability of obtaining k sets of consecutive successes in n trials.
Math. Student, Vol. 10, pp. 83–4 1942.

(563) NAKAGAMI, M., and OHNO, M. On the resultant intensity of a number of vibrations whose phases are at random.
Nippon Elec. Commun. Engrg., No. 19, pp. 129–37, 1940.

(564) NAKAGAMI M. Study on the resultant amplitude of many vibrations whose phases and amplitudes are at random.
Nippon Elec. Commun. Engrg., No. 22, pp. 69–92, 1940.

(565) NARASIMHA MURTI V. On a problem of arrangements. I.
J. Indian Math. Soc. (*N.S.*), 4, 39–43, 1940.

(566) NARASIMHA MURTI, V. A problem in combinations.
Math. Student, Vol. 10, pp. 85–6, 1942.

(567) OLDS, E. G. On a method of sampling.
Annals Math. Statistics, Vol. XI, pp. 355–8, 1940.

(568) OLMSTEAD, P. S. Note on theoretical and observed distributions of repetitive occurrences.
Annals Math. Statistics, Vol. XI, pp. 363–6, 1940.

(569) ONICESCU, O. La définition de la probabilité et le problème de la roulette.
Acad. Roum. Bull. Sect. Sci., Vol. 21, pp. 106–9, 1939.

(570) ONICESCU, O. La probabilité d'un événement isolé.
Acad. Roum. Bull. Sect. Sci., Vol. 22, pp. 280–6, 1940.

CC-0. In Public Domain. Gurukul Kangri Collection, Hāridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(571) ORNSTEIN, L. S., and MILATZ, J. M. W. The analogy between statistics of numbers and statistical mechanics.
Nederl. Akad. Wetensch. Proc., Vol. 44, pp. 163–72, 1941.

(572) RADHAKRISHNA RAO, C. On the volume of a prismoid in n-space and some problems in continuous probability.
Math. Student, Vol. 10, pp. 68–74, 1942.

(573) RIBEIRO, H. On a problem of Tchebycheff.
Univ. Lisboa. Revista Fac. Ci. 2, pp. 89–92, 1940.

(574) ROSENBLATT, A. Sur les théorèmes des petits nombres de Poisson, de Bortkiewicz et G. Pólya. Application aux phénomènes rares. I. Propagation des maladies contagieuses: peste bubonique au Brésil.
Actas Acad. Ci. Lima, Vol. 3, pp. 160–7, 1940.

(575) ROSENBLATT, A. Sur le concept de contagion de M. G. Polya dans le calcul des probabilités. Divers schèmes. Application à la peste bubonique au Pérou.
Actas Acad. Ci. Lima, Vol. 3, pp. 186–204, 1940.

(576) RUARK A. The time distribution of so-called random events.
Phys. Rev., Vol. 56, pp. 1165–7, 1939.

(577) SANTALÓ, L. A. Integral geometry 31. On mean values and geometrical probabilities.
Abh. Math. Sem. Hansischen Univ., Vol. 13, pp. 284–94, 1940.

(578) SANTALÓ, L. A. A system of mean values in the theory of geometric probabilities.
Rev. Ci., Lima, Vol. 43, pp. 147–54, 1941.

(579) SAVKEVITCH, V. Sur le schéma des urnes à composition variable.
C. R. Acad. Sci. U.R.S.S. (*N.S.*), Vol. 28, pp. 8–12, 1940.

(580) v. SCHRUTKA, L. Eine neue Einteilung der Permutationen.
Math. Ann., Vol. 118, pp. 246–50, 1941.

(581) SEITZ, B. Sur une équation diophantienne en rapport avec le calcul des probabilités.
Comment. Math. Helv., Vol. 12 pp. 323–5, 1940.

(582) SILBERSTEIN, L. On a hystero-differential equation arising in a probability problem.
Phil. Mag., Vol. 29, pp. 75–84, 1940.

(583) STEPHAN, F. F. An iterative method of adjusting sample frequency tables when expected marginal totals are known.
Annals Math. Statistics, Vol. XIII, pp. 166–78, 1942.

(584) STEVENS, W. L. Distribution of groups in a sequence of alternatives.
Annals of Eugenics, Vol. IX, pp. 10–18, 1939.

(585) TACKLIND, S. Sur le risque de ruine dans des jeux inéquitables.
Skandinavisk Actuarietidskrift, Vol. XXV, pp. 1–42, 1942.

(586) TODD, H. Note on random associations in a square point lattice.
J. Roy. Stat. Soc., Suppl., Vol. VII, pp. 78–82, 1940–1.

(587) VOLBERG, O. Problème de la queue stationnaire et nonstationnaire.
C. R. Acad. Sci. U.R.S.S., Vol. 24, pp. 657–61, 1939.

(588) WOODBURY, M. A. Rank correlation when there are equal variates.
Annals Math. Statistics, Vol. XI, pp. 358–62, 1940.

XII. GROUPING AND CURVE FITTING

(589) ANDERSON, W. A general formula for the normal mean errors of the coefficients in parabolic least squares graduation.
Skandinavisk Actuarietidskrift, 1940, pp. 44–53, 1940.

(590) ANDERSSON, W. The binomial type of Gram's series.
Skandinavisk Actuarietidskrift, Vol. XXIV, pp. 203–13, 1941.

(591) ANDERSSON, W. On the Gram series on Pearson's system of frequency functions.
Skandinavisk Actuarietidskrift, Vol. XXV, pp. 141–9, 1942.

(592) BEALE, F. S. On a certain class of orthogonal polynomials.
Annals Math. Statistics, Vol. XII, pp. 97–103, 1941.

(593) BHATTACHARYYA, B. C. The use of McKay's Bessel function curves for graduating frequency distributions.
Sankhyā, Vol. 6, pp. 175–82, 1942.

CC-0. In Public Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(594) BLEICK, W. E. A least squares accumulation theorem.
Annals Math. Statistics, Vol. XI, pp. 225–56, 1940.

(595) BULA, C. A. Theory and evaluation of central moments in two dimensions. Sheppard's corrections. The simpler method of Mitropolsky.
Revista Union Mat. Argentina, Vol. 5, pp. 1–97, 1940. (Spanish.)

(596) BULA, C. A. Calculation of frequency surfaces. Experimental verification and comparison of the method of marginal functions with that of the 15 constants of Karl Pearson.
Union Mat. Argentina, Publ. No. 10, 109, pp., 1940. (Spanish. French summary.)

(597) COX, G. J., and MATUSCHAK, M. C. An abbreviation of the method of least squares.
J. Phys. Chem., Vol. 45, pp. 362–9, 1941.

(598) CRAIG, CECIL C. A note of Sheppard's correction.
Annals Math. Statistics, Vol. XII, pp. 339–45, 1941.

(599) DANIELS, H. E. A method of improving certain routine measurements.
J. Roy. Stat. Soc., Suppl., Vol. VII, pp. 146–50, 1941.

(600) DWYER, P. S. The cumulative numbers and their polynomials.
Annals Math. Statistics, Vol. XI, pp. 66–71, 1940.

(601) DWYER, P. S. Grouping methods.
Annals Math. Statistics, Vol. XIII, pp. 138–55, 1942.

(602) EDDINGTON, A. S. The correction of statistics for accidental error.
Monthly Not. Roy. Astron. Soc., Vol. 100, pp. 354–61, 1940.

(603) FERGUSON, G. A. The application of Sheppard's correction for grouping.
Psychometrika, Vol. 6, pp. 21–7, 1941.

(604) FISHER, R. A. The negative binomial distribution.
Annals of Eugenics, Vol. XI, pp. 182–7, 1941.

(605) GARCIA-MATA, C. A practical method of smoothing statistical curves by hand.
Bull. Pan-Amer. Un., Vol. 75, pp. 276–81, 1941.

(606) HALDANE, J. B. S. The fitting of binomial distributions.
Annals of Eugenics, Vol. XI, pp. 179–81, 1941.

(607) HULME, H. R. Statistical theory of errors.
Monthly Not. Roy. Astron. Soc., Vol. 100, pp. 303–14, 1940.

(608) JOHNSON, E. Estimates of parameters by means of least squares.
Annals Math. Statistics, Vol. XI, pp. 453–6, 1940.

(609) JONES, H. L. The use of grouped measurements.
J. Amer. Stat. Ass., Vol. 36, pp. 525–9, 1941.

(610) KAVANAGH, A. J. Note on the adjustment of observations.
Annals Math. Statistics, Vol. XII, pp. 111–14, 1941.

(611) KEYFITZ, N. Graduation by a truncated normal.
Annals Math. Statistics, Vol. X, pp. 66–7, 1939.

(612) KIMBALL, B. F. Orthogonal polynomials applied to least square fitting of weighted observations.
Annals Math. Statistics, Vol. XI, pp. 348–52, 1940.

(613) MALMQUIST, K. G. Elimination of the effects of accidental errors of measurements in statistical investigations.
Ark. Mat. Astr. Fys., Vol. 27A, No. 24, pp. 13, 1941.

(614) MULLER, E. Genauigkeit der bei Reduktion von Fehlergleichungen eliminierten Unbekannten.
Z. Vermessungswesen, Vol. 71, pp. 186–90, 1942.

(615) NAIR, K. R., and SHRIVASTAVA, M. P. On a simple method of curve fitting.
Sankhyā, Vol. 6, 121–32, 1942.

(616) ONO, K. Eine Ausgleichungsmethode der statistischen Reihen.
Jap. J. Math., Vol. 17, pp. 117–26, 1940.

(617) ONO, K. Über eine Art der Ausgleichung der statistischen Reihen.
Jap. J. Math., Vol. 17, pp. 513–15, 1941.

(618) PIERCE, J. A. A study of a universe of n finite populations with application to moment-function adjustments for grouped data.
Annals Math. Statistics, Vol. XI, pp. 311–34, 1940.

CC-0. In Public Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(619) SELBERG, H. L. Über die Darstellung willkürlicher Funktionen durch Charliersche Differenzreihen.
Skandinavisk Aktuarietidskrift, Vol. XXV, pp. 228–46, 1942.

(620) VERNOTTE, P. Formule pour la quadrature empirique d'une fonction expérimentale.
C. R. Acad. Sci. Paris, Vol. 214, pp. 107–10, 1942.

(621) STONER, P. M. Fitting the exponential function and the Gompertz function by the method of least squares.
J. Amer. Stat. Ass., Vol. 36, pp. 515–18, 1941.

(622) SYZRANTZEV, P. I. Basic coefficients for mathematical treatment of experimental data by the method of averages.
Doklady Vsesoiuznoi Akademii S.-KH. Nauk 1941, pp. 45–8, 1941.

(623) TILDESLEY, M. L. Extent to which the grouping of the data effects the accuracy of mean value.
Man, Vol. 40, pp. 132–6, 1940.

C. APPLICATIONS

XIII. AGRICULTURE AND FORESTRY

(624) AKEMINE, M. Analysis of variance as applied to the intervarietal difference of sterility in paddy rice.
Jour. Soc. Agric. and Forest. (*Sapporo*), Vol. 31, pp. 1–14, 1939. (Japanese, with English summary.)

(625) ANDREWS, W. B. The elimination of differences in investment in the evaluation of fertilizer analyses.
J. Amer. Soc. Agron., Vol. 32, pp. 495–502, 1940.

(626) ANGELO, E., and POTTER, G. F. The error of sampling in studying distribution of the root systems of tung trees by means of the Viehmeyer soil tube.
Proc. Amer. Soc. Hort. Sci., Vol. 37, pp. 518–20, 1939 (1940).

(627) AYYANGAR RANGASWAMI, G. N. Border effect on the crop in irrigated sorghum.
Madras Agric. J., Vol. 27, pp. 112–18, 1939.

(628) AYYANGAR, G. N. R., AYYAR, M. A., and RAJABHOOSHANAM, D. S. Border effect on the crop in irrigated sorghum.
Madras Agric. Jour., Vol. 27, pp. 112–18, 1939.

(629) BAILEY, B. A study of sampling in cross-section measurements of wool fibre.
J. Agric. Res., Vol. 63, pp. 407–15, 1941.

(630) BAKER, G. A. Mathematical model of embryo abortion in Philips Cling peaches.
J. Agric. Res., Vol. 64, pp. 173–8, 1942.

(631) BAKER, G. O., and Klages, K. H. W. Crop rotation studies.
Idaho Agric. Exp. Sta. Bull., Vol. 227, pp. 1–34, 1938.

(632) BAILEY, B. Diameter relationships of wool fibres from five breeds of sheep raised in South Dakota.
J. Agric. Res., Vol. 60, pp. 415–26, 1940.

(633) BARKWORTH, H. Lay-out of experiments.
Proc. Soc. Agric. Bact., 1941, p. 50, 1941.

(634) BATEN, W. D. How to determine which of two variables is better for predicting a third variable.
J. Amer. Soc. Agron., Vol. 33, pp. 695–9, 1941.

(635) BATEN, W. D., NORTHAM, J. I., and YEAGER, A. F. Grouping of strains or varieties by use of a Latin square.
J. Amer. Soc. Agron., Vol. 33, pp. 616–22, 1941.

(636) BEALL, G. The technique of randomization in field work.
Canadian Ent., Vol. 72, pp. 45–8, 1940.

(637) BILLINGS, W. D. Quantitative correlations between vegetational changes and soil development.
Ecology, Vol. 22, pp. 448–56, 1941.

CC-0. In Public Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(638) BLISS, C. I., and DEARBORN, R. B. The efficiency of lattice squares in corn selection tests in New England and Pennsylvania.
Proc. Amer. Soc. Hort. Sci., Vol. 41, pp. 324–42, 1942.

(639) BOSE, S. S., and KHANNA, K. L. Statistical notes for agricultural workers. Note on the optimum shape and size of plots for sugarcane experiments in Bihar.
Indian Jour. Agric. Sci., Vol. 9, 807–16, 1939.

(640) BRANDT, A. E. Practical difficulties met in the use of experimental designs.
J. Amer. Stat. Ass., Vol. 35, pp. 101–6, 1940.

(641) BUSH, H. L. Further studies in newer designs for large-scale variety tests.
Proc. Amer Soc. Sugar Beet Technol., 1942, pp. 365–72, 1942.

(642) CANFIELD, R. H. Application of the line interception method in sampling range vegetation.
J. Forest., Vol. 39, pp. 388–94, 1941.

(643) CHESTER, K. S. The probability law in cotton seedling disease.
Phytopath., Vol. 31, pp. 1078–88, 1941.

(644) CHRISTIDIS, B. G. Variability of plots of various shapes as affected by plot orientation.
Empire J. Exp. Agric., Vol. VII, pp. 330–42, 1939.

(645) LECLERG, E. L. Relation of field plot design to seed-source tests of Irish potatoes in the south.
Amer. Potato J., Vol. 19, pp. 75–9, 1942.

(646) LECLERG, E. L., and HENDERSON, M. T. Relative efficiency of the two-dimensional quasi-factorial design as compared with a randomized-block arrangement when concerned with yields of Irish potatoes.
Amer. Potato J., Vol. 17, pp. 279–82, 1940.

(647) COCHRAN, W. G. Long-term agricultural experiments.
J. Roy. Stat. Soc. Suppl., Vol. VI, pp. 104–40, 1939.

(648) COCHRAN W. G. The estimation of the yields of cereal experiments by sampling for the ratio of grain to total produce.
J. Agric. Sci., Vol. XXX, pp. 262–75, 1940.

(649) COCHRAN, W. G. Lattice designs for wheat variety trials.
J. Amer. Soc. Agron., Vol. 33, pp. 351–60, 1941.

(650) COCHRAN, W. G. An examination of the accuracy of lattice and lattice square experiments on corn.
Iowa Agric. Exp. Sta. Res. Bull., Vol. 289, pp. 397–15, 1941.

(651) CRAFTS, E. C. Height–volume distribution in range grasses.
J. Forest., Vol. 36, pp. 1182–5, 1938.

(652) CRIST, J. W. Bi-serial *r* for horticultural research.
Proc. Amer. Soc. Hort. Sci., Vol. 37, pp. 269–71, 1939.

(653) CRIST, J. W. Correlation from ranks for horticultural research.
Proc. Amer. Soc. Hort. Sci., Vol. 38, pp. 593–5, 1941.

(654) CRIST, J. W. Tetrachoric correlation for horticultural research.
Proc. Amer. Soc. Hort. Sci., Vol. 40, pp. 549–51, 1942.

(655) DAVEY, R. The number of sample trees required for determining volumes of woods with various degrees of accuracy.
Forestry, Vol. 16, pp. 52–5, 1942.

(656) DAVIS, J. F. The relationship between leaf area and yield of the field bean with a statistical study of methods for determining leaf area.
J. Amer. Soc. Agron., Vol. 32, pp. 323–9, 1940.

(657) DAVIS, J. F., COOK, R. L., and BATEN, W. D. A method of statistical analysis of a factorial experiment involving influence of fertilizer analyses and placement of fertilizer on stand and yield of cannery peas.
J. Amer. Soc. Agron., Vol. 34, pp. 521–32, 1942.

(658) DEMING, G. W., and COLEMAN, O. H. Comparative efficiency of lattice and random-block designs for a sugarbeet variety test.
Proc. Amer. Soc. Sugar Beet Technol., 1942, pp. 181–3, 1942.

(659) DOWN, E. E., and THAYER, J. W. Plot technic studies with navy beans.
J. Amer. Soc. Agron., Vol. 34, pp. 919–22, 1942.

CC-0. In Public Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(660) EMMERT, E. M. Partial elimination of experimental error from data by the use of significance tests.
Proc. Amer. Soc. Hort. Sci., Vol. 37, pp. 272–8, 1939 (1940).

(661) EMSWELLER, S. L., BLODGETT, C. O., and PRYOR, R. L. A statistical study of doubleness in nasturtium.
Proc. Amer. Soc. Hort. Sci., Vol. 36, p. 826, 1938 (1939).

(662) FIPPIN, E. O. The objectives and methods of field plot fertilizer tests and a proposed improvement of methods.
Soil Sci. Soc. Amer. Proc., Vol. 5, pp. 274–80, 1940 (1941).

(663) FOLSOM, D. Sample size and reliability.
Amer. Potato J., Vol. 19, pp. 197–9, 1942.

(664) GARNER, H. V., and WEIL, J. W. The standard errors of field plots at Rothamsted and outside centres.
Empire J. Exp. Agric., Vol. VII, pp. 369–79, 1939.

(665) GLOCK, W. S. Growth rings and climate.
Botanical Rev., Vol. 7, pp. 649–713, 1941.

(666) GOBEIL, R. Importance des statistiques dans les travaux forestiers.
Forêt Québecoise, Vol. 2, pp. 31–48, 1940.

(667) HARRINGTON, J. B. The effect of having rows different distances apart in rod row plot tests of wheat, oats and barley.
Scientific Agriculture, Vol. 21, pp. 589–606, 1941.

(668) HARTMAN, J. D., and STAIR, E. C. Field plot technique studies with tomatoes.
Proc. Amer. Soc. Hort. Sci., Vol. 41, pp. 315–20, 1942.

(669) HASEL, A. A. Estimation of vegetation-type areas by linear measurement.
J. Forest., Vol. 39, pp. 34–40, 1941.

(670) HASEL, A. A. Estimation of volume in timber stands by strip sampling.
Annals Math. Statistics, Vol. XIII, pp. 179–206, 1942.

(671) HASEL, A. A. Sampling error of cruises in the California pine region.
J. Forest., Vol. 40, pp. 211–17, 1942.

(672) HENRY, G. F., DOWN, E. E., and BATEN, W. D. An adequate sample of corn plots with reference to moisture and shelling percentages.
J. Amer. Soc. Agron., Vol. 34, pp. 777–81, 1942.

(673) HOPKINS, J. W. Estimation of leaf area in wheat from linear dimensions.
Canadian Jour. Res. C., Vol. 16, pp. 300–4, 1939.

(674) HORTON, J. S. The sample plot as a method of quantitative analysis of chaparral vegetation in southern California.
Ecology, Vol. 22, pp. 457–68, 1941.

(675) IMMER, F. R., TYSDAL, H. M., and STEECE, H. M. Bibliography of field experiments.
J. Amer. Soc. Agron., Vol. 31, pp. 1049–52, 1939.

(676) JACOB, W. C. The importance of border effect in certain kinds of field experiments with potatoes.
Proc. Amer. Soc. Hort. Sci., Vol. 37, pp. 866–70, 1939 (1940).

(677) JOLLY, A. L. Uniformity trials on estate cacao fields in Grenada, B.W.I.
Trop. Agric., Vol. 19, pp. 167–74, 1942.

(678) KING, A. J., MCCARTH, D. E., and MCPEEK, M. An objective method of sampling wheat fields to estimate production and quality of wheat.
U.S. Dept. Agric. Tech. Bull., Vol. 814, pp. 1–87, 1942.

(679) KRAUSS, G. Zur forstlichen Standortsbeschreibung.
Forstarchiv, Vol. 15, pp. 85–93, 1939.

(680) LATHROP, F. H., and PLUMMER, B. E. A simplified method of sampling known areas of apple leaves for chemical analysis.
J. Economic Entomology, Vol. 35, p. 961, 1942.

(681) LEGGATT, C. W. Contributions to the study of statistics of seed testing. VII. Further studies on the distribution of particles differing in specific gravity or size.
Proc. Internat. Seed Testing Assoc., Vol. 11, pp. 25–39, 1939.

CC-0. In Public Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(682) LEGGATT, C. W. Contribution to the study of the statistics of seed testing.
Proc. Internat. Seed Testing Assoc., Vol. 11, pp. 40–3, 1939.

(683) LEGGATT, C. W. A study of the relative efficiency of seed sampling methods.
Canadian Jour. Res. C., Vol. 19, pp. 156–62, 1941.

(684) LEXEN, B. The application of sampling to log scaling.
J. Forest., Vol. 39, pp. 624–31, 1941.

(685) LIVERMORE, J. R. Report of the committee on standardization of field plot technique.
Amer. Potato J., Vol. 17, pp. 114–23, 1940.

(686) LYNCH, D. W., and SCHUMACHER, F. X. Concerning the dispersion of natural regeneration.
J. Forest., Vol. 39, pp. 49–51, 1941.

(687) LYNCH, P. B. Sampling methods for the estimation of grain yields in cereal trials.
New Zealand J. Sci. and Tech., Vol. 22, pp. 151A–7A, 1940.

(688) MA, R. H., and KAO, L. H. A factorial experiment on rice culture.
Empire J. Exp. Agric., Vol. VIII, pp. 23–33, 1940.

(689) MAHALANOBIS, P. C. A review of the application of statistical theory to agricultural experiments in India.
Indian Jour. Agric. Sci., Vol. 10, pp. 192–212, 1940.

(690) MAHALANOBIS, P. C. Statistical definition of standard yield of crops.
Sankhyā, Vol. 6, pp. 97–8, 1941.

(691) MARINO, A. E. Eficiencia de ensayos planeados por el método de las parcelas divididas. (Efficiency (or accuracy) of trials calculated by the divided-plot method.)
Rev. Argentina Agron., Vol. 7, pp. 69–88, 1940.

(692) MARINO, A. E., and LUNA, J. T. Planeos de ensayos en blocks incompletos (lattice y lattice balanceado). Analisis de resultados en ensayos con maices. (Methods of experiments in incomplete blocks—lattice and balanced lattice. Analyses of results in experiments with corn.)
Rev. Argentina Agron., Vol. 8, pp. 281–316, 1941.

(693) MEYER, H. A. A mathematical expression for height curves.
J. Forest., Vol. 38, pp. 415–20, 1940.

(694) OSBORNE, J. G. A design for experiments in thinning forest stands.
J. Forest., Vol. 37, pp. 296–304, 1939.

(695) PAPADAKIS, J. S. Comparison de différentes méthodes d'expérimentation phytotechnique. (A comparison of different methods of phytotechnical experimentation.)
Rev. Argentina Agron., Vol. 7, pp. 297–362, 1940.

(696) PARKER, E. R. Adjustment of yields in an experiment with orange trees.
Proc. Amer. Soc. Hort. Sci., Vol. 41, pp. 23–33, 1942.

(697) PECHANEC, J. F. Sampling error in range surveys.
J. Forest., Vol. 39, pp. 52–4, 1941.

(698) PECHANEC, J. F. Application of analysis of covariance to range research data.
U.S. Forest Serv., Inter-mountain Forest and Range Exp. Sta. Tech. Note 1, pp. 1–21, 1941.

(699) PECHANEC, J. F., and STEWART, G. Sagebrush-grass range sampling studies: variability of native vegetation and sampling error.
J. Amer. Soc. Agron., Vol. 33, pp. 1057–71, 1941.

(700) PETRESCU, T. Teoria erorilor si verificarea ei asupra fenomenelor silvice. (Application of the theory of errors to silvical phenomena.)
Rev. Pădurilor (*Bucharest*), Vol. 52, pp. 433–42, 1940.

(701) PETRESCU, T. Teoria erorilor si verificarea ei asupra fenomenelor silvice. (Theory of errors and its application to forest phenomena.)
Rev. Pădurilor (*Bucharest*), Vol. 52, pp. 693–7, 1940.

(702) PETRESCU, T. Teoria erorilor si verificarea ei asupra fenomenelor silvice. (The theory of errors as applied to forestry phenomena.)
Rev. Pădurilor (*Bucharest*), Vol. 53, pp. 1–5, 1941.

(703) PRICE, W. A., and VAUGHAN, E. C. An experiment in mixing and sampling Kentucky Bluegrass.
Proc. Assoc. Offic. Seed Analysts N. Amer., Vol. 32, pp. 77–80, 1940 (1941).

CC-0. In Public Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(704) PROEBSTING, E. L. The relative yield of border fruit trees.
Proc. Amer. Soc. Hort. Sci., Vol. 41, pp. 34–6, 1942.

(705) RAMDAS, L. A., and KALAMKAR, R. J. Statistical investigations on crop–weather relationships in India.
Sankhyā, Vol. 3, pp. 285–90, 1938.

(706) SALLANS, H. R., and ANDERSON, J. A. Observations on the study of varietal differences in the malting quality of barley. IV.
Canadian Jour. Res. C., Vol. 17, pp. 57–71, 1939.

(707) SALMON, S. C. The use of modern statistical methods in field experiments.
J. Amer. Soc. Agron., Vol. 32, pp. 308–20, 1940.

(708) SARLE, C. F. The possibilities and limitations of objective sampling in strengthening agricultural statistics.
Econometrica, Vol. VIII, pp. 45–61, 1940.

(709) SCHUMACHER, F. X., and CHAPMAN, R. A. Sampling methods in forestry and range management.
Duke Univ. Sch. Forest. Bull., Vol. 7, pp. 1–213, 1942.

(710) SNEDECOR, G. W. The use of tests of significance in an agricultural experiment station.
J. Amer. Stat. Ass., Vol. 37, pp. 383–6, 1942.

(711) SNEDECOR, G. W., and KING, A. J. Recent developments in sampling for agricultural statistics.
J. Amer. Stat. Ass., Vol. 37, pp. 95–102, 1942.

(712) SORNAY, A. DE. Estimates of cane yields by means of random rows and stools.
Rev. Agric. Ile Maurice, Vol. 21, pp. 107–13, 1942.

(713) STOFFELS, A. Die Beurteilung eines Ertragsunterschiedes bei Feldversuchen.
J. Landw., Vol. 86, pp. 181–6, 1939.

(714) STOFFELS, A. De berekening van middelbare fouten by niet homogene proefvelden. (Calculating standard errors in connection with non-homogeneous experimental fields.)
Landbouwk. Tijdschr., Vol. 52, pp. 165–74, 1940.

(715) TAYLOR, A. J. Grading and sampling of wattle bark.
J. S. African Forest. Assoc., Vol. 8, pp. 103–11, 1942.

(716) THARP, W. H., WADLEIGH, C. H., and BARKER, H. D. Some problems in handling and interpreting plant disease data in complex factorial design.
Phytopath., Vol. 31, pp. 26–48, 1941.

(717) VAGHOLKAR, B. P., APTE, V. N., and IYER, S. S. A study on plot size and shape technique for field experiments on sugarcane.
Indian Jour. Agric. Sci., Vol. 10, pp. 388–403, 1940.

(718) VAIDYANATHAN, M., and IYER, S. S. A note on the analysis of 3^3 and 3^4 designs (with three-factor interactions confounded) in field experiments in agriculture.
Indian Jour. Agric. Sci., Vol. 10, pp. 213–36, 1940.

(719) VARIOUS AUTHORS. Handledning i försöksteknik. (Handbook in the technique of field experiments.)
Lantbrukshögskolan Jordbruksförsöksanstalten Meddel., Vol. 1, pp. 1–207, 1939.

(720) WADLEIGH, C. H., and THARP, W. H. Factorial design in plant nutrition experiments in the greenhouse.
Arkansas Agric. Exp. Sta. Bull., Vol. 401, pp. 1–67, 1940.

(721) WALKER, M. G. A mathematical analysis of the distribution in maize in *Heliothis armigera* Hb. (*obsoleta* F.).
Canadian Jour. Res. D., Vol. 20, pp. 235–61, 1942.

(722) WALLRABENSTEIN, P. P. The accuracy of district and county estimates of yield per acre, 1929 and 1934.
Ohio State Univ. Absts. of Doctoral Dissert., Vol. 37, pp. 291–7, 1941 (1942).

(723) WATSON, D. J. Field experiments on the effect of applying a nitrogenous fertilizer to wheat at different stages of growth.
J. Agric. Sci., Vol. XXIX, pp. 379–98, 1939.

(724) WEIDHAAS, H. Experimentelle Studien an Gemuse über die Entnahme von Durchschnittsproben zur chemischen Qualitätsbestimmung unter Anwendung stätistischer Methoden.
Bodenk. u. Pflanzenernähr., Vol. 30, pp. 1–35, 1942.

CC-0. In Public Domain. Gurukul Kangri Collection, Haridwar

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(725) WELHAUSEN, E. J. The accuracy of incomplete block designs in varietal trials in West Virginia.
J. Amer. Soc. Agron., Vol. 35, pp. 66–76, 1943.

(726) WILCOX, O. W. A critique of field experiments with plant nutrients.
Amer. Fertilizer, Vol. 95, pp. 8–11, 24, 25, 1941.

(727) WILSON, E. A simple technique for laying out sample plots.
J. Forest., Vol. 39, 650–1, 1941.

(728) WISHART, J. Field trials: their lay-out and statistical analysis.
Tech. Communic. Imp. Bur. Plant Breeding and Genetics, 1940, pp. 1–36, 1940.

(729) YOUDEN, W. J. Experimental designs to increase accuracy of greenhouse studies.
Contr. Boyce Thompson Inst., Vol. 11, pp. 219–28, 1940.

(730) ZUBER, M. S. Relative efficiency of incomplete block designs using corn uniformity trial data.
J. Amer. Soc. Agron., Vol. 34, pp. 30–47, 1942.

XIV. GENETICS

(731) BERNSTEIN, S. Solution of a mathematical problem connected with the theory of heredity.
Annals Math. Statistics, Vol. XIII, pp. 53–61, 1942.

(732) BUCHHOLZ, J. T., and KAEISER, M. A statistical study of two variables in the Sequoias—pollen grain size and cotyledon number.
Amer. Nat., Vol. 74, pp. 279–83, 1940.

(733) COMSTOCK, R. E., and WINTERS, L. M. Design of experimental comparisons between lines of breeding in livestock.
J. Agric. Res., Vol. 64, pp. 523–32, 1942.

(734) CORBETT, R. A. Uniform annual reports for artificial breeding associations, including standardized methods of calculating conception rates.
J. Dairy Sci., Vol. 24, pp. 490–1, 1941.

(735) COTTERMAN, C. W. The biometrical approach in human genetics.
Amer. Nat., Vol. 76, pp. 144–55, 1942.

(736) CRAMPTON, E. W. The design of animal husbandry experiments.
J. Animal Sci., Vol. 1, pp. 263–76, 1942.

(737) DAVENPORT, C. B. Analysis of variance applied to human genetics.
Proc. Nat. Acad. Sci. U.S.A., Vol. 26, pp. 1–3, 1940.

(738) DAY, B. B., and LLOYD, A. A three-dimensional lattice design for studies in forest genetics.
J. Agric. Res., Vol. LIX, pp. 101–19, 1939.

(739) DICKERSON, G. E. Experimental design for testing inbred lines of swine.
J. Animal Sci., Vol. 1, pp. 326–41, 1942.

(740) DUMON, A. G. Correlatiestudie en genetisch onderzodk bij kippen. (Correlation studies and genetical investigations on chickens.)
Natuurwetenschap. Tijdschr., Vol. 21, pp. 393–9, 1940.

(741) ETHERINGTON, I. M. H. Non-associative algebra and the symbolism of genetics.
Proc. Roy. Soc. Edinb., Vol. 61B, pp. 24–42, 1941.

(742) FINNEY, D. J. The detection of linkage.
Annals of Eugenics, Vol. 2, pp. 171–214, 1940.

(743) FINNEY, D. J. The detection of linkage. II. Further mating types; scoring of Boyd's data.
Annals of Eugenics, Vol. XI, pp. 10–30, 1941.

(744) FINNEY, D. J. The detection of linkage. III. Incomplete parental testing.
Annals of Eugenics, Vol. XI, pp. 115–35, 1941.

(745) FINNEY D. J. The detection of linkage. IV. Lack of parental records and the use of empirical estimates of information.
J. Heredity, Vol. 33, pp. 157–60, 1942.

(746) FINNEY, D. J. The detection of linkage. V. Supplementary tables. VI. The loss of information from incompleteness of parental testing.
Annals of Eugenics, Vol. XI, pp. 224–44, 1941–2.

(747) FISHER, R. A. The estimation of the proportion of recessives from tests carried out on a sample not wholly unrelated.
Annals of Eugenics, Vol. X, pp. 160–70, 1940.

CC-0. In Public Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(748) FISHER, R. A. Average excess and average effect of a gene substitution.
Annals of Eugenics, Vol. XI, pp. 53–63, 1941.

(749) FRYER, H. C. An analysis of group differences arising from a Poisson distribution of observations obtained from irradiation experiments.
Iowa State Coll. J. Sci., Vol. 16, pp. 49–51, 1941.

(750) GEPPERT, M.-P. Über die Alterskorrektur von Merkmalshäufigketien in der Erbstatistik.
Arch. Math. Wirtsch.-Sozialforschg., Vol. 6, pp. 80–102, 1940.

(751) GINI, C. Ressemblance parentale et ressemblance fraternelle.
Metron, Vol. XIII, pp. 59–74, 1939.

(752) GINI, C. The relative importance of hereditary and non-hereditary factors in determining the heterogeneity of a generation.
Annals of Eugenics, Vol. X, pp. 42–7, 1940.

(753) GLIVENKO, V. I. Studies on mathematical genetics.
Bull. Acad. Sci. U.R.S.S., Sér. Biologique, Vol. 1939, pp. 615–35, 1939.

(754) HALDANE, J. B. S. The theory of the evolution of dominance.
J. Genetics, Vol. 37, pp. 365–74, 1939.

(755) HALDANE, J. B. S. Selection against heterozygosis in man.
Annals of Eugenics, Vol. XI, pp. 333–40, 1942.

(756) HAZEL, L. N., and LUSH, J. L. The efficiency of three methods of selection.
J. Heredity, Vol. 33, pp. 393–9, 1942.

(757) KADANOFF, D. Untersuchungen über die Norm.
Zeitschr. Rassenk., Vol. 10, pp. 68–79, 1939.

(758) KOLMAN, E. Misinterpretation of mathematics for the service of Mendelism.
Iarovizatsiia (*Yarovisation*), Vol. 3, pp. 70–4, 1939.

(759) KOLMOGOROFF, A. N. On a new confirmation of Mendel's laws.
C. R. Acad. Sci. U.R.S.S. (*N.S.*), Vol. 27, pp. 37–41, 1940.

(760) KOSTITZIN, V. A. Sur la loi logistique et ses généralisations.
Acta Bioth. Ser. A., Vol. 5, pp. 155–9, 1940.

(761) KOSTITZIN, V. A. Sur la ségrégation physiologique et la variation des espèces.
Acta Bioth., Ser. A., Vol. 5, pp. 160–8, 1940.

(762) LOTKA, A. J. The progeny of an entire population.
Annals Math. Statistics, Vol. XIII, pp. 115–26, 1942.

(763) MALÉCOT, G. Mendélisme et consanguinité.
C. R. Acad. Sci. Paris, Vol. 215, pp. 313–14, 1942.

(764) MARGOLIS, O. S. The facet frequency distribution in Bar-eyed Drosophila.
Amer. Nat., Vol. 73, pp. 472–6, 1939.

(765) MAUNG, K. Measurement of association in a contingency table with special reference to the pigmentation of hair and eye colours of Scottish school children.
Annals of Eugenics, Vol. XI, pp. 189–223, 1941–2.

(766) MCMILLAN, J. R. A. The use of the coefficient of correlation of quantitative characters as a measure of gene linkage.
Australia Counc. Sci. and Indust. Res. J., Vol. 11, pp. 311–16, 1938.

(767) MITTMANN, OTFRID. Funktionale Zusammenhänge zwischen Zygotenwahrscheinlichkeiten.
Deutsch Math., Vol. 5, pp. 563–70, 1941.

(768) MYERS, R. J. The validity and significance of male net reproduction rates.
J. Amer. Stat. Ass., Vol. 36, pp. 275–82, 1941.

(769) OLBRYCHT, T. M. Statistical analysis of black colour in Wessex Saddleback breed.
Annals of Eugenics, Vol. XI, pp. 80–8, 1941.

(770) PANSE, V. G. A statistical study of quantitative inheritance.
Annals of Eugenics, Vol. X, pp. 76–105, 1940.

(771) PANSE, V. G. Methods in plant breeding.
Indian J. Genetics and Pl. Breeding, Vol. 2, pp. 151–8, 1942.

(772) RIFE, D. C. A common misconception concerning the behavior of linked factors.
Human Biology, Vol. 11, pp. 546–8, 1939.

(773) SCHOLZ, EDMUND. Ein methodischer Beitrag zur Berechnung des Erbgefüges.
Deutsche Math., Vol. VI, pp. 100–4, 1941.

CC-0. In Public Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(774) SCHOLZ, EDMUND. Die Auflösung eines gewissen linear homogenen Systems von Rückschlussformeln und ihre Anwendung auf Probleme der Inzucht.
Deutsche Math., Vol. VI, pp. 104–7, 1941.

(775) STEVENS, W. L. Tables of the recombination fraction estimated from the product ratio.
J. Genetics, Vol. 39, pp. 171–80, 1939.

(776) STEVENS, W. L. Accuracy of mutation rates.
J. Genetics, Vol. 43, pp. 301–7, 1942.

(777) SUKHATME, P. V. On Bernstein's improved method of estimating blood-group gene frequencies.
Sankhyā, Vol. 6, pp. 85–92, 1942.

(778) TIMOFEEFF-RESSOVSKY, N. W. Genetik und Evolution.
Zeitschr. Indukt. Abstamm.-u. Vererbungsl. B. 76, pp. 158–219, 1939.

(779) WRIGHT, S. Statistical genetics and evolution.
Bull. Amer. Math. Soc., Vol. 48, pp. 223–46, 1942.

XV. ACTUARIAL AND POPULATION STUDIES

(780) ACKERMANN, W. G. Eine Erweiterung des Poissonschen Grenzwertsatzes und ihre Anwendung auf die Risikoprobleme in der Sachversicherung.
Inst. Math. Berlin Univ. Dissert., 1939.

(781) AMMETER, H. Das Zufallsrisiko bei kleinen Versicherungsbestanden.
Mitt. Verein. Schweiz. Versich.-Math., Vol. 42, pp. 155–82, 1942.

(782) ANDERSEN, N. E. De la durée moyenne d'une série de prestations en capitaux arbitraires en fonction du taux instantané d'intérêt et des demi-invariants de la répartition.
Trans. 12th International Congress of Actuaries, Lucerne, 1940. Vol. III, pp. 437–57. Orell Füssli, Zurich, 1941.

(783) ANDERSON, R. D. On the application of quantum mechanics to mortality tables.
J. Inst. Actuaries, Vol. LXXI, pp. 228–58, 1942.

(784) ANONYMOUS. 1837–1937: A retrospect of vital statistics.
Brit. Med. Jour., 1940, p. 547, 1940.

(785) BAPTIST, J. Le calcul des probabilités dans le domaine de l'assurance.
Trans. 12th International Congress of Actuaries, Lucerne, 1940. Vol. I, pp. 117–36. Orell Füssli, Zurich, 1941.

(786) BERGER, A. Welche Hypothesen liegen der Versicherungsmathematik zugrunde und wie kann die Anwendung der Wahrscheinlichkeitstheorie und der Risikotheorie im Versicherungswesen begrundet werden?
Trans. 12th International Congress of Actuaries, Lucerne, 1940. Vol. IV, pp. 9–26. Orell Füssli, Zurich, 1941.

(787) BLACKMAN, G. Wachstum und Sterblichkeit.
Lunds Univ. Arsskr. Vol. 35, 11, pp. 1–50, 1939.

(788) BOSCHAN, P. Some considerations concerning probability in actuarial science and the foundation of the extended life table.
Trans. 12th International Congress of Actuaries, Lucerne, 1940. Vol. I, pp. 159–69. Orell Füssli, Zurich, 1941.

(789) BURKHARDT, FELIX. Uber Stand und Wandlungen von bevölkerungs- und versicherungsstatistischen Personengesamtheiten.
Bl. Versich.-Math., Vol. 5, pp. 212–27M, 1941.

(790) BUSTAMANTE, M. E., and ALVARO ALDAMA, C. Tablas de vida de los habitantes do los Estados Unidos Mexicanos. (Mortality and life expectancy tables for Mexico.)
Rev. Inst. Salubridad y Enferm. Trop., Vol. 1, pp. 131–50, 1940.

(791) CODY, D. D. Actuarial note. The standard deviation in the rate of mortality by amounts.
Trans. Actuar. Soc. America, Vol. 42, pp. 69–73, 1941.

(792) CONSAEL, R. Sur une généralisation des formules d'ajustement de E. T. Whittaker.
Mitt. Verein. Schweiz. Versich.-Math., Vol. 41, pp. 95–107, 1941.

(793) CORNFIELD, J. On certain biases in samples of human populations.
J. Amer. Stat. Ass., Vol. 37, pp. 63–8, 1942.

CC-0. In Public Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(794) DASEN, E. Note sur l'approximation du taux effectif des emprunts par obligations amortissables par le système de l'annuité constante.
Mitt. Verein. Schweiz. Versich.-Math., Vol. 41, pp. 201–4, 1941.

(795) DEMING, W. E., and GEOFFREY, L. On sample inspection in the processing of census returns.
J. Amer. Stat. Ass., Vol. 36, pp. 351–60, 1941.

(796) DEMING, W. E., and STEPHAN, F. F. On the interpretation of censuses as samples.
J. Amer. Stat. Ass., Vol. 36, pp. 45–9, 1941.

(797) EDWARDS, T. I. The coding and tabulation of medical and research data for statistical analysis.
Publ. Health Repts., Vol. 57, pp. 7–21, 1942.

(798) FÉRAUD, L. Le renouvellement, quelques problèmes connexes et les équations intégrales du cycle fermé.
Mitteilungend Vereiningung Schweiz Versicherungs-Math., H. 41, pp. 81–93, 1941.

(799) DE FINETTI, B. Il calcolo delle probabilità nel dominio dell'assicurazione.
Trans. 12th International Congress of Actuaries, Lucerne, 1940. Vol. I, pp. 253–61. Orell Füssli, Zurich, 1941.

(800) FISCHER, K. Beiträge zur Tarifanalyse.
Bl. Versich. Math., Vol. 4, pp. 491–506, 1939.

(801) FRANCK, E. I capitali di sopravvivenza.
Giorn. Ist. Ital. Attuari, Vol. X, pp. 213–20, 1939.

(802) FRANKEL, L. R., and STOCK, J. S. On the sample survey of unemployment.
J. Amer. Stat. Ass., Vol. 37, pp. 77–80, 1942.

(803) GILLETT, R. L. Problems and progress in State statistics.
J. Amer. Stat. Ass., Vol. 35, pp. 335–40, 1940.

(804) GÖRING, E. Eine Erweiterung der Mises'schen Kollektive und der entsprechende Ausbau der Theorie der Wahrscheinlichkeitsrechnung.
Trans. 12th International Congress of Actuaries, Lucerne, 1940. Vol. I, pp. 329–48. Orell Füssli, Zurich, 1941.

(805) GREVILLE, T. N. E. "Census" methods of constructing mortality tables and their relation to "insurance" methods.
Rec. Amer. Inst. Actuar., Vol. 31, pp. 367–73, 1942.

(806) GUERNSEY, P. D. The modern public opinion poll. A means of defining and appraising community health problems.
Amer. J. Publ. Health, Vol. 32, pp. 973–8, 1942.

(807) HADWIGER, H. Eine analytische Reproduktionsfunktion für biologische Gesamtheiten.
Skandinavisk Actuarietidskrift, 1940, pp. 101–13, 1940.

(808) HADWIGER, H. Natürliche Ausscheidefunktionen fur Gesamtheiten und die Lösung der Erneuerungsgleichung.
Mitteilungend Vereiningung schweiz Versicherungs-Math., H. 40, pp. 31–9, 1940.

(809) HADWIGER, H. Eine Formel der mathematischen Bevölkerungstheorie.
Mitteilungend Vereiningung schweiz Versicherungs-Math., H. 41, pp. 67–73, 1941.

(810) HADWIGER, H. Über eine Funktionalgleichung der Bevölkerungstheorie und eine spezielle Klasse analytischer Lösungen.
Bl. Versich.-Math., Vol. 5, pp. 181–8, 1941.

(811) HADWIGER, H., and RUCHTI, W. Ueber eine specielle Klasse analytischer Geburtenfunktionen.
Metron, Vol. XIII, pp. 17–26, 1939.

(812) HADWIGER, H. Ueber die Integralgleichung der Bevölkerungstheorie.
Mitteilungen d. Vereinigung schweiz. Versierungs Math., H. 38, pp. 1–14, 1939.

(813) HADWIGER, H., and RUCHTI, W. Über eine spezielle Klasse analytischer Geburten funktionen.
Metron, Vol. XIII, pp. 17–26, 1939.

(814) HADWIGER, H., and WEGMÜLLER, W. Entwicklung und Umschichtung von Personengesamtheiten.
Trans. 12th International Congress of Actuaries, Lucerne, 1940. Vol. III, pp. 369–86. Orell Füssli, Zurich, 1941.

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Digitized by Arya Samaj Foundation Chennai and eGangotri

(815) HAFERL, E. Die Bestimmung der Selbstbehalte in der Lebensversicherung.
Trans. 12th International Congress of Actuaries, Lucerne, 1940. Vol. I, pp. 349–76. Orell Füssli, Zurich, 1941.

(816) HAGSTROEM, K. G. Rilievi sulla teoria del deprezzamento.
Giorn. Ist. Ital. Attuari, Vol. X, pp. 193–212, 1939.

(817) HAGSTROEM, K.-G. Quelques réflexions sur le rôle de la théorie des probabilités dans l'assurance pratique.
Trans. 12th International Congress of Actuaries, Lucerne, 1940. Vol. I, pp. 321–8. Orell Füssli, Zurich, 1941.

(818) HAGSTROEM, K.-G. Note sur l'incertitude du risque.
Skandinavisk Aktuarietidskrift, Vol. XXV, pp. 150–68, 1942.

(819) HAMMON, P. H., and CLARKE, R. D. Some effects upon insurance problems of modern criticisms of the frequency theory of probability.
Trans. 12th International Congress of Actuaries, Lucerne, 1940. Vol. I, pp. 207–24. Orell Füssli, Zurich, 1941.

(820) HANSEN, M. H., and HURWITZ, W. N. Relative efficiencies of various sampling units in population enquiries.
J. Amer. Stat. Ass., Vol. 37, pp. 89–94, 1942.

(821) HAUSER, P. M. The use of sampling in the census.
J. Amer. Stat. Ass., Vol. 369–75, 1941.

(822) HAUSER, P. M. Proposed annual sample census of population.
J. Amer. Stat. Ass., Vol. 37, pp. 81–8, 1942.

(823) HENDERSON, R. Actuarial notes on (*i*) Sheppard's corrections, (*ii*) χ^2 test, (*iii*) difference equation graduation.
Trans. Actuar. Soc. America, Vol. 41, pp. 480–6, 1940.

(824) HENTIG, H. v. A statistical test of causal significance.
Amer. Sociol. Rev., Vol. 5, pp. 930–6, 1940.

(825) HERBOSE, M. M., and CRUZ, A. H. The law of small chances applied to methods of measuring the health of Philippine communities.
Acta Med. Philippina, Vol. 1, pp. 69–80, 1939.

(826) HOHAUS, R. Actuarial problems in social insurance.
J. Amer. Stat. Ass., Vol. 35, pp. 37–46, 1940.

(827) HORNICH, H. Zur Theorie des Risikos.
Monatsh. Math. Phys., Vol, 50, pp. 142–50, 1941.

(828) HULTMAN, K. Einige numerische Untersuchungen auf Grund der kollektiven Risikotheorie. Erster Teil.
Skandinavisk Aktuarietidskrift, Vol. XXV, pp. 84–119, 1942.

(829) HULTMAN, K. Einige numerische Untersuchungen auf Grund der kollektiven Risikotheorie. Zweiter Teil.
Skandinavisk Aktuarietidskrift, Vol. XXV, pp. 169–99, 1942.

(830) HUNTINGTON, E. V. Methods of Apportionment in Congress. A survey of methods of apportionment in Congress.
Senate Document No. 304, *76th Congress, 3rd Session*. U.S. Govt. Printing Office, Washington, 1940.

(831) JACOB, M. Su di un metodo d'approssimazione per il calcolo del rischio quadratico medio.
Trans. 12th International Congress of Actuaries, Lucerne, 1940. Vol. I, pp. 285–307. Orell Füssli, Zurich, 1941.

(832) JECKLIN, H. The theory of probability in insurance.
Trans. 12th International Congress of Actuaries, Lucerne, 1940. Vol. I, pp. 71–115. Orell Füssli, Zurich, 1941.

(833) JECKLIN, H. Ist die Anwendbarkeit der Wahrscheinlichkeitstheorie in der Lebensversicherung besser fundiert als in der Sachversicherung?
Trans. 12th International Congress of Actuaries, Lucerne, 1940. Vol. I, pp. 377–93. Orell Füssli, Zurich, 1941.

(834) KARPINOS, B. D. Towards determining the true rate of growth of the Negro population.
Amer. Jour. Hyg., Vol. 34, pp. 24–7, 1941.

CC-0. In Public Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(835) KOŁODZIEJCZYK, S. Sur les événements à probabilité élevée.
Trans. 12th International Congress of Actuaries, Lucerne, 1940. Vol. I, pp. 309–19. Orell Füssli, Zurich, 1941.

(836) KREIS, H. Zerfällung einer Gesamtheit in Aktiven- und Invalidengruppen.
Mitt. Verein. Schweiz. Versich.-Math., Vol. 41, pp. 205–9, 1941.

(837) KRIEHL, L. Das Problem der Rassenvermischung und seine Bedeutung für die dynamische Sterbetafel.
Arch. Math. Wirtsch. u. Sozialforsch., Vol. 5, pp. 166–88, 1939.

(838) LAH, I. Wahrscheinlichketisrechnung und Versicherungswesen.
Trans. 12th International Congress of Actuaries, Lucerne, 1940. Vol. I, pp. 409–34. Orell Füssli, Zurich, 1941.

(839) LANG, K. Analysis of net premium formulas for the income endowment policy.
Rec. Amer. Inst. Actuar., Vol. 31, pp. 398–405, 1942.

(840) LEGRAS, P. Über das asymptotische Verhalten der Erneuerungsfunktion.
Mitteilungen d. Vereinigung schweiz Versicherungs-Math. H. 42, pp. 183–204, 1942.

(841) LEWIS, E. G. On the generation and growth of a population.
Sankhyā, Vol. 6, pp. 85–92, 1941.

(842) LINDMAN, E. L. The adequacy of follow-up samplings.
Occupations, Vol. 19, pp. 33–5, 1940.

(843) LOTKA, ALFRED J. Sur une équation intégrale de l'analyse démographique et industrielle.
Mitteilungend Vereinigung schweiz Versicherungs-Math., H. 40, pp. 1–16, 1940.

(844) LOTKA, A. J. The progeny of an entire population.
Annals Math. Statistics, Vol. XI, pp. 115–25, 1942.

(845) LUKACS, E. Zur theorie der Selekttafeln.
Skandinavisk Actuarietidskrift, Vol. 1939, pp. 223–6, 1939.

(846) LUKACS, E. Wahrscheinlichkeitstheoretischer Aufbau der Theorie des mittleren Risikos.
Trans. 12th International Congress of Actuaries, Lucerne, 1940. Vol. I, pp. 171–205. Orell Füssli, Zurich, 1941.

(847) LUNDBERG, O. On random processes and their application to sickness and accident statistics.
University of Stockholm Thesis, Uppsala, 1940.

(848) LUNDBERG, O. On the importance of regrading risk premiums in voluntary sickness and accident insurance. A theoretical basis for regrading.
Trans. 12th International Congress of Actuaries, Lucerne, 1940. Vol. II, pp. 543–60. Orell Füssli, Zurich, 1941.

(849) MACDOUGALL, G. G. A. Inter-war population changes in town and country.
J. Roy. Stat. Soc., Vol. CIII, pp. 30–51, 1940.

(850) MALMQUIST, F. Abfindung beim Rücktritt vom Lebensversicherungsvertrag.
Skandinavisk Actuarietidskrift, Vol. XXIV, pp. 70–6, 1941.

(851) MEIDELL, B. Zur theorie und Praxis der Berechnung des effektiven Zinsfusses bei Anleihen.
Skandinavisk Actuarietidskrift, Vol. 1939, pp. 122–51, 1939.

(852) MEISSNER, W. Das Zinsfussproblem bei der Leibrente.
Bl. Versich. Math., Vol. 5, pp. 467–91, 1939.

(853) NOLFI, P. Die jährlichen Sterblichkeitsschwankungen und ihre wahrscheinlichkeitstheoretische Erfassung.
Trans. 12th International Congress of Actuaries, Lucerne, 1940. Vol. I, pp. 395–408. Orell Füssli, Zurich, 1941.

(854) PARRAN, T., and AUSTIN, W. L. A diagnosis code for use in tabulating morbidity statistics.
Publ. Health Repts., Vol. 55, pp. 1558–75, 1940.

(855) TEN PAS, W. G. J. Wahrscheinlichkeitsrechnung und Statistik in der Versicherungsmathematik.
Trans. 12th International Congress of Actuaries, Lucerne, 1940. Vol. I, pp. 263–83. Orell Füssli, Zurich, 1941.

(856) DEPORTE, J. V. Effectiveness of different systems of collecting vital statistics data.
Amer. J. Publ. Health, Vol. 29, pp. 856–60, 1939.

CC-0. In Public Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(857) REDINGTON, F. M., and MICHAELSON, R. L. An aspect of the " a priori" probability theory of mortality.
Trans. 12th International Congress of Actuaries, Lucerne, 1940. Vol. I, pp. 225–39. Orell Füssli, Zurich, 1941.
(858) RHODES, E. C. Population mathematics. I.
J. Roy. Stat. Soc., Vol. CIII, pp. 61–89, 1940.
(859) RHODES, E. C. Population mathematics. II.
J. Roy. Stat. Soc., Vol. CIII, pp. 218–45, 1940.
(860) RHODES, E. C. Population mathematics. III.
J. Roy. Stat. Soc., Vol. CIII, pp. 362–87, 1940.
(861) RHODES, E. C. Secular changes in death rates.
J. Roy. Stat. Soc., Vol. CIV, pp. 15–33, 1941.
(862) RICE, S. A. Vital statistics and national defense.
Amer. J. Publ. Health, Vol. 31, pp. 251–5, 1941.
(863) RICH, C. D. A general theory of mortality.
J. Inst. Actuaries, Vol. LXX, pp. 314–63, 1940.
(864) RICHTER, H. Die Konvergenz der Erneuerungs funktion.
Bl. Versich.-Math., Vol. 5, pp. 21–35, 1940.
(865) RICHTER, H. Untersuchungen zum Erneuerungsproblem.
Math. Ann., Vol. 118, pp. 145–94, 1941.
(866) RIEBESELL, P. Die mathematischen Grundlagen der Sachversicherung.
Trans. 12th International Congress of Actuaries, Lucerne, 1940. Vol. IV, pp. 27–36. Orell Füssli, Zurich, 1941.
(867) ROPER, E. Sampling public opinion.
J. Amer. Stat. Ass., Vol. 35, pp. 325–34, 1940.
(868) RUSTING, F. H. A property concerning the computation of premiums with abnormal risks.
Verzekerings Arch., Vol. 20, pp. 84–100, 1939.
(869) SCHÄRF, H. Über einige Variationsprobleme der Versicherungsmathematik.
Mitt. Verein. Schweiz. Versich.-Math., Vol. 41, pp. 163–96, 1941.
(870) SEAL, H. L. Tests of mortality table graduation.
J. Inst. Actuaries, Vol. 71, pp. 5–48, discussion, pp. 48–67, 1941.
(871) SEGERDAHL, C.-O. Über einige risikotheoretische Fragestellungen.
Skandinavisk Aktuarietidskrift, Vol. XXV, pp. 43–83, 1942.
(872) SHANNON, S. Some assumptions and hypotheses underlying actuarial calculations.
Trans. 12th International Congress of Actuaries, Lucerne, 1940. Vol. I, pp. 137–58. Orell Füssli, Zurich, 1941.
(873) SHAPIRO, H. L. Variations in samples of identical populations.
Anthropol. Briefs, Vol. 1, pp. 17–28, 1942.
(874) SPOERL, C. A. The Whittaker-Henderson graduation formula A. The mixed difference case.
Trans. Actuar. Soc. America, Vol. 42, pp. 292–313, 1941.
(875) SPOERL, C. A. The Whittaker-Henderson graduation formula A. The mixed difference case.
Trans. Actuar. Soc. America, Vol. 43, pp. 68–80, 1942.
(876) STELLER, E. T. Critical remarks on some important points in actuarial science.
Verzekerings Arch., Vol. 20, pp. 101–30, 1939.
(877) STEPHAN, F. F., DEMING, W. E., and HAUSEN, M. H. The sampling procedure of the 1940 population census.
J. Amer. Stat. Ass., Vol. 35, pp. 615–30, 1940.
(878) STERN, E. Kursformeln für Anleihen mit gleichmässig gestaffelten Zins.
Verzekerings-Arch., Vol. 20, pp. 131–46, 1939.
(879) STERN, E. Leibrenten und veränderliche Todesfallversicherungen.
Verzekerings-Arch., Vol. 22, pp. 285–318, 1941.
(880) SWARTOUT, H. O., and WEBSTER, R. G. To what degree are mortality statistics dependable?
Amer. J. Publ. Health, Vol. 30, pp. 811–915, 1940.

CC-0. In Public Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(881) TARJAN, R. Untersuchungen über den Kapitalbedarf des Lebensversicherungsgeschäftes.
Trans. 12th International Congress of Actuaries, Lucerne, 1940. Vol. III, pp. 335–51. Orell Füssli, Zurich, 1941.

(882) TRUESDALL, L. E. New features in the 1940 population census.
J. Amer. Stat. Ass., Vol. 36, pp. 361–8, 1941.

(883) VAJDA, S. Die erweiterte Sterbetafel und ihre wahrscheinlichketistheoretische Verwendung.
Trans. 12th International Congress of Actuaries, Lucerne, 1940. Vol. I, pp. 241–51. Orell Füssli, Zurich, 1941.

(884) WOLFENDEN, H. H. On the formulae for calculating the " exposed to risk " in constructing mortality and other tables from the individual records of insured lives.
Trans. Actuar. Soc. America, Vol. 43, pp. 234–77, 1942.

(885) ZAPFF, J. Die Ausgleichung von Sterbetafeln unter besonderer Berücksichtigung der Gewichte der Einzelbeobachtungen.
Bl. Versich.-Math., Vol. 5, pp. 1–21 and 49–68, 1940.

(886) ZAPPA, G. Sulla lege di estinzione delle famiglie.
Metron, Vol. XIII, pp. 75–93, 1939.

(887) ZWINGGI, E. Study of internal variation in groups of persons.
Trans. 12th International Congress of Actuaries, Lucerne, 1940. Vol. III, pp. 263–303. Orell Füssli, Zurich, 1941.

(888) ZWINGGI, E. Leben und Sterben in mathematischer Darstellung. (Über Sinn und Herkommen der " Sterblichkeitgesetze.") (Life and death mathematically represented.)
Mitteil. Vereinig. schweiz. Versicherungsmathematiker, Vol. 42, pp. 77–95, 1942.

XVI. PSYCHOLOGY

(889) ANON. Quantitative methods in psychology.
Nature, Vol. 141, p. 613, 1938.

(890) BAKER, G. The fetish of statistics.
J. Education Res., Vol. 34, pp. 458–59, 1941.

(891) BARDECKI, A. (Statistical formulae for the use of psychologists.)
Trav. Inst. Psychol. Univ. Jean Casimir, Vol. 1, p. 22, 1938.

(892) BAXTER, B. An application of factorial design to a psychological problem.
Psychol. Bull., Vol. 37, p. 421, 1940.

(893) BAXTER, B. The application of factorial design to a psychological problem.
Psych. Rev., Vol. 47, pp. 494–500, 1940.

(894) BAXTER, B. Problems in the planning of psychological experiments.
Amer. J. Psychol., Vol. 54, pp. 270–80, 1941.

(895) BECKNELL, E. A. Probability: a function of ideology.
Amer. J. Psychol., Vol. 53, pp. 604–9, 1940.

(896) BLOOM, B. S. Test reliability for what?
J. Educ. Psych., Vol. XXXIII, pp. 517–26, 1942.

(897) BLOOMERS, P., and LINDQUIST, E. F. Experimental and statistical studies: application of newer statistical techniques.
Rev. Educ. Res., Vol. 12, pp. 501–20, 1942.

(898) BOISSEVAIN, C. H. Distribution of abilities depending upon two or more independent factors.
Metron, Vol. XIII, pp. 49–58, 1939.

(899) BORDIN, E. S. Factor analysis—art or science?
Psych. Bull., Vol. 38, pp. 520–1, 1941.

(900) BORING, E. G. Statistical frequencies as dynamic equilibria.
Psych. Rev., Vol. 48, pp. 279–300, 1941.

(901) BUTLER, J. M. A ratio for estimating the reliability of test scores.
J. Educ. Psych., Vol. XXXIII, pp. 391–5, 1942.

(902) CASANOVA, T. Analysis of the effect upon the reliability coefficient of changes in variables involved in the estimation of test reliability.
J. exp. Educ., Vol. 9, pp. 219–28, 1941.

(903) COBB, P. W. The limit of usefulness of accident rate as a measure of accident proneness.
J. Appl. Psychol., Vol. 24, pp. 154–9, 1940.

CC-0. In Public Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(904) DEEMER, W. L. A method of estimating accuracy of test scoring.
Psychometrika, Vol. 7, pp. 65–73, 1942.
(905) DELGER, J. Das Gauss'sche Streuungsgesetz in der praktischen Psychologie. (The Gaussian law of distribution in practical psychology.)
Industr. Psychotech., Vol. 17, pp. 1–20, 1940.
(906) DRESSEL, P. L. Some remarks on the Kuder-Richardson reliability coefficient.
Psychometrika, Vol. 5, pp. 305–10, 1940.
(907) ELLSON, D. G. A criticism of Dr. Pratt's use of Chapman's " Statistics of the method of correct matchings " in the evaluation of ESP in drawings.
J. Parapsychol., Vol. 4, pp. 329–36, 1940.
(908) FELLER, W. K. Statistical aspects of ESP.
J. Parapsychol., Vol. 4, pp. 271–98, 1940.
(909) FERGUSON, G. A. The factorial interpretation of test difficulty.
Psychometrika, Vol. 6, pp. 323–9, 1941.
(910) FERGUSON, G. A. A bi-factor analysis of reliability coefficients.
Brit. J. Psych., Vol. XXXI, pp. 172–82, 1941.
(911) FERGUSON, L. W., and LAWRENCE, W. R. An appraisal of the validity of the factor loadings employed in the construction of the Primary Social Attitude Scales.
Psychometrika, Vol. 7, pp. 135–8, 1942.
(912) FLANAGAN, J. C. Statistical methods related to test construction and evaluation.
Rev. Educ. Res., Vol. 11, pp. 109–30, 1941.
(913) FOSTER, A. A. A perception ratio statistic for ESP tests.
J. Parapsychol., Vol. 4, pp. 320–4, 1940.
(914) GHISELLI, E. E. Estimating the minimal reliability of a total test from the inter-correlations among, and the standard deviations of, the component parts.
J. appl. Psychol., Vol. 26, pp. 332–7, 1942.
(915) GOODFELLOW, L. D. The human element in probability.
J. gen. Psychol., Vol. 23, pp. 201–5, 1940.
(916) GREENWOOD, J. A. A caution on the use of the method of correct matchings.
Amer. J. Psychol., Vol. 53, pp. 614–5, 1940.
(917) GREENWOOD, J. A. The rôle mathematics has played in ESP research.
J. Parasitol., Vol. 6, pp. 268–83, 1942.
(918) GREENWOOD, J. A., and STUART, C. E. A review of Dr. Feller's critique.
J. Parapsychology, Vol. 4, pp. 299–319, 1940.
(919) GRIFFIN, C. H. An empirical study of the validity of the Thurstone multiple factor analysis technique.
Penn. St. Coll. Stud. Educ., No. 22, pp. 25–6, 1940. (Abstract.)
(920) GUILFORD, J. P. Human abilities.
Psychol. Rev., Vol. 47, pp. 367–94, 1940.
(921) GUILFORD, J. P., and LYONS, T. C. On determining the reliability and significance of a tetrachoric coefficient of correlation.
Psychometrika, Vol. 7, pp. 243–6, 1942.
(922) HEINLEIN, C. P. Basic postulates neglected in the application of standard parameters to empirical samples of biotic data.
Psych. Bull., Vol. 39, pp. 601–8, 1942. (Abstract.)
(923) HARMAN, H. H. On the rectilinear prediction of oblique factors.
Psychometrika, Vol. 6, pp. 29–35, 1941.
(924) HARSH, C. M. Constancy and variation in patterns of factor loadings.
J. educ. Psych., Vol. 31, pp. 335–59, 1940.
(925) HOLZINGER, K. J. Why do people factor?
Psychometrika, Vol. 7, pp. 147–56, 1942.
(926) HOUSEHOLDER, A. S. Studies in the mathematical theory of excitation.
Bull. Math. Biophys., Vol. 1, pp. 129–41, 1939.
(927) JACKSON, R. W. B. Application of the analysis of variance and covariance method to educational problems.
Univ. Toronto Dep. Educ. Res. Bull., No. 11, 1940.

CC-0. In Public Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(928) JACKSON, R. W. B. Some pitfalls in the statistical analysis of data expressed in the form of IQ scores.
J. Educ. Psych., Vol. XXXI, pp. 677–85, 1940.

(929) JACKSON, R. W. B. Some difficulties in the application of the analysis of covariance method to educational problems.
J. educ. Psychol., Vol. 32, pp. 414–22, 1941.

(930) JACKSON, R. W. B. Note on the relationship between internal consistency and test-retest estimates of the reliability of a test.
Psychometrika, Vol. 7, pp. 157–64, 1942.

(931) JACKSON, R. W. B., and FERGUSON, G. A. Studies on the reliability of tests.
Univ. Toronto Dep. educ. Res. Bull., No. 12, p. 132, 1941.

(932) JOHNSON, H. M. Pre-experimental assumptions as determiners of experimental results.
Psych. Rev., Vol. 47, pp. 338–46, 1940.

(933) JOHNSON, H. M. General rules for predicting the selectivity of a test.
Amer. J. Psychol., Vol. 55, pp. 436–42, 1942.

(934) KELLEY, T. L. The fields to which factor analysis applies.
Psych. Bull., Vol. 39, pp. 454, 1942. (Abstract.)

(935) KENDALL, W. E. The relative efficiency of three types of factorial design.
Psych. Bull., Vol. 38, p. 605, 1941. (Abstract.)

(936) KOENKER, R. H., and HANSEN, C. W. Steps for the application of the Johnson-Neyman technique—a sample analysis.
J. exp. Educ., Vol. 10, pp. 164–73, 1942.

(937) LANDAHL, H. D. Contributions to the mathematical biophysics of the central nervous system.
Bull. Math. Biophys., Vol. 1, pp. 95–118, 1939.

(938) LANDAHL, H. D. A contribution to the mathematical biophysics of psychophysical discrimination. II, III.
Bull. Math. Biophys., Vol. 1, pp. 159–76, 1939.

(939) LANDAHL, H. D. Studies in the mathematical biophysics of discrimination and conditioning. II: Special case: Errors, trials, and number of possible responses.
Bull. Math. Biophys., Vol. 3, pp. 71–7, 1941.

(940) LAWLEY, D. N. The estimation of factor loadings by the method of maximum likelihood.
Proc. Roy. Soc. Edinb., Vol. 60, pp. 64–82, 1940.

(941) LAWSHE, C. H. A nomograph for estimating the validity of test items.
J. appl. Psychol., Vol. 26, pp. 846–9, 1942.

(942) LAZARSFELD, P. F. The statistical analysis of reasons as research operation.
Sociometry, Vol. 5, pp. 29–47, 1942.

(943) LINDQUIST, E. F. Sampling in educational research.
J. Educ. Psych., Vol. XXXI, pp. 561–74, 1940.

(944) LOEVINGER, J. Some common errors in the use of the critical ratio.
Psych. Bull., Vol. 37, p. 592, 1940.

(945) MCCLOY, C. H. The factor analysis as a research technique.
Res. Quart. Amer. Assoc. Health Phys. Educ., Vol. 12, pp. 22–33, 1941.

(946) MCNEMAR, Q. Sampling in psychological research.
Psych. Bull., Vol. 37, pp. 331–65, 1940.

(947) MORRISON, A. W. A graphical device for comparing the form of a given distribution of test scores with the form of the standard distribution for the test.
J. Education Res., Vol. 36, pp. 218–20, 1942.

(948) MOSIER, C. I. A modification of the method of successive intervals.
Psychometrika, Vol. 5, pp. 101–7, 1940.

(949) MOSIER, C. I. A short cut in the estimation of the split-halves coefficients.
Educ. Psychol. Measmt., Vol. 1, pp. 407–8, 1941.

(950) MUKHERJI, N. P. Why and what in factors.
Indian J. Psychol., Vol. 17, pp. 41–7, 1942.

(951) PETERS, C. C. A technique for correlating measurable traits with freely observed social behaviors.
Psychometrika, Vol. 6, pp. 209–19, 1941.

CC-0. In Rublic Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(952) PETERS, C. C. A method of matching groups for experiment with no loss of population.
J. Education Res., Vol. 34, pp. 606–12, 1941.
(953) PRESTON, M. G. Concerning the determination of trait variability.
Psychometrika, Vol. 5, pp. 275–81, 1940.
(954) RICHARDSON, M. W. Possible applications of factor analysis to public personnel work.
Publ. Person. Rev., Vol. 1, pp. 18–22, 1940.
(955) RICHARDSON, M. W. The combination of measures.
Soc. Sci. Res. Coun. Bull., No. 48, pp. 377–402, 1941.
(956) ROYER, E. B. A machine method for computing the biserial correlation coefficient in item validation.
Psychometrika, Vol. 6, pp. 55–9, 1941.
(957) SCATES, D. E. Assumptions underlying research data.
J. Education Res., Vol. 34, pp. 241–53, 1940.
(958) SCATES, D. E. Correlation coefficients and other constants for combined and separated distributions.
J. exp. Educ., Vol. 9, pp. 275–84, 1941.
(959) SCHUWER, C. Sur la notion psychologique du hasard. (On the psychological signification of chance.)
J. Psychol. norm. path., Vol. 36, pp. 199–221, 1939.
(960) SHEN, E. Experimental design and statistical treatment in educational research.
J. exp. Educ., Vol. 8, pp. 346–53, 1940.
(961) TAYLOR, W. S. The use of interactions in analysing psychological data.
Brit. J. Psych., Vol. XXXII, pp. 248–58, 1942.
(962) THOMSON, G. H. The use of the Latin square in designing educational experiments.
Brit. J. Educ. Psych., Vol. XI, pp. 135–7, 1941.
(963) THORNDIKE, R. L. Regression fallacies in the matched groups experiment.
Psychometrika, Vol. 7, pp. 85–102, 1942.
(964) THORNDIKE, R. L. Regression fallacies in the matched-group experiment.
Psych. Bull., Vol. 38, pp. 606–7, 1941. (Abstract.)
(965) THURSTONE, L. L. Experimental study of simple structure.
Psychometrika, Vol. 5, pp. 153–68, 1940.
(966) TOOPS, H. A. The L-method.
Psychometrika, Vol. 6, pp. 249–66, 1941.
(967) TURNER, W. D. A method for the analysis and interpretation of intro-group changes in measurements.
J. gen. Psych., Vol. 23, pp. 343–65, 1940.
(968) WALKER, H. M. Degrees of freedom.
J. educ. Psychol., Vol. 31, pp. 253–69, 1940.
(969) ZIEVE, L. Note on the correlation of initial scores with gains.
J. educ. Psych., Vol. 31, pp. 391–94, 1940.
(970) HOLZINGER, K. J. A synthetic approach to factor analysis.
Psychometrika, Vol. 5, pp. 235–50, 1940.
(971) JOHNSON, H. M. General rules for predicting the selectivity of a test when the standardizing population and the parent population are not necessarily homogeneous.
Psych. Bull., Vol. 39, pp. 577, 1942. (Abstract.)
(972) KELLEY, T. L. Comment on Wilson and Worcester's " Note on factor analysis ".
Psychometrika, Vol. 5, pp. 117–20, 1940.
(973) KELLEY, T. L. The reliability coefficient.
Psychometrika, Vol. 7, pp. 75–83, 1942.
(974) LAWLEY, D. N. Further investigations in factor estimation.
Proc. Roy. Soc. Edinb., Vol. 61, pp. 176–85, 1942.
(975) LUNDBERG, G. A. Some problems of group classification and measurement.
Amer. sociol. Rev., Vol. 5, pp. 351–60, 1940.
(976) MCNEMAR, Q. On the number of factors.
Psychometrika, Vol. 7, pp. 9–18, 1942.

CC-0. In Public Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(977) PENROSE, L. S. On the assignment of precise normal scale values to frequency distributions.
Bull. Canadian Psychol. Ass., Vol. 2, pp. 1–2, 1942.

(978) SWINEFORD, F. Some comparisons of the multiple-factor and the bi-factor methods of analysis.
Psychometrika, Vol. 6, pp. 375–82, 1941.

(979) SWINEFORD, F., and HOLZINGER, K. J. Selected references on statistics, the theory of test construction, and factor analysis.
Sch. Rev., Vol. 48, pp. 460–6, 1940.

(980) SWINEFORD, F., and HOLZINGER, K. J. Selected references on statistics, the theory of test construction, and factor analysis.
Sch. Rev., Vol. 49, pp. 461–7, 1941.

(981) SWINEFORD, F., and HOLZINGER, K. J. Selected references on statistics, the theory of test construction, and factor analysis.
Sch. Rev., Vol. 50, pp. 456–65, 1942.

(982) TUCKER, L. R. The rôle of correlated factors in factor analysis.
Psychometrika, Vol. 5, pp. 141–52, 1940.

(983) WHERRY, R. J. An approximation method for obtaining a maximized multiple criterion.
Psychometrika, Vol. 5, pp. 109–15, 1940.

(984) WILSON, E. B., and WORCESTER, J. The resolution of six tests into three general factors.
Proc. Nat. Acad. Sci. U.S.A., Vol. 25, pp. 73–7, 1939.

(985) WOLFE, D. Factor analysis to 1940.
Psychometr. Monogr., No. 3, 1940.

(986) YOUNG, G. Maximum likelihood estimation and factor analysis.
Psychometrika, Vol. 6, pp. 49–53, 1941.

(987) YOUNG, G., and HOUSEHOLDER, A. S. Factorial invariance and significance.
Psychometrika, Vol. 5, pp. 47–56, 1940.

(988) BABITZ, M., and KEYS, N. A method of approximating the average intercorrelation coefficient by correlating the parts with the sum of the parts.
Psychometrika, Vol. 5, pp. 283–8, 1940.

(989) BLAKEY, R. A re-analysis of a test of the theory of two factors.
Psychometrika, Vol. 5, pp. 121–36, 1940.

(990) COOMBS, C. H. A criterion for the number of factors in a table of intercorrelations.
Psychol. Bull., Vol. 37, p. 455, 1940.

(991) COOMBS, C. H. A criterion for significant common factor variance.
Psychometrika, Vol. 6, pp. 267–72, 1941.

(992) DARMOIS, G. Les mathématiques de la psychologie.
Memor. Sci. Math. f. 98, 51 pp., 1940.

(993) DODD, S. C. Analyses of the interrelation matrix by its surface and structure.
Sociometry, Vol. 3, pp. 133–43, 1940.

(994) DWYER, P. S. The evaluation of multiple and partial correlation coefficients from the factorial matrix.
Psychometrika, Vol. 5, pp. 211–32, 1940.

(995) GUILFORD, J. P. The phi coefficient and chi square as indices of item validity.
Psychometrika, Vol. 6, pp. 11–19, 1941.

(996) GUTTMAN, L. Multiple rectilinear prediction and the resolution into components.
Psychometrika, Vol. 5, pp. 75–99, 1940.

XVII. Other Biological Applications

(997) BACKMAN, G. Methodik der theoretischen Wiedergabe beobachteter Wachstumsserien.
Lunds Univ. Arsskrift, Vol. 35, No. 8, 20 pp., 1939.

(998) BAKER, G. A., and GUILBERT, H. R. Non-randomness of variations in daily weights of cattle.
J. Animal Sci., Vol. 1, pp. 293–9, 1942.

(999) BARBENSI, G. Note Biometriche.
Arch. Ital. Anat. e Embriol., Vol. 41, pp. 550–91, 1939.

CC-0. In Rublic Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(1000) BEALL, G. The fit and significance of contagious distributions when applied to observations on larval insects.
Ecology, Vol. 21, pp. 460–74, 1940.

(1001) BEALL, G. Method of estimating the population of an agricultural pest over areas of many square miles.
Canadian Jour. Res. D., Vol. 19, pp. 167–77, 1941.

(1002) BERG, W. N. The law of small numbers as applied to virulence measurement.
Amer. Rev. Tuberc., Vol. 43, pp. 685–91, 1941.

(1003) BLISS, C. I. Factorial design and covariance in the biological assay of vitamin D.
J. Amer. Stat. Ass., Vol. 35, pp. 498–506, 1940.

(1004) BLISS, C. I. Quantitative aspects of biological assay.
J. Amer. Phar. Assoc., Sci. Ed., Vol. XXIX, pp. 465–75, 1940.

(1005) BLISS, C. I. Biometry in the service of biological assay.
Indust. and Engineer. Chem. Analyt. Ed., Vol. 13, pp. 84–8, 1941.

(1006) BLISS, C. I. Statistical problems in estimating populations of Japanese beetle larvae.
J. Economic Entomology, Vol. 34, pp. 221–31, 1941.

(1007) BLISS, C. I., and MARKS, H. P. The Biological Assay of Insulin. I. Some general considerations directed to increasing the precision of the curve relating dosage and graded response.
Quart. J. Pharmacy and Pharmacology, Vol. 12, pp. 82–110, 1939.

(1008) BLISS, C. I., and MARKS, H. P. The biological assay of insulin. II. The estimation of drug potency from a graded response.
Quart. J. Pharmacy and Pharmacology, Vol. 12, pp. 182–211, 1939.

(1009) BLUM, H. F., GRADY, H. G., and KIRBY-SMITH, J. S. Limits of accuracy in experimental carcinogenesis as exemplified by tumor induction with ultraviolet radiation.
J. Nat. Cancer Inst., Vol. 3, pp. 83–9, 1942.

(1010) BONET, MAURY P. Méthodes statistiques de titrage des ultravirus.
Ann. Inst. Pasteur, Vol. 68, pp. 491–4, 1942.

(1011) BUCHANAN-WOLLASTON, H. J. On tests of the significance of differences in degree of pollution by coliform bacteria and the estimation of such differences.
J. Hyg., Vol. 41, pp. 139–68, 1941.

(1012) CHATTOPADHYAY, K. P. Application of statistical methods to anthropological research.
Sankhyā, Vol. 6, pp. 99–104, 1941.

(1013) COUNT, E. W. Some uses of co-ordinate geometry in craniometry analysis: A second contribution.
Zeitschr. Rassenk., Vol. 12, pp. 87–95, 1941.

(1014) FELLER, W. On the logistic law of growth and its empirical verifications in biology.
Acta Bioth., Vol. 5, pp. 51–66, 1940.

(1015) FENWICK, D. W., and FRANKLIN, M. T. Identification of Heterodera species by larval length. Technique for estimating the constants determining the length variations within a given species.
J. Helminthol., Vol. 20, pp. 67–114, 1942.

(1016) FIELLER, E. C. The biological standardisation of insulin.
J. Roy. Stat. Soc., Suppl., Vol. VII, pp. 1–53, 1940–1.

(1017) FIELLER, E. C., IRWIN, J. O., MARKS, H. P., and SHRIMPTON, E. A. G. The dosage-response relation in the cross-over rabbit test for insulin.
Quart. J. Pharmacy and Pharmacology, Vol. 12, pp. 206–11, 724–42, 1939.

(1018) FINNEY, D. J. The analysis of toxicity tests on mixtures of poisons.
Annals Appl. Biol., Vol. 29, pp. 82–94, 1942.

(1019) GAFAFER, W. M. The measurement and comparison of dental caries experience.
J. Dental Res., Vol. 21, pp. 443–53, 1942.

(1020) GARWOOD, F. The application of maximum likelihood to dosage-mortality problems.
Biometrika, Vol. XXXII, pp. 46–58, 1940–1.

(1021) GINSBURG, I. Divergence and probability in taxonomy.
Zoologica, Vol. 25, pp. 15–31, 1940.

(1022) GLASGOW, H. A plot arrangement for timing the applications in a control program.
J. Economic Entomology, Vol. 33, pp. 357–61, 1940.

CC-0. In Public Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(1023) GREENSLADE, R. M., and PEARCE, S. C. Field-sampling for the comparison of infestations of strawberry crops by the aphis *Capitophorus fragariae* Theob.
Jour. Pomol. and Hort. Sci., Vol. 17, pp. 308–17, 1940.

(1024) HALDANE, J. B. S. Sampling errors in the determination of bacterial or virus density by the dilution method.
J. Hyg., Vol. 39, pp. 289–93, 1939.

(1025) HALVORSON, H. O., and MOEGLEIN, A. Application of statistics to problems in bacteriology. V. The probability of occurrence of various experimental results.
Growth, Vol. 4, pp. 157–68, 1940.

(1026) HANKS, J. H., and JAMES, D. F. The enumeration of bacteria by the microscopic method
J. Bact., Vol. 39, pp. 297–305, 1940.

(1027) HUFFAKER, C. B., and BACK, R. C. A comparative study of several methods of sampling adult mosquito populations.
J. Economic Entomology, Vol. 35, p. 955, 1942.

(1028) HVIDSTEN, H. Beregning av vekta hos storfe etter mal. (Calculations of the weight of cattle by measuring.)
Norges Landbrukshøgskole Foringsforsøkene Beretning, Vol. 49, pp. 1–39, 1940.

(1029) IRWIN, J. O. The distribution of the logarithm of survival times when the true law is exponential.
J. Hyg., Vol. 42, pp. 328–33, 1942.

(1030) JOHN, K. Über ein neuartiges Integriergerät und seine Anwendung insbesondere in der Biologie.
Zeitschr. wiss. Mikrosk. u. mikrosk. Tech., Vol. 52, pp. 163–8, 1940.

(1031) JORDAN, P. Statistische Analyse biologischer Elementarreaktionen.
Arch. ges. Virusforsch., Vol. 1, pp. 171–214, 1939.

(1032) KLAUBER, L. M. The frequency distribution of certain herpetological variables.
Bull. Zool. Soc. San Diego, Vol. 17, 5–31, 1941.

(1033) KLAUBER, L. M. Illustrations of the relationship between populations and samples.
Bull. Zool. Soc. San Diego, Vol. 17, pp. 33–71, 1941.

(1034) LASCH, C. H. Krebskrankenstatistik. Beginn und Aussicht.
Aeitschr. Krebsforsch., Vol. 50, pp. 245–98, 1940.

(1035) LOUTTIT, C. M., and SAPERO, J. J. The use of statistics in the solution of a biological problem.
Proc. Indiana Acad. Sci., Vol. 51, p. 261, 1942.

(1036) LURIA, S. Méthodes statistiques appliquées à l'étude des ultraviruses.
Ann. Inst. Pasteur (Paris), Vol. 64, pp. 415–38, 1940.

(1037) MARKS, H. P. The mouse method for the assay of insulin.
Quart. J. Pharmacy and Pharmacology, Vol. 13, pp. 344–8, 1940.

(1038) MATHER, K. The design and significance of synergic action tests.
J. Hyg., Vol. 40, pp. 513–31, 1940.

(1039) MELLEY, A. Une méthode capable de caractériser la variabilité en biologie.
Thesis: Lausanne, 1941.

(1040) VON MISES, R. On the correct use of Bayes' formula.
Annals Math. Statistics, Vol. XIII, pp. 156–65, 1942.

(1041) MITTMAN, O. Funktionale Zusammenhange in erbbiologischen Gesamtheiten.
Arch. Math. Wirtsch.-Sozialforschg., Vol. 6, pp. 70–80, 1940.

(1042) MOROZOV, A. V. On the method of Oscar Sund and on the utilization of this method for prognosis of anticipated fish hauls.
Trudy Biologicheskogo Nauchno-Issledovatel'skogo Instituta, Vol. 4, pp. 137–51, 1937.

(1043) MORRELL, C. A. The application of statistical analysis with special reference to the assay of vitamins.
Chem. and Indust., Vol. 61, pp. 35–9, 1942.

(1044) MOTTLEY, C. McC. Experimental designs for developing and testing a stocking policy.
Trans. North Amer. Wildlife Conf., Vol. 7, pp. 224–38, 1942.

(1045) NAIR, K. R. Efficiency of the adjustment for concomitant characters in biological experiments.
Sankhyā, Vol. 6, pp. 167–74, 1942.

CC-0. In Rublic Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(1046) PARKER-RHODES, A. F. Studies on the mechanism of fungicidal action. II. Elements of the theory of variability.
Ann. Appl. Biol., Vol. 29, pp. 126–35, 1942.

(1047) PEARL, E., PARK, T., and MINER, J. R. Experimental studies on the duration of life. XVI. Life tables for the flour beetle *Tribolium confusum* Duval.
Amer. Nat., Vol. 75, pp. 5–19, 1941.

(1048) PETERS, B. G. Dilution egg-counts and the Poisson series.
J. Helminthol., Vol. 19, pp. 59–62, 1941.

(1049) PHLEGER, F. B. Relative growth and vertebrate phylogeny.
Amer. J. Sci., Vol. 238, pp. 643–62, 1940.

(1050) RASHEVSKY, N. Mathematical Biophysics in its relation to the cancer problem.
Acta Bioth., Vol. 5, pp. 139–54, 1940.

(1051) RASHEVSKY, N. On the so-called plasticity of the central nervous system.
Bull. Math. Biophys., Vol. 1, pp. 93–4, 1939.

(1052) RASHEVSKY, N. Mathematical biophysics of growth.
Bull. Math. Biophys., Vol. 1, pp. 119–27, 1939.

(1053) REINER, J. M. Diffusion in colloidal media.
Bull. Math. Biophys., Vol. 1, pp. 143–9, 1939.

(1054) SENEVET, G. Quelques notions pratiques de statistique médicale.
Arch. Inst. Pasteur Algérie, Vol. 20, pp. 172–97, 1942.

(1055) SCOTT, J. A. The natural pattern of dilution counts of helminth eggs.
Amer. J. Trop. Med., Vol. 22, pp. 647–54, 1942.

(1056) SHOCK, N. W., and MORALES, M. F. A fundamental form for the differential equation of colonial and organism growth.
Bull. Math. Biophys., Vol. 4, 63–71, 1942.

(1057) SHRIMPTON, E. A. G. The error of vitamin D assay by the ash content of bone method.
Quart. J. Pharmacy and Pharmacology, Vol. 13, pp. 97–108, 1940.

(1058) SWAROOP, S. A. A modification of the routine dilution tests and tables showing the most probable number of organisms and the standard error of this number.
Indian J. Med. Res., Vol. 29, pp. 499–510, 1941.

(1059) SWAROOP, S. A consideration of the accuracy of estimation of the most probable number of organisms by dilution tests.
Indian J. Med. Res., Vol. 29, pp. 511–22, 1941.

(1060) THOMPSON, W. R. Biological control and the theories of the interactions of populations.
Parasitology, Vol. 31, pp. 299–388, 1939.

(1061) UPHOLT, W. M. The use of the square root transformation and analysis of variance with contagious distributions.
J. Economic Entomology, Vol. 35, pp. 536–43, 1942.

(1062) UPHOLT, W. M., and CRAIG, R. A note on the frequency distribution of black scale insects.
J. Economic Entomology, Vol. 33, pp. 113–4, 1940.

(1063) VARGAS, L. Aplicación taxonómica de un método estadístico.
Rev. Inst. Salubridad y Enferm. Trop., Vol. 2, pp. 123–28, 1941.

(1064) WAERDEN, B. L. VAN DER. Biologische Konzentrationsauswertung.
Ber. Verh. Sächs. Akad. Wiss. Leipzig, Vol. 92, pp. 41–4, 1940.

(1065) WEINBERG, A. M., and HOUSEHOLDER, A. S. Statistical distribution of impedance elements in biological systems.
Bull. Math. Biophys., Vol. 3, pp. 129–35, 1941.

(1066) WHITNEY, J. S. (*edited by*). Statistical data.
Amer. Rev. Tuberc., Vol. 42, pp. 287–91, 1940.

(1067) WOODRUFF, N. H. Mathematics used in biology.
J. Tennessee Acad. Sci., Vol. 15, pp. 177–266, 1940.

XVIII. ECONOMIC AND INDUSTRIAL PROBLEMS

(1068) ALGER, P. L. The importance of the statistical viewpoint in high production manufacturing.
J. Amer. Stat. Ass., Vol. 36, pp. 50–2, 1941.

CC-0. In Public Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(1069) AMOROSO, L. The transformation of value in the productive process.
Econometrica, Vol. VIII, pp. 1–11, 1940.

(1070) BROWN, T. H. Business approaches to rearmament production control.
J. Amer. Stat. Ass., Vol. 36, pp. 27–35, 1941.

(1071) CHAMBERS, E. G., and YULE, G. U. Theory and observation in the investigation of accident causation.
J. Roy. Stat. Soc., Suppl., Vol. VII, pp. 89–101; discussion, pp. 101–9, 1941.

(1072) DODD, EDWARD L. The cyclic effects of linear graduations persisting in the differences of the graduated values.
Annals Math. Statistics, Vol. XII, pp. 127–36, 1941.

(1073) DODD, E. L. The problem of assigning a length to the cycle to be found in a simple moving average and in a double moving average of chance data.
Econometrica, Vol. IX, pp. 25–37, 1941.

(1074) DUDDING, B. P. Statistical methods as aids to production efficiency.
Trans. Manchester Stat. Soc., Session 1941–2.

(1075) EPSLEY, D. C. Harmonic analysis by the method of central differences.
Phil. Mag., Vol. 28, pp. 338–52, 1939.

(1076) FISHER, I. Mathematical method in the social sciences.
Econometrica, Vol. IX, pp. 185–97, 1941.

(1077) FLOOD, M. M. Recursive methods in business-cycle analysis.
Econometrica, Vol. VIII, pp. 333–53, 1940.

(1078) FROLOW, V. Utilisation du coefficient de corrélation dans l'analyse harmonique.
C. R. Acad. Sci. Paris, Vol. 213, pp. 56–7, 1941.

(1079) GALBRAITH, V. L., and THOMAS, D. S. Birth rates and the interwar business cycles.
J. Amer. Stat. Ass., Vol. 36, pp. 465–76, 1941.

(1080) GINI, C. Sulla determinazione dell'indice di cograduazione. (Determination of the index of cograduation.)
Metron, Vol. XIII, pp. 41–8, 1939.

(1081) GIRSHICK, M. A. The application of the theory of linear hypotheses to the coefficient of elasticity of demand.
J. Amer. Stat. Ass., Vol. 37, pp. 233–7, 1942.

(1082) GLOCK, W. S. A rapid method of correlation for continuous time series.
Amer. J. Sci., Vol. 240, pp. 437–42, 1942.

(1083) HAAVELMO, T. The inadequacy of testing dynamic theory by comparing theoretical solutions and observed cycles.
Econometrica, Vol. VIII, pp. 312–21, 1940.

(1084) HAAVELMO, T. A note on the variate difference method.
Econometrica, Vol. IX, pp. 74–9, 1941.

(1085) HAFSTAD, L. R. On the bartels technique for time-series analysis, and its relation to the analysis of variance.
J. Amer. Stat. Ass., Vol. 35, pp. 347–61, 1940.

(1086) HORROBIN, S., and DAVIES, O. L. Statistics; a technical tool in the chemical industry.
Trans. Manchester Stat. Soc., Session 1939–40.

(1087) KALECKI, M., and TEW, B. A new method of trend elimination.
Econometrica, Vol. VIII, pp. 117–29, 1940.

(1088) KAPLAN, M. A method of analyzing the elements of foreclosure risk.
J. Amer. Stat. Ass., Vol. 37, pp. 247–55, 1942.

(1089) KENDALL, M. G. The effect of the elimination of trend on oscillations in time-series.
J. Roy. Stat. Soc., Suppl., Vol. CIV, pp. 43–52, 1941.

(1090) KERSHNER, R. Note on compound interest.
American Math. Monthly, Vol. XLVII, pp. 196–8, 1940.

(1091) KNUDSEN, L. F. Interdependence in a series.
J. Amer. Stat. Ass., Vol. 35, pp. 507–14, 1940.

(1092) KONÜS, A. On the theory of means.
Acta Univ. Asiae Mediae. Ser. V-a., *fasc.* 24, 1939. (Russian. English summary.)

CC-0. In Public Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(1093) KOOPMANS, T. The degree of damping in business cycles.
Econometrica, Vol. VIII, pp. 79–89, 1940.
(1094) KOOPMANS, T. The logic of econometric business-cycle research.
J. Polit. Econ., Vol. 49, 157–81, 1941.
(1095) KORMES, M. Statistical methods as applied to casualty insurance.
J. Amer. Stat. Ass., Vol. 35, pp. 47–54, 1940.
(1096) KOSAMBI, D. D. Correlation and time series.
Current Sci., Vol. 10, pp. 372–4, 1941.
(1097) KOZLIK, A. The use of *per capita* figures for demand curves.
J. Amer. Stat. Ass., Vol. 36, pp. 417–22, 1941.
(1098) LANGE, O. Theoretical derivation of elasticities of demand and supply: the direct method.
Econometrica, Vol. 10, pp. 193–214, 1942.
(1099) LATANÉ, H. A. Seasonal factors determined by difference from average of adjacent months.
J. Amer. Stat. Ass., Vol. 37, pp. 517–22, 1942.
(1100) LEWIS, H. G. On the distribution of the partial elasticity coefficient.
J. Amer. Stat. Ass., Vol. 36, pp. 413–16, 1941.
(1101) LORGE, I. A statistician looks at time study, motion study, and wage incentive methods.
Inst. Mgmt. Ser., Amer. Mgmt. Ass., No. 18, 1937.
(1102) LOTKA, A. J. The theory of industrial replacement. A commentary.
Skandinavisk Actuarietidskrift, 1940, pp. 1–14, 1940.
(1103) MOSTELLER, FREDERICK. Note on an application of runs to quality control charts.
Annals Math. Statistics, Vol. XII, pp. 228–32, 1941.
(1104) NICHOL, A. J. Probability analysis in the theory of demand, net revenue and price.
J. Polit. Econ., Vol. 49, pp. 637–61, 1941.
(1105) OGBURN, W. F. Statistical trends.
J. Amer. Stat. Ass., Vol. 35, pp. 252–60, 1940.
(1106) PAGE, L. F. An example of periodic analysis of a persistent series.
Bull. Amer. Meteorol. Soc., Vol. 21, pp. 196–99, 1940.
(1107) PREINREICH, G. A. D. The theory of industrial replacement.
Skand Aktuarietidskrift, Vol. 1939, pp. 1–9, 1939.
(1108) PREINREICH, G. A. D. The economic life of industrial equipment.
Econometrica, Vol. VIII, pp. 12–44, 1940.
(1109) REIERSØL, O. Confluence analysis by means of lag moments and other methods of confluence analysis.
Econometrica, Vol. IX, pp. 1–24, 1941.
(1110) SALVEMINI, T. L'indice di cograduazione del Gini nel caso di serie statistiche con ripetizioni. (Gini's index of co-graduation in the case of statistical series with repetitions.)
Metron, Vol. XIII, pp. 27–39, 1939.
(1111) SCHUMANN, T. E. W. On Yule's method of investigating periodicities of disturbed series. The motion of a pendulum in a turbulent fluid.
Phil. Mag., Vol. 33, pp. 138–50, 1942.
(1112) SIEGEL, I. H. The difference between the Paasche and Laspeyres index-number formulas.
J. Amer. Stat. Ass., Vol. 36, pp. 343–50, 1941.
(1113) SIEGEL, I. H. Further notes on the difference between index-number formulas.
J. Amer. Stat. Ass., Vol. 36, pp. 519–24, 1941.
(1114) SIMON, L. E. On the initiation of statistical methods for Quality Control in industry.
J. Amer. Stat. Ass., Vol. 36, pp. 53–60, 1941.
(1115) SIMON, L. E. Application of statistical methods to ordnance engineering.
J. Amer. Stat. Ass., Vol. 37, pp. 95–102, 1942.
(1116) SINHA, H. Rôle of mathematics in economic statistics.
Science and Culture, Vol. 6, pp. 255–8, 1940.
(1117) SMITH, E. D. Market sampling.
J. Marketing, Vol. 4, pp. 45–50, 1939.

CC-0. In Public Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(1118) STANTON, F. Problems of sampling in market research.
J. consult. Psychol., Vol. 5, pp. 149–53, 1941.

(1119) SUBRAMANIAN, S. Compatibility of Fisher's tests for index number formulae.
Math. Student, Vol. 8, pp. 124–7, 1940.

(1120) THESEN, G. Über periodische Funktionen, ihre Observation und Ausgleichung.
Skandinavisk Aktuarietidskrift, 1940, pp. 168–95.

(1121) TINTNER, G. The analysis of economic time series.
J. Amer. Stat. Ass., Vol. 35, pp. 93–100, 1940.

(1122) TINTNER, G. The variate-difference method: a reply.
Econometrica, Vol. IX, pp. 163–4, 1941.

(1123) TINTNER, G. The theory of choice under subjective risk and uncertainty.
Econometrica, Vol. IX, pp. 298–304, 1941.

(1124) TINTNER, G. The pure theory of production under technological risk and uncertainty.
Econometrica, Vol. IX, pp. 305–12, 1941.

(1125) UGGÉ, A. Sul metodo di eliminazione della costruzione dei numeri indici dei prezzi.
Pon. Acad. Acta, Vol. 5, pp. 67–71, 1941.

(1126) WALD, A. The approximate determination of indifference surfaces by means of Engel curves.
Econometrika, Vol. VIII, pp. 144–75, 1940.

(1127) WILKINSON, R. I. The combination of probability curves in engineering.
Trans. Amer. Inst. Elec. Engrs., Vol. 61, pp. 953–63, 1942.

XIX. COMPUTATION

(1128) ANDERSON, R. L., and HOUSEMAN, E. E. Tables of orthogonal polynomial values extended to N = 104.
Agric. Exp. Stat., Iowa State Coll. of Agric. Mech. Arts, Stat. Sect., Res. Bull., No. 297, pp. 595–627, 1942.

(1129) ANDREWS, J. M. A nomogram for computation of probable errors of differences.
Human Biology, Vol. 11, pp. 543–6, 1939.

(1130) ANONYMOUS. Calculator.
Indust. Equip. News, Vol. 9, p. 52, 1941.

(1131) AROIAN, L. A. Continued fractions for the incomplete Beta function.
Annals Math. Statistics, Vol. XII, pp. 218–23, 1941.

(1132) BANACHIEWICZ, T. An outline of the Cracovian algorithm of the method of least squares.
Astr. J., Vol. 50, pp. 38–41, 1942.

(1133) BATEN, W. D. A solution of normal equations giving the standard errors of the constants.
J. Agric. Res., Vol. 61, pp. 237–40, 1940.

(1134) BECKERLEY, JAMES G. The calculation of arg $\Gamma(ia + 1)$.
Indian J. Phys., Vol. 15, pp. 229–32, 1941.

(1135) BERJMAN, E. A solution of the problem of least square adjustment by Gauss polynomials, using the computation and tabulation of the parametric coefficients of parabolic functions from the first to the fifth order for series having up to one hundred elements.
An. Soc. Ci. Argentina, Vol. 132, pp. 34–48, 104–17, 212–17, 1941; Vol. 133, pp. 208–15, 442–5, 1942.

(1136) BERKSON, J. A punch card designed to contain written data and coding.
J. Amer. Stat. Ass., Vol. 36, pp. 535–8, 1941.

(1137) BINGHAM, M. D. A new method for obtaining the inverse matrix.
J. Amer. Stat. Ass., Vol. 36, pp. 530–4, 1941.

(1138) BLOOM, B. S., and LUBIN, A. Use of the test scoring machine and the graphic item-counter for statistical work.
Psychometrika, Vol. 7, pp. 233–41, 1942.

(1139) DU BOIS, P. H. Some statistical operations on the counting sorter.
Psychometrika, Vol. 6, pp. 383–90, 1941.

(1140) DU BOIS, P. H. A note on the computation of biserial *r* in item validation.
Psychometrika, Vol. 7, pp. 143–6, 1942.

CC-0. In Public Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(1141) BULA, CLOTILDE A. Calculation of frequency surfaces. Experimental verification and comparison of the method of marginal functions with that of the 15 constants of Karl Pearson.
Union Mat. Argentina, Publ., No. 10, pp. 109, 1940; *Revista Union Mat. Argentina*, Vol. 6, pp. 109, 1940.

(1142) CARPENTER, A. Short cuts in working out intercorrelations.
Res. Quart. Amer. Assoc. Health Phys. Educ., Vol. 11, pp. 32–7, 1940.

(1143) CHEN, C. S. (A new method for calculating the arithmetic mean.)
Educ. Rev. (Chinese), Vol. 27, pp. 87–92, 1937.

(1144) COCHRAN, W. G. Note on an approximate formula for the significance levels of z.
Annals Math. Statistics, Vol. XI, pp. 93–5, 1940.

(1145) COHEN, A. C. The numerical computation of the product of conjugate imaginary gamma functions.
Annals Math. Statistics, Vol. XI, pp. 213–18, 1940.

(1146) COLLATZ, L. Fehlerabschatzung für das Iterationsverfahren zur Auflösung linearer Gleichungssysteme.
Z. Angew. Math. Mech., Vol. 22, pp. 357–61, 1942.

(1147) COMRIE, L. J., and HARTLEY, H. O. Table of Lagrangian coefficients for harmonic interpolation in certain tables of percentage points.
Biometrika, Vol. XXXII, pp. 183–6, 1941.

(1148) DANIEL, C. Statistically significant differences in observed per cents.
J. appl. Psychol., Vol. 24, pp. 826–30, 1940.

(1149) DEMING, W. E., TIPPING, B. J., and GEOFFREY, L. Errors in card punching.
J. Amer. Stat. Ass., Vol. 37, pp. 525–36, 1942.

(1150) DODD, E. L. A transformation of Tippett random sampling numbers into numbers normally distributed.
Bol. Mat., Vol. 15, pp. 73–7, 1942.

(1151) DODGE, H. F., and ROMIG, H. G. Single sampling and double sampling inspection tables.
Bell Syst. Tech. J., Vol. 20, pp. 1–61, 1941.

(1152) DUNLAP, J. W. Note on the computation of tetrachoric correlation.
Psychometrika, Vol. 5, pp. 137–40, 1940.

(1153) DWYER, P. S. The Doolittle technique.
Annals Math. Statistics, Vol. XII, pp. 449–58, 1941.

(1154) DWYER, P. S. The solution of simultaneous equations.
Psychometrika, Vol. 6, pp. 101–29, 1941.

(1155) DWYER, P. S. The evaluation of determinants.
Psychometrika, Vol. 6, pp. 191–204, 1941.

(1156) DWYER, P. S. The evaluation of linear forms.
Psychometrika, Vol. 6, pp. 355–365, 1941.

(1157) DWYER, P. S. The calculation of correlation coefficients from ungrouped data with modern calculating machines.
J. Amer. Stat. Ass., Vol. 35, pp. 671–3, 1940.

(1158) DWYER, P. S. The implicit evaluation of linear forms and the solution of simple matrix equations.
Psychometrika, Vol. 6, pp. 355–65, 1941.

(1159) EMDE, F. Tafeln elementarer Funktionen.
B. G. Teubner, Leipzig, 1941.

(1160) FISCHER, E. Das Zinsfussproblem der Lebensversicherungsrechnung als Interpolationsaufgabe.
Mitt. Verein. Schweiz. Versich.-Math., Vol. 42, pp. 205–307, 1942.

(1161) FLOOD, M. M. A computational procedure for the method of principal components.
Psychometrika, Vol. 5, pp. 169–72, 1940.

(1162) FROCHT, M. M., and LEVEN, M. M. A rational approach to the numerical solution of Laplace's equation.
J. Appl. Phys., Vol. 12, pp. 596–604, 1941.

CC-0. In Public Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(1163) FULCHER, J. S., and ZUBIN, J. The item analyzer: a mechanical device for treating the four-fold table in large samples.
J. appl. Psychol., Vol. 26, pp. 511–23, 1942.

(1164) GABRIEL, F. Graphisch-rechnerisches Verfahren zur schnellen Ermittlung von Trefferprozenten unter einfachen und erschwerten Bedingungen. "Trefferspinne."
Luftfahrtforschung, Vol. 19, pp. 231–5, 1942.

(1165) GRAY, P. A simplified method for computing the theoretical class frequencies of a binomial expansion.
Growth, Vol. 5, pp. 267–71, 1941.

(1166) GREEN, H. D. Square root extractor.
Rev. Sci. Instruments, Vol. 11, pp. 262–4, 1940.

(1167) GREENSPAN, M. Approximation to a function of one variable from a set of its mean values.
J. Research Nat. Bur. Standards, Vol. 23, pp. 309–17, 1939.

(1168) GUTTMAN, L. Mathematical and tabulation techniques.
Soc. Sci. Res. Coun. Bull., No. 48, pp. 251–364, 1941.

(1169) HANCOCK, R. T. The analysis of compound exponential curves.
Engineer, Vol. 168, p. 492, 1939.

(1170) HART, B. I. Tabulation of the probabilities for the ratio of the mean square successive difference to the variance. Note by John von Neumann.
Annals Math. Statistics, Vol. XIII, pp. 207–14, 1942.

(1171) HART, B. I. Significance levels for the ratio of the mean square successive difference to the variance.
Annals Math. Statistics, Vol. XIII, pp. 445–7, 1942.

(1172) HARTKEMEIER, H. P., and MILLER, H. E. Obtaining differences from punched cards.
J. Amer. Stat. Ass., Vol. 37, pp. 285–7, 1942.

(1173) HARTLEY, H. O. The probability integral of the range in samples of n observations from a normal population. II. Numerical evaluation of the probability integral.
Biometrika, Vol. XXXII, pp. 309–10, 1942.

(1174) HOEL, P. G. On methods of solving normal equations.
Annals Math. Statistics, Vol. XII, pp. 354–9, 1941.

(1175) HOOGLAND, J. J. Note on the numerical calculation of the orthogonal polynomials.
Annals of Eugenics, Vol. XI, pp. 77–9, 1941.

(1176) HORST, P. A non-graphical method for transforming an arbitrary factor matrix into a simple structure factor matrix.
Psychometrika, Vol. 6, pp. 79–99, 1941.

(1177) HOYT, C. J. Note on a simplified method of computing test reliability.
Educ. psychol. Measmt., Vol. 1, pp. 93–5, 1941.

(1178) HUGERSHOFF, R. Graphisch-mechanische Bestimmung der Mittelwerte und der m. Schwankung bei grösseren Beobachtungsreihen.
Tharandter forstl. Jahrb., Vol. 90, pp. 717–23, 1939.

(1179) JOLLIFE, E. T. Fundamental principles in tabulating machine methods of statistical analysis.
J. exp. Educ., Vol. 9, pp. 254–84, 1941.

(1180) KERAWALA, S. M. A rapid method for calculating the least squares solution of a polynomial of degree not exceeding the fifth.
Indian J. Phys., Vol. 15, pp. 241–76, 1941.

(1181) KNUDSEN, L. F. A punched card technique to obtain coefficients of orthogonal polynomials.
J. Amer. Stat. Ass., Vol. 37, pp. 496–506, 1942.

(1182) LANCASTER, O. E. Machine method for the extraction of cube root.
J. Amer. Stat. Ass., Vol. 37, 112–15, 1942.

(1183) LAURILA, E. Ueber das Nyströmsche Stieltjesplanimeter.
Soc. Sci. Fennica Comment. Phys.-Math., Vol. 10, No. 7–9, 1940.

(1184) LIDSTONE, G. J. Notes on interpolation. Part 2. III. (i) continued. The origin of the throw-back device. IV. Aitken's new method of inverse interpolation. V. The connexion of the throw-back with Stirling's and Bessel's formulae.
J. Inst. Actuaries, Vol. LXXI, pp. 68–95, 1941.

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Digitized by Arya Samaj Foundation Chennai and eGangotri

(1185) LONSETH, A. T. Systems of linear equations with coefficients subject to error.
Annals Math. Statistics, Vol. XIII, pp. 332–7, 1942.

(1186) MACLAURY, D. A " slide rule " for determining time intervals.
Poultry Sci., Vol. 21, pp. 462–63, 1942.

(1187) MAXWELL, L. R. An electrical method for compounding sine functions.
Rev. Sci. Instruments, Vol. 11, pp. 47–54, 1940.

(1188) MCPHERSON, J. C. On mechanical tabulation of polynomials.
Annals Math. Statistics, Vol. XII, pp. 317–27, 1941.

(1189) MCPHERSON, J. C. Mathematical operations with punched cards.
J. Amer. Stat. Ass., Vol. 37, pp. 275–81, 1942.

(1190) MERRINGTON, M. Numerical approximations to the percentage points of the χ^2 distribution.
Biometrika, Vol. XXXII, pp. 200–2, 1941.

(1191) MERRINGTON, M. Table of percentage points of the t-distribution.
Biometrika, Vol. XXXII, p. 300, 1942.

(1192) MILLANG, A., and SPRAGUE, G. F. The use of punched card equipment in predicting the performance of corn double crosses.
J. Amer. Soc. Agron., Vol. 32, pp. 815–16, 1940.

(1193) MIKELADZE, S. Numerische Integration der Gleichungen vom elliptischen und parabolischen Typus.
Bull. Acad. Sci. U.R.S.S., Sér. Math., Vol. 5, pp. 57–74, 1941.

(1194) MORRIS, J. Frequency equations.
Aircraft Engrg., Vol. 14, pp. 108–10, 1942.

(1195) MORRIS, J., and HEAD, J. W. Lagrangian frequency equations. An " escalator " method for numerical solution.
Aircraft Engrg., Vol. 14, pp. 312–14, 316, 1942.

(1196) MULLER, E. Genauigkeit der bei Reduktion von Fehlergleichungen eliminierten Unbekannten.
Z. Vermessungswesen, Vol. 71, pp. 186–90, 1942.

(1197) NAKAGAMI, M. A method of calculating probability by means of differential operator.
Nippon Elec. Commun. Engrg., No. 21, pp. 28–34, 1940.

(1198) OKAYA, TOKIHARU. Numerical integration by Tchebychef's q-functions.
Proc. Phys.-Math. Soc., Japan, III, Vol. 19, pp. 273–82, 1941.

(1199) OSTROWSKI, A. Sur la convergence et l'estimation des erreurs dans quelques procédés de résolution des équations numériques.
Memorial volume dedicated to D. A. Grave (Sbornik posyjaščenii pamjati D. A. Grave), Moscow, pp. 213–34, 1940.

(1200) PÄTAU, K. Eine neue χ^2 Tafel.
Zeitschr. Indukt. Abstamm.-u. Vererbungsl., Vol. 80, pp. 558–64, 1942.

(1201) PEARSON, E. S. The probability integral of the range in samples of n observations from a normal population. I. Foreword and tables.
Biometrika, Vol. XXXII, pp. 301–8, 1942.

(1202) PEATMAN, J. G., and SCHAFER, R. A table of random numbers from Selective Service numbers.
J. Psychol., Vol. 14, pp. 295–305, 1942.

(1203) RAMSEY, E. W. Use of the marginal punch card in tabulating vital statistics data.
Amer. J. Publ. Health, pp. 907–9, 1939.

(1204) REIERSØL, O. A method of recurrent computation of all the principal minors of a determinant, and its application in confluence analysis.
Annals Math. Statistics, Vol. XI, pp. 193–8, 1940.

(1205) SAMUELSON, P. A. A method of determining explicitly the coefficients of the characteristic equation.
Annals Math. Statistics, Vol. XIII, pp. 424–9, 1942.

(1206) SANDOMIRE, M. M. Accumulating cubes with punch cards.
J. Amer. Stat. Ass., Vol. 36, pp. 507–14, 1941.

CC-0. In Public Domain. Gurukul Kangri Collection, Haridwar

Digitized by Arya Samaj Foundation Chennai and eGangotri

(1207) SCHMIDT, R. J. On the numerical solution of linear simultaneous equations by an iterative method.
Phil. Mag., Vol. 32, pp. 369–83, 1941.

(1208) SCHUMANN, T. E. W. A mechanical appliance for the smoothing of time series.
Phil. Mag., Vol. 30, pp. 39–48, 1940.

(1209) SCHUMANN, T. E. W. The principles of a mechanical method for calculating regression equations and multiple correlation coefficients and for the solution of simultaneous linear equations.
Phil. Mag., Vol. 29, pp. 258–73, 1940.

(1210) SCHUMANN, T. E. W., and HOFMEYER, W. L. The problem of auto-correlation of meteorological time-series.
Q. J. Met. Soc., Vol. 68, pp. 177–188, 1942.

(1211) SIMAIKA, J. B. Interpolation for fresh probability levels between the standard table levels of a function.
Biometrika, Vol. XXXII, pp. 263–76, 1942.

(1212) SMITH, H. G. Some practical uses for hyperbolic grids.
J. Amer. Stat. Ass., Vol. 36, pp. 293–8, 1941.

(1213) SPOERL, C. A. Solving equations in the machine age.
Rec. Amer. Inst. Actuar., Vol. 31, pp. 129–49, 490–506, 1942.

(1214) Table of 5-Point Lagrangean Interpolation Coefficients.
Marchant Calculating Machine Company, Oakland, Calif., 1942.

(1215) TAYLOR, W. S. Partialling out sums of squares and products in calculating correlations with non-homogeneous data.
Brit. J. Psych., Vol. XXXII, pp. 318–23, 1942.

(1216) THOMPSON, C. M. Tables of percentage points of the incomplete beta-function. Prefatory note by E. S. Pearson; description of the calculation by L. J. Comrie and H. O. Hartley; methods of interpolation by H. O. Hartley.
Biometrika, Vol. XXXII, pp. 151–81, 1941.

(1217) THOMPSON, CATHERINE M. Table of percentage points of the χ^2 distribution.
Biometrika, Vol. XXXII, pp. 187–91, 1941.

(1218) TUCKER, L. R. A matrix multiplier.
Psychometrika, Vol. 5, pp. 289–94, 1940.

(1219) TUCKERMAN, L. B. On the mathematically significant figures in the solution of simultaneous linear equations.
Annals Math. Statistics, Vol. XII, pp. 307–316, 1941.

(1220) VASILEVSKIS, S. Scheme for the solution of normal equations on the calculating machine.
Acta Univ. Latviensis (3), No. 11, pp. 3–12, 1940 = *Publ. Observ. Astr. Univ. Lettonie*, No. 4, 1940. (English. Latvian summary.)

(1221) VIETORIS, L. Zur Theorie der Integraphen.
Über. Deutsch. Math. Verein, Vol. 52, pp. 71–4, 1942.

(1222) WHERRY, R. J. An extension of the Doolittle method to simple regression problems.
J. educ. Psychol., Vol. 32, pp. 459–64, 1941.

D. MISCELLANEOUS

(1223) BECKENBACH, E. F. Convexity properties of generalized mean value functions.
Annals Math. Statistics, Vol. XIII, pp. 88–90, 1942.

(1224) BLUMENSTOCK, G. Drought in the United States analyzed by means of the theory of probability.
U.S. Dept. Agric. Tech. Bull., Vol. 819, pp. 1–63, 1942.

(1225) BOUCHMAN, E. N. Problems of congestion in telephony.
J. Appl. Math. Mech., Vol. 6, pp. 247–56, 1942.

(1226) BREITINGER, E. Die Berechnung der Streuung zusammengesetzter Gruppen. (The calculation of the dispersion of combined groups.)
Anthrop. Anz., Vol. 16, pp. 107–11, 1939.

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Digitized by Arya Samaj Foundation Chennai and eGangotri

(1227) DOOB, J. L. The Brownian movement and stochastic equations.
Annals of Mathematics, Vol. 43, pp. 351–69, 1942.

(1228) GORDON, S. A sampling technique for the determination of hunters' activities and the economics thereof.
J. Wildlife Management, Vol. 5, pp. 260–78, 1941.

(1229) GUMBEL, E. J. The return period of flood flows.
Annals Math. Statistics, Vol. XII, pp. 163–90, 1941.

(1230) GUMBEL, E. J. Probability-interpretation of the observed return-periods of floods.
Trans. Amer. Geophys. Union, 1941, pp. 836–850, 1941.

(1231) GUMBEL, E. J. On the frequency distribution of extreme values in meteorological data.
Bull. Amer. Meteorol. Soc., Vol. 23, pp. 95–105, 1942.

(1232) GUMBEL, E. J. Statistical control-curves for flood-discharges.
Trans. Amer. Geophys. Union, 1942, pp. 489–509, 1942.

(1233) HALL, F. A. Statistical measurement in group work.
U.S. Child. Bur. Publ., No. 248, 1939.

(1234) HORST, P. Mathematical contributions.
Soc. Sci. Res. Coun. Bull., No. 48, pp. 403–47, 1941.

(1235) KIMBALL, B. F. Limited type of primary probability distribution applied to annual maximum flood flows.
Annals Math. Statistics, Vol. XIII, pp. 318–25, 1942.

(1236) KOSAMBI, D. D. On valid tests of linguistic hypotheses.
New Indian Antiquary, Vol. 5, pp. 21–4, 1942.

(1237) LÉVY, P. Le mouvement brownien plan.
Amer. J. Math., Vol. LXII, pp. 487–550, 1940.

(1238) LORENZ, P. Darstellung statistischer Übersichten mit zwei Eingängen durch orthogonale ganze rationale Funktionen (Flächendarstellung).
Arch. Math. Wirtsch.-Sozialforschg., Vol. 6, pp. 57–70, 1940.

(1239) OCCHIALINI, G., and DE SOUZA SANTOS, M. D. On a method of recording random events.
Ann. Acad. Brasil Sci., Vol. 13, pp. 57–62, 1941.

(1240) POTOČEK, J. La diffusion et la notion de réversibilité de M. Kolmogoroff.
Bull. Int. Acad. Sci. Bohême, Vol. 1939, pp. 1–10, 1939.

(1241) PROUDFOOT, M. J. Sampling with transverse traverse lines.
J. Amer. Stat. Ass., Vol. 37, pp. 265–70, 1942.

(1242) RIEBESELL, P. Einige grundsätzliche Bemerkungen zur Frage des mittleren Fehlers.
Mitteilungen Math. Ges. Hamburg, Vol. 8, pp. 31–3, 1940.

(1243) SIMPSON, G. G., and ROE, A. A standard frequency distribution method.
Amer. Mus. Novitates 1190, pp. 1–19, 1942.

(1244) SMITH, B. B. Note on an alternant suggested by statistical theory.
Edinburgh Math. Notes, No. 32, pp. 19–22, 1941.

(1245) SOLTERER, J. A sequence of historical random events: do Jesuits die in threes?
J. Amer. Stat. Ass., Vol. 36, pp. 477–84, 1941.

(1246) THOM, H. C. S. On statistical analysis of rainfall data.
Trans. Amer. Geophys. Union, 1940, pp. 490–9, 1940.

(1247) VILLE, J. Sur un problème de géométrie suggéré par l'étude du mouvement brownien.
C. R. Acad. Sci. Paris, Vol. 215, pp. 51–2, 1942.

(1248) WALKER, G. Selection of factors in statistical investigations.
Quart. J. Roy. Met. Soc., Vol. 67, pp. 261–2, 1941.

(1249) WIENER, N., and WINTNER, A. Harmonic analysis and ergodic theory.
Amer. J. Math., Vol. LXIII, pp. 415–26, 1941.

(1250) WILLIAMS, C. B. A note on the statistical analysis of sentence length as a criterion of literary style.
Biometrika, Vol. XXXI, pp. 356–61, 1940.

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Dot and Scatter Diagrams made by means of a Typewriter

By Th. Busk, Actuary

The Danish Cancer Registry, Copenhagen

In statistical practice it very often happens that a drawing is needed of a certain frequency distribution either to express the result of the investigation or for the further handling of the data. For this purpose histograms or frequency curves are drawn, but these drawings have the disadvantage that it is very difficult to distinguish the single elements in the drawing. An immediate impression

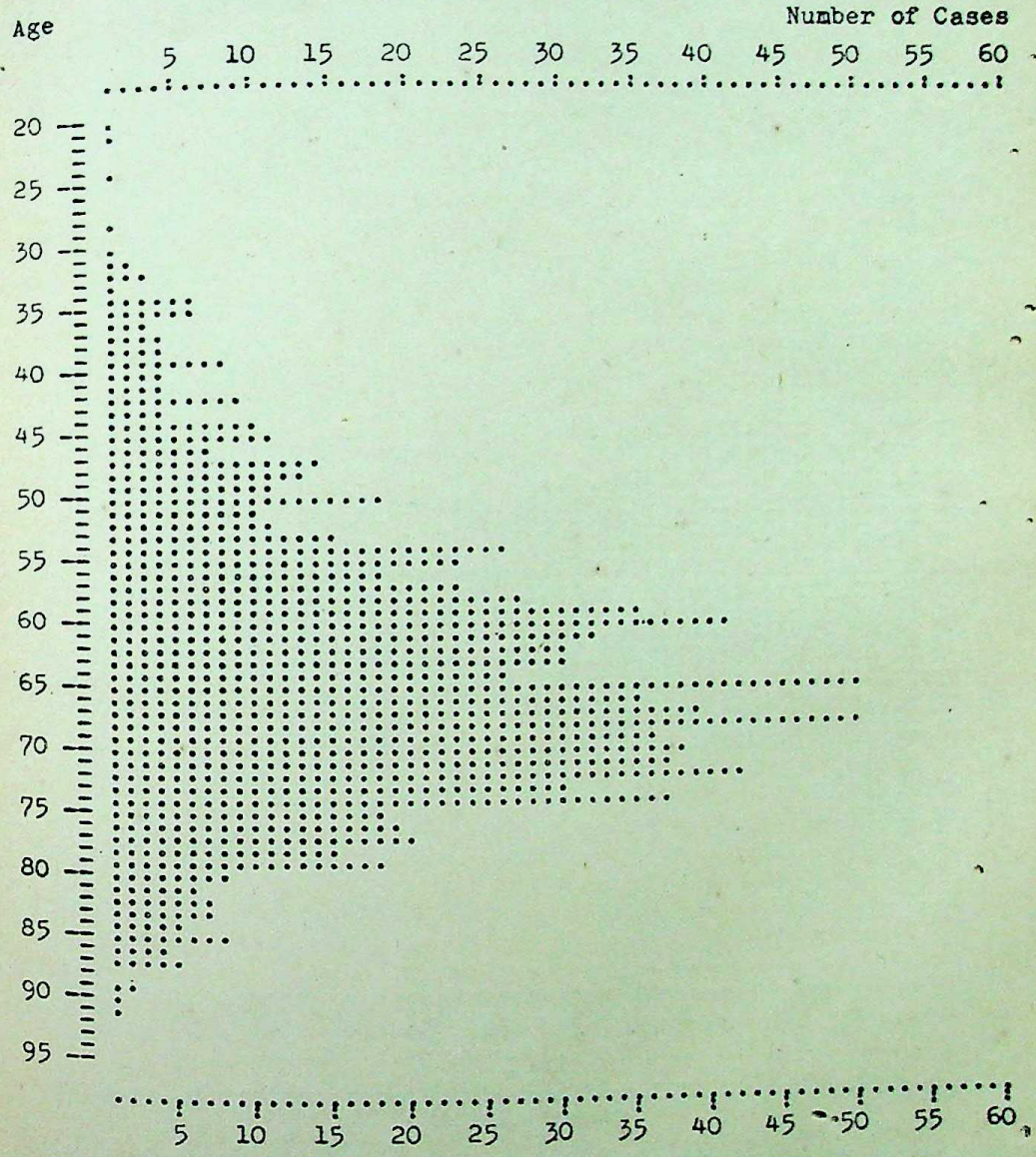

Fig. 1.

Frequency Distribution of 1,032 Cases of Cancer of the Stomach according to the Age at first Treatment in Hospital.

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of the number of elements distributed upon the different groups cannot be obtained by means of histograms or frequency curves, as a reading on the axis of the ordinate is almost always necessary. To avoid this a dot diagram can be drawn, each dot representing one element of the distribution. But when dealing with a more extensive material the drawing of some hundreds or thousands of

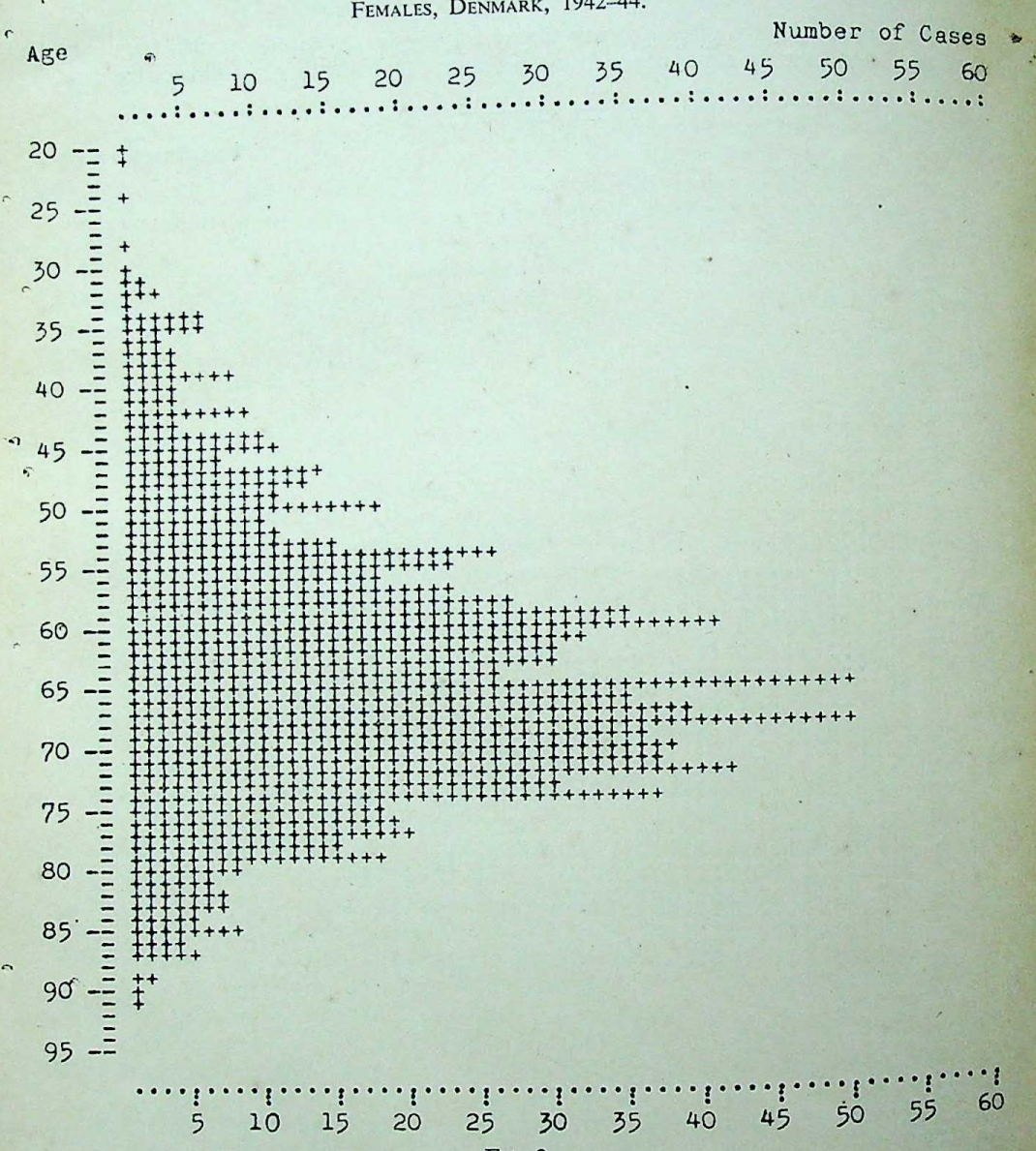

FIG. 2.

Frequency Distribution of 1,032 Cases of Cancer of the Stomach according to the Age at first Treatment in Hospital.

dots is laborious and difficult, when the work is carried out by hand. These difficulties can, however, be overcome by means of a common typewriter (in good condition) as demonstrated in Figs. 1–4, which show four different types.

The scales are made in the following way:

The horizontal scale is marked by full-stops, every fifth (or tenth, etc.) of which is replaced by a colon. The numbers can easily be written above or below each colon by counting the spaces.

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It is practical to fit the scale on the paper to the scale on the slide of the typewriter, in order to facilitate further work.

Contrary to the horizontal scale, the vertical scale is made by means of dashes little by little (in order to avoid moving the lines up and down, which may cause inexactitudes, particularly in the copies), the figures on the scale being designated by an extra dash.

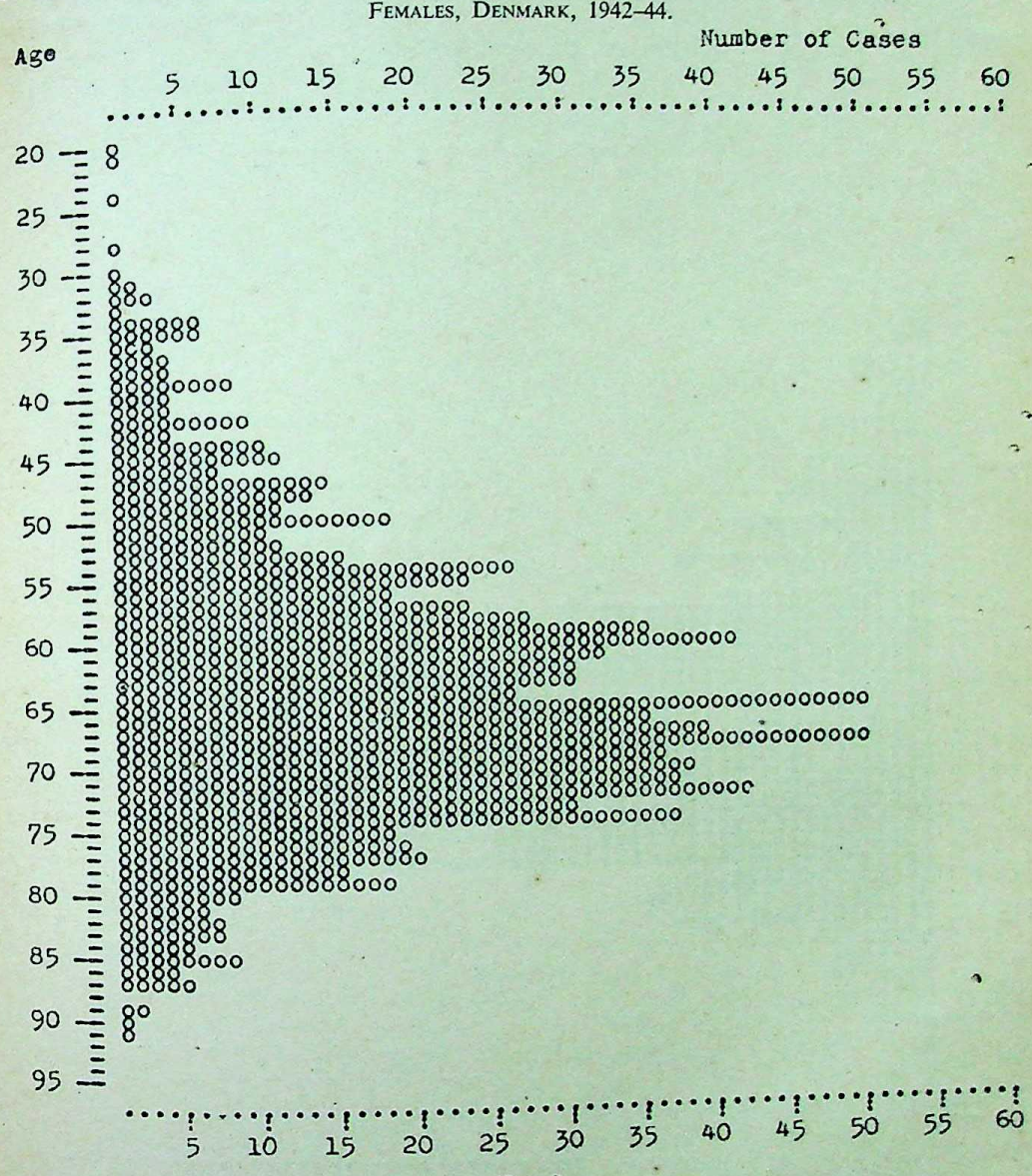

FIG. 3.
Frequency Distribution of 1,032 Cases of Cancer of the Stomach according to the Age at first Treatment in Hospital.

The dot diagram itself is most easily made by using the scale on the slide of the typewriter. It can be made in different ways as the figures show:

Fig. 1 is made by means of full stops.
" 2 " " " plus-signs.
" 3 " " " the letter o.
" 4 " " " " " " filled with some suitable material.

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A material which fulfils all the following claims:

(*a*) it should be easy to fix in the o and
(*b*) easy to remove, and, furthermore,
(*c*) it should not fall out whilst the o is used, and finally
(*d*) it must not affect the colour-tape of the typewriter,

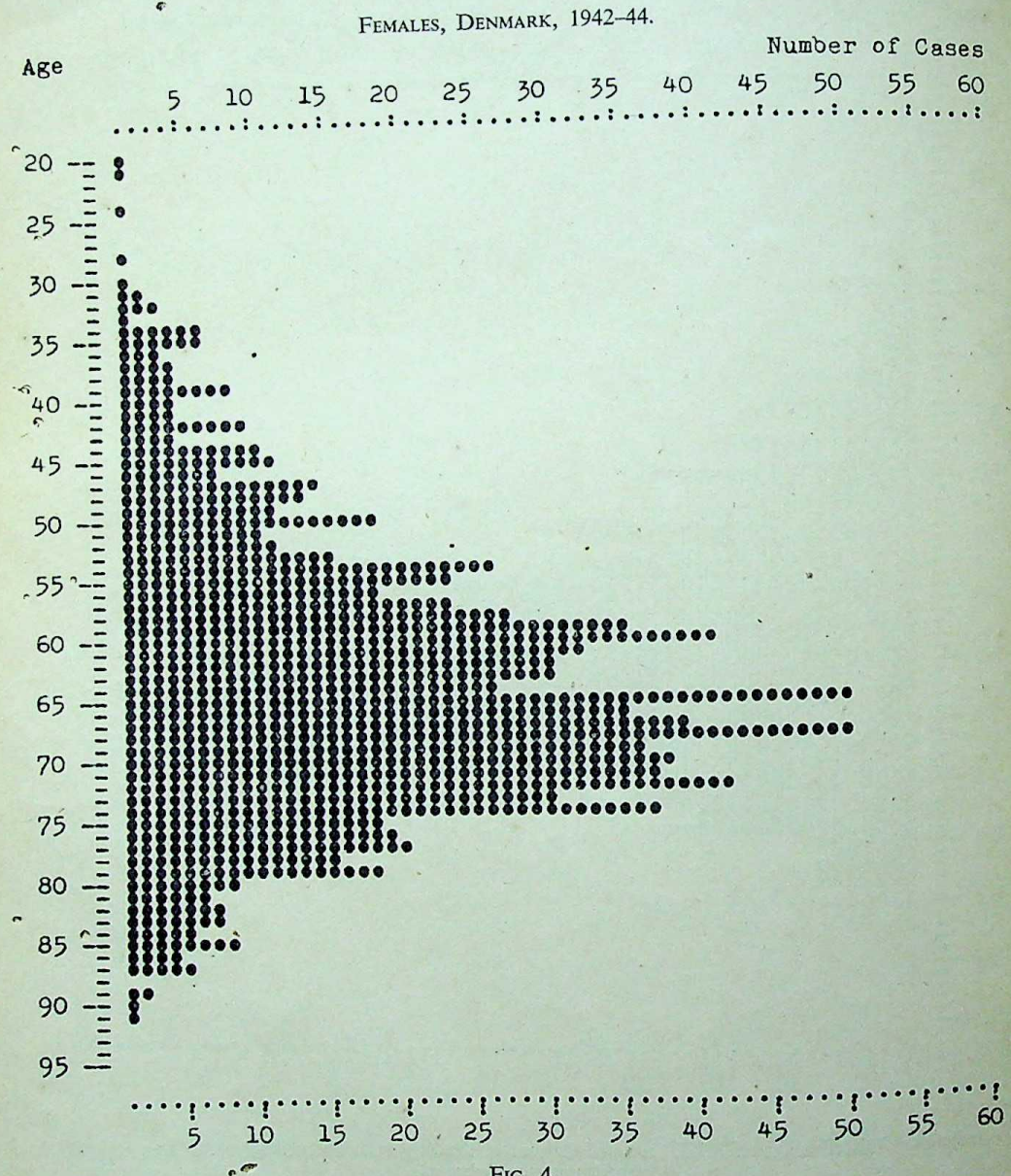

FIG. 4.
Frequency Distribution of 1,032 Cases of Cancer of the Stomach according to the Age at first Treatment in Hospital.

is perhaps not so easy to find. For want of a better material common shaving-soap has been applied for writing Fig. 4.

Fig. 5 shows an example of a dot diagram made by means of different types on the typewriter (the letters o and v and the plus-sign) in order to make it possible to sum different frequency distri-

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butions and still distinguish between these distributions. The choice of types naturally depends on the sort of typewriter. In the examples shown in Figs. 1–5 the choice has been made to suit the chosen distance between the lines of ½. If the diagram is not to be reproduced, the two colours of the tape may also be applied.

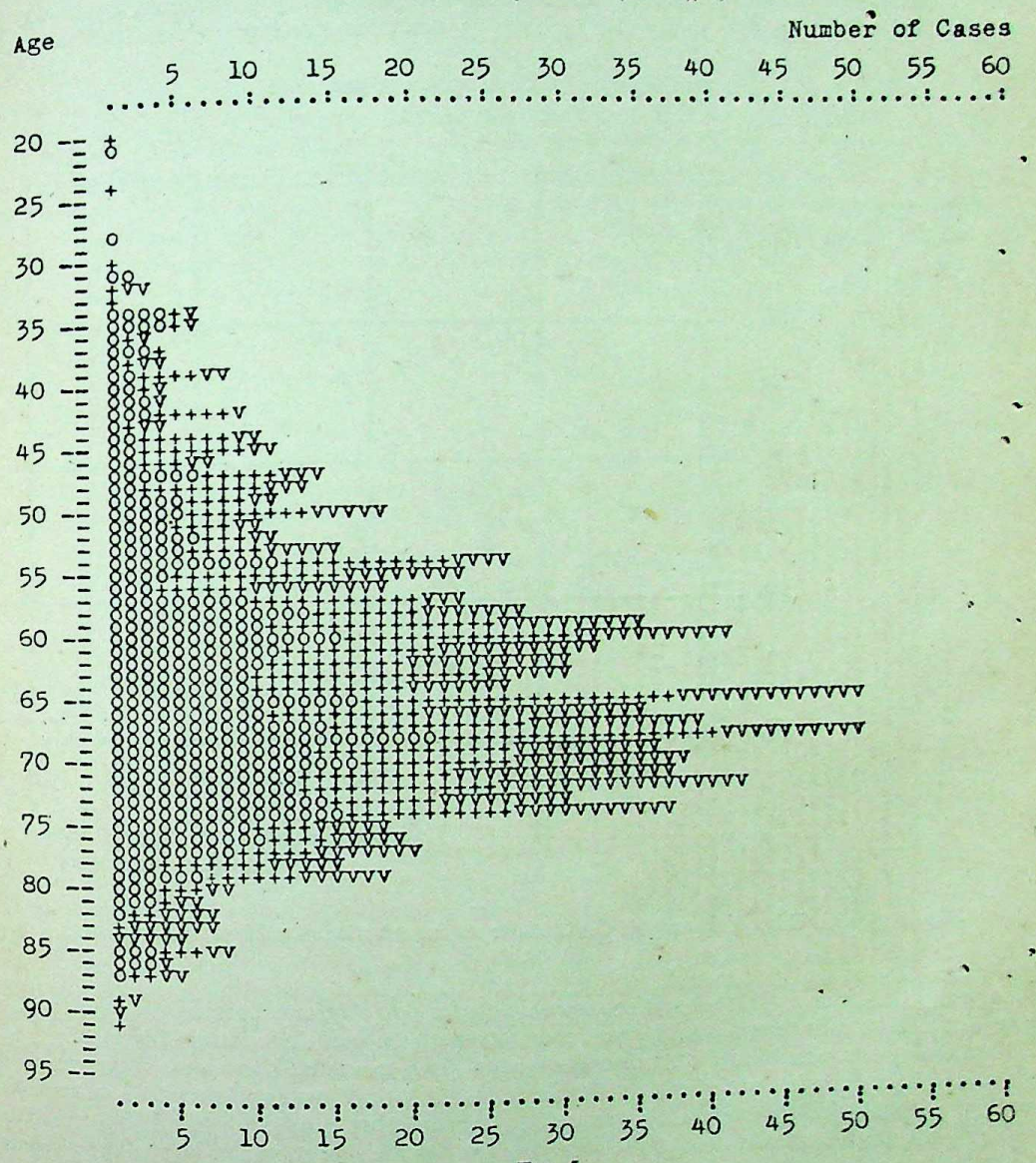

FIG. 5.

Frequency Distribution of 1,032 Cases of Cancer of the Stomach according to the Age at first Treatment in Hospital.

If the number of elements given in the frequency distribution is very great (*e.g.*, several hundreds in each group) an approximate description of the data can be obtained by determining that each dot represents—*e.g.*, two, three, four, five, ten or one hundred elements, and the figures in the original table are then shortened for the purpose.

The typewriter can also be used for drawing scatter diagrams, as shown in Fig. 6, the example

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having been taken from the text-book of G. Udny Yule and M. G. Kendall: *An Introduction to the Theory of Statistics*, 12th ed., p. 205. The horizontal and vertical lines are made by means of dashes and colons respectively, and the scales can be made in the same way as for dot diagrams.

Before making a scatter diagram it is most practical to arrange the values of the variate of the vertical scale, in order of decreasing magnitude, and write the corresponding values of the other variate against these figures. This is done to avoid moving the lines up and down.

If various scatter diagrams have to be compared in the same figure different types may be used—*e.g.*, full-stop, plus-sign and the letters o and v, which latter in many typewriters resembles

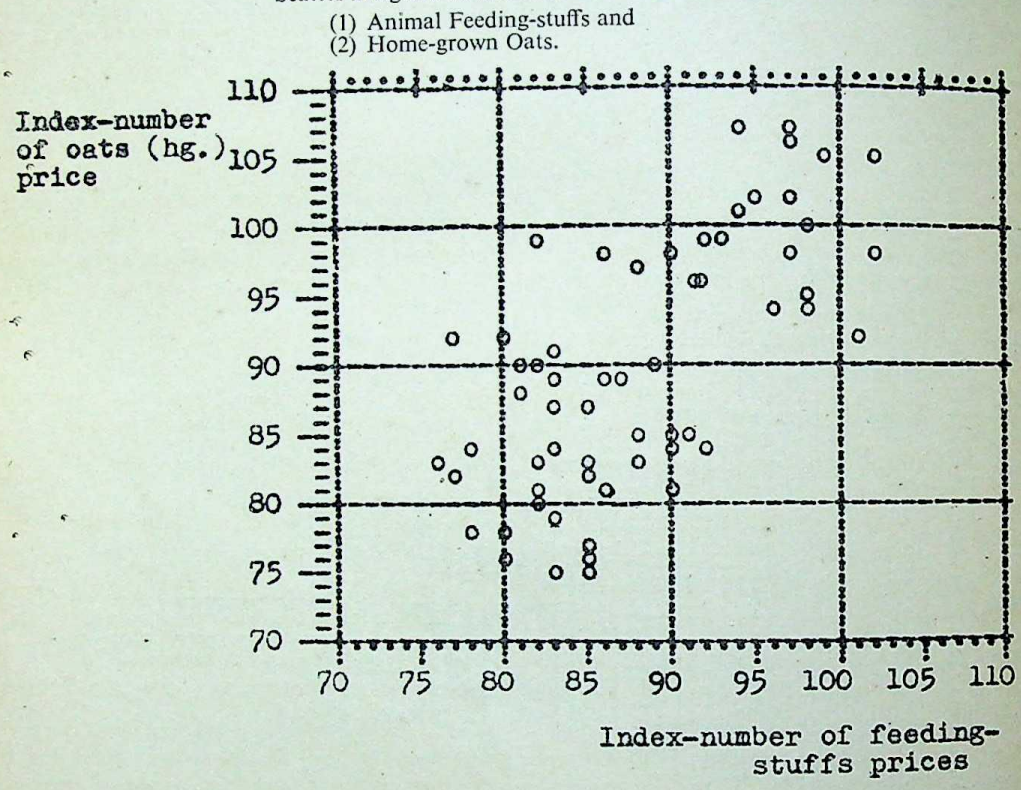

FIG. 6.

Fig. 11.4 in G. Udny Yule and M. G. Kendall: *An Introduction to a Theory of Statistics*, 12th ed., p. 205.

a triangle. The letter x, which is not recommendable for dot diagrams because the result will be more like a histogram than a dot diagram, may, however, be used for scatter diagrams.

Concerning the possibilities for reproduction the given examples show that diagrams made by means of a typewriter are nearly as clear as China-ink drawings, and therefore suitable for printing. In statistical practice it also very often happens that a diagram is not only used for preparing the numerical treatment of the data, but also for the report, which very often is needed in five to ten copies. Therefore, it is convenient that copies can be made on the typewriter with tracing-paper, and, if more copies are needed, a stencil can be produced. If the diagrams are reproduced by photo-lithography the best results will be obtained if tracing-paper is laid on each side of the transparent copy, when this is placed in the typewriter.

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Literature on Testing the Equality of Variances and Covariances in Normal Populations

By R. L. Plackett, M.A.

(Communication from the National Physical Laboratory)

Introduction

This survey represents an attempt to enumerate the tests of various hypotheses concerning equality of variances and covariances in normal populations and to describe the way in which some of these tests have developed over the past twenty years or so. The criteria employed have in most cases been obtained by the likelihood-ratio method introduced by Neyman and Pearson (1928) although in certain instances they have been suggested on intuitive or other considerations. Generally speaking, the difficulties in testing such hypotheses lie not so much in deriving criteria (a straightforward process if the likelihood-ratio principle be adopted), but in finding their exact distributions when the hypotheses are true and determining the best critical region to adopt; occasionally the exact distributions are known in some standard form but it is more usual to obtain a complicated expression or be forced to approximate to the distribution of some function of the criterion (almost always by a Pearsonian Type I curve because the function usually lies between 0 and 1).

The classification adopted is hierarchical: the three main groups are univariate, bivariate and multivariate populations (not mutually exclusive; but notation is thereby simplified); each of these is subdivided into the cases of one, two and k populations (also not mutually exclusive, since k not necessarily greater than two); finally each sub-group consists of hypotheses of three types:

(i) Equality of variances and covariances with no assumptions about means.
(ii) Equality of variances, covariances and means.
(iii) Equality of variances and covariances assuming means known or having interrelations of equality.

This subdivision is very broad since there is no limit to the number of hypotheses which ingenuity can contrive by making assumptions. Evidently two important classes have been omitted:

(iv) Equality of means with no assumptions about variances and covariances.
(v) Equality of means assuming variances and covariances known (or specified respectively equal).

(ii) is obviously common to the problems treated here and those not treated; it is hoped that (iv) and (v) will be the subject of a future survey, but they are excluded here for reasons of space and time. The terms "critical region," "biased," "power," etc., will not be defined; for the theory of testing statistical hypotheses reference should be made elsewhere—*e.g.*, Neyman (1942).

The two papers of Nair (1941) in *Sankhyā* it has not been possible to consult and details of their contents are taken from *Mathematical Reviews*: if our information is correct, the copies of Part 2 of Volume 5 despatched to this country were lost by enemy action. Apologies are due to those authors whose work has been inadvertently omitted and readers are invited to notify us concerning such lacunæ.

Basic notation. It is convenient to define here the following distributions:

(A) The distributionof $\chi^2 = x$ on f degrees of freedom has the probability density function

$$[2^{\frac{1}{2}f}\Gamma(\tfrac{1}{2}f)]^{-1}\, x^{\frac{1}{2}(f-2)} e^{-\frac{1}{2}x} \qquad 0 \leqslant x \leqslant \infty .$$

The cumulative distribution function is tabulated in *Tables of the Incomplete Gamma-function* (edited by Karl Pearson, 1934) but those calculated by Thompson (1941a) will be more useful in the applications here. There are many elsewhere.

(B) The simplest form of Pearson's Type I distribution has the probability density function

$$[B(p,q)]^{-1}\, x^{p-1}(1-x)^{q-1}, \qquad 0 \leqslant x \leqslant 1.$$

The cumulative distribution function, *i.e.*, $\int_0^x$ of the above, is usually denoted by $I_x(p,q)$ and is

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tabulated in *Tables of the Incomplete Beta-function* (edited by Karl Pearson, 1934). Again, tables calculated by Thompson (1941b) will be more useful in applications.

(C) The variance-ratio F with f_1 degrees of freedom in the numerator and f_2 in the denominator, has the probability density function

$$[B(\tfrac{1}{2}f_1,\tfrac{1}{2}f_2)]^{-1}(f_1/f_2)^{\frac{1}{2}f_1}F^{\frac{1}{2}f_1-1}\left(1+\frac{f_1}{f_2}F\right)^{-\frac{1}{2}(f_1+f_2)} \qquad 0 \leqslant F \leqslant \infty.$$

Tables of F and $z = \frac{1}{2}\log_e F$ are widespread. If x is defined by

$$F = \frac{f_2}{f_1}\cdot\frac{x}{1-x}$$

then x has the distribution (B), with $p = \frac{1}{2}f_2$ and $q = \frac{1}{2}f_1$.

From now on the term probability density function will be contracted to p.d.f.; cumulative distribution function to c.d.f.; likelihood-ratio to LR; uniformly most powerful to UMP; and common best critical to CBC. $p(x|H)$ will denote the p.d.f. of the statistic x when hypothesis H is true. Other notation is defined under the main group headings.

Section 1. Univariate Populations

There are k populations of which the t-th ($t = 1,2, \ldots,k$) has mean μ_t, standard deviation σ; from it a sample size n_t is drawn, the values of the observations being x_{ti} ($i = 1,2, \ldots,n_t$).

$$N = \sum_{t=1}^{k} n_t; \quad n_t\bar{x}_t = \sum_{i=1}^{n_t} x_{ti}; \quad N\bar{x} = \sum_{t=1}^{k} n_t\bar{x}_t$$

$$n_t s_t^2 = \sum_{i=1}^{n_t}(x_{ti}-\bar{x}_t)^2; \quad n_t v_t^2 = \sum_{i=1}^{n_t} x_{ti}^2$$

$$Ns_a^2 = \sum_{t=1}^{k} n_t s_t^2; \quad Ns_0^2 = \sum_{t=1}^{k}\sum_{i=1}^{n_t}(x_{ti}-\bar{x})^2.$$

α denotes the significance level adopted and $n_t' = n_t - 1$.

1.1. *One Univariate Population*

This is effectively the same as the case of two univariate populations with an infinite sample size from the second, the general problem with $n_2 < \infty$ being treated in section 1.2. Here $t = 1$ and the appropriate suffix is dropped.

1.1.1. H_1: $\sigma = \sigma_0$ *where* σ_0 *is specified.* μ *is unknown*, i.e., $-\infty < \mu < \infty$.

(*a*) The alternative hypotheses specify $\sigma > \sigma_0$ (or $0 < \sigma < \sigma_0$). As shown by Neyman and Pearson (1933) a CBC region exists with regard to either of these classes and consists of those values of s^2 for which

$$s^2 \geqslant s_u^2 \ (\leqslant s_l^2)$$

where $$\alpha = \int_{s_u^2}^{\infty} p(s^2|H_1)ds^2 \left[= \int_0^{s_l^2} p(s^2|H_1)ds^2\right]$$

and $p(s^2|H_1)$ is obtained by putting $x = ns^2/\sigma_0^2$, $f = n-1$ in distribution (A).

(*b*) The alternative hypotheses specify $0 < \sigma \neq \sigma_0$. From the foregoing no UMP test exists. Letting $n_2 \to \infty$ in the discussion of section 1.2.2. leads to the unbiased test whose critical region is specified by $s^2 \geqslant s_u^2$ and $s^2 \leqslant s_l^2$ where

$$1-\alpha = \int_{s_l^2}^{s_u^2} p(s^2|H_1)ds^2$$

and $$(s_l^2)^{\frac{1}{2}(n-1)}\exp\left(-\tfrac{1}{2}\cdot\frac{ns_l^2}{\sigma_0^2}\right) = (s_u^2)^{\frac{1}{2}(n-1)}\exp\left(-\tfrac{1}{2}\cdot\frac{ns_u^2}{\sigma_0^2}\right).$$

1.1.2. H_2: $\sigma = \sigma_0$, $\mu = \mu_0$, *where* σ_0 *and* μ_0 *are specified.*

A test for this hypothesis has been given by Neyman and Pearson (1928), the LR being

$$\lambda(H_2) = \left(\frac{s}{\sigma_0}\right)^n \exp\left[-\frac{n}{2}\left\{\frac{(\bar{x}-\mu_0)^2+s^2}{\sigma_0^2}-1\right\}\right].$$

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The exact distribution of this criterion is not known but tables and diagrams have been calculated, appearing in the same paper; the former are reproduced as Tables 35 and 36 in Part II of *Tables for Statisticians and Biometricians* (edited by Karl Pearson, 1931). With regard to alternative hypotheses specifying $\sigma \neq \sigma_0$, $\mu \neq \mu_0$, Neyman and Pearson (1933) showed there was no CBC region.

1.1.3. H_3: $\sigma = \sigma_0$ *where* σ_0 *is specified.* μ *is assumed known and may be taken zero without loss of generality.*

(*a*) The alternative hypotheses specify $\sigma > \sigma_0$ (or $0 < \sigma < \sigma_0$). Neyman and Pearson (1933) have shown that a CBC region exists with regard to either of these classes and consists of those values of v^2 for which $v^2 \geqslant v_u^2$ ($\leqslant v_l^2$) where

$$\alpha = \int_{v_u^2}^{\infty} p(v^2|H_3)dv^2 \left[= \int_0^{v_l^2} p(v^2|H_3)dv^2\right]$$

and $p(v^2|H_3)$ is obtained by putting $x = nv^2/\sigma_0^2, f = n$ in distribution (A).

(*b*) The alternative hypotheses specify $0 < \sigma \neq \sigma_0$. Since no UMP test exists, Neyman and Pearson (1936) defined an unbiased critical region of Type A whose power function had a minimum at $\sigma = \sigma_0$, showing it was defined by the equations

$$v^2 \geqslant v_u^2 \quad \text{and} \quad v^2 \leqslant v_l^2$$

$$1 - \alpha = \int_{v_l^2}^{v_u^2} p(v^2|H_3)dv^2$$

$$(v_l^2)^{\frac{1}{2}n} \exp\left(-\tfrac{1}{2} \cdot \frac{nv_l^2}{\sigma_0^2}\right) = (v_u^2)^{\frac{1}{2}n} \exp\left(-\tfrac{1}{2} \cdot \frac{nv_u^2}{\sigma_0^2}\right).$$

Fertig and Proehl (1937) calculated tables useful in determining this region.

1.2. *Two Univariate Populations*

1.2.1. H_4: $\sigma_1 = \sigma_2$. $\quad -\infty < \mu_1, \mu_2 < \infty$.

Fisher (1915) first introduced $z = \frac{1}{2}\log(n_1 n_2' s_1^2/n_1' n_2 s_2^2)$ and in 1924 suggested it as a criterion for testing H_4, giving its distribution. It seems more usual now to consider the statistic $(n_1 n_2' s_1^2/n_1' n_2 s_2^2)$ (named F by Snedecor (1934)). On H_4 the p.d.f. of F is obtained by putting $f_1 = n_1', f_2 = n_2'$ in the distribution (C). For the power function

$$p(F|\sigma_1 = \theta^{\frac{1}{2}}\sigma_2) = p(F/\theta|\sigma_1 = \sigma_2).$$

Pearson and Neyman (1930) derived F by the LR method and later (1933) showed there was a CBC region for the set of alternatives $\sigma_1 > \sigma_2 > 0$ (and for the set $0 < \sigma_1 < \sigma_2$) defined by $F \geqslant F_u$ ($\leqslant F_l$) where

$$\alpha = \int_{F_u}^{\infty} p(F|H_4)dF \left[= \int_0^{F_l} p(F|H_4)dF\right].$$

Consequently no UMP test exists. Cochran (1940) suggested an improvement on Fisher's approximation formulæ for significance levels of z when n_1, n_2 are large (*cf.* Fisher and Yates (1943)). Nair (1941a) (quoted from *Maths. Rev.*, 4, 222 (1943)) compared the power functions of five alternative tests, using the statistic $x = n_1 s_1^2/(n_1 s_1^2 + n_2 s_2^2)$ whose c.d.f. is $I_x(\frac{1}{2}n_1', \frac{1}{2}n_2')$. The first two of these tests were those already considered, using one tail of the distribution, the remainder having critical regions determined by

(iii) $$x \geqslant x_u' \quad \text{and} \quad x \leqslant x_l'$$

where $$I_{x_l'}(\tfrac{1}{2}n_1', \tfrac{1}{2}n_2') = 1 - I_{x_u'}(\tfrac{1}{2}n_1', \tfrac{1}{2}n_2') = \tfrac{1}{2}\alpha;$$

the much-used equal-tail test.

(iv) $$x \geqslant x_u'' \quad \text{and} \quad x \leqslant x_l''$$

where x_u'' and x_l'' are selected to make the test unbiased, the condition of unbiasedness being

$$(x_u'')^{n_1'}(1 - x_u'')^{n_2'} = (x_l'')^{n_1'}(1 - x_l'')^{n_2'}$$

(v) $$x \geqslant x_u''' \quad \text{and} \quad x \leqslant x_l'''$$

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where x_u''' and x_l''' are the limits resulting from the LR test discussed in section 1.3.1. and satisfy

$$(x_u''')^{n_1}(1 - x_u''')^{n_2} = (x_l''')^{n_1}(1 - x_l''')^{n_2}.$$

Nair concluded that all tests except (iv) were biased, with (iii) more satisfactory than the others. Arpian (1941) studied in detail the distribution of z and approximations thereto, while Paulson (1942) gave a function of F whose distribution was nearly normal for $n_2 \geqslant 3$. A survey of the whole problem occurs in Scheffé (1942a), who expresses the conditions on the limits of (iii), (iv) and (v) in terms of F and adds another test whose critical region is defined by

(vi) $$F \geqslant F_u \quad \text{and} \quad F \leqslant F_l$$

where $F_u \,.\, F_l = 1$. If in fact the limits for F are $F_u(n_1', n_2')$ and $F_l(n_1', n_2')$, he points out that all these two-tail tests satisfy $F_u(n_2', n_1').F_l(n_1', n_2') = 1$, a symmetry condition which means that the criterion $(n_1 n_2' s_1^2 / n_1' n_2 s_2^2)$ will lead to the same conclusion as $(n_1' n_2 s_2^2 / n_1 n_2' s_1^2)$, *i.e.*, the allocation of suffices is irrelevant. Test (iv) is regarded as based on logarithmically shortest confidence intervals (*i.e.*, minimizing F_u/F_l): the condition on the limits is the same and in terms of F is

$$(F_u)^{n_1'}\left(1 + \frac{n_1'}{n_2'}F_u\right)^{-(n_1' + n_2')} = (F_l)^{n_1'}\left(1 + \frac{n_1'}{n_2'}F_l\right)^{-(n_1' + n_2')}$$

giving limits which are hard to compute. Scheffé concludes that (iii) suffers little in general from loss of efficiency. In the second half of the paper he shows the critical region of the unbiased test (iv) is of Type (B) (Scheffé (1942b)), providing the shortest unbiased set of confidence intervals.

When $n_1 = n_2$, another test for H_4 has been provided by the approach leading to the test of H_9 in section 2.1.2. when it is assumed known that $\rho = 0$. The F-test appears to be slightly more powerful (see section 2.1.1.).

1.2.2. H_5: $\sigma_1 = \sigma_2$ *and* $\mu_1 = \mu_2$.

Attempts to solve the problem of testing this hypothesis were made by Pearson (1926) and Romanovsky (1927). The latter considered four criteria, three being of the form of Student's t and the fourth being s_1^2/s_2^2; their distributions were obtained. Pearson and Neyman (1930) gave reasons for rejecting these criteria and proposed one based on the LR. The resulting statistic is

$$\lambda(H_5) = \left(\frac{s_1}{s_0}\right)^{n_1}\left(\frac{s_2}{s_0}\right)^{n_2}.$$

The moments of the distribution of $\lambda(H_5)$ when H_5 is true were given and it was shown that as $n_1, n_2 \to \infty$ $\lambda(H_5)$ tends to be distributed in rectangular form. When $n_2 \to \infty$ the criterion of section 1.1.2. and its moments result. They proposed to fit the distribution of $\lambda(H_5)$ by a Type I curve and gave numerical evidence for the validity of this approximation. Table 37 in Part II of *Tables for Statisticians and Biometricians* is useful for applications and Mahalanobis (1933) has also given tables. Sukhatme (1935) derived the sampling distribution of $L(H_5) = [\lambda(H_5)]^{2/N}$ in terms of incomplete beta-functions and showed that when $n_1 = n_2$ the p.d.f. could be expressed in terms of elliptic functions of the first order. Using a Type I fit to $L(H_5)$, tables of the 5 per cent. and 1 per cent. levels were calculated; these are very nearly $(\cdot 05)^{1/m_1}$ and $(\cdot 01)^{1/m_2}$ respectively, where m_1 and m_2 are the parameters of the curve calculated from the moments of the L-distribution. Approximations depending only on N were obtained for large sample sizes suggesting that here n_1 and n_2 can both be replaced by their mean without much effect on the L-distribution. Applicability to non-normal data was discussed, leading him to conclude that the test could safely be applied except with extreme non-normality.

1.3. k *Univariate Populations*

1.3.1. H_6: $\sigma_1 = \sigma_2 = \ldots = \sigma_k$. $\quad -\infty < \mu_t < \infty$.

Neyman and Pearson (1931) gave the LR criterion:

$$\lambda(H_6) = \prod_{t=1}^{k}\left(\frac{s_t^2}{s_a^2}\right)^{n_t/2}.$$

Hence $L(H_6) = [\lambda(H_6)]^{2/N}$ is the ratio of the weighted geometric mean of the sample variances to their weighted arithmetic mean; it is usually denoted by L_1, a precedent which will be followed in this section. The moments of the $\lambda(H_6)$-distribution when H_6 is true were derived and it was proved that $-2\log[\lambda(H_6)] = -N\log L_1$ is for large n_t distributed approximately in the form (A)

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with $f = k - 1$. The moments of L_1 were also given and it was proposed that the L_1-distribution be fitted by a Type I curve, some numerical evidence being provided in substantiation. Mahalanobis (1933) used the Type I approximation to provide tables of the 5 per cent. and 1 per cent. levels of L_1, obtaining these via the tabulated levels of z. Welch (1935) showed that a criterion of the L_1 form could be employed to test the equality of k residual variances, each being about the regression line of one variate on another. Moments were derived and a Type I fit used. Nayer (1936; written 1934) pointed out that Mahalanobis had not investigated the adequacy of the Type I approximation; also that the argument intervals in his tables were not always convenient for interpolation. After detailed examination, Nayer concluded that the approximation was satisfactory; concentrating attention on the case $n_t = n$, the exact distribution known for $k = 2$ and the limiting χ^2 form were compared with the approximate method (being in good agreement) and he gave tables of 5 per cent. and 1 per cent. levels, entered with n and k. For n_t not all the same, moderately large and nearly equal, he gave evidence for believing his tables could be entered with the mean of the n_t. Welch (1936) extended his 1935 results to the regressions of one variate on any number of others, and put forward a criterion obtained by replacing n_t by arbitrary weights w_t in the expression for L_1, deriving its moments. Bartlett (1937) approached the problem afresh from considerations of sufficiency. In his view exact tests of significance concerning the sample variances T must be based on the conditional variation $T|U$ where U is a sufficient statistic for the unknown common value of σ_t^2. Thus if L denotes the logarithm of the likelihood, the following property of sufficient statistics:

$$L(T|U) = L(T) - L(U)$$

leads to the criterion

$$M = f_0 \log (\Sigma f_t S_t^2/f_0) - \Sigma f_t \log S_t^2$$

where S_t^2 is an unbiased estimate of σ_t^2 based on f_t degrees of freedom and $f_0 = \Sigma f_t$. Thus in our (special) case of k samples from univariate populations, $f_t = n'$ and $n'S_t^2 = n_t s_t^2$, *i.e.*, $f_0 = N - k$. Due to the limit property of the likelihood, M in large samples is distributed nearly in the form (A) with $f = k - 1$; Bartlett also obtained a correcting factor for smaller samples, showing that M/K where

$$K = 1 + \frac{1}{3(k-1)}\left[\sum_{t=1}^{k}\left(\frac{1}{f_t}\right) - \frac{1}{f_0}\right]$$

follows the same distribution. In very small samples the use of K tends to over-correct the otherwise exaggerated significance levels. Wilks and Thompson (1937) gave the first two moments of the distribution of L_1 when the σ_t were not all equal and proposed to approximate to the distributions by Type III curves. In some particular examples these gave excellent agreement with empirical distributions. Nair (1939) expressed the exact distribution of L_1 as a complicated infinite series when $n_t = n$. Calculations made on the basis of this were reproduced in Bishop and Nair (1939): these showed the probability levels in the tables of Nayer (1936) were in excellent agreement with the true values. Bishop and Nair also discussed the relation between L_1 and the criteria of Welch (1936) and Bartlett (1937), remarking that the latter two were equivalent if $w_t = f_t$ and that L_1 and M were equivalent when $n_t = n$ and $f_t = f$. They concluded that Bartlett's approximation to the probability levels of his criterion would give excellent results, provided the individual f_t were not too small, and in several practical cases their calculations led them to suppose that L_1 could be badly biased, especially when the differences between n_t and f_t were large; they recommended Bartlett's test in preference. Brown (1939) proved that if $n = n$ the L_1 test was unbiased; that it was in general biased; and that if $n_t = a_t m$ where a_t are fixed and $m \to \infty$, the significance level remaining constant, the power of the test with the respect to alternative hypotheses tended to unity (*i.e.*, the test is "asymptotically unbiased"). The power function was computed in various cases when $k = 2$. Here L_1 is equivalent to F, being

$$\frac{N \cdot (n_1' n_2 F/n_1 n_2')^{n_1'/N}}{n_2\left(1 + \frac{n_1'}{n_2'}F\right)}$$

and the use of L_1 implies a certain choice of critical region for F (see section 1.2.1.). Mood (1939), by consideration of the semi-invariants of $-2 \log [\lambda(H_0)]$ when $n_t = n$, showed that the degree of approximation of this quantity to the distribution (A) with $f = k - 1$ was mainly dependent on n and for practical purposes independent of k for moderate n. Pitman (1939a) defined a general

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type of unbiased test: on application to testing H_6 Bartlett's 1937 criterion resulted and was in this elegant manner shown to be unbiased. Independently of Brown (1939) he showed the L_1 test to be unbiased only when $n_t = n$ and transformed its distribution into a multiple integral which could be evaluated in certain instances by reduction to elliptic integrals. Hartley (1940), using the moments of Bartlett's criterion obtained from the work of Welch (1936), applied Mellin's inversion formula to derive its distribution, showing there was an approximation of sufficient accuracy depending only on k, $c_1 = \Sigma\left(\frac{1}{f_t}\right) - \frac{1}{f_0}$ and $c_3 = \Sigma\left(\frac{1}{f_t}\right)^3 - \left(\frac{1}{f_0}\right)^3$. Neyman (1941) considered the problem when $n_t = n$ from the point of view of power. He showed there was no UMP test for testing H_6 against all simple alternatives of a particular type he considered appropriate, but by further assumptions about the alternatives, an unbiased test was obtained, and its use indicated for large k. Thompson and Merrington (1946), using the work of Hartley (1940), gave tables of M, entered with n, k and c_1; there are two values of M, corresponding to the approximate limits of c_3.

1.3.2. H_7: $\sigma_1 = \sigma_2 = \ldots = \sigma_k$; $\mu_1 = \mu_2 = \ldots = \mu_k$.

Neyman and Pearson (1931) gave the LR criterion:

$$\lambda(H_7) = \prod_{t=1}^{k} \left(\frac{s_t^2}{s_0^2}\right)^{n_t/2}$$

Hence $L(H_7) = [\lambda(H_7)]^{2/N}$ is the ratio of the weighted geometric mean of the sample variances to the variance obtained by combining all the k samples together into one sample size N. The moments of $\lambda(H_7)$ and $L(H_7)$ were given, and it was proved that $-2\log[\lambda(H_7)] = -N\log[L(H_7)]$ was for large n_t distributed approximately in the form (A) with $f = 2k - 2$. They proposed to approximate to the distribution of $L(H_7)$ by a Type I curve, reasons being given, and hence significance levels could be got from tables of z. Mahalanobis (1933) provided significance levels by this method. Welch (1935) tested hypotheses concerning the slopes, positions and residual variances of k regression lines by LR methods, obtaining similar criteria. Simpson (1943) proposed to adapt Student's t to test whether k samples were random and drawn from the same normal population, although the use of t for $k = 2$ was discarded by Pearson and Neyman in 1930, strong reasons for this being given by Hsu (1938). Kempthorne (1944) pointed out that Simpson's criterion does not have the distribution stated in his paper.

Section 2. Bivariate Populations

This section deals with papers concerned specifically with bivariate populations and refers to other sections for p-variate problems applied to the special case $p = 2$. There are k bivariate populations of which the t-th ($t = 1,2, \ldots, k$) has means μ_{xt}, μ_{yt}, standard deviations σ_{xt}, σ_{yt}, correlation coefficient ρ; from it a sample size n_t is drawn, the values of the observations being x_{ti}, y_{ti} ($i = 1, 2, \ldots, n_t$).

$$N = \sum_{t=1}^{k} n_t; \quad n_t\bar{x}_t = \sum_{i=1}^{n_t} x_{ti}; \quad N\bar{x} = \sum_{t=1}^{k} n_t\bar{x}$$

$$n_t s_{xt}^2 = \sum_{i=1}^{n_t} (x_{ti} - \bar{x}_t)^2; \quad \text{similarly for } y.$$

$$N s_{xa}^2 = \sum_{t=1}^{k} n_t s_{xt}^2; \quad N s_{x_0}^2 = \sum_{t=1}^{k}\sum_{i=1}^{n_t} (x_{ti} - \bar{x})^2$$

$$n_t s_{xt} s_{yt} r_t = \sum_{i=1}^{n_t} (x_{ti} - \bar{x}_t)(y_{ti} - \bar{y}_t); \quad |v_{ijt}| = s_{xt}^2 s_{yt}^2 (1 - r_t^2)$$

$$N s_{xa} s_{ya} r_a = \sum_{t=1}^{k} n_t s_{xt} s_{yt} r_t; \quad |v_{ija}| = s_{xa}^2 s_{ya}^2 (1 - r_a^2)$$

$$N s_{x_0} s_{y_0} r_0 = \sum_{t=1}^{k}\sum_{i=1}^{n_t} (x_{ti} - \bar{x})(y_{ti} - y); \quad |v_{ij_0}| = s_{x_0}^2 s_{y_0}^2 (1 - r_0^2)$$

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2.1. *One Bivariate Population*

$t = 1$ and the appropriate suffix is dropped.

2.1.1. H_8: $\sigma_x = \sigma_y$. $-1 \leqslant \rho \leqslant 1$; $-\infty < \mu_x, \mu_y < \infty$.

Finney (1938) adapted his test for H_9 (section 2.1.2.) using a method suggested by Hirschfeld (and given in section 2.2.4), thus attaining an approximate test of H_8. Morgan (1939) gave the LR criterion

$$\lambda(H_8) = \left[\frac{4s_x^2 s_y^2(1-r^2)}{(s_x^2+s_y^2)^2 - 4r^2 s_x^2 s_y^2}\right]^{n/2}$$

showing that the c.d.f. of $w = [\lambda(H_8)]^{2/n}$ was $I_w\left(\frac{n-2}{2}, \frac{1}{2}\right)$. Morgan tabulated the power of this test in various particular cases, showing that when ρ is known zero the ordinary F-test is slightly better. With ρ known, he compared it with Finney's criterion for H_9, finding it better for σ_x/σ_y near 1, less sensitive elsewhere; in general ρ is unknown and his test is preferable. Pitman (1939b) independently and by a different method arrived at Morgan's criterion.

2.1.2. H_9: $\sigma_x = \sigma_y$. *It is known* $\rho = \rho_0$. $-\infty < \mu_x, \mu_y < \infty$.

Bose (1935) derived the distribution of the ratio $w = s_x/s_y$ by summing an infinite series. The compact form of the solution led Finney (1938) to give a more direct derivation, and he proceeded to use w to test H_9: choosing x and y so that $w \geqslant 1$ the probability of exceeding the sample value is $I_u\left(\frac{n-1}{2}, \frac{n-1}{2}\right)$ where

$$u = \tfrac{1}{2}\left[1 - \frac{w - w^{-1}}{\sqrt{(w+w^{-1})^2 - 4\rho_0^2}}\right]$$

Nair (1941b) has also considered this problem; our information does not, however, go beyond this fact.

2.1.3. H_{10}: $\sigma_x = \sigma_y$ *and* $\mu_x = \mu_y$. $-1 \leqslant \rho \leqslant 1$.

Hsu (1940) gave the LR criterion:

$$[\lambda(H_{10})]^{2/n} = L(H_{10}) = \frac{4s_x^2 s_y^2(1-r^2)}{\{s_x^2 + s_y^2 + \frac{1}{2}(\bar{x}-\bar{y})^2\}(1-t^2)}$$

where

$$t = \frac{2rs_x s_y - \frac{1}{2}(\bar{x}-\bar{y})^2}{s_x^2 + s_y^2 + \frac{1}{2}(\bar{x}-\bar{y})^2}$$

showing that the p.d.f. was $\frac{1}{2}(n-2)L^{\frac{1}{2}(n-4)}$ $(0 \leqslant L \leqslant 1)$ and providing tables.

2.1.4. H_{11}: $\sigma_x = \sigma_y$ *and* $\rho = \rho_0$. $-\infty < \mu_x, \mu_y < \infty$.

When $\rho_0 = 0$ see section 3.1.1. Hsu (1940) gave the LR criterion for ρ_0 not necessarily zero.

$$[\lambda(H_{11})]^{2/n} = L(H_{11}) = \frac{4s_x^2 s_y^2(1-\rho_0^2)(1-r^2)}{(s_x^2+s_y^2)(1-\rho_0 u)^2}$$

where

$$u = \frac{2rs_x s_y}{s_x^2 + s_y^2}$$

showing that the p.d.f. was $\frac{1}{2}(n-2)L^{\frac{1}{2}(n-4)}$ $(0 \leqslant L \leqslant 1)$ and providing tables. Villars and Anderson (1943) gave tables of an equivalent criterion.

2.1.5. H_{12}: σ_x, σ_y, ρ, μ_x, μ_y *all have specified values.*

See section 3.1.2.

2.1.6. H_{13}: $\rho = \rho_0$. $-\infty < \mu_x, \mu_y < \infty$; $0 < \sigma_x, \sigma_y < \infty$.

The distribution of r when $\rho_0 = 0$ was given by "Student" (1908) and for general ρ_0 by Fisher (1915). Tables have been calculated by David (1938) where a complete survey of the literature may be found.

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2.1.7. H_{14}: $\rho = \rho_0$. *It is assumed* $\sigma_x = \sigma_y$. $-\infty < \mu_x, \mu_y < \infty$.

De Lury (1938) put forward the criterion

$$u = \frac{2rs_xs_y}{s_x^2 + s_y^2}$$

deriving its distribution. When $\rho_0 = 0$ the distribution of u is identical with that of r for a sample size $n + 1$. A transformation to a nearly normal deviate $\tan h^{-1}u$ was discussed, being analogous to the transformation on r. Pitman (1939b) showed that if

$$w = \frac{1 - \rho_0}{1 + \rho_0} \cdot \frac{1 + u}{1 - u}$$

then $v = w/(1 + w)$ has the c.d.f. $I_v\left(\frac{n-1}{2}, \frac{n-1}{2}\right)$. Hsu (1940) gave the LR criterion equivalent to Pitman's. Nair (1941b) has discussed this problem and Villars and Anderson (1943) gave a more complicated criterion when $\rho_0 = 0$, providing tables.

2.1.8. H_{15}: $\rho = \rho_0$. *It is assumed* $\mu_x = \mu_y$, $\sigma_x = \sigma_y$.

Pitman (1939b) showed that if

$$w = \frac{1 - \rho_0}{1 + \rho_0} \cdot \frac{1 + t}{1 - t}$$

where

$$t = \frac{2rs_xs_y - \frac{1}{2}(\bar{x} - \bar{y})^2}{s_x^2 + s_y^2 + \frac{1}{2}(\bar{x} - \bar{y})^2}$$

then $v = w/(1 + w)$ has the c.d.f. $I_v\left(\frac{n-1}{2}, \frac{n}{2}\right)$. Hsu (1940) gave the LR criterion, which is equivalent, and verified that the tests for $\rho = \rho_0$ increased in power as more assumptions were made. The gain in sensitivity is, however, small.

2.2. *Two Bivariate Populations*

2.2.1. H_{16}: $\sigma_{x_1} = \sigma_{x_2}, \sigma_{y_1} = \sigma_{y_2}, \rho_1 = \rho_2$. $-\infty < \mu_{xt}, \mu_{yt} < \infty$.

See section 2.3.1.

2.2.2. H_{17}: $\sigma_{x_1} = \sigma_{x_2}, \sigma_{y_1} = \sigma_{y_2}, \rho_1 = \rho_2, \mu_{x_1} = \mu_{x_2}, \mu_{y_1} = \mu_{y_2}$.

See section 2.3.2.

2.2.3. H_{18}: $\rho_1 = \rho_2$. $-\infty < \mu_{xt}, \mu_{yt} < \infty$; $0 < \sigma_{xt}, \sigma_{yt} < \infty$.

Fisher (1921) discussed the transformation to a nearly normal quantity $\tan h^{-1}r_i$ which can be used in this connection (see R. A. Fisher (1944) § 35). Brandner (1933) gave the LR criterion, showing that when $n_1 = n_2 = n$

$$[\lambda(H_{18})]^{1/2n} = \sec h\tfrac{1}{2}(\tan h^{-1}r_1 - \tan h^{-1}r_2)$$

so that $\tan h^{-1}r_1 - \tan h^{-1}r_2$, which is Fisher's criterion, can be used. Otherwise the LR is complicated, but on making some approximations Fisher's criterion again results. Bartlett (1937) remarked that since no function of r_1 and r_2 was a sufficient statistic for the unknown common value of ρ_1 and ρ_2, he could not apply the method he used to modify the L_1 criterion (section 1.3.1.).

2.2.4. H_{19}: $\rho_1 = \rho_2$. *It is assumed* $\sigma_{x_1} = \sigma_{x_2}, \sigma_{y_1} = \sigma_{y_2}$. $-\infty < \mu_{xt}, \mu_{yt} < \infty$.

Hirschfeld (1937) tested H_{19} by determining the distribution of $r_1s_{x_1}s_{y_1}/r_2s_{x_2}s_{y_2}$. The c.d.f. of the ratio is a complicated expression involving incomplete beta-functions and the common value ρ of ρ_1 and ρ_2. Since ρ is unknown there are three types of result:

(*a*) If for all values of ρ the test gives a significant result, then the ratio is certainly significant.

(*b*) If for all ρ the test gives a non-significant result, then the ratio is non-significant.

(*c*) If for an interval I_1 of the values of ρ the test gives a significant result, and for I_2 non-significant, he suggested testing the significance of the best estimate of ρ from the nearest value in I_2. The test is then significant or not according as this deviation is significant or not. (If the estimate lies in I_2 then the result is non-significant.)

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2.2.5. H_{20}: $\rho_1 = \rho_2$. *It is assumed* $\sigma_{x_1} = \sigma_{y_1}$, $\sigma_{x_2} = \sigma_{y_2}$. $-\infty < \mu_{xt}, \mu_{yt} < \infty$.

Hsu (1941) gave the LR criterion and showed that when $n_1 = n_2$ the p.d.f. of an equivalent criterion could be expressed in terms of a hypergeometric function.

2.2.6. H_{21}: $\rho_1 = \rho_2$. *It is assumed* $\mu_{xt} = \mu_{yt}$, $\sigma_{xt} = \sigma_{yt}$.

Hsu (1941) gave the LR criterion, showing an equivalent criterion was

$$\frac{(s_{x_1}^2 + s_{y_1}^2 + 2r_1 s_{x_1} s_{y_1})(s_{x_2}^2 + s_{y_2}^2 - 2r_2 s_{x_2} s_{y_2} + [\bar{x}_2 - \bar{y}_2]^2)}{(s_{x_2}^2 + s_{y_2}^2 + 2r_2 s_{x_2} s_{y_2})(s_{x_1}^2 + s_{y_1}^2 - 2r_1 s_{x_1} s_{y_1} + [\bar{x}_1 - \bar{y}_1]^2)}$$

which, with $n_1 = n_2 = n$, is distributed in the form (C) with $f_1 = f_2 = 2(n-1)$.

2.3. k *Bivariate Populations*

2.3.1. H_{22}: $\sigma_{xt} = \sigma_x$, $\sigma_{yt} = \sigma_y$, $\rho_t = \rho$. $-\infty < \mu_{xt}, \mu_{yt} < \infty$.

Pearson and Wilks (1933) gave the LR criterion, a special case of that of section 3.3.1. This is

$$\lambda(H_{22}) = \prod_{t=1}^{k} (|v_{ijt}|/|v_{ijo}|)^{n_t/2}.$$

The moments were given and the exact distribution for $k = 2$, $n_t = n$. In general they suggested fitting a Type I curve to $L(H_{22}) = [\lambda(H_{22})]^{1/N}$. Nair (1939) found the exact distribution for $n_t = n$ and all k in the form of an involved infinite series.

2.3.2. H_{23}: $\sigma_{xt} = \sigma_x$, $\sigma_{yt} = \sigma_y$, $\rho_t = \rho$, $\mu_{xt} = \mu_x$, $\mu_{yt} = \mu_y$.

Pearson and Wilks (1933) gave the LR criterion

$$\lambda(H_{23}) = \prod_{t=1}^{k} (|v_{ijt}|/|v_{ijo}|)^{n_t/2}$$

finding the exact distribution for $k = 2$, $n_t = n$. Otherwise a Type I fit to $L(H_{23}) = [\lambda(H_{23})]^{1/N}$ was suggested.

Section 3. *p*-Variate Populations

There are k p-variate populations of which the t-th ($t = 1,2, \ldots, k$) has means μ_{qt} ($q = 1, 2, \ldots, p$), standard deviations σ_{qt}, correlation coefficients ρ_{qrt} ($q,r = 1,2, \ldots, p$); from it a sample size n_t is drawn, the values of the observations being x_{qti} ($i = 1,2, \ldots, n_t$).

$$N = \sum_{t=1}^{k} n_t; \quad n_t \bar{x}_{qt} = \sum_{i=1}^{n_t} x_{qti}; \quad N\bar{x}_q = \sum_{t=1}^{k} n_t \bar{x}_{qt}$$

$$n_t v_{qrt} = \sum_{i=1}^{n_t} (x_{qti} - \bar{x}_{qt})(x_{rti} - \bar{x}_{rt})$$

$$Nv_{qra} = \sum_{t=1}^{k} n_t v_{qrt}; \quad Nv_{qr_o} = \sum_{t=1}^{k} \sum_{i=1}^{n_t} (x_{qti} - \bar{x}_q)(x_{rti} - \bar{x}_r).$$

$v_{qrt}|$ is the determinant of the v_{qrt} when t is fixed ($q,r = 1,2, \ldots, p$). Similarly $|v_{qra}|$ and $|v_{qr_o}|$ are determinants of v_{qra} and v_{qr_o} respectively ($q,r = 1,2, \ldots, p$).

3.1. *One* p-*Variate Population*

$t = 1$ and the appropriate suffix is dropped.

3.1.1. H_{24}: $\sigma_q = \sigma$, $\rho_{qr} = 0$. $-\infty < \mu_q < \infty$.

Mauchly (1940) gave the LR criterion:

$$\lambda(H_{24}) = \left[\frac{|v_{qr}|}{(v_0)^p}\right]^{n/2}$$

where $p \cdot v_0 = \sum_{q=1}^{p} v_{qq}$. With $L(H_{24}) = [\lambda(H_{24})]^{1/n}$ the p.d.f. of L when $p = 2$ is $(n-2)L^{n-3}$ ($0 \leqslant L \leqslant 1$). For $p = 3$ a Type I curve was fitted and from results of Wilks (1938) it appeared that $-2n \log [L(H_{24})]$ is for large n distributed in the form (A) with $f = \frac{1}{2}p(p-1)$.

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3.1.2. H_{25}: σ_q, ρ_{qr}, μ_q *all have specified values.*

Fertig and Leary (1936) gave the LR criterion and found the moments using Wilks' (1932) results. Type I curves were fitted and significance levels calculated for $p = 1, 2, 3, 4$ and a range of n.

3.1.3. H_{26}: $\sigma_q = \sigma$, $\rho_{qr} = \rho$. $\quad -\infty < \mu_q < \infty$.

Wilks (1946) gave the LR criterion. With

$$p.v = \sum_{q=1}^{p} v_{qq} \quad \text{and} \quad t = \Big(\sum_{q \neq r=1}^{p} v_{qr}\Big)/(p-1)\sum_{q=1}^{p} v_{qq}$$

$$\lambda(H_{26}) = \frac{|v_{qr}|}{(v)^p(1-t)^{p-1}(1+\overline{p-1}t)}.$$

The exact (Type I) distribution of this criterion was obtained for $p = 2$ and 3; in other cases results of Tukey and Wilks (1946) were used to calculate approximate significance levels. As $n \to \infty$ the distribution of $-n \log [\lambda(H_{26})]$ tends to the form (A) with $f = \frac{1}{2}p(p+1) - 2$.

3.1.4. H_{27}: $\sigma_q = \sigma$, $\rho_{qr} = \rho$, $\mu_q = \mu$.

Wilks (1946) gave the LR criterion. With

$$n.p.\bar{x} = \sum_{q=1}^{p}\sum_{i=1}^{n} x_{qi}; \quad n.s_{qr} = \sum_{i=1}^{n}(x_{qi}-\bar{x})(x_{ri}-\bar{x})$$

$$p.v_0 = \sum_{q=1}^{p} s_{qq}; \quad t_0 = \Big(\sum_{q \neq r=1}^{p} s_{qr}\Big)/(p-1)\sum_{q=1}^{p} s_{qq}$$

$$\lambda(H_{27}) = \frac{|v_{qr}|}{(v_0)^p(1-t_0)^{p-1}(1+\overline{p-1}t_0)}.$$

The exact (Type I) distribution of this criterion was obtained for $p = 2$ and 3; in other cases results of Tukey and Wilks (1946) were used to calculate approximate significance levels. As $n \to \infty$ the distribution of $-n \log [\lambda(H_{27})]$ tends to the form (A) with $f = \frac{1}{2}p(p+3) - 3$.

3.2. *Two* p-*Variate Populations*

3.2.1. H_{28}: $\sigma_{q_1} = \sigma_{q_2}$, $\rho_{qr_1} = \rho_{qr_2}$. $\quad -\infty < \mu_{qt} < \infty$.

See also section 3.3.1. Roy (1939) proposed to test H_{28} by criteria which he called p-statistics: these were defined as the roots of the determinantal equation

$$|v_{qr_1} - \theta v_{qr_2}| = 0$$

and their joint distribution found, having applications in canonical correlation theory. Bartlett (1941) considered that Roy's criteria were insufficient to cope with the problem. Roy (1942) later derived the distribution of his criteria when H_{28} was not true.

3.2.2. H_{29}: $\sigma_{q_1} = \sigma_{q_2}$, $\rho_{qr_1} = \rho_{qr_2}$, $\mu_{q_1} = \mu_{q_2}$.

Lengyel (1939), using results of Wilks (1932), gave the LR criterion when $n_1 = n_2 = n$, viz.

$$\lambda(H_{29}) = |v_{qr_1}|^{\frac{1}{2}n}|v_{qr_2}|^{\frac{1}{2}n}/|v_{qr_0}|^n.$$

A Type I curve was fitted to $L(H_{29}) = [\lambda(H_{29})]^{1/n}$, after investigation of the adequacy of this approximation, and significance levels calculated for $p = 2, 3, 4$ and a range of n.

3.3. k p-*Variate Populations*

3.3.1. H_{30}: $\sigma_{qt} = \sigma_q$, $\rho_{qrt} = \rho_{qr}$. $\quad -\infty < \mu_{qt} < \infty$.

Wilks (1932) gave the LR criterion, viz.

$$\lambda(H_{30}) = \prod_{t=1}^{k} (|v_{qrt}|/|v_{qra}|)^{n_t/2}$$

and its moments.

Bishop (1939) studied the case $n_t = n$, fitting a Type I curve to $L(H_{30})$ and $[\lambda(H_{30})]^{1/N}$: he found this was satisfactory for all k and $p = 1,2, \ldots, 6$ by investigating the adequacy of the approximation, comparing with the exact distributions when known. For large samples, $-2N \log [L(H_{30})]$ is distributed approximately in the form (A) with $f = \frac{1}{2}(k-1)p(p+1)$, and he gave a correcting

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factor for smaller samples (*cf.* Bartlett (1937)). George (1945) further investigated approximations by working out the exact distribution in special cases; she concluded that the Type I approximation seemed inferior for $p \geqslant 3$ and that for $-2N \log [L(H_{30})]$ with a correcting factor was better.

3.3.2. H_{31}: $\sigma_{qt} = \sigma_q$, $\rho_{qrt} = \rho_{qr}$, $\mu_{qt} = \mu_q$.

Wilks (1932) gave the LR criterion, viz.

$$\lambda(H_{31}) = \prod_{t=1}^{k} (|v_{qrt}|/|v_{qra}|)^{n_t/2}$$

and its moments.

This concludes the survey and a list of the papers mentioned is appended in alphabetical order. The survey has been carried out as part of the research programme of the National Physical Laboratory, and this paper is published by permission of the Director of the Laboratory.

References.

Aroian, L. A. (1941). A study of R. A. Fisher's z distribution and the related F distribution. *Ann. Math. Stat.*, **12**, 429–448.

Bartlett, M. S. (1937). Properties of sufficiency and statistical tests. *Proc. Roy. Soc.*, A., **160**, 268–282.

Bartlett, M. S. (1941). The statistical significance of canonical correlations. *Biometrika*, **32**, 29–37.

Bishop, D. J. (1939). On a comprehensive test for the homogeneity of variances and covariances in multivariate problems. *Biometrika*, **31**, 31–55.

Bishop, D. J., and Nair, U. S. (1939). A note on certain methods of testing for the homogeneity of a set of estimated variances. *Supp. J. Roy. Stat. Soc.*, **6**, 89–99.

Brown, G. W. (1939). On the power of the L_1 test for the equality of several variances. *Ann. Math. Stat.*, **10**, 119–128.

Bose, S. (1935). On the distribution of the ratio of variances of two samples drawn from a given normal bivariate correlated population. *Sankhyā*, **2**, 65–72.

Brandner, F. A. (1933). A test of the significance of the difference of the correlation coefficients in normal bivariate samples. *Biometrika*, **25**, 102–109.

Cochran, W. G. (1940). Note on an approximate formula for the significance levels of z. *Ann. Math. Stat.*, **11**, 93–95.

David, F. N. (1938). *Tables of the correlation coefficient.* Biometrika Office, University College, London.

De Lury, D. B. (1938). Note on correlations. *Ann. Math. Stat.*, **9**, 149–151.

Fertig, J. W., and Leary, M. V. (1936). On a method of testing the hypothesis that an observed sample of n variables and of size N has been drawn from a specified population of the same number of variables. *Ann. Math. Stat.*, **7**, 113–121.

Fertig, J. W., and Proehl, E. A. (1937). A test of a sample variance based on both tail ends of the distribution. *Ann. Math. Stat.*, **8**, 193–205.

Finney, D. J. (1938). The distribution of the ratio of estimates of the two variances in a sample from a normal bivariate population. *Biometrika*, **30**, 190–192.

Fisher, R. A. (1915). Frequency distribution of the values of the correlation coefficient in samples from an indefinitely large population. *Biometrika*, **10**, 507–521.

Fisher, R. A. (1921). On the " probable error " of a coefficient of correlation deduced from a small sample. *Metron*, **1**, No. 4, 3–32.

Fisher, R. A. (1924). On a distribution yielding error functions of several well-known statistics. *Proc. Int. Cong. of Math.*, Toronto, 805–813.

Fisher, R. A., and Yates, F. (1943). *Statistical Tables for Biological, Agricultural and Medical Research.* Oliver and Boyd, Edinburgh.

Fisher, R. A. (1944). *Statistical Methods for Research Workers.* 9th Edition. Oliver and Boyd, Edinburgh.

George, A. (1945). On the accuracy of the different approximations to the L_1-distribution. *Sankhyā*, **7**, 20–26.

Hartley, H. O. (1940). Testing the homogeneity of a set of variances. *Biometrika*, **31**, 249–255.

Hirschfeld, H. O. (1937). The distribution of the ratio of covariance estimates in two samples drawn from normal bivariate populations. *Biometrika*, **29**, 65–79.

Hsu, C. T. (1940). On samples from a normal bivariate population. *Ann. Math. Stat.*, **11**, 410–426.

Hsu, C. T. (1941). Samples from two bivariate normal populations. *Ann. Math. Stat.*, **12**, 279–292.

Hsu, P. L. (1938). Contribution to the theory of " Student's " t-test as applied to the problem of two samples. *Statistical Research Memoirs*, **2**, 1–24.

Kempthorne, O. (1944). Comments on the note " On a theorem concerning sampling." *J. Roy. Stat. Soc.* (*N.S.*), **107**, 58.

Lengyel, B. A. (1939). On testing the hypothesis that two samples have been drawn from a common normal population. *Ann. Math. Stat.*, **10**, 365–375.

Mahalanobis, P. C. (1933). Tables for the application of L-tests. *Sankhyā*, **1**, 109–122.

Mauchly, J. W. (1940). Significance test for sphericity of a normal n-variate distribution. *Ann. Math. Stat.*, **11**, 204–209.

Mood, A. M. (1939). On the L_1 test for many samples. *Ann. Math. Stat.*, **10**, 187–190.

Morgan, W. A. (1939). A test for the significance of the difference between the two variances in a sample from a normal bivariate population. *Biometrika*, **31**, 13–19.

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Nair, U. S. (1939). The application of the moment function in the study of distribution laws in statistics. *Biometrika*, **30**, 274–294.
Nair, U. S. (1941a). A comparison of tests for the significance of the difference between two variances. *Sankhyā*, **5**, 157–164.
Nair, U. S. (1941b). Probability statements regarding the ratio of standard deviations and correlation coefficient in a bivariate normal population. *Sankhyā*, **5**, 151–156.
Nayer, P. P. N. (1936). An investigation into the application of Neyman and Pearson's L_1 test, with tables of percentage points. *Statistical Research Memoirs*, **1**, 38–56.
Neyman, J. (1941). On a statistical problem arising in routine analyses and in sampling inspections of mass production. *Ann. Math. Stat.*, **12**, 46–76.
Neyman, J. (1942). Basic ideas and some recent results of the theory of testing statistical hypotheses. *J. Roy. Stat. Soc.* (*N.S.*), **105**, 292–327.
Neyman, J., and Pearson, E. S. (1928). On the use and interpretation of certain test criteria for purposes of statistical inference. *Biometrika*, **20**, A, 175–240.
Neyman, J., and Pearson, E. S. (1931). On the problem of k samples. *Bull. Acad. Polonaise Sci. Lett., Série A*, 460–481.
Neyman, J., and Pearson, E. S. (1933). On the problem of the most efficient tests of statistical hypotheses. *Phil. Trans.*, A., **231**, 289–338.
Neyman, J., and Pearson, E. S. (1936). Contributions to the theory of testing statistical hypotheses. (I). Unbiased critical regions of Type A and Type A_1. *Statistical Research Memoirs*, **1**, 1–37.
Paulson, E. (1942). An approximate normalization of the analysis of variance distribution. *Ann. Math. Stat.*, **13**, 233–235.
Pearson, E. S., and Neyman, J. (1930). On the problem of two samples. *Bull. Acad. Polonaise Sci. Lett., Série A*, 73–96.
Pearson, E. S., and Wilks, S. S. (1933). Methods of statistical analysis appropriate for k samples of two variables. *Biometrika*, **25**, 353–378.
Pearson, K. (1926). On the coefficient of racial likeness. *Biometrika*, **18**, 105–117.
Pearson, K. (Editor) (1931). *Tables for Statisticians and Biometricians. Part II.* Biometrika Office, University College, London.
Pearson, K. (Editor) (1934). *Tables of the Incomplete Beta-function.* Biometrika Office, University College, London.
Pearson, K. (Editor) (1934). *Tables of the Incomplete Gamma-function.* Biometrika Office, University College, London.
Pitman, E. J. G. (1939a). Tests of hypotheses concerning location and scale parameters. *Biometrika*, **31**, 200–215.
Pitman, E. J. G. (1939b). A note on normal correlation. *Biometrika*, **31**, 9–12.
Romanovsky, V. (1927). On the criteria that two given samples belong to the same normal population. *Metron*, **7**, No. 3, 3–46.
Roy, S. N. (1939). p-statistics, or some generalizations in analysis of variance appropriate to multivariate problems. *Sankhyā*, **4**, 381–396.
Roy, S. N. (1942). The sampling distribution of p-statistics and certain allied statistics on the non-null hypothesis. *Sankhyā*, **6**, 15–34.
Scheffé, H. (1942a). On the ratio of the variances of two normal populations. *Ann. Math. Stat.*, **13**, 371–388.
Scheffé, H. (1942b). On the theory of testing composite hypotheses with one constraint. *Ann. Math. Stat.*, **13**, 280–293.
Simpson, H. (1943). On a theorem concerning sampling. *J. Roy. Stat. Soc.* (*N.S.*), **106**, 266–267.
Snedecor, G. W. (1934). *Calculation and interpretation of analysis of variance and covariance.* Collegiate Press, Ames, Iowa.
" Student " (1908). Probable error of a correlation coefficient. *Biometrika*, **6**, 302–310.
Sukhatme, P. V. (1935). A contribution to the problem of two samples. *Proc. Ind. Acad. Sci.*, Section A, **2**, No. 6, 584–604.
Thompson, C. M. (1941a). Tables of percentage points of the χ^2 distribution. *Biometrika*, **32**, 188–189.
Thompson, C. M. (1941b). Tables of percentage points of the incomplete beta-function. *Biometrika*, **32**, 168–181.
Thompson, C. M., and Merrington, M. (1946). Tables for testing the homogeneity of a set of estimated variances. *Biometrika*, **33**, 302–304.
Tukey, J. W., and Wilks, S. S. (1946). Approximation of the distribution of the product of beta variables by a single beta variable. *Ann. Math. Stat.*, **17**, 318–324.
Villars, D. S., and Anderson, T. W. (1943). Some significance tests for normal bivariate distributions. *Ann. Math. Stat.*, **14**, 141–148.
Welch, B. L. (1935). Some problems in the analysis of regression among k samples of two variables. *Biometrika*, **27**, 145–160.
Welch, B. L. (1936). Note on an extension of the L_1 test. *Statistical Research Memoirs*, **1**, 52–56.
Wilks, S. S. (1932). Certain generalizations in the analysis of variance. *Biometrika*, **24**, 471–494.
Wilks, S. S. (1938). The large-sample distribution of the likelihood ratio for testing composite hypotheses. *Ann. Math. Stat.*, **9**, 60–62.
Wilks, S. S. (1946). Sample criteria for testing equality of means, equality of variances, and equality of covariances in a normal multivariate distribution. *Ann. Math. Stat.*, **17**, 257–281.
Wilks, S. S., and Thompson, C. M. (1937). The sampling distribution of the criterion λ_{B1} when the hypothesis tested is not true. *Biometrika*, **29**, 124–132.

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The Stock-carrying Capacity of Farms

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Introduction

An examination has been made of the relationship between the numbers of livestock and the cropping of farms, and between stock-carrying capacity and rental value of land. The work was largely a by-product of the analysis made for the report of the National Farm Survey.[1] In order to carry out that analysis, the Farm Survey data of a sample of farms, together with corresponding data (including rent) from the June 4th Returns of 1941, were punched on cards, and the results described in the present paper may be regarded as an illustration of the secondary uses to which such punched-card data can be put. The main purpose of the investigation was exploratory, but the data, although subject to some limitations, were sufficient to give a reliable indication of the relation between the factors examined.

In considering the results of the present investigation, it should be borne in mind that the data refer to a year in which the numbers of some classes of stock fell substantially; the relationship between stocking and acreage of crops should not, however, be seriously affected. On the other hand, the rationing of cattle feeding-stuffs which was then in force should improve the accuracy of farm-to-farm comparisons, as interchange of food between farms was smaller in that year than in pre-war years. It should also be noted that the cropping and livestock of the farms relate to a particular day of the year and do not necessarily represent the average stocking over the whole year. The measure of stock-carrying capacity adopted is consequently subject to a certain amount of regional bias, since, for example, the stocking of farms in the uplands of Wales must be considerably greater in the summer, while the rate of stocking in Norfolk is probably less in summer than in autumn and winter. There will also be some exaggeration of the carrying capacity of the upland areas with their small sheep and low-yielding dairy cows. Again, in the Midland counties and Northumberland, "other" cattle are mainly over two years, while in the west of England and Wales the average age does not greatly exceed one year; this difference is largely compensated for by the fact that, in the Midlands, the stock of over two years are fattened on grass during the summer. If more complete data were available, an analysis taking account of these factors would be fruitful; but it is unlikely that the results given in this paper would be altered appreciably.

The Measure of Stock-carrying Capacity

An index of the total stock carried on the farm was obtained by weighting the numbers of the different types of stock by factors representing their energy requirements (in terms of starch equivalent) to give the total number of "stock units" per farm. The weighting factors were based on Woodman's feeding standards (*Min. Agric. Bull.*, No. 48). In the case of dairy cows, allowance was made for rationed feeding-stuffs brought on to the farm, it being estimated that these would provide about one-third of the average production requirements per cow. For poultry it was assumed that about half the total energy requirements were obtained from rationed foods, kitchen waste, scavenging, etc. The unadjusted weights are shown in the first column of Table 1, and the weights adjusted for these other sources of food are shown in the second column.

Table 1.

Weights for each class of stock

	Unadjusted	Adjusted
Cows	9	7
Other cattle	5·5	5·5
Sheep	1	1
Pigs	2·5	2·5
Poultry	0·1	0·05
Agricultural horses	9·5	9·5

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The acreage of each farm which was available to carry this stock—the "acreage" units—was taken to be the sum of the acreages of permanent grass, temporary grass, oats, beans, peas, mixed corn, fodder crops, rough grazing and half the acreage of hops and orchards.

The stock-carrying capacity of a farm was defined as the "stock" units divided by the "acreage" units.

General Results.

The sample of farms was obtained from the complete survey data using the following sampling fractions:

Size of farm group (acres)	Sampling fraction
5–24·9	1/20
25–99·9	1/10
100–299·9	1/4
300–699·9	1/2
700 and over	1

Table 2 gives the mean stock-carrying capacity for each farm-size group in each county weighted according to acreage units: the group of farms of over 700 acres is rather small in many counties and has been amalgamated with the size group immediately below.

County means standardised for size of farm have been calculated by taking a weighted mean of the index of the three groups of farms of over 25 acres, the weights being proportional to the total areas in England and Wales in each group. County-to-county comparisons of this "standardised" mean will be unaffected by differences in the distribution by size of farms within counties. The farms of under 25 acres have been omitted entirely from this standardised mean because the farm-to-farm variation of the index is very considerable for such farms, and they cover only a small part of the total acreage. The standardised means are given in Table 2, and are shown cartographically in Fig. 1.

The rates of stocking actually obtained vary between 402 units per 100 acres (equivalent to one cow per 1¾ acres of grass and fodder crops) in Chester to 194 units (or one cow per 3½ acres) in Merioneth. The highest rates of stocking occur in the dairy counties of the north-west Midlands (Chester, Flint, Stafford and Salop). Although districts within some other counties (*e.g.* the clay vales of north-west Wiltshire and east Somerset) may be equally heavily stocked, their contribution to the aggregate county figures is outweighed by the considerable areas of low productivity in those counties. The remainder of the dairying counties of the west and north-west Midlands and the principal cattle-feeding counties have a somewhat lower level of production.

The high values obtained for Holland and the Isle of Ely are based on low acreages of grass and fodder crops and low numbers of stock, and while these two counties are undoubtedly of higher fertility than the neighbouring counties, the stock indices must be regarded as suspect. The figure for Middlesex cannot be taken as a reliable indication of the stock-carrying capacity of this county.

The greater part of the chalk country of southern England together with the poor gravelly soils of the Hampshire Basin, has a very low stock-carrying capacity. The only English counties having a lower rate of stocking are Northumberland and Durham which have relatively little intensively stocked land to counterbalance large areas of moorland and poor grass. The average rate of stocking is higher in Cumberland and Westmorland in spite of the low production of the Lakeland mountains.

A third area of low productivity is formed by west and central Wales; these counties have a substantial proportion of mountain and hill land, and much of the grassland at lower levels is only moderately productive; the counties of south Wales and Anglesey and Denbigh have a rather higher stocking.

Density of Stock and Rental Value of Land.

While the rental value of a farm is affected by the condition and adequacy of the permanent equipment other than land—farmhouse, buildings, farm roads and so on—it is to be expected that there will be some relationship between rent per acre and the index of stocking within counties. For the examination of this relationship, the rent per acre was taken to be the total rent divided by the acreage of crops and grass, rough grazing being neglected. In computing the stock index, the

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TABLE 2

Stock-carrying capacity by size of farm groups.

County	Stock units per 100 acres				
	Size group (acres)				Standardised mean (see text)
	5 –	25 –	100 –	300 +	
Bedford	583	345	262	260	283
Berkshire	364	366	261	233	281
Buckingham	436	333	288	282	298
Cambridge	980	399	253	235	287
Isle of Ely	629	450	361	326	375
Chester	524	419	408	372	402
Cornwall	431	350	291	242	294
Cumberland	226	260	295	264	278
Derby	407	329	305	256	298
Devon	390	340	289	273	298
Dorset	391	306	230	209	244
Durham	339	259	238	235	243
Essex	399	301	277	270	281
Gloucester	489	352	300	245	299
Hampshire	465	315	242	207	252
Isle of Wight	395	385	259	205	278
Hereford	474	385	334	319	343
Hertford	377	332	262	240	275
Huntingdon	459	412	252	229	288
Kent	535	315	291	292	298
Lancaster	508	405	331	273	335
Leicester	331	344	327	306	326
Holland	649	487	381	320	393
Kesteven	440	359	283	253	295
Lindsey	477	338	299	290	307
Middlesex and London	1720	504	319	287	359
Norfolk	597	340	280	263	291
Northampton	457	360	315	305	324
Soke of Peterborough	450	308	255	254	269
Northumberland	118	171	207	229	203
Nottingham	393	331	280	269	291
Oxford	475	329	276	260	286
Rutland	488	360	314	261	312
Salop	484	413	356	329	364
Somerset	557	342	309	277	309
Stafford	459	415	336	339	358
East Suffolk	415	317	294	277	296
West Suffolk	307	360	292	263	302
Surrey	516	375	234	250	275
East Sussex	317	287	247	263	262
West Sussex	422	292	256	224	257
Warwick	436	381	311	300	327
Westmorland	355	340	298	274	303
Wiltshire	542	325	245	202	255
Worcester	519	363	304	312	322
Yorkshire East Riding	570	407	285	241	306
Yorkshire North Riding	466	266	281	231	264
Yorkshire West Riding	565	336	267	232	276
Anglesey	337	321	303	272	300
Brecon	319	287	301	299	297
Caernarvon	205	268	203	312	249
Cardigan	241	219	215	227	219
Carmarthen	325	282	257	262	265
Denbigh	361	322	277	202	269
Flint	482	445	407	351	402
Glamorgan	231	276	286	288	284
Merioneth	225	197	213	157	194
Monmouth	421	338	309	308	316
Montgomery	418	277	268	186	249
Pembroke	355	278	240	262	256
Radnor	330	236	240	186	225
England and Wales	448	336	288	265	295

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value attributed to rough grazing was reduced—six and a half acres of rough grazing being assumed equivalent to one acre of permanent grass. The analysis has, however, been confined to sixteen representative English counties; in all of these rough grazing was of little importance.

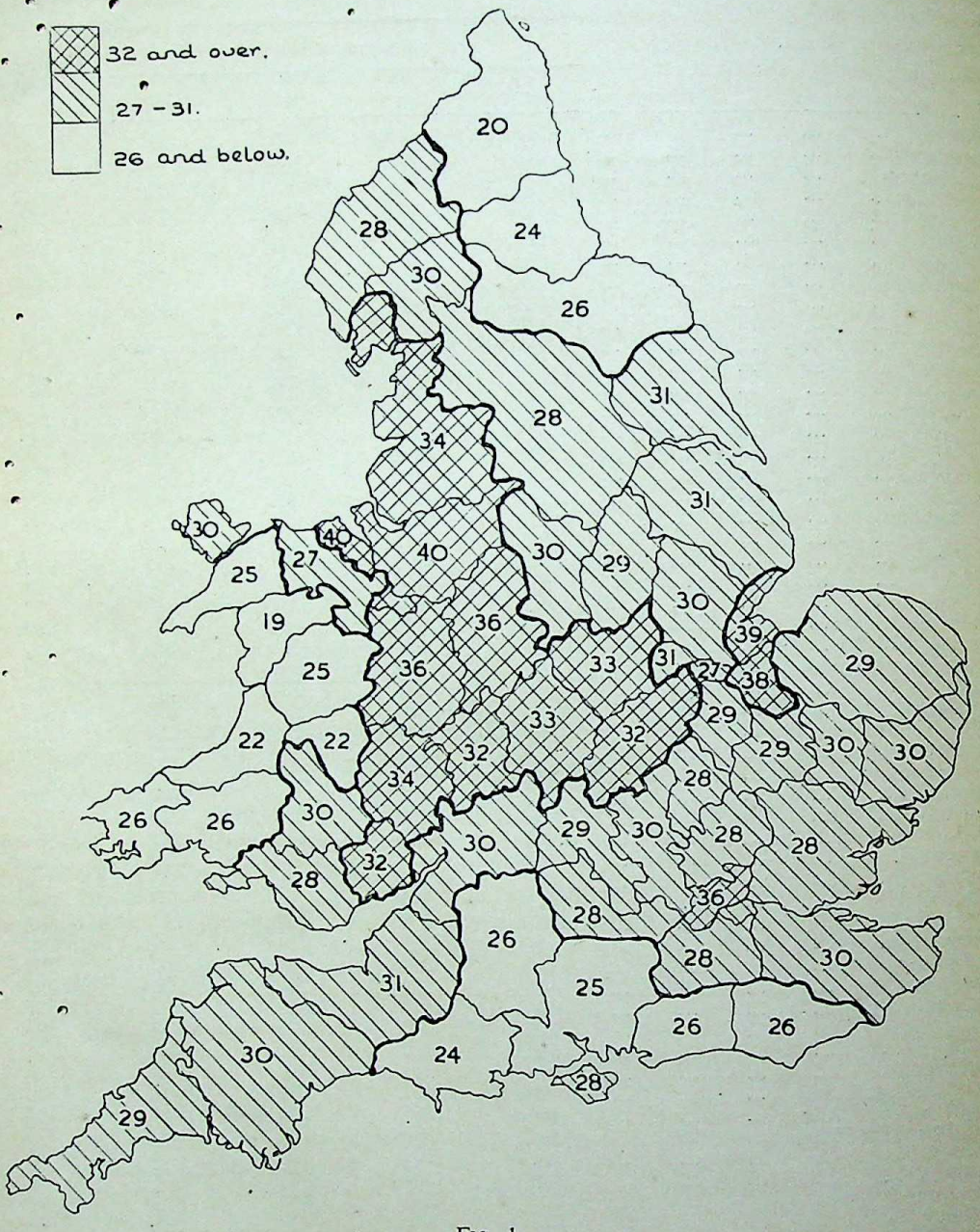

FIG. 1.
Stock-carrying capacity: stock units per 10 acres (standardised for size of farm).

The average stocking at different levels of rent per acre for farms of size 25–100 and 100–300 acres is shown in Fig. 2. The two groups show a similar increase for rents below 40*s.* equal to about 5 stock units per 100 acres for each shilling increase in rent. It is not to be expected that this rate of increase will be maintained at very high levels of rent, since these rents can be borne only by intensive arable and market-garden farms, except in unusually favoured districts or where farms

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are in close proximity to a town. The rate of increase of rent was related to the profit to be expected at pre-war prices and costs from the corresponding increases in density of stocking, and it was found that an increase of one stock unit per acre would give an increase in profit of about 28*s*. in the case of dairy cows and 24*s*. in the case of sheep. From Figure 2, the increase in rent corresponding to one stock unit per acre is equal to 20*s*. Bearing in mind that the costs on which the calculation was based were those prevailing in the pre-war years when the farming industry was not making large profits, the figures are in substantial agreement.

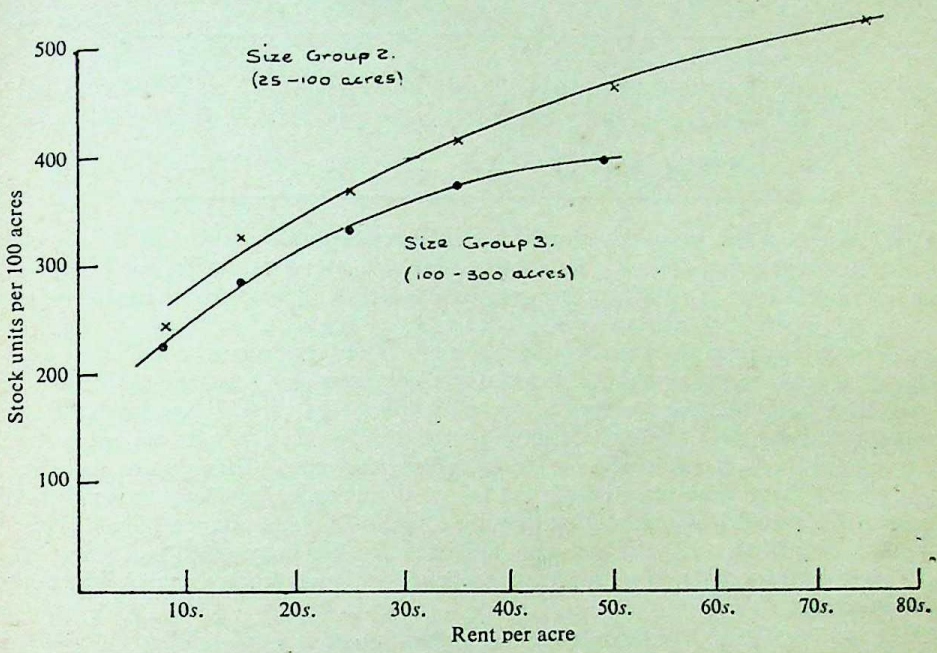

FIG. 2.
Stock-carrying capacity and rent.

More detailed examination was made of the data for three counties, Cambridgeshire, Hampshire and Monmouth. This consisted in the evaluation of the regression of stock density on rent per acre for tenant-occupied farms. The results obtained are as follows: R being the rent per acre (total rent in pounds, divided by acreage of crops and grass, rough grazing being of negligible importance in these counties) and S the stock units per acre:

County	Size group (acres)	Regression equation
Cambridge	100–300	$S = 1{\cdot}36 + 1{\cdot}07\,R$
Monmouth	25–100	$S = 3{\cdot}33 + 0{\cdot}86\,R$
	100–300	$S = 2{\cdot}34 + 0{\cdot}84\,R$
Hampshire	25–100	$S = 2{\cdot}21 + 0{\cdot}91\,R$
	100–300	$S = 1{\cdot}57 + 1{\cdot}00\,R$

The similarity between the regression coefficients is striking. They confirm that an increase in rent of £1 per acre corresponds to an increase in stock density of the order of one stock unit per acre.

Size of Farm

Table 2 shows the number of stock units per 100 acres for each county according to size of farm (the grouping of farms is by acreage of crops and grass and not by the acreage supporting stock).

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In practically all counties, the density of stock decreases with increasing size of farm, the results for the whole country being as follows:

TABLE 3

Stock-carrying capacity by size of farm

Size of farm (excluding rough grazing)	Stock units per 100 acres	
	Actual	Adjusted for rent
5–24·9	448	308
25–99	336	306
100–299	288	288
300 and over	265	287
All farms	302	—

The table indicates that very small farms carry about 70 per cent more stock per acre of crops and grass than that carried by large farms. Such farms, however, comprise only about one-twentieth of the total agricultural area. Combining the first two and last two groups, it is found that the farms of over 100 acres have an average stock-carrying capacity of about 280 units per hundred acres, and the farms of under 100 acres 355 units or approximately 25 per cent more.

Taking the stock-carrying capacity of farms of 100–299 acres as a base, the figures for the other size groups have been adjusted for the considerable differences in their average rents per acre. These adjusted values, given in Table 3, show that the differences in rent appear to be almost entirely accounted for by the differences in stock-carrying capacity. If this is the case, the higher rents of the small farms arise mainly from the fact that the better land is in general partitioned into smaller units, though it is possible that they are also due to some extent to the greater value of buildings per acre of land. Some of the small difference between large and small farms which still remain after allowing for rent may be due to the tendency for small farms to buy in more foods not subject to rationing.

Other Measures of Productivity

It is possible to make an assessment of the productivity of different counties by two general methods of approach, either by determining the stock-carrying capacity as in the present paper, or by integrating the average yields of the various crops. The second method of approach was adopted by Kendall,[2] and it is of interest to consider the various assessments of productivity made by Kendall and to compare his results with those described in this paper.

Kendall obtained four distinct measures of productivity, using available statistics of the ten major crops—wheat, barley, oats, beans, peas, potatoes, turnips and swedes, mangolds, hay from temporary grass and hay from permanent grass:

(1) *The Productivity Coefficient.* The productivity coefficient was obtained by a complicated statistical technique, which led to a linear function in the yields per acre of the ten crops. Each county gave a point in 10-dimensional space by regarding the yields of the ten crops as co-ordinates. The line which best fitted these points was used to grade the counties.

(2) *The Ranking Coefficient.* Each county was ranked for the average yield per acre of each of the ten crops and the coefficient of each county was taken to be the mean of these rankings.

(3) *The Money-Value Coefficient.* This coefficient was obtained from the average value produced per acre from the ten crops, weighting each crop by its acreage.

(4) *The Energy Coefficient.* This coefficient was obtained from the average gross digestible energy produced per acre, again weighting each crop by its acreage.

Both the productivity coefficient and the ranking coefficient take no account of the area devoted to each crop in the counties. As there are obviously large differences between counties in this respect, the use of equal weights for all crops gives an unfair comparison between counties, particularly as the correlations of the yields of grass and fodder crops with those of cereal crops are sometimes negative. The money-value coefficient has some advantages over the two previous ones: for example, in that account is taken of the volume of production. A serious disadvantage, however, is that certain parts of the country are better suited to the more profitable crops because of

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proximity to markets, and the coefficient will measure this factor to some extent. Kendall's last coefficient—the energy coefficient—has more to recommend it; but it is unfortunate that he chose to work in terms of gross digestible energy instead of starch equivalent. (His use of the term "starch equivalent" to imply total digestible nutrients is misleading.) This has the effect of favouring the wheat-growing counties, since about one-half of the value attributed to wheat is due to the straw. Again, pasture which is hayed is only taken account of in so far as hay is produced; as hay is only about half the total yearly output of such pasture, the counties which have a large acreage of hay will have low coefficients. This is probably the reason for the differences in the grading of Somerset, which was given a high grade by the productivity and rank methods but a low grade by the other methods.

On the basis of the four coefficients for each of four years Kendall classified the counties as excellent, good, moderate, indifferent or poor. There are considerable differences between his results and those given in the present map, of which the following are worth quoting:

(*a*) Kendall classed Salop as only moderate, and most of the other Midland counties (Hereford, Worcester, Warwick, Leicester and Northampton) as poor or indifferent. As Kendall himself pointed out, this was due mainly to the fact that his indices referred principally to arable land. On the basis of stock-carrying capacity, these counties are well above the average; this is supported by the average rent per acre, which was 28*s.* per acre for these counties compared with the average for England of 26*s.* per acre.

(*b*) Most of the eastern counties were classed as good by him—largely because, as pointed out above, his indices favour arable counties.

(*c*) Northumberland and Durham were classed as average by Kendall while their stock-carrying capacity and rent per acre (under 20*s.*) are both low.

(*d*) His classification of the south-western counties placed Wiltshire and Cornwall above Devon and Somerset which were classed as "indifferent". The present map reverses this assessment and is supported by the average rent per acre, Somerset having the highest rent and stocking per acre and Wiltshire the lowest.

(1) "National Farm Survey of England and Wales." A Summary Report. Ministry of Agriculture and Fisheries. H.M.S.O. 1946.

(2) Kendall, M. G. "The Geographical Distribution of Crop Productivity in England." *Journal of the Royal Statistical Society*, 1939, vol. cii, 21–48.

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REPORT OF THE COUNCIL

For the FINANCIAL YEAR *ended December 31st, 1945, and for the* SESSIONAL YEAR *ending June 27th, 1946, presented at the* ONE HUNDRED AND TWELFTH ANNUAL GENERAL MEETING *of the* ROYAL STATISTICAL SOCIETY, *held at the London School of Hygiene and Tropical Medicine, W.C.1, on June 27th, 1946.*

THE Council has the honour to submit its One Hundred and Twelfth Report.

Number of Fellows

The number of Fellows on December 31st, 1945, was 1417, an increase of nearly 150 over the total at the end of 1944, and the largest number in the history of the Society. There is no doubt that this number is slightly swollen by the inclusion of some defaulters, since the Council has been reluctant to take final action against Fellows whose subscriptions are in arrears until normal communications throughout the world were restored and prisoners of war released. This factor probably contributes to the relatively low "exit" rate in the past two years revealed in the figures given below. The most marked feature in the trend is, however, the high and increasing number of elections of new Fellows in the past three years.

Calendar Year	Lost by death, withdrawal or default	Elected, or restored to the roll	On the roll at December 31st
1935	53	67	1051
1936	64	62	1049
1937	45	59	1063
1938	56	76	1083
1939	39	64	1108
1940	76	47	1079
1941	65	60	1074
1942	69	74	1079
1943	61	121	1139
1944	29	159	1269
1945	29	177	1417

In addition, there were 18 Honorary Fellows, including seven who are Presidents for the time being of other Societies.

The Council hopes to resume next session the publication of the list of Fellows. This list has been in abeyance during the war years both through lack of paper (which may still delay its issue) and the lack of stability in Fellows' addresses.

Losses by death

The Council regrets to report that during the sessional year (to May 31st, 1946) the Society lost by death the undermentioned Fellows:—

	Date of Election
Anning, Edward H., F.R.G.S.	1897
*cdp*Ashley, Sir Percy, K.B.E., C.B., M.A.	1908
*Bagnall, Bernard T. S., F.C.A.	1909
*d*Bastable, Prof. C. F., M.A., L.L.D., F.B.A.	1889
Besso, S. L.	1912
*d*Ceadel, Albert E.	1927
Clark, Prof. Arch. Brown	1904
*Duveen, Edward J.	1910
Hanna, Wm. G. C., O.B.E., C.A., F.C.I.S.	1921
Harding, A. Seymour	1919
Harris, Frank George	1917
Harris, W. F., C.B.E., F.C.A.	1897
Jones, Owen	1924
*cp*Keynes, The Lord, C.B.	1909
*d*Plender, The Rt. Hon. Lord, G.B.E., LL.D., J.P., F.C.A.	1901

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	Date of Election
Rathbone, Miss Eleanor F., M.P.	1903
*d*Samuel, George A. H.	1908
*cdp*Smith, Sir H. Llewellyn, G.C.B., B.Sc.	1888
Strong, W. R., F.I.A., F.I.C.S.	1926
Tocher, James Fowler, D.Sc.	1929
*Watt, James, L.L.D., F.F.A.	1910
Williams, T. Taliesin	1911
Wilson, Herbert W.	1898
cdp†Wood, George H.	1897

* Life Fellow.
† Guy Medallist.

c Served on Council.
d Donor to the Library.
p Contributed to *Proceedings*.

Sir Percy Ashley was elected a Fellow of the Society in 1908, served for many years on the Council, and was a Vice-president from 1938 to 1941 and from 1943 to 1945. After a few years of teaching and research at the then new London School of Economics, he took up his life work when he joined in 1907 Sir Hubert Llewellyn Smith's staff at the Board of Trade. After the first world war he was mainly concerned with the reconstruction of British industry and its relations with Government. In 1923 he took charge of the newly-formed Industries and Manufactures Department, and to this he brought not only a wide knowledge of industry, but a gift of attracting the trust and confidence of business men, who readily laid their many problems before him. In 1932 he was appointed Secretary to the Import Duties Advisory Committee. When that body ceased to function at the outbreak of war he became Cotton Controller and Chairman of the Cotton Board. He retired from Government service in 1941, but then undertook the chairmanship of the British Rayon Federation.

Sir Hubert Llewellyn Smith was elected a Fellow of the Society in 1888, and served on the Council in 1896–1900 and 1901–05. His name will always be associated with the part he played in Charles Booth's classical investigation of the Life and Labour of the People and with his own re-investigation of its problems when, soon after his retirement from official work, he undertook in 1928 the direction of the New Survey of London Life and Labour. In between these great contributions to knowledge lay his work at the Board of Trade, to which he was appointed in 1893. Here, in the Commercial, Labour and Statistical Department, he was concerned not only with the building up of the means for the collection and publication of information on problems of labour and wages, but also with the early development of negotiations in labour disputes. Having succeeded Sir Robert Giffen as Controller-General of this Department, he subsequently became Permanent Secretary to the Board, and played an important part in the establishment of Labour Exchanges and in the first Unemployment Scheme. Though throughout his career his special interest always lay in labour conditions and their improvement, he was a man of very wide interests and great and varied abilities.

Mr. George Wood had been a Fellow of the Society for nearly fifty years, had served on the Council and was a Guy medallist (in silver). He, too, had served in his early life at the Board of Trade, to which he had been appointed following his researches, alone and in association with Professor Bowley, into the course of real wages. Here he made many enquiries into wages and working hours, employment, cost of living and labour questions generally. He left the Board in 1908 to become Secretary to the Huddersfield Woollen Manufacturers' and Spinners' Association, a post which in 1917 developed into the Secretaryship of the Woollen and Worsted Trades Federation. To that administrative work he brought his statistical interests and acumen, and built up a system of collecting and reporting which covered many aspects of the industry. He had an enthusiasm for cricket, and many Fellows will remember the galaxy of cricket stars he attracted to a meeting of the Society in November 1944 when he made his final contribution to the Society's proceedings.

Dr. James Tocher joined the Society late in life, at the age of 65, and, living in Aberdeen, was unable to take any active part in its work. To a wide knowledge of chemistry and pharmacy he brought an ability in statistical reasoning and methodology cultivated under Karl Pearson. From 1911 to 1943 he was lecturer on statistics at Aberdeen University, and took a generally active part

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in the academic life there. His main statistical publications lay in the field of anthropometry—surveys of Scottish children and adults—and in his monumental studies of the milk yield of cows.

Lord Plender joined the Society in 1901, at an early stage in a career of public service which has rarely been equalled in length, importance or variety. He was always ready to place his great gifts at the disposal of the nation, and his pre-eminence in his own profession was marked by his election on three occasions as President of the Institute of Chartered Accountants, in 1910–11 and 1911–12 and again in 1929–30, when he was felt to be the most fitting person to preside during the Jubilee year of the Institute. When the Society decided to appoint a professional auditor in 1939, he at once accepted its invitation to be the first holder of that office, and placed himself at the disposal of the Society not only in the annual audits until his death in January 1946, but also in carrying out a revision of the form of the Society's Accounts.

Lord Keynes was elected a Fellow of the Society in 1909, at the age of 26, being proposed by his father, Dr. John Neville Keynes, who had himself been elected in 1883, and is still a Fellow sixty-three years later. At that time John Maynard Keynes was lecturer in economics at King's College, Cambridge, whither he had returned after two years' work at the India Office and by which he had been awarded a prize fellowship for his dissertation on "Probability." In the development of the Economics Faculty of the University he later played an important part. On behalf of the Society his activities included three terms of service as a member of the Council—from 1915 to 1919, 1933 to 1937 and 1941 to 1945, his retirement at the end of the present session being required by the current bye-laws on grounds of seniority. He read one paper to the Society, in 1936—namely, an appreciation of the life and work, economic and statistical, of William Stanley Jevons—and he was an occasional contributor to Miscellanea. In the brief compass of the Council's Annual Report it would clearly be impossible to do justice even briefly to the career of one of the Society's most distinguished Fellows. In mourning his death at the age of 63 the Council has, therefore, limited attention to Lord Keynes' associations with the Society. An appreciation of his life and work will be published in the *Journal*.

Miss Eleanor Rathbone joined the Society in 1903, her interests lying in the field of social conditions and their reform, to which she brought for nearly half a century an unchanging devotion and a complete lack of self-interest. From her early studies of family budgets, housing and poverty, and from her work during the war of 1914–18 on the financial conditions of women whose husbands were serving in the Forces, flowed much of her long-fought case for Family Allowances. By this, now finally accepted, she will always be remembered.

Elections and Restorations

From June 1945 to May 1946, inclusive, 251 Fellows were elected or restored to the roll, and at the end of May 1946 the total number of Fellows was 1,567. The names of Fellows elected from June 1945 to May 1946 are listed in Appendix A.

Acting under Bye-law 7, the Council unanimously agreed to invite the Presidents for the time being of the Institute of Actuaries and of the Faculty of Actuaries to become Honorary Fellows of the Society. The two Presidents in office have accepted the invitation and expressed their appreciation of the Council's action.

Meetings of the Society

The large increase in the Fellowship of the Society in the last three years has naturally been reflected in the attendance at its Ordinary Meetings. At the Presidential Address delivered by Lord Woolton nearly 250 Fellows and guests were present, and at other meetings the attendance has usually exceeded 150, often by a substantial amount. All meetings have been held at the London School of Hygiene and Tropical Medicine, where these larger numbers can be admirably accommodated, and the Council is greatly indebted to the School for placing these facilities at its disposal.

Following last year's procedure, the Council decided that no paper should be read at the Ordinary Meeting to be held in conjunction with the Annual General Meeting in June 1946. This will give ample time for the discussion of its Annual Report and for the general consideration of the Society's affairs. Advantage, however, has been taken of the presence in this country of Professor Mahalanobis, F.R.S., to arrange for him to read a paper to the Society in July.

It may be noted that if the Society accepts a proposed revision in the relevant Bye-laws (to be

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submitted to a Special General Meeting in the autumn) the Council will be empowered to arrange Ordinary Meetings in any month, as it considers advisable, and not be limited to the months of November to June. It will be enabled to hold the Annual General Meeting either in June or July.

The following papers have been read:—

1945.
November 22nd ... WOOLTON, The Rt. Hon. Lord, P.C., C.H. Some Aspects of the Use of Statistics in Government, with Special Reference to the Human Budget.

1946.
January 23rd ... YATES, F., Sc.D. A Review of Recent Statistical Developments in Sampling and Sampling Surveys.

February 27th ... GREENWOOD, Prof. MAJOR, F.R.S. The Statistical Study of Infectious Diseases.

March 28th ... ... MANDEVILLE, JOHN, A.M.I.Mech.E. Improvements in Methods of Census and Survey Analysis.

April 25th ... ... BLAU, Miss GERDA, Ph.D. Wool in the World Economy.

May 29th ... ... ALLEN, Prof. R. G. D. Mutual Aid between the U.S. and the British Empire, 1941–45.

The Research Section

Early in the session the Council was able to revive the activities of the former Industrial and Agricultural Research Section. It decided, however, to replace the original body by two distinct sections, to deal separately with research and the industrial applications of statistical methods. The Research Section, as its name implies, is primarily concerned with research into the theory of statistics and statistical methods and with the development of their applications. Dr. J. Wishart was elected chairman of this Section for the session, and four meetings have been held. The papers and discussions at these meetings, as well as other contributions, including those from the Industrial Applications Section, are being published in the *Supplement*. The issue of this is now being resumed, editorial arrangements being in the hands of the Committee of the Research Section.

The following have been the papers read:—

1945.
December 4th ... Sequential Tests in Industrial Statistics, by Mr. G. A. Barnard.

1946.
January 29th ... Symposium on Auto-Correlation in Time Series:
1. On the theoretical Specification and Sampling properties of Auto-correlated Time Series, by Dr. M. S. Bartlett.
2. Some instruments for the analysis of Time Series and their application to Textile Research, by Mr. G. A. R. Foster.
3. Random processes in problems of Air Warfare, by Dr. L. B. C. Cunningham and Mr. L. R. B. Hynd.

March 26th ... ... Statistical methods in the selection of Navy and Army personnel, by Dr. P. E. Vernon.

June 5th ... ... The application of some commercial calculating Machines to certain Statistical Calculations, by Dr. H. O. Hartley.

The Industrial Applications Section

During the session Local Groups of the Industrial Applications Section have been formed in Birmingham, London and Sheffield. The officers are as follows:—

Group.	Chairman.	Honorary Secretary.
Birmingham and district	Dr. J. W. Rodgers.	Mr. D. J. Desmond.
London	Mr. H. Rissik.	Mr. W. J. Jennett.
Sheffield	Mr. A. Dodd.	Mr. F. C. Turner.

For co-ordinating the work of the groups and uniformly developing their status within the Society, a Section Committee has been formed consisting of representatives appointed by the Council and by each of the Local Groups. Dr. B. P. Dudding and Mr. W. J. Jennett have been elected Chairman and Honorary Secretary of this Committee. The Committee has drafted regulations governing the activities of the Section, and these have been approved by the Council. Notes on the formation of the Section and notices of meetings have been published in many scientific and technical journals, thus bringing its activities to the notice of interested persons. Early in the present year the membership already numbered some 300 persons in addition to about 100 Fellows of the Society.

During the session seven meetings have been held in Birmingham, seven in London and four in Sheffield.

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The subjects of discussion have been as follows:—

Birmingham and District Group

1945.
September 26th ... Statistics in Engineering—Dr. B. P. Dudding.
October 31st ... A New Dodge Method of Sampling—Capt. A. H. R. Grimsey.
November 28th ... Sampling—Mr. D. Newman.
1946.
January 30th ... Statistics in the War Effort—Mr. Van Rest.
February 27th ... Impressions of Statistics in America—Mr. Bennett and Mr. Milbourn.
March 27th ... ... From Quality Control to Analysis of Variance—Mr. D. J. Desmond.
April 24th ... ... Quality Control Economics—Mr. W. A. Bennett.

London Group

1945.
September 21st ... The Difficulties of Introducing Quality Control into a Factory—Mr. W. Kellett.
October 26th ... Some Applications of Statistical Methods to Chemical and Spectrographic Analysis—Mr. H. Singleton.
November 30th ... An Engineer's Approach to the Analysis of Variance—Mr. A. W. Swan.
1946.
January 25th ... A Symposium on the Reliability of 100% Inspection—Mr. D. Newman and others.
March 1st ... ... Some Applications of Statistics in the War Effort—Mr. J. Womersley and Mr. Van Rest.
April 5th ... ... Statistics in Radio Valve Component Manufacture—Mr. A. Gait.
May 10th ... ... The Accuracy of Automatic Lathes—Mr. P. J. Stanley.

Sheffield Group

1945.
December 6th ... Everyday Works Problems—Mr. A. H. Dodd.
1946.
February 28th ... Examples of the Application of Statistical Methods to Steel Works' Problems—Mr. N. H. Bacon.
March 28th ... ... An Application of Regression Analysis to Experimental Metallurgical Data—Mr. J. I. Morley.
May 2nd ... ... Costs, Overhead Expenses and Effort Assessment in Practice—Mr. F. C. Lawrence.

The Study Group

During the session the Council appointed a Committee to consider the revival of the Study Group. On the advice of the Committee, the Council has circulated a letter to Fellows in London and the Home Counties so that it can measure the degree of support that would be forthcoming.

Award of Guy Medal

Under the regulations governing the award of a Guy Medal in gold, such an award can be considered only at intervals of five years (beginning in 1935) unless the Council by special resolution directs that an award should be discussed at an intermediate date. Such a resolution was passed by the Council during the past session, and it has unanimously agreed upon the award of a Guy Medal in gold to Professor R. A. Fisher, F.R.S., for his distinguished contributions to the theory and practice of statistics and to the design of experiments, contributions which have had a profound effect upon scientific methodology in many fields.

Frances Wood Memorial Prize

During the war no competition for the Frances Wood Memorial Prize was held. The Council has decided to offer the prize, of £30, in 1946 for the best investigation on statistical lines of any problem affecting the economic or social conditions of the people. This revival has been advertised and brought to the attention of University departments and many other interested bodies. Papers, which may be submitted by Fellows or non-Fellows, must be sent to the Honorary Secretaries not later than December 31st, 1946.

The Election of the Council and Honorary Officers

Acting upon a resolution passed at the Annual General Meeting held on June 26th, 1945, the Council appointed a Committee, consisting of six members of the Council and six other Fellows, to consider the present bye-laws relating to the election of the Council and Honorary Officers and to recommend such revision in the present procedure as they thought desirable.

The Committee unanimously agreed that changes in the present electoral procedure were desirable, and that such changes should give Fellows freedom to nominate persons to serve as the

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Honorary Officers of the Society or as ordinary members of the Council, and also greater opportunity to elect those they wish to serve in these capacities. To this end they submitted to the Council three draft clauses to replace the present bye-laws Nos. 12, 13 and 14. In broad outline the principal changes suggested by these Clauses are as follows:—

(*a*) that of past Presidents only three should be members of the Council in virtue of their position as Honorary Vice-Presidents;

(*b*) that the Honorary Treasurer and Honorary Secretaries should not hold office as such for more than ten consecutive sessions.

(*c*) that annually at least two Fellows should be elected to the Council who have never served on it previously;

(*d*) that Fellows should be annually invited to make suggestions to the Council for the composition of the ensuing Council;

(*e*) that Fellows, if they are then not satisfied with the Council's proposals, should be enabled to nominate candidates for election as President, Honorary Officers or other members of Council.

(*f*) that in the event of such nominations being received a postal ballot should be held.

The Council has accepted these proposals and the new bye-laws (Nos. 15, 16 and 17) have been incorporated in the general revision of the bye-laws to be placed before a Special General Meeting to be held later in the year.

For the nomination and election of the President, Council and Officers for the session 1946–47 the Council has, of course, been bound by the present Bye-law No. 14. It has as usual secured advice from some Fellows not of its own number and has included in the list of those it recommends for election several Fellows who have never previously served on the Council.

The Society's Bye-laws

In July 1945 the Council appointed a Committee to consider a revision of the Society's bye-laws and to advise on such changes and additions as they considered necessary in view of the development of the Society in the last fifty years. This Committee has placed before the Council a draft of new bye-laws, which the Council has accepted, and will place before the Society at a Special General Meeting. The Council is much indebted to the members of this Committee and in particular to its Chairman, Dr. Heron, for the large amount of work they have performed on its behalf.

Teaching of Statistics

The Council's Committee on the Teaching of Statistics is charged with the duty of considering and reporting on the present position of the teaching of statistics in Universities, Technical Colleges, Schools and other bodies, and with making general recommendations regarding the future scope and organisation of such teaching. For this purpose it has circulated widely an enquiry into the present position, and is at present engaged upon an analysis of the replies.

The Society's Petition to the Privy Council

It was briefly reported to the Annual General Meeting, held on June 26th, 1945, that the Counter-petition to the Society's petition to the Privy Council for powers to conduct examinations and award diplomas and certificates had subsequently been supported by the Royal Economic Society. The Council of the latter body lodged objections to the Society's proposals with the Privy Council. During the year a joint meeting of representatives of the two Societies took place in the hope that some agreement on the matter could be reached. The Council has to report that no satisfactory agreement is possible, that the Royal Economic Society appears to be irrevocably opposed to the Society's proposals regarding diplomas, and is not prepared to withdraw its objections. In view of this situation the Council concluded that it would be idle to pursue the matter in its original form, and it appointed its Honorary Officers, Dr. E. C. Snow, and Professor Greenwood as a Committee to consider any alternative scheme that might be in conformity with the Society's expressed wishes.

The Journal

After a close study of the advantages and disadvantages of changing the format of the Society's *Journal*, the Council was agreed that a larger page was desirable for the proper presentation of tables, diagrams and mathematical formulæ. It has, therefore, increased the page size from

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$8\frac{1}{2} \times 5\frac{1}{2}$ inches to $9\frac{3}{4} \times 6\frac{7}{8}$ inches. The selling price to non-Fellows has at the same time been raised for 1946 onwards to 15*s.* per issue.

The *Supplement* will be of the same size and priced at 10*s.* to members of the Research Section and 13*s.* to other purchasers and subscribers.

To overcome the delay in the issue of the *Journal*, the Council authorised the Honorary Editors to issue two parts in one for the latter half of 1944, and also during the publication of the volume for 1945. This procedure has done much to overtake the arrears which the war had unfortunately brought about, but printing difficulties still remain considerable.

The Library

Early in the war the Council removed many of the older volumes in the Library to places of greater safety, but retained in London such books and records as were likely to be in demand for work connected with the war effort. In all, over 6,000 volumes were dispersed, mainly to the New Bodleian Library at Oxford. For these facilities the Society is deeply indebted to the New Bodleian Library, and the Council would in particular express its gratitude to Mr. F. W. Dubber, who gave much valuable advice and assistance. By the end of 1945 all the books had been brought back and, thanks to the Society's Librarian, the arrangements made for their dispersal and subsequent return worked so smoothly that there was little interruption in the normal working of the Library. Fortunately the Society lost very few books by enemy action, and most of those so lost have already been replaced.

The end of the war will also alleviate other difficulties which have handicapped the Library in the past six years. Deficiencies caused by the interruption in the supply of books from overseas, losses in transit and the shortage of supplies during the war will take time to remedy, but the Library Committee has already had some success in making good these deficiencies.

The departure from London of the exiled Allied Government Departments, who made extensive use of the Library, lightened the load upon the limited accommodation of the Reading Room, but with the rapidly growing Fellowship, and the high proportion who regularly use the Library, it is, however, once more frequently filled to capacity. During the calendar year 1945, 284 Fellows (20 per cent. of the total Fellowship) borrowed books from the Library. These Fellows made 1,204 effective applications in person or by post and borrowed 2,458 volumes. The number of visits to the Reading Room was 2,004, as measured by signatures in the Register of Fellows and others authorized to use the Library. (The corresponding figures for the previous year were 249 Fellows borrowing books, 1,232 effective applications, 2,778 volumes borrowed and 2,213 signatures in the Register.)

Non-serial works added to the Library in 1945 amounted to 363 volumes compared with 317 in 1944.

Finance

The abstract of the Hon. Treasurer's Accounts, viz., the Statement of Income and Expenditure for the year 1945 and the Balance Sheet as at December 31st, 1945, together with the report of the Auditors thereon, are given in Appendices B and C.

The death of Lord Plender, referred to above, caused a vacancy in the office of Auditor of the Society, and the Council is glad to be able to announce that Sir Alan Rae Smith, of Messrs. Deloitte, Plender, Griffiths and Co., Chartered Accountants, has consented to accept that office. At the Annual General Meeting, Fellows will be asked to confirm that appointment.

Income other than Life Composition Fees increased from £3,029 in 1944 to £3,818 in 1945, and included increases of £359 in Annual Subscriptions of Fellows and of £419 in *Journal* and *Supplement* Sales. Expenditure increased from £2,994 in 1944 to £3,299 in 1945, and included increases of £174 in Salaries and Wages and £158 in Publication and Distribution Expenses, but there was no expenditure in 1945 on Fire Watching and Legal Expenses which cost £54 and £146 respectively in 1944. The excess of income over expenditure was £519, as compared with £34 in 1944.

At the end of 1945, the Life Composition Fee Fund amounted to £3,669, an increase of £766 in the year, and the Accumulated Fund of the Society amounted to £8,257.

During 1945, £2,000 3% Savings Bonds, 1965/75, were purchased at par, and the book value of the Society's investments at the end of the year was £11,479, as compared with a market value of £17,478.

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The Retirement of Miss Thorburn

The retirement of Miss Thorburn from the Assistant Secretaryship has already been reported in the *Journal*, but the Council cannot conclude without expressing in its Annual Report the deep feelings of gratitude for all she has done on its behalf and for the Society as a whole over the long period of twenty-seven years. At the Ordinary Meeting held on November 22nd, 1945, the President presented Miss Thorburn with a cheque for £350 subscribed by Fellows of the Society to the Testimonial fund.

The Council has appointed Miss U. M. Croker to be Assistant Secretary.

The Council and Officers

The Fellows named below, nominated in accordance with Bye-law No. 14, are recommended for election as President, Council and Officers of the Society for the Session 1946–47:—

President

The Rt. Hon. Lord Woolton, P.C., C.H.

Council

*R. G. D. Allen.
*M. S. Bartlett.
Sir Alexander Carr-Saunders.
Henry Clay.
L. R. Connor.
*†H. E. Daniels.
B. P. Dudding.
*†D. J. Finney.
C. Oswald George.
R. F. George.
Sir Gwilym Gibbon.
D. V. Glass.
*R. Glenday.
P. N. Harvey.
R. G. Hawtrey.
D. Heron.
A. Bradford Hill.
J. O. Irwin.
*†W. J. Jennett.
M. G. Kendall.
G. Rae.
E. C. Rhodes.
R. Stone
L. H. C. Tippett.
*†B. A. Welch.
G. R. White.
A. S. Windett.
J. Wishart.
J. R. Womersley.
F. Yates.

Honorary Treasurer

David Heron.

Honorary Secretaries

A. Bradford Hill. C. Oswald George. M. G. Kendall.

Honorary Foreign Secretary

A. Bradford Hill.

On behalf of the Council,
WOOLTON,
President.

A. BRADFORD HILL
C. OSWALD GEORGE
M. G. KENDALL
} *Honorary Secretaries.*

June, 1946.

Those marked * were not Members of Council during the preceding session and those marked † have never previously served on the Council.

APPENDIX A

From June 1945, to May 1946, inclusive, the candidates named below were elected Fellows of the Society.

June 1945 to April 1946:—

Allen, D. A. V.
Amery, George Douglas, M.C., M.A.
Aston, Arthur Harold.
Bailey, Walter George, F.I.A.
Banister, Reginald Francis Hugh, B.Sc.
Bayley, Gordon Vernon, Major.
Beilby, Owen John.
Bett, Walter Reginald, M.R.C.S., L.R.C.P.
Bevan, Ronald William.
Bhatnagar, Om Prakash, B.Sc.
Billingham, Leonard Richard.
Bishop, David James.
Black, John Howard.
Bloch, Victor, Dr.jur.
Blundell, Jack Edric, M.Com.
Bodkin, Kathleen Sarah.
Boxer, Sydney Cyril, B.Sc.
Bray, Mrs. Pleasance Clara.

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Bridger, Marcus.
Brookes, Bertram C.
Buckatzsch, Eric John Metius, M.A.
Burdon-Cooper, A. M., Major, M.A.
Burman, John Peter.
Burt, Archibald Reid Gunn, B.Com.
Butler, John Hugh.
Campbell, Hubert.
Carter, Charles Frederick.
Carter, William Leslie.
deCastro, Diego, Professor.
Charles, Ivor.
Cheeseman, Eric Arthur.
Chinloy, Thomas.
Clark, Dudley Albert, B.A.
Clarke, Roland David, F.I.A.
Cohen, Kenneth Leon.
Coley, Kenneth David.
Collier, George Metcalfe.
Collingwood, Ronald, Major.
Conn, John Farquhar Christie, D.Sc., M.Inst.N.A.
Cooper, Joseph Rock.
Corlett, Wilfred John.
Cornell, James Alfred.
Creed, Leslie Edwin.
Cunliffe, Stella Vivian.
Cunningham, Leslie Bennet Craigie.
Dalberg, Rudolph, Dr.phil., Dr.jur.
Datta, Kalyan Kumar, B.Sc.
Daw, Raymond, F.I.A.
Deeks, Herbert William George.
Derksen, Johannes Bernardus Dirk.
Dixon, William John.
Dobeyn, Maurice William John.
Dowbekin, Henry.
Dowman, John Parker Gilbert.
Drinkwater, Thomas Leonard.
Druiff, Philip Derek Jessel.
Durant, Henry William.
Dyke, A. J., B.Sc.
Dyke-Hart, Henry Algernon Thomas.
Dymond, Archibald William John.
Emery, Edward Thomas George.
Enters, Johan Hendrik.
Forecast, Kenneth George.
George, Phyllis Ann.
Gollop, Leslie John.
Goodier, Herbert.
Gough, Vernon Eric.
Greenwood, John Baur.
de Guingand, Noel, Major.
Halliday, Ronald James Rivers.
Halpern, Detlev Bruno, Dr.
Hammersley, John Michael, Major.
Hanks, Frederick Horace.
Harvey, Stanley Richard.
Hassan Sharib, Zahural, M.A., Ph.D., LL.B.
Hertz, Frederick.
Hilton, Peter John, B.A.
Hogben, Lancelot, Professor, F.R.S.
Holmes, Alec Richard, B.Sc.
Houchin, Leslie Frederick.
Howarth, Arthur, M.Sc.
Howell, Reginald Walter.
Hull, Philip Heber, B.A.
Hutchins, Aldred Charles.
Hynd, William Robert Brough.
Illingworth, James Ronald, B.Sc.
Jackson, Edward Francis.
Jennett, William Joseph.
Jones, Margaret.
Jones, R. D. H., Lieut.-Col.
Jowett, Geoffrey Harcourt.
Kelly, Thomas Herbert, Professor.
Kempsell, William Potter.
Kennewell, Herbert.
Kirstein, Per.
Knowles, Elsie Anna Grace.
Kora, Popatlal Dahyabhai.
Lawley, Derrick Norman.
Leppard, Anthony Herbert.
Levy, Walter, B.Sc.
Lewis, Frederick Arthur, F.I.A.
Liston, David Joel.
Lodge, Tom Stewart.
Longden, John.
Lydall, Harold French.
Lyle, Philip.
Lyne, James Frank.
McLean, George Osborne.
MacLeod, John Ratcliffe.
Madge, John Hylton.
Majumdar, Byomkes, B.Sc.
Mantell, Jack Edward.
Marley, Joan Grace, B.A.
Mars, John, M.A., M.Com., B.Litt.
Martin, Bernard John Albert.
Mason, John Isaac, M.A.
Mathison, Irena.
Mayne, George William, Major.
Meade, James Edward.
Meadows, Thomas Scruton.
Medlicott, Edward Samuel.
Merani, S. T., B.A., Ph.D.
Michaelson, Ronald Lionel, F.I.A.
Michelsen, James Edward.
Moore, Harold.
Morant, Geoffrey Miles.
Morgenstern, George, B.Sc.
Morris, Gomer Hanworth.
Mullins, A. F. J.
Musk, Alfred Stephen.
Myburgh, Corneles Albert Lloyd.
Nair, Kesavan Raghaven.
Naylor, Philip Edward.
Neat, Charles Philip Guy.
Nevard, Lawrence.
Nye, Phyllis Mary.
Oram, Arthur Talbot.
Osborn, Harold Ernest.
Outridge, Percival Albert, B.Sc., F.I.A.
Owen, George Reuben Bertram.
Owen, Kenneth, Major.
Pachachi, Nadim, Ph.D.
Padley, Richard.
Perceval, John Francis George.
Plackett, Robert Lewis.
Polwart, John Thomas Armstrong.
Pooley, Frederick Bernard.
Pratt, Harry.
Purslow, Eric Tressler, B.Sc.
Quenouille, Maurice Henri.
Rajalakshman, Devarakonda Venkata.
Rawes, George Romney, Major.
Ray, Ram Hohan.
Reisz, Andrew Bandi.
Rose, Joseph.
Royes, Osmond M.
Russell, R. H. L., B.Sc.
Safkin, Joseph.
Secord, Campbell.
Sen, Amode Ranjan.
Seng, You Poh, B.Sc.
Seymour, Thomas Hewitson.
Shanahan, Dorothy Ruth.
Sheppard, Edmund Fleetwood.

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Shimy, Reza, B.Sc.
Shooter, Nina, B.Sc.
Simpson, Edward Hugh, B.Sc.
Singer, Hans Wolfgang.
Sittig, Jan.
Slater, Patrick.
Smith, Cedric Austen Bardell.
Smith, David Hammond.
Smith, Frederick Gordon, M.A., F.I.A.
Solomon, Leon.
Spicer, Clive Colquhoun.
Spieler, Michael Henry.
Steer, Edward Thomas.
Steiner, Ernst.
Stephens, Gwyn Owen.
Sumner, Edwin Frederick Winchester, F.I.A.
Sutherland, Ian.
Suttie, Thomas Ronald, F.I.A.
Taylor, Ernest Edward.
Taylor, Jack.
Teleky, Agnes, Ph.D.
Tessler, Andrew.
Thompson, Henry Cooper.
Tomkins, George.
Tutt, Leslie William Godfrey, Major.
Twine, Sydney Walter.
Vale, Maurice Arthur Warneford, Major.
Vincent, George Kenneth, B.Sc.
Viswanathan, T. V., M.A., B.Sc.
Wakefield, David.
Walker, Andrew Morris.
Wallace, James, M.A.
Walton, John Percy Hartley, B.Sc.
Watson, Eric William.
Weatherburn, Julia Rule.
Weinberg, Sidney.
Welch, Bernard Lewis, Ph.D.
West, John Harold.
Wilkins, Leslie Thomas.
Williams, Clive Bertram.
Williams, Gwilym Ogwen.
Williams, William Idris, Capt.
Wilson, Thomas.
Worrall, Joseph Alfred.
Young, Basil.

Corporate Representatives.

Bruce, Charles Edward Rhodes, *representing* the British Electrical and Allied Industries Research Association.
Cranch, Arthur Graeme, *representing* Mather & Crowther, Ltd.
Goodman, Bruce Wilfred, *representing* Marks and Spencer, Ltd.
Hollowood, Albert Bernard, *representing* The Council of Industrial Design.
Holmes, Oscar Claude Ronald, *representing* Photo-Union.
Fraser-Stephen, Mrs. Elspet, B.A., *representing* Adprint, Ltd.
Fuller, Horace Arthur, *representing* the Commonwealth Bank of Australia.
Jennings, Leonard Godfrey, *representing* the Timber Development Association, Ltd.
Klaar, Dr. Rosi M., *representing* the Manchester Oil Refinery, Ltd.
Macneill, Malcolm Alexander Campbell, *representing* the Anglo-Iranian Oil Company, Ltd.
Orr, Mrs. Elinor, *representing* the Industrial Welfare Society, Inc.
Seland, Johan, *representing* the Norwegian Shipping & Trade Mission.
Sen, Dinendu Mohan, B.Sc., B.A. *representing* A. C. Nielsen Co., Ltd.
Taplin, Walter, *representing* the Economist Newspaper.
Todd, George Frederick, M.A., B.Litt., C.A., *representing* the Imperial Tobacco Company, Ltd.
Winter, Felix, *representing* the Economist Newspaper.

May 1946:—

Bailey, James Arthur.
Bluett, Denis Guy.
Doncaster, Alan Clement Barber.
Emmens, Clifford Walter.
Farmer, Alan James.
Gloyne, Ronald Westlake.
Gore, Muriel.
Johnstone, Muriel Macintosh, M.A.
Jones, Merlin Stephens, M.Sc., D.I.C., A.R.C.S.
Kaye, George Donald.
Keller, Norman Grace.
Mehra, Guru Datta.
Mitchell, John Robert.
Pollard, A. H.
Ray, John Hubert, A.C.I.S.
Sharp, Arthur, M.A.
Sorrell, Alec Albert, B.Sc.
Vernon, James William.
Watts, Frederick Clement, M.A.
Wheble, C. L., B.Com., M.Sc.
Woods, George Arthur.
Wootton, Ian David Phimester.

Corporate representatives.

de Launay, C.B., representing the Daily Mirror Newspapers, Ltd.
Tewson, Geoffrey E., representing the Economist Newspaper.

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APPEN

STATEMENT OF INCOME AND EXPENDITURE

EXPENDITURE.

1944 £ s. d.	1944 £ s. d.		1945 £ s. d.	1945 £ s. d.
	380 0 0	Rent		380 0 0
	229 0 6	House Expenses		231 15 6
	989 13 6	Salaries and Wages (including contribution to Staff Superannuation Scheme)		1,164 0 5
	39 0 0	Allowance		39 0 0
	157 15 3	Meetings—Ordinary and General		161 2 3
		Expenses of Sections:—		
10 1 3		Research	8 2 6	
— — —		Industrial Applications	34 1 1	
	10 1 3			42 3 7
		Publication and Distribution Expenses:—		
	632 17 0	Journal and Reprints		791 0 10
		Library:—		
43 3 4		Books	102 9 5	
39 6 1		Binding	17 19 0	
	82 9 5			120 8 5
	21 6 4	Insurance		20 16 1
	99 9 3	Stationery and Miscellaneous Printing		150 12 4
	57 3 0	Postage and Telephone		80 19 4
	— — —	Guy Medals		42 3 11
	3 5 4	Miscellaneous Items		2 14 5
	36 15 0	Auditor's Fee [1944]		36 15 0
	48 18 4	War Damage Insurance		4 9 4
	5 10 0	Emergency Transport and Storage		31 10 0
	54 12 0	Fire-watching		— — —
	146 16 2	Legal Expenses in respect of Petition for Supplemental Charter		— — —
	2,994 12 4			3,299 11 5
	34 10 11	Balance carried to Accumulated Fund: Excess of Income over Expenditure for the year 1945		519 4 8
	3,029 3 3			3,818 16 1
	388 10 0	Amount carried to Life Composition Fee Fund		766 10 0
	£3,417 13 3			£4,585 6 1

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DIX B.

FOR THE YEAR ENDED 31ST DECEMBER, 1945

INCOME.

1944 £ s. d.	1944 £ s. d.		1945 £ s. d.	1945 £ s. d.
		Annual Subscriptions:—		
1,906 16 0		Fellowship	2,265 18 0	
5 0 0		Research Section	— — —	
	1,911 16 0			2,265 18 0
	572 1 1	Sales of Journal and Reprints		948 14 2
	— — —	Journal Advertisements		3 0 0
	25 14 0	Sales of Supplement		68 16 5
	1 4 8	Sales of other Publications		1 16 0
	50 0 0	Contribution from Royal Economic Society ...		50 0 0
	457 17 6	Dividends and Interest (gross)		480 11 6
	10 10 0	Special Donation		— — —
	3,029 3 3			3,818 16 1
	388 10 0	Life Composition Fees		766 10 0
	£3,417 13 3			£4,585 6 1

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APPEN

BALANCE SHEET AT

LIABILITIES.

1944 £ s. d.	1944 £ s. d.		1945 £ s. d.	1945 £ s. d.
		Annual Subscriptions received in advance:—		
98 14 0		Fellowship	117 12 0	
— — —		Research Section	10 0	
— — —		Industrial Applications Section	99 10 0	
	98 14 0			217 12 0
	253 15 7	Payments for Journal and Supplement received in advance		509 0 10
	361 15 10	Sundry Creditors		251 9 0
		Life Composition Fee Fund:—		
2,514 15 0		Balance at December 31st, 1944	2,903 5 0	
388 10 0		*Add:* Life Composition Fees received in 1945	766 10 0	
	2,903 5 0			3,669 15 0
		Accumulated Fund of the Society:—		
7,703 5 0		Balance at December 31st, 1944, per last Accounts	7,737 15 11	
34 10 11		*Add:* Excess of Income over Expenditure for the year 1945, per annexed Statement	519 4 8	
	7,737 15 11			8,257 0 7
	11,355 6 4			12,904 17 5
		Building Fund (per contra):—		
1,027 16 0		Balance at December 31st, 1944	1,066 19 7	
39 3 7		*Add:* Income for year 1945	40 9 11	
	1,066 19 7			1,107 9 6
		Frances Wood Memorial Fund (per contra):—		
455 2 2		Balance at December 31st, 1944	483 2 11	
28 0 9		*Add:* Income for year 1945	28 13 9	
	483 2 11			511 16 8
	£12,905 8 10			£14,524 3 7

Note.—No value is placed in the Accounts on (1) Journals and other Publications in stock, (2) Books in Library, and (3) Pictures, Furniture and Equipment.

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DIX C

31ST DECEMBER, 1945

ASSETS.

1944 £	s.	d.	£	s.	d.		1945 £	s.	d.	£	s.	d.
						Investments, at cost or under:—						
5,580	*0*	*0*				£10,527 12*s.* 3*d.* 2½% Consols (Guy Bequest)	5,580	0	0			
1,185	*0*	*0*				£2,236 11*s.* 3*d.* 2½% Consols	1,185	0	0			
1,324	*0*	*0*				£1,864 14*s.* 1*d.* 3½% Conversion Loan ...	1,324	0	0			
490	*0*	*0*				£500 3½% War Loan	490	0	0			
800	*0*	*0*				£1,169 17*s.* 6*d.* 3% Local Loans Stock ...	800	0	0			
100	*0*	*0*				£666 4% 2nd Prefd. Stock, London and North-Eastern Railway	100	0	0			
—	—	—				£2,000 3% Savings Bonds, 1965–75	2,000	0	0			
			9,479	*0*	*0*					11,479	0	0
						(Market value, December 31st, 1945, less Interest accrued, £17,478)						
			102	*3*	*10*	Interest accrued on Investments (gross) ...				117	3	10
			1,457	*3*	*7*	Cash at Bank and in hand				1,011	7	6
						Arrears of Subscriptions recoverable (estimated)				84	0	0
			84	*0*	*0*							
			70	*3*	*11*	Sundry Debtors and amounts paid in advance				188	14	11
			162	*15*	*0*	Stock of Journal Paper				24	11	2
			11,355	*6*	*4*					12,904	17	5
						Building Fund:—						
			1,066	*19*	*7*	£1,185 14*s.* 7*d.* 3½% Conversion Loan ...				1,107	9	6
						(Market value, December 31st, 1945, £1,257)						
						Frances Wood Memorial Fund:—						
300	*0*	*0*				£600 4% Preference Stock, London, Midland and Scottish Railway	300	0	0			
						(Market value, December 31st, 1945, £462)						
171	*2*	*11*				Post Office Savings Bank Deposit	199	16	8			
12	*0*	*0*				Income Tax Refund due from Society ...	12	0	0			
			483	*2*	*11*					511	16	8
			£*12,905*	*8*	*10*					£14,524	3	7

REPORT OF THE AUDITORS

We have examined the foregoing Statement of Income and Expenditure and Balance Sheet with the Books and Records of the Society. We have verified the Investments and Cash appearing in the Balance Sheet. We report that the above Balance Sheet is, in our opinion, properly drawn up so as to exhibit a true and correct view of the state of the affairs of the Society, according to the best of our information and the explanations given to us, and as shown by the Books and Records.

A. RAE SMITH, *Chartered Accountant*, Auditor.
A. S. WINDETT } Honorary
F. E. RICHMOND } Auditors.

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PROCEEDINGS OF THE ONE HUNDRED AND TWELFTH ANNUAL GENERAL MEETING OF THE ROYAL STATISTICAL SOCIETY, HELD AT THE LONDON SCHOOL OF HYGIENE AND TROPICAL MEDICINE ON THURSDAY, JUNE 27TH, 1946, AT 5 P.M.

The Chair was taken by the PRESIDENT, THE RT. HON. LORD WOOLTON, P.C., C.H.

The PRESIDENT presented the Report of the Council for the financial year 1945 and the Session 1945–46, drawing attention to items of special interest. He referred to the award of the Guy Medal in gold to Professor R. A. Fisher, F.R.S., who was unfortunately unable to be present that day, but would be presented with the medal at the Ordinary Meeting in November.

He said that the new Bye-laws of the Society would be laid before a Special General Meeting in the autumn. There would also be presented to that Meeting the report of the Council on an alternative scheme for conducting examinations.

The PRESIDENT then moved the adoption of the Report.

The HONORARY TREASURER, Dr. David Heron, presented the accounts, and seconded the motion for the adoption of the Report which, on being put to the Meeting, was carried unanimously.

A cordial vote of thanks to the President and retiring Council was proposed by Mr. D. J. Finney, seconded by Dr. Isserlis, and carried unanimously. On the proposal of Sir Gwilym Gibbon, special tribute was paid to Professor Bradford Hill, as an Honorary Secretary, for his work during the session.

The PRESIDENT announced that as a result of the ballot taken during the Meeting the President, Council and Officers nominated for the ensuing session were all declared elected.

The HONORARY TREASURER proposed that Sir Alan Rae Smith be appointed Auditor for the next session and that confirmation be given to the action of the Council in appointing him to succeed the late Lord Plender.

Sir William Elderton seconded the motion, which was carried unanimously.

The Meeting then terminated.

PROCEEDINGS OF A SPECIAL GENERAL MEETING OF THE ROYAL STATISTICAL SOCIETY, HELD AT THE LONDON SCHOOL OF HYGIENE AND TROPICAL MEDICINE ON THURSDAY, NOVEMBER 7TH, 1946, AT 5.15 P.M.

The Chair was taken by the PRESIDENT, THE RT. HON. LORD WOOLTON, P.C., C.H.

The PRESIDENT, in referring to the documents * before the Meeting, said that it was a matter of considerable disappointment to the Council that he had now to ask the Meeting to withdraw the Petition for a Supplemental Charter to enable the Society to hold examinations and to award Diplomas and Certificates as a result of those examinations. The Council were fully satisfied that the Society had no alternative but to take this course.

He then moved

> "That the Petition of the Society for a Supplemental Charter, submitted to His Majesty in Council by authority of the Special General Meeting of the Society held on October 17th, 1944, be withdrawn."

Dr. E. C. Snow seconded the resolution which, on being put to the Meeting, was carried unanimously.

The HONORARY TREASURER, Dr. David Heron, introduced the new Bye-laws to the Meeting, drawing attention to the main changes. Under Bye-law No. 3 candidates might be proposed from general or from personal knowledge, and corporate bodies are permitted only one representative. In Bye-law No. 5 a new scale of composition fees was proposed, and the number of annual subscriptions which, in special circumstances, the Council might remit, was increased from five to ten. It was recommended, by deleting the present Bye-law No. 6, that the formal admission of new

* Memorandum on the Proposal to be laid before the Special General Meeting of November 7th, 1946. *See* pp. 492 *et seq.*

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Fellows at Ordinary General Meetings be abolished. In the new Bye-law No. 7 the Council was given power to deal with defaulters, and, in No. 8, to reinstate in approved cases. In No. 9 it was proposed that British subjects resident in this country might now be eligible for election as Honorary Fellows.

The procedure proposed for the election of the Council, given in the new Bye-laws 13, 14 and 15, was the outcome of recommendations made by a committee of six members of Council and six other Fellows which was appointed to consider the matter, as the result of a suggestion made at the Annual General Meeting in 1945.

In regard to Bye-law No. 16, the Council recommended the abolition of the office of Honorary Auditor. In Bye-law No. 36 the Council asked authority to pay the travelling expenses of those attending meetings of the Council and Committees. Bye-law No. 37 recognized the establishment of Sections for special purposes. In Bye-law No. 38 the Council asked authority to hold examinations for the purpose of testing the proficiency of such Fellows as might care to submit themselves for examination. Bye-law No. 42 set out the procedure for altering the Society's Bye-laws when this proved necessary.

Dr. Heron then moved

> "That the Bye-laws contained in the printed documents submitted to the Meeting and for the purpose of identification subscribed by the Chairman of the meeting be approved and adopted as the Bye-laws of the Society in substitution for, and to the exclusion of, all the existing Bye-laws thereof."

Mr. M. G. Kendall seconded the resolution.

The new Bye-laws were then submitted *seriatim*, and discussed.

They were formally adopted by the Meeting with one verbal amendment proposed by Mr. Day and seconded by the President (the deletion of the words "present and" from the phrase "present and voting" in Bye-law No. 42).

The Proceedings then terminated.

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MEMORANDUM ON THE PROPOSAL LAID BEFORE THE SPECIAL GENERAL MEETING OF 7th NOVEMBER, 1946, THAT THE SOCIETY'S PETITION FOR A SUPPLEMENTAL CHARTER BE WITHDRAWN.

FOLLOWING the resolution approved by a Special General Meeting held on October 17th, 1944, the Royal Statistical Society presented a Petition to his Majesty in Council praying for the grant of a Supplemental Charter to allow the Society to hold examinations and to award Diplomas and Certificates. At the Special General Meeting to be held on 7th November, 1946, the Society will be invited to pass a resolution in favour of withdrawing this Petition. In inviting Fellows to take this step the Council feels it essential to explain to them in full the circumstances which have led up to it and to place before them all the relevant documents upon which its decision has been based.

Following the Society's Petition for the necessary Supplemental Charter (reproduced in Document A) a Counter-Petition was presented to the Privy Council by fourteen signatories (Document B). This Counter-Petition was subsequently supported in a letter addressed to the Clerk to the Privy Council on behalf of the Council of the Royal Economic Society (Document C).

The Council of the Royal Statistical Society disagreed fundamentally with the argument set forth in the Counter-Petition and its then President (Dr. E. C. Snow) and the other Honorary Officers drafted a proposed reply to it (Document D). These observations were, however, not placed before the Council of the Society nor otherwise proceeded with. They should, therefore, be read merely as illustrative of the form of answer which, in the opinion of the then President and Honorary Officers, would have been appropriate. No further action was taken in this respect as the Council was advised that it was most unlikely that the Society would be granted the Supplemental Charter desired so long as any serious opposition to it was maintained. The Council, therefore, turned its attention to an endeavour to reach some compromise with the signatories to the Counter-Petition and the Royal Economic Society which would allow the Royal Statistical Society's main proposals to go forward unopposed. In this it was unsuccessful. A joint meeting between representatives of the two Societies showed that the Royal Economic Society was irrevocably opposed to the Royal Statistical Society's scheme. In view of this the Council of the Royal Statistical Society reluctantly concluded that it would be idle, as well as extremely costly, to press any further the matter of its Petition. It, therefore, directed its Honorary Officers, Dr. E. C. Snow, and Professor Major Greenwood, serving as a Committee of the Council, to consider possible alternative schemes which would be in conformity with, and at least partially fulfil, the Society's expressed wishes.

This Committee, after taking legal advice, has advised the Council that the Society has powers under its present Charter to set examinations for any of its own Fellows who wish to submit themselves to such a test of their statistical proficiency (Document E). They accordingly recommended that a new Bye-law to read as follows be laid before a Special General Meeting:—

> "The Council may at any time cause examinations to be held for the purpose of testing the statistical proficiency of such Fellows as may desire to submit themselves for examination. The Council shall determine the subjects to be comprised in such examinations, the manner in which such examinations shall be conducted, the fees to be paid to examiners whether Fellows or not and the fees to be paid by examinees. Such examinations shall be open to every Fellow and every successful examinee shall be granted a certificate stating the nature of the examination which he has passed."

The Council unanimously adopted this report, believing that even this limited scheme does much to meet the wishes of the Society. They have included this Bye-law in the draft new Bye-laws to be submitted to the Special General Meeting on 7th November, 1946, a full copy of which is enclosed. If this procedure should have the approval of the Special General Meeting it will be necessary also to pass a formal resolution withdrawing the Petition for a Supplemental Charter.

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DOCUMENT A

The Humble Petition of the Royal Statistical Society

Sheweth as follows :—

1. That Your Petitioners are The Royal Statistical Society incorporated by Royal Charter bearing date the 31st day of January, 1887 (hereinafter referred to as " the Society ").

2. That by the said Royal Charter (hereinafter referred to as " the Royal Charter ") Her late Majesty Queen Victoria did grant and declare that the persons who were then members of The Statistical Society or who shall from time to time be elected Fellows of the Society thereby incorporated should form one body politic and corporate for the purpose of the Original Charter under the name of The Royal Statistical Society with perpetual succession and a common seal.

3. That the Society has continuously throughout its existence of 57 years carried out the objects of the Original Charter.

4. That the field of work which can properly be described as statistical has widened considerably since the granting of the Original Charter and will become wider. During the last 20 years there have been striking developments in the use of statistical methods in the business of Government, in Science, and in Industry and Commerce which have resulted in a large increase in the number of those engaged in statistical work for a livelihood.

5. That there is at present no recognised status in the profession of statistics and no standard by which the attainments or qualifications of a statistician can be judged, nor any generally recognised divisions or descriptions of the various types of statisticians.

6. That this is unsatisfactory for employers, employees and the general public. The employer who requires a secretary, an accountant or an actuary has clear and well defined standards of reference. If he needs a statistician he has no guide but has to rely upon personal recommendations or attempt a judgment of statistical ability by relation to other standards, *e.g.*, academic distinction in mathematics. It is unsatisfactory to the employee when there are no standards by which he can judge the level of his own attainments and no general approved method of distinguishing between a genuinely accomplished man and one who is not. During discussions with Your Majesty's Treasury on the Society's Report on Official Statistics, official representatives emphasised strongly the importance they would attach to the existence of approved qualifications if it were decided to set up a grade of professional statistician in the Civil Service. They alluded to the accepted qualifications in the field of accountancy and pointed out how useful it would be to Departments considering the appointment within the Civil Service of officers to responsible positions in statistical branches, if some similar statistical qualifications were in existence.

7. That the absence of such a recognised status is unsatisfactory for the general public who are affected far more than they realize by bad statistical work. Where a man lacking the necessary statistical knowledge can gain an employer's confidence important decisions on his reports may be made and grave damage to the public welfare result.

8. That most Universities offer degrees for which statistics is an optional subject but in only a few cases is it a compulsory one, and the syllabuses in statistics for these examinations vary greatly in scope. At some Universities an advanced standard is required in the examination in statistics, whereas in others the examinations are intended to be a test of only an elementary knowledge of statistical methods. There are no professional examinations which provide for statisticians as a whole. While the Universities must continue to be the main source of training in statistics there is a group of persons who do not become interested in statistics until they have made considerable progress in their Civil Service, business or other careers and are thus generally prevented from attending University courses with a view to graduating. Many of those who are most anxious to obtain evidence of proficiency in statistics are not in a position to obtain the wider educational qualifications indicated by a University degree. For such persons there is need for adequate test and certificate of purely technical and professional attainments relating to their particular profession of statistics.

9. That the profession of statistics has come to take such an important part in the business of Government and in the progress of Science and the Manufactures, Industries and Commerce of the British Empire that any obstacle which tends to discourage a proper appreciation of its importance

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in modern conditions should be removed and provision should be made for the proper recognition of the professional status of statistics.

10. That for these and for other reasons it is desirable that the Society should be empowered to cause examinations to be held of candidates desirous of obtaining a recognised status in the profession of statistics (whether Fellows of the Society or not) and to confer on the candidates who pass the examinations to the approval of the Society a diploma in statistics and to issue a certificate to those candidates who pass a part of such examinations and that a successful candidate should be permitted to describe himself in his professional or scientific work as the possessor of a Certificate or Diploma and that the holder of the Diploma may use after his name the letters " Dip. Stats." (representing the words Diploma in Statistics of the Royal Statistical Society).

11. That at the special meeting of the Society duly convened constituted and held on the 17th day of October, 1944, at Keppel Street, London, W.C.1, the draft of the Petition now being presented and the draft of the proposed Supplemental Charter which accompanies the said Petition (both of which drafts were initialled by Ernest Charles Snow, C.B.E., D.Sc., President, for the purposes of identification) were adopted and approved and it was resolved that the seal of the Society be affixed to the said Petition and that the said Petition be presented to Your Majesty.

12. That Your Petitioners being fully aware of the many advantages which have resulted from the Original Charter and being desirous of securing the means for obtaining a recognised status for the profession of statistics desires that Your Majesty may be graciously pleased to command that as and from a certain date to be fixed by Your Majesty the Council of the Society by the grant of a Supplemental Charter may be entitled to:—

(i) Cause examinations to be held of all candidates applying for a certificate of the Society or the Society's Diploma in Statistics, whether Fellows of the Society or not.

(ii) Appoint examiners for that purpose and determine the conditions of these appointments and their remuneration from time to time be prescribed by such bye-laws.

(iii) Upon the report of the examiners grant to every person who shall satisfactorily pass an examination for the Society's Certificate or such other examination as the Council may approve a certificate that he has passed the same.

(iv) Confer on every person who has satisfactorily passed the Diploma examination of the Society or such other examination as the Council may approve the Society's Diploma in statistics in such form as the bye-laws of the Society may prescribe.

(v) Permit a person on whom the Society's Diploma in statistics has been conferred to write after his name the letters " Dip. Stats." (referring to the Diploma in Statistics of the Royal Statistical Society).

(vi) Provide and collaborate with others in providing suitable tuition for the said examination.

13. That by the Supplemental Charter Your Majesty may be graciously pleased to extend the purposes for which bye-laws of the Society may be made under the Original Charter to include:—

For regulating the manner and times and places of holding examinations for a certificate and for a diploma in statistics and the subjects for and the manner of conducting all or any such examinations and for fixing reasonable fees to be paid by candidates and the conditions on which the Examiners shall hold office and their remuneration and for providing or collaborating with others in providing suitable tuition for the said examinations.

14. The Society most humbly prays that Your Majesty will be graciously pleased to confer by a Supplemental Charter upon the Society the powers and privileges mentioned in this Petition and in the draft humbly submitted to Your Majesty or such other power and privileges as to Your Majesty may seem fit and in other respects to conform to the Original Charter.

AND YOUR PETITIONERS as in duty bound will ever pray, etc.

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DOCUMENT B

COUNTER-PETITION TO THE PRIVY COUNCIL

IN THE MATTER *of the Petition presented to His Majesty in Council by the* ROYAL STATISTICAL SOCIETY *Praying for the Grant of a Supplemental Charter.*

To

THE KING'S MOST EXCELLENT MAJESTY IN COUNCIL.

THE HUMBLE PETITION of the undersigned University Teachers, Statisticians and Economists

Sheweth as follows :—

1. Your Counter-Petitioners are University Teachers, Statisticians and Economists and for the reasons hereafter assigned oppose the prayer of the Petitioners.

2. Your Counter-Petitioners do not dispute the statements of fact contained in paragraphs 1, 2, 3, 4, 5, 6, and 11 of the Petition but as to paragraph 11 observe that the said meeting was held on a weekday afternoon in war time when not more than 12 per cent. of the Fellows were present and no proxies were allowed and no written communication read. Many of the most eminent Fellows were either precluded by their duties from being present or were abroad on State service.

3. Your Counter-Petitioners recognize that the petitioners are actuated by the desire to advance the professional status of certain of their Fellows and others.

4. Your Counter-Petitioners consider that the steps to be taken to achieve the ends envisaged in paragraphs 6, 7, 8 and 9 of the Petition involve many other learned bodies, especially the Universities, whose interests are involved but who have not been consulted.

5. Your Counter-Petitioners feel it their duty to place before Your Majesty certain considerations for a reason which becomes apparent on referring to the suggested syllabus for the proposed examinations. Part I has to be taken before a certificate is granted. Part I consists of a paper in a subject in Group A and in Group B. Group B includes papers designed to show that the candidate has "a knowledge of the application of statistical methods" to one or other of the following inter alia: (3) Actuarial Science (4) Medical Science (5) Agricultural Science (6) Biometry and Genetics (7) Economics (12) Meteorology (13) Psychology (14) Sociology and so forth.

6. It is not disputed that statisticians are concerned with such matters nor is it disputed that any examination which sought to cover all the subjects statisticians might have to be acquainted with would have to include not merely questions designed to test "a knowledge of the application of statistical methods to" a particular science or art but questions designed to test a knowledge of the subjects in question, *e.g.*, Actuarial Science or Economics or Psychology. Thus there are other learned bodies concerned.

7. Your Counter-Petitioners are of the opinion that unless the matter is very carefully considered from all points of view the result might be unfortunate and the Society's Certificates and Diplomas be the cause of persons being misled not through any fault of the Petitioners but from the nature of the case.

8. The term "Statistician" is not a definite expression. Originally statistics denoted the collection of data and the formulation of estimates relating to social and economic matters involved in State policy but now a statistician may be concerned with matters having nothing whatever to do with the State. Some statisticians are hardly to be distinguished from pure mathematicians; others (equally eminent but concerned with an entirely different kind of statistical enquiry) are hardly concerned with mathematics at all, the object of their work being the collection and tabulation of quantitative data. Such men's eminence is due to skill, experience and knowledge in fields entirely different from that of the first-named class. Yet both classes are referred to as statisticians. The second class might include men whose real ability would not lie in mathematical or even primarily in statistical enquiry but in a knowledge of subjects such as those referred to in paragraph 5. Others would need a wide acquaintance with business or international finance and so forth.

9. Your Counter-Petitioners are of the opinion that an examination designed to test competence

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in such a wide field would involve the testing of knowledge in fields remote from that with which the Petitioners are concerned; they are further of the opinion that a certificate or a diploma of a purely statistical body might be most misleading unless the person accepting it as a guide of competence appreciated that a statistician might be an admirable and experienced statistician for one purpose and quite incompetent and inexperienced for another purpose even though both purposes would normally be referred to as a "statistical enquiry."

10. The Counter-Petitioners therefore most humbly pray that Your Majesty will be graciously pleased to refer back the Petition to the Petitioners for further enquiry and consultation with the other learned bodies concerned before conferring the powers prayed for.

AND YOUR COUNTER-PETITIONERS as in duty bound will ever pray, etc.

JOHN JEWKES
Professor of Social Economics
University of Manchester.

LIONEL ROBBINS
Professor of Economics in the
University of London

E. A. G. ROBINSON
Lecturer in Economics in the
University of Cambridge

S. R. DENNISON
Professor of Economics

R. R. KUCZYNSKI
Statistician

A. K. CAIRNCROSS
Lecturer in Political Economy in the
University of Glasgow

E. DEVONS
Statistical Economist (Temporary Civil Servant)

RICHARD M. TITMUSS
Statistician

H. C. B. MYNORS
An Adviser to the Governors
Bank of England

GEOFFREY CROWTHER
Editor of The Economist

D. G. CHAMPERNOWNE
Reader in Statistics at
Oxford University

F. A. HAYEK
Tooke Professor of Economic Science and Statistics
in the
University of London

J. E. MEADE
Civil Servant

HUGH WEEKS
Statistical Manager (Temporary Civil Servant).

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DOCUMENT C

COPY OF LETTER FROM SECRETARY OF THE ROYAL ECONOMIC SOCIETY TO THE CLERK TO THE PRIVY COUNCIL

16th June 1945.

Dear Sir,

At a recent meeting of the Council of the Royal Economic Society the Council had under consideration the Petition that has been made to the Privy Council by the Royal Statistical Society for a Supplemental Charter which shall empower the Society (inter alia) to hold examinations and confer diplomas or certificates. The Council instructed me to write to you on their behalf, asking that, before such Supplemental Charter be granted, full investigation be made, and full opportunity be given both to the Universities and to other Learned Bodies (including the Royal Economic Society) to examine the possible consequences of any such grant.

In making such a request, the Council of the Royal Economic Society has a two-fold interest: first, as a body representing a very large number of persons working in the field of statistics, as applied to problems of economics, industry and commerce; second, as a body in close touch with the Teaching Faculties in Economics in the Universities of this country.

The ground of the Royal Statistical Society's application is that "there is at present no recognized status in the profession of statistics and no standard by which the attainment or qualifications of a statistician can be judged, nor any generally recognised divisions or descriptions of the various types of statisticians," and "that this is unsatisfactory for employers, employees and the general public."

The term "statistician" has been applied in the past, and whatever may now be decided is likely to continue to be applied, to persons with a very wide range of qualifications and aptitudes. In the early part of the nineteenth century (and the use persists even today) the term was almost synonymous with that of "applied economist." The great contributors to our system of national statistics possessed few of the mathematical and technical qualifications on which the Royal Statistical Society would wish to insist. We feel doubtful whether it is right or useful to attempt to grade statisticians as if they constituted one homogeneous profession.

The term covers a great many workers in industry and commerce who are applying to economic problems relatively simple techniques of measurement and comparison. The necessary qualification for many of these posts in addition to judgment, common sense, experience, administrative ability, is a theoretical and practical knowledge of the subject with which they have to deal, rather than knowledge of mathematical techniques which may have little or no relevance to the tasks that they have to perform. We fear lest, if the Charter be granted, it becomes a means of excluding from posts in the Civil Service and in other spheres persons who are more, and not less, qualified than those who may receive these diplomas or certificates. In particular, we fear lest they may debar from the opportunity of most useful employment a number of persons with a good general training in economics, who are fully competent to handle the statistical problems of the posts in question, and more competent to interpret and analyse the statistics.

We fear also lest the application of a single standard, differentiated only in detail, may take insufficient account of the varied needs of the various types of statisticians to whom the Petition alludes. The field of application of the scientific methods of measurement and statistical analysis is extending constantly. We feel some doubts whether a body representative primarily of the general techniques of statistical investigation rather than of the subjects to which such techniques may be applied, can be so fully sensitive to the varied and developing needs of all the branches of learning (including the physical and biological sciences) as other bodies (including the University Faculties) more directly concerned in the application of statistical methods to particular fields. Nor, in our view, would it be right to lay undue stress on present difficulties of assessing the qualifications of statisticians in the higher branches of statistical work. Many Universities already have examinations which test, inter alia, statistical qualifications, and information regarding these is fairly readily available.

We have viewed this problem also from the angle of those concerned with the teaching of economics in the Universities and elsewhere. A large number of those who study economics

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obtain their first employment in the statistical organisations of the Civil Service or industry. If the possession of a certificate or diploma became the key to such a career, the Universities, however much they might disagree with the form and content of the schedule of the examinations conducted by the Royal Statistical Society, or the emphasis placed on statistical technique as compared with other more general training in economics or in the kindred subjects which contribute to a broad education, might find themselves obliged to provide teaching for these examinations. We observe that the Royal Statistical Society has made no reference to the problem of teaching for its proposed examinations; it is our view that the whole question of their relation to University teaching requires to be most carefully examined by the Universities themselves before any final conclusion is reached.

For these reasons we ask that, before the Privy Council grant the Supplemental Charter, full opportunity is given both for the Universities and for other learned bodies (including ourselves) to be satisfied that their common interest in improving the standards of statistical teaching and competency will be promoted by the grant.

Yours truly,

E. A. G. ROBINSON

Secretary to the Royal Economic Society.

Clerk to the Privy Council,
Whitehall,
London, S.W. 1.

DOCUMENT D

ROYAL STATISTICAL SOCIETY

DRAFT OBSERVATIONS ON THE COUNTER-PETITION DRAWN UP BY THE HONORARY OFFICERS AND THE PRESIDENT THEN IN OFFICE (DR. E. C. SNOW)

1. The Petitioners have carefully considered the observations in the Counter-Petition which is signed by fourteen persons (University Teachers, Statisticians and Economists) who acted in their individual capacities and not as representing any learned or other bodies.

2. The grounds of objection are :—

(a) that the petition was approved at a Special General Meeting of the Society which "was held on a week-day afternoon in war time when not more than 12 per cent. of the Fellows were present and no proxies were allowed and no written communication read." The argument appears to be that the approval given to the submission of the petition was not representative of the views of the Royal Statistical Society as a whole. (Para. 2 of the Counter-Petition)

(b) that other learned bodies are affected by the petition and should have been consulted (Paras. 4–6 of the Counter-Petition)

(c) that as the word "statistician" covers a wide range of attainments the granting of Diplomas or Certificates might be misleading (Paras. 7–9 of the Counter-Petition).

The Petitioners' comments on these subjects are as follows :—

3. The procedure for seeking an amendment to the Society's Charter is laid down in the Charter itself, and the associated Bye-laws :—

(a) The Petition was circulated to all Fellows resident in the United Kingdom three weeks in advance of the meeting at which it was approved. There is no power to admit votes by proxy.

(b) Meetings of the Society have always been held on a week-day in the late afternoon and the Council saw no reason (and still sees no reason) for expecting any greater attendance by holding this Special Meeting in the morning or evening or at a week-end.

(c) The attendance of Fellows at the meeting was greater than at any Ordinary or Annual General Meeting held in the last 25 years.

(d) At the meeting of the Society on 17th October, 1944, the voting was very strongly in

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favour of the Petition. An amendment which would have negatived the resolution to submit the Petition was defeated by 105 votes to 16 and a second amendment, which would have deferred action until there had been full consultation with (i) other learned bodies with common interests and (ii) the Universities in the United Kingdom, was defeated by 90 votes to 20. On the main motion to submit the Petition, the number of dissentients fell to 10.

4. The Petitioners accordingly strongly deny any suggestion that the submission of the Petition was not in accordance with the wishes of the great majority of Fellows of the Society.

5. Paras. 5 and 6 of the Counter-Petition, which refer to the draft syllabus,* appear to be written under a misapprehension. Thus in Para. 5 it is stated that in the suggested Syllabus "Part I consists of a paper in a subject in Group A and in Group B." Actually the suggested Syllabus would require candidates for Part I to take one paper on each of three sections of Group A and one paper on one section of Group B. Further Para. 6 says "nor is it disputed that any examination which sought to cover all the subjects statisticians might have to be acquainted with would have to include not merely questions designed to test 'a knowledge of the application of statistical methods to' a particular science or art but questions designed to test a knowledge of the subjects in question." The Petitioners do not make any claim of this kind, and would certainly dispute the implication of this quoted sentence. It is not part of the proposed system that they should examine in subjects outside their competence, and it is specifically stated in the suggested syllabus that the examination would be directed to the application of statistical methods to various subjects. The Counter-Petitioners' argument appears to be that one cannot examine in such methods without examining in the subject to which they are applied. If this is a correct interpretation (and the Petitioners can see no other) the argument is not admitted. It might as well be claimed that one cannot examine on the Law of Contract without examining on the economics of commercial practice.

6. The Petitioners have recognized that in examining on the application of statistical methods they would have to enlist the assistance not only of statisticians versed in the subject concerned but also of experts in that subject. In dealing with the application of statistical methods to economics, for example, they had in mind that the examiners would include statisticians whose primary interest is economics.

7. As to the points raised by the Counter-Petitioners concerning consultation with other learned bodies :

(a) In the first place, the Petitioners wish to emphasize that they are not primarily concerned with persons who have had a University education. They are endeavouring to cater for any person who wishes to take up statistics as a profession at any stage of his career, even though he has not had the opportunity of obtaining a University degree. Part I of the proposed examinations (which would qualify the successful candidate for a certificate) was mainly designed for the junior employee of a firm or a junior Civil Servant who goes to work immediately on leaving school. No such technical qualification is anywhere available at the present time and the Petitioners think that there would be a much larger number of candidates for this Part, which is intended for those who normally do not aspire to a University degree, than for Part II.

(b) It is recognized that students who read Statistics at a University may not require to take the Diploma, which is regarded as complementary to, rather than in competition with, equivalent University qualifications. The Petitioners, indeed, hope that more men and women will in future study statistics as such at Universities and that the facilities for doing so will be extended.

(c) As regards the Universities themselves, careful consideration was given by the Council to the question of formal consultation. It was felt to be unnecessary for two reasons apart from those indicated in previous sub-paragraphs. First, practically every University teacher of statistics is a Fellow of the Society and could express his views direct. Second, a number of prominent University teachers were on the Committee which framed the Report on the

* The syllabus referred to did not form part of the Petition but was appended to a report of a Committee of the Council which recommended the proposals embodied in the Petition. The draft syllabus was given for purposes of exemplification rather than as a final document and has always been regarded as such. This, however, does not materially affect the Counter-Petitioners' point or the Petitioners' reply.

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Status of Statisticians, and a number of others (such as Dr. A. L. Bowley, Professor Emeritus of Statistics of the University of London, and Mr. G. Udny Yule, F.R.S., formerly reader in Statistics at the University of Cambridge) were consulted and indicated their complete agreement.

8. Paras. 7 to 9 of the Counter-Petition appear to argue that because statisticians are of various different types and cover a wide field of interests and attainments, they are not entitled to any professional qualifications. This argument proves too much. It is equally valid (or rather invalid) against qualifications for any profession with a diversity of application, such as law or medicine.

The fact that some lawyers specialize in criminal practice, in marine law or in company law has never been regarded as a reason why they all should not undergo examinations on the general theory of law and its various branches before being allowed to practise. Further a man might be a Bachelor or Doctor of Law but could not practise as a solicitor since the Law Society insists on practical experience although it gives certain exemption to graduates in the same way as the Society proposes to do. The fact that a medical man may be a general practitioner, a surgeon, an ear-nose-and-throat specialist, or a Medical Officer of Health, does not exempt him from the necessity for general qualifications. Yet the diversity among statisticians is no greater than in these two cases. There is a certain minimum of knowledge which every statistician should have, and in addition he should have special knowledge of the statistical methods of the field or fields in which he proposes to practise. The Petitioners are not seeking authority to limit the practice of statistics to persons with special qualifications nor have they any intention of doing so and they do not in any way claim that the qualifications they propose should be the only ones in existence. All that they are proposing is that a person with such knowledge should be allowed to obtain a Certificate or Diploma from the Royal Statistical Society—the most competent body for the purpose—that he possesses it.

9. The Counter-Petitioners claim that the issue of such a certificate or Diploma might be misleading, though they do not say to whom. On this it is only necessary to say:—

(a) that employers are not nearly so likely to be misled with certificates of proficiency in evidence as they are at the present time, when any person can claim to be a statistician on no evidence at all;

(b) that it is provided in the proposed examinations that certificates or diplomas shall be endorsed with the special subjects taken, so that an employer has only to ask to see a candidate's diploma or certificate to find out what qualifications he has;

(c) that it is proposed to make practical experience a condition of the issue of a Diploma so that an employer would always know what type of work the applicant was used to.

10. In fact, this objection also appears to the Petitioners to be not so much a specific objection to their Petition as a general objection to any sort of professional qualification and *a fortiori* to an academic qualification. It further appears to contemplate a degree of ignorance or incompetence on the part of employers which the Petitioners do not believe to exist.

11. The Petitioners would like to add two further observations:—

(a) that every employer of statisticians who has expressed an opinion to them on this subject, including H.M. Treasury, has been very definitely in favour of the proposals for professional qualifications; indeed H.M. Treasury emphasized the assistance which they would derive from any system which conferred qualifications acceptable by employers; and

(b) that the Institution of Professional Civil Servants, in a memorandum issued on February 10th, 1945, have warmly welcomed the Petitioners' proposals.

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DOCUMENT E

ROYAL STATISTICAL SOCIETY

REPORT OF COMMITTEE ON ALTERNATIVE EXAMINATION PROPOSALS

At its meeting on 13th March, 1946, the Council received a report of the joint meeting between the Society's representatives and those of the Royal Economic Society, held on 12th February, 1946, which showed that the Royal Economic Society was irrevocably opposed to the Royal Statistical Society's scheme for holding examinations as set out in its petition for a Supplemental Charter. The Council thereupon appointed a Committee consisting of the three Honorary Secretaries, the Honorary Treasurer and Dr. Snow to consider possible alternative schemes which would be in conformity with the Society's expressed wishes. Professor Greenwood was subsequently added to the Committee.

It was intended that the proposals of the Committee, if approved by the Council, should be submitted to the 1946 Annual General Meeting, but the Committee is only now able to submit its report.

Your Committee felt that, in view of the very large majority at a Special General Meeting in favour of the Council's proposal to institute examinations for statisticians, it was necessary to make every possible effort to obtain a satisfactory alternative scheme. It was encouraged in this view by the statement in the Treasury Circular on "Statisticians in the Civil Service" (E.O.C 21/46 of 21st March, 1946) to the effect (para. 13) that if the Society's proposal to institute a specific qualification for statisticians who have not taken a University degree were carried into effect, "the Treasury would wish to consider training policy in that light." A copy of that circular is attached.*

Your Committee has concluded that a Supplementary Charter is most unlikely to be granted if it is opposed by the Royal Economic Society, by any body of similar standing or, indeed, by a comparatively small number of persons who are not even Fellows of the Society. It therefore feels that any alternative scheme must be such as could be carried into effect under the powers granted by the present Charter.

Three proposals which, it was hoped, might satisfy this condition were considered by your Committee. These were:—

(a) the Society to hold examinations for the purpose of testing the statistical proficiency of such Fellows as may desire to submit themselves for examination,

(b) to throw open the examinations referred to in (a) to candidates for Fellowship and to authorize the Council to recommend for election as Fellows candidates who have passed such examinations, and

(c) to establish a class of Associate (and/or Licentiate) with privileges less than those enjoyed by Fellows and to test by appropriate examinations candidates for election as Associates (and/or Licentiates).

After considering these proposals your Committee felt that it was essential to obtain the opinion of Counsel in regard to them. A copy of the case prepared by the Society's Solicitors for submission to Counsel and a copy of the opinion expressed by Mr. R. T. Monier-Williams of the Inner Temple are attached.* It will be seen that Counsel is of opinion that the Society has power under its present Charter to carry out proposals (a) and (b) but that if it desires to give effect to proposal (c) a Supplemental Charter would be required.

Your Committee therefore rejected proposal (c) and considered whether it should recommend the adoption of proposals (a) and (b) or proposal (a) alone. Although the late Lord Keynes and other representatives of the Royal Economic Society had stated at the joint meeting that that Society would have no grounds for objecting to examinations for admission to the Royal Statistical Society, it was felt that candidates who passed an examination of the Society under proposal (b) could be under no compulsion to proceed to election as Fellows and, indeed, might fail to secure election. There was, therefore, some danger that under such a scheme the Society might be considered to be merely evading the issue raised in the opposition to the application for a Supplemental Charter and to be introducing in only another form a national examination in statistical methods. It

* Omitted here.

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was also felt that sometimes difficulties might arise over candidates for election who had passed the examination but were regarded by the Council on other grounds as not suitable for election. Your Committee therefore decided to recommend that only proposal (a) should be adopted and that only Fellows of the Society should be permitted to take the examinations. Even this limited scheme will, it believes, do much to meet the expressed wishes of the Society.

Your Committee also considered carefully a suggestion made at the joint meeting with representatives of the Royal Economic Society that the Ministry of Education should be asked to institute a National Certificate in Statistics and Mr. Kendall had an interview on the subject with an official of the Ministry of Education. It appeared, however, that candidates for such a National Certificate must have attended a course of instruction, usually for three years, at an approved centre; this would make the certificate inapplicable to a large class for whom the Society is anxious to cater. Further, although there would normally be consultation between the Ministry and the Society in regard to the syllabus and to the conduct of the examinations, it would not be possible for the Society itself to conduct the examinations. Your Committee therefore feels that a National Certificate could not be accepted as a satisfactory alternative to the examinations which the Society desires to carry on.

If your Committee's recommendation that proposal (a) be adopted is approved by the Council, it is suggested (1) that No. 1 of the draft Bye-laws submitted for the opinion of Counsel be inserted in the Bye-laws in the group of Clauses dealing with the powers of the Council,* (2) that the draft Bye-laws so revised be submitted to a Special General Meeting in the autumn, and (3) that at this meeting Fellows be asked to pass a formal resolution withdrawing the petition for a Supplemental Charter.

* It reads as follows :—

38. "The Council may at any time cause examinations to be held for the purpose of testing the statistical proficiency of such Fellows as may desire to submit themselves for examination. The Council shall determine the subjects to be comprised in such examinations, the manner in which such examinations shall be conducted, the fees to be paid to examiners whether Fellows or not and the fees to be paid by examinees. Such examinations shall be open to every Fellow and every successful examinee shall be granted a certificate stating the nature of the examination which he has passed."

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REVIEWS OF STATISTICAL AND ECONOMIC BOOKS

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1.—*Fundamentals of the Theory of Statistics*. By J. G. Smith and A. J. Duncan. Vol. 1. *Elementary Statistics and Applications*. 1944. x + 720 pp. Vol. 2. *Sampling Statistics and Applications*. 1945. $8\frac{1}{4}'' \times 5\frac{1}{2}''$. xii + 498 pp. New York and London: McGraw-Hill.

It must be assumed that the authors would not have undertaken the tremendous task of writing well over 1,000 pages on statistics if they had felt that there were already books available which met their requirements. The Preface states that "the approach in this book is that of the teacher," and it is clear that the authors had to cover some syllabus which is reflected in the variety of contents of the two volumes. Part I of the first volume, headed "Introduction," deals with such subjects as "Sources of Statistics" and "Presentation of Statistics," before Frequency Distributions are investigated in Part II. Two further Parts follow, on "The Normal Frequency Curve" and "Study of Bivariates and Multivariates," respectively. The sequel to these subjects is found in volume II, with "General Theory of Frequency Curves," "Elementary Theory of Random Sampling" and finally "Advanced Random Sampling Problems." The two final Parts in vol. I which we have not yet mentioned are a "Study of Dynamic Variability," including index numbers and time series, and a very chatty Part VI on "Forecasting." They are strange intruders in an otherwise systematic exposition.

Stress is laid throughout on modern developments, and it is welcome to find the theory of testing, of confidence intervals and of point estimation treated as different aspects of the same subject—namely, statistical inference. It is perhaps just as well in a textbook not to mention partisan conflicts (the term "fiducial" does not appear), but one may doubt the wisdom of omitting any references whatsoever to inverse probability and Bayes's problem. Some readers will find the authors too anxious to reach quickly the normal distribution, which according to them has "its greatest usefulness in the theory of random sampling" (I, 307), and it is surprising to find Tchebychef's and similar inequalities hidden away in the very last chapter, on the problem of non-normality. Mathematical developments are, as a rule, put at the end of chapters, and one must sympathize with the authors' difficulty of separating fundamentals from details. Thus, for example, the description of samples of two from a normal population (II, 221 *et seq.*) need not have been repeated in the appendix, the less so as the general case of larger samples is also treated there.

Having thus given a general view of the contents, let us look more closely at the presentation. Does it encourage independent thought, or is the stress laid rather on giving answers to examination questions? The somewhat dogmatic treatment suggests the latter: "There are five kinds of charts" (I, 101), "The simplest type of functional relationship is expressed by a polynomial" (I, 256), "The coefficient of risk, as it is called"—meaning the level of significance—"is commonly, but not necessarily, set at 0·05." Why? Then again on p. 11 of vol. II we learn that R. A. Fisher has suggested the use of certain k-statistics "because of their theoretical advantage," but the reader can hardly guess what kind of advantage this may be. Incidentally, the text suggests that only four k-statistics exist. (The same applies to p. 243, but the formula given there for k_4 differs from that on p. 11 and is incorrect.)

In a book for students statements which are misleading cannot be excused by saying that they are correct as far as they go. Anyone not familiar with the subject will take a passage on I, 316 to mean that the well-known relation between the standard deviation of the population and that of the sampling distribution of the mean only holds because the latter tends to be normal. The statement on II, 242, that the sampling distribution of the median from a normal population is itself normal, is, of course, wrong. The reviewer is unable to attach any meaning to the assertion that the normal bivariate surface of correlated variates is "narrower" than that of uncorrelated ones (I, 492), and on the same page the double integral of $dP(x_1x_2)$ should certainly not be denoted by $\iint P(x_1x_2)dx_1dx_2$.

The main merit of the books consists in their dealing with modern theory. But here again

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something could be gained by more attention to logical principles. On II, 163 a clear and convincing description is given of the notions of errors of the first and second kind (surprisingly enough, without mentioning either J. Neyman or E. S. Pearson, who are only introduced as the originators of the likelihood ratio test), but it is at least misleading to state (II, 162) that "the possibility that . . . may be due to chance . . . may be determined by probability theory," without adding "assuming the null hypothesis to be correct" or some words to this effect. On p. 163 the hypothesis "not more than 10 per cent. substandard" is confused with "10 per cent. substandard," and it is incorrect to describe the limits of the confidence belt as "boundary lines for which it can be stated that the chances are, say, 95 out of 100 that this boundary includes the true population" (II, 174). An analogous mistake occurs on p. 179 in connection with an example. On p. 183 we are told that maximum likelihood estimates are sufficient, without any warning that sufficient estimates may not always exist, and without an appropriate explanation what sufficiency means. The footnote on p. 198 should contain a hint that the power of a test is a function of alternative values of a parameter, so that most powerful tests cannot always be devised. There is, moreover, no need to introduce (II, 202 n.) unbiased confidence intervals after the previous introduction of unbiased critical regions. On the other hand, unbiased estimates are not mentioned (unless the reviewer has overlooked them), possibly because only maximum likelihood estimates are considered.

It may be thought ungenerous to dwell mainly on the shortcomings of the books. The reviewer would prefer his remarks to be taken as modest suggestions for improvements in later editions. It is for the same reason that he mentions a few misprints, although he recognizes their minor importance. H. L. Rietz is called Reitz throughout the second volume, including the index, but excluding the very last chapter, and I, 563 refers to R. A. Fischer (*sic.*). The note on II, 93 has omitted the plus-sign in $e^a e^b = e^{a+b}$. On II, 211 the well-known author of the reference to the Prussian horse-kicks is misspelt as Borthewich, and if a straight line has direction ratios $-a$ and b, then that perpendicular to it has b and a, not a and b (II, 253). On the next page gamma is twice misprinted as T, and Table VIII in vol. I should have the same heading as the identical Table in II—viz., Hyperbolic Tangents. The reference on p. 291 of vol. II should be to equation (2), not (3) of Chapter X, and this is an unfortunate mistake, because it adds to the difficulty of students to understand the relation between the frequency distributions of σ and σ^2 respectively.

In volume II the description of figures 46 and 47 should refer to $N = 25$, not $= 5$, figure 35*b* should have the inscription ·341 in the shaded portion and figure 89*a* has $\sigma^2 = 180$ in the legend and 200 in the text. This reference to figures gives an opportunity, welcome to the reviewer, to praise the graphs and diagrams. They are quite admirably selected and printed. Much thought has gone into their construction, and if it were for nothing else, the perusal of the books must be recommended for their sake. The reader who already knows something of the subject will find them stimulating, and teachers will be convinced that geometry can be used for teaching statistics without climbing up to the *n*th dimension. The tables are well chosen, and the clarity of type is above criticism. American publishers are fortunate indeed in being able to produce such books "in full compliance with their governments regulations."

S. V.

2.—*Contributions to the Study of Oscillatory Time-series.* By M. G. Kendall. National Institute of Economic and Social Research: Occasional Papers IX. Cambridge University Press. 1946. $9\frac{1}{2}'' \times 6\frac{1}{4}''$. 76 pp. 7*s.* 6*d.*

In this account of the analysis of oscillatory time-series Mr. Kendall presents some extensive numerical researches, with which we are already familiar in part from his paper to the Society in 1945. After describing the methods of investigating oscillations in observed series by correlogram analysis, periodogram analysis, counting peaks, and variate differencing, he proceeds to try out these methods on four artificially constructed autoregressive series, and also on some actual economic series. The autoregressive structure chosen for the artificial series seems a reasonable one, and the series obtained show much the same features as do observed economic series. The value of trying out the usual methods of analysis on such artificial series is that some insight is gained into what to expect from series of this structure. The investigation has yielded striking and valuable results; and it is interesting that Mr. Kendall's observations on the failure of the correlogram to damp out have already been confirmed by subsequent theoretical researches of Dr. M. S. Bartlett. The treatment is complementary to the account of the mathematical theory of time series given in the last chapter of the same author's *Advanced Theory of Statistics*, Vol. 2. Our thanks and congratulations are due to Mr. Kendall and the National Institute of Economic and Social Research (which supplied computational assistance) for carrying through an arduous programme.

An appendix gives tables to facilitate harmonic analysis.

That Mr. Kendall's little book is lucid and pleasant to read goes without saying. It will interest both the economist and the mathematician—the economist for the light cast on fundamental ideas of time-series analysis, the mathematician for the wide field of research suggested.

F. Y. A.

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3.—*Sequential Analysis of Statistical Data: Applications.* Report prepared by the Statistical Research Group, Columbia University, for the Applied Mathematics Panel, National Defense Research Committee, Office of Scientific Research and Development (U.S.A.). Columbia University Press. Geoffrey Cumberlege, Oxford University Press. 1945. $11\frac{1}{2}'' \times 10\frac{1}{2}''$. xxix + 287 pp. 42*s*.

Abraham Wald's sequential analysis is the most important new idea in statistical theory of the last half-dozen years or so. Others before Wald had considered sequential tests, but he was the first to find a fertile principle by which to construct such tests. The idea is very simple. To discriminate between two alternative hypotheses we continue to take samples until the ratio of the likelihoods (in R. A. Fisher's sense) of the accumulated observations on the two hypotheses is either very large or very small, and we then accept one or other hypothesis accordingly. The two target values for the likelihood ratio are fixed in advance, taking account of the risks that we are willing to run of making a wrong decision. The principle can be applied to a variety of problems, and leads to tests that are appropriate when our object is primarily to make a decision between two alternative courses of action, such as accepting or rejecting a batch of materials, or pursuing or not pursuing some experimental line of development. (Although it is possible to estimate from a sequential test the magnitude of effects, the calculations are rather laborious, and if such estimation were the main object of the test Wald's procedure would usually be inadvisable.)

Wald's tests have been applied in a number of connexions. In particular, they have proved really valuable in industry, in the inspection of batches of goods to determine whether they reach an acceptable standard of quality—a fact of some consequence in these days when manpower is reputedly short and greater efficiency in production is urgently required. For statisticians in general, perhaps the most important effect of Wald's researches is that we have been taught to think freely of sequential procedures, and it is likely that in the near future many new sequential methods will be developed, not all on the principle of the likelihood ratio.

The book under review explains simply how Wald sequential tests are operated. It is a revised version of a restricted report issued in 1944, about a year after Wald first developed his theory. Authorship is stated to have been shared by H. A. Freeman, M. A. Girshick, and W. Allen Wallis, with the co-operation of Kenneth J. Arnold, Milton Friedman, Edward Paulson, and others. The book consists of seven pamphlets reproduced from typescript, set in a ring binder so that they can be read and used individually. The first section is introductory, and the following five give sequential tests for the fraction defective of a batch (binomial population), for comparing the fractions defective of two batches (double dichotomy), for the mean of a normal population with known variance, both when we wish to discriminate between a high and a low mean and when we wish to test a departure of the mean in either direction from a standard, and lastly for the variance of a normal population. These five sections are independent of each other, so that the reader may turn to whichever interests him. There is therefore a good deal of repetition of description and even of tables. The last section of the book consists of appendices giving various general formulæ and including some remarks on truncation.

That such an account of sequential sampling methods should have been produced so soon after the development of the theory indicates a creditable spirit of enterprise on the part of the Columbia Research Group. One can, indeed, find a few faults here and there. Thus there is a confident statement that "truncation of any kind ought generally to be avoided", which is presumably based only on ignorance of the effect of truncation. And Wald's elegant theorem concerning the minimum sample sizes incurred in sequential tests is stated without the necessary conditions, and becomes thereby untrue. In any case, I believe no such theorem has ever been proved regarding the test given here for double dichotomies. These inaccuracies, however, are not likely seriously to mislead anyone who becomes interested in the subject. The main facts of sequential analysis are presented clearly, and the book will do more than instruct the practical man in methods. It will prove a helpful introduction to the theoretical literature in *Annals of Mathematical Statistics* and in the Supplement to this *Journal*.

F. J. A.

4.—*Statistical Methods Applied to Experiments in Agriculture and Biology.* By George W. Snedecor. Iowa State College Press, Ames, Iowa. Fourth Edition, 1946. 9″ × 6″. Pp. xvi + 485. $4.50.

This book first appeared (1937) when there had been time to settle down to the new conceptions of statistics as applied generally by biologists, which date from the appearance twelve years earlier of R. A. Fisher's epoch-making *Statistical Methods for Research Workers*. The author is head of the Statistical Laboratory of Iowa State College, and is both a mathematician and an essentially practical worker. The emphasis throughout was laid on methodology, and infinite pains were taken, even at the cost of seeming wordy, to explain everything in simple terms, and to illustrate profusely by means of examples.

The present edition is largely a re-written text, in which the scope has been widened in a number of directions. As explained in the preface, greater emphasis has been placed on the theoretical conditions in which the methods have validity, and on the conduct of the experiment; further, estimates and fiducial inference have been brought into equal prominence with tests of hypotheses.

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The treatment of correlation and of experimental design has been expanded, and the methods for dealing with disproportionate sub-class numbers have been extended.

The impression left on the reviewer's mind is that the book has been very much improved. It will startle the reader accustomed to the orthodox treatment of the subject as addressed to students of economics and commerce to find the χ^2 test in Chapter 1, fiducial inference in Chapter 2, and the t-distribution in Chapter 3, while, on the other hand, the binomial and Poisson distributions are relegated to Chapter 16, and are left out of the short course of reading in the elements of statistical method suggested by the author. But the deliberately conceived plan is to use all that is known of the logic of statistical inference from the beginning, and in even the simplest classes of observational data. Mathematical proofs are necessarily absent from a book conceived as this one is, but the plan followed is logical. To take one simple case: if the classical methods of testing for significance in large samples must be replaced for small samples by different methods, it is clear that these new methods also apply to the former case. If, as is usually the case, they are not more difficult to apply, one can forget all about the classical methods. Most such tests require tables, even those for large samples; it is then only a case of replacing one set of tables by another.

It will be clear from what has been said that there is much in the book which can be studied with profit by any worker in statistics. Having said this, it remains to be added that students of agriculture and biology have to go much further into details of experimental design and analysis of variance applications than workers in other branches of the subject. That is done in this book. Chapters are devoted to analysis of variance in two or more groups of measurement data (the general extension of the t-test for the comparison of two groups), and with two or more criteria of classification (leading to the common designs). Further chapters cover the analysis of covariance, first as an extension of simple regression in two variables, and later, in association with multiple regression, where there are two, three or more independent variates. In the same sequence there is a chapter on individual degrees of freedom, covering the methods applicable to special experimental designs of factorial character. Apart from the special emphasis given to the biological applications there is little of the content of the modern methodological text-book on statistics that does not find a place; the economist will look in vain for a discussion on index numbers or time-series, but he will profit by study of the chapter on sampling. J. W.

5.—*Statistische Methoden für Naturwissenschafter, Mediziner und Ingenieure*. von Arthur Linder. Basel, 1945. $9\frac{1}{2}'' \times 6\frac{1}{2}''$. 150 pp. Price: bound, 18·50 fr. (Swiss); stitched, 15·50 fr.

The justification of Professor Linder's ambitious title is that he has taken arithmetical examples from a wide field, so that biologists, pathologists, engineers and industrialists will *all* know what *some* of the examples mean. It must be said that some of the examples are very old friends and that the author confines himself strictly to arithmetic and algebra.

In the first 42 pages we have a very elementary account of the common statistical indices and in the next 38 illustrations of methods of assessing errors of sampling. In illustrating the χ^2 technique, Professor Linder does not explicitly warn the practitioner who eschews algebra that it is a large sample test. Possibly a convenient way of doing this, as the practitioner does know what a variance is, might be to point out that for a table with m cells and given *a priori* p's ($S\,p = 1$) the variance is:—

$$\frac{2(m-1)(s-1)}{s} + \frac{m^2}{sH}\left(\frac{1}{m} - H\right)$$

where s is the number in sample and H the harmonic mean of the p's. The practitioner will then observe that, if one or more of the p's is very small, even with a respectable-looking s, the variance will differ much from $2(m-1)$.

The last 56 pages are intended for students with a fair knowledge of algebra. This section is concise and clear. In his deduction of the χ^2 distribution the author begins with the sum of squares of n stochastically independent variables each normally distributed. This seems the most logical order of approach. For historical reasons most textbook writers begin with a table of m cells subject to the restriction that the total contents are fixed.

It would not be true to say that an English reader will learn much from this book which he cannot find at home, but to read it would be excellent practice in scientific German, and its scope is intermediate between a mere manual of instructions and an advanced treatise. M. G.

6.—*A First Course in Mathematical Statistics*. By C. E. Weatherburn. Cambridge University Press. 1946. $8\frac{1}{2}'' \times 5\frac{1}{2}''$. xv + 271 pp. 15*s*.

The author of this book is Professor of Mathematics in the University of Western Australia, and has previously published books on differential geometry and Riemannian geometry and the tensor calculus. He has not, so far as is known, published any papers on the theory of statistics, and it is thus interesting to see the attitude of a mathematician, with some ability as a text-book writer, who has now turned to statistics as his theme. Perhaps the best-known previous books written for the professional mathematician are Aitken's well known small work on statistical

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mathematics, Kendall's *Advanced Theory of Statistics* and Wilks's *Mathematical Statistics*. The author has a number of references to Aitken and Kendall (vol. 1), but has evidently been uninfluenced by Wilks, although a reference to the work is inserted at the end.

The book is based on a course of lectures given by the author, and first impressions are that the subject has been dealt with competently and well, so far as it is taken. A short first chapter discusses frequency distributions in a general way, introducing mean, variance, moments in general and the grouped distribution. This is followed by the discussion of probability, and the notions of a probability distribution and the expected value of a variate, or of a function of variates, later expanded to deal with the moment generating and cumulant functions. The next chapter deals with the binomial, Poisson and normal distributions (with mathematical notes added). Two further chapters are given to bi-variate analysis, introducing regression and correlation, and ending with the simple fitting of curved regression lines. The author goes on to discuss the theory of simple sampling, for attributes as well as values of a variable, and ends what may be regarded as the first section of his work with a chapter on large sample standard errors of statistics.

A chapter in which the Gamma function and the two forms of the Beta function are used to define three general distributions, the Gamma distribution and the Beta distributions of the first and second kind, together with their properties, leads to the proof of the χ^2-distribution, which is illustrated in "goodness of fit" and used to derive the usual "small sample" distributions. The use of orthogonal linear transformations is explained. A further chapter gives the small sample significance tests, classified as the distributions of "Student" and of the Variance Ratio and Fisher's transformation of the correlation coefficient. There is a chapter on analysis of variance and covariance, and the author concludes with multivariate distributions, illustrating by means of three, and gives the partial and multiple correlation definitions and tests. There is a list of references, and the usual tables are provided, mostly Fisher's.

A very good idea of what the subject is about generally, and of how it is dealt with by modern exponents, can be had from this book by anyone who has a moderate mathematical knowledge and who possibly finds Kendall's two volumes too formidable and time-consuming. It would be a good idea if mathematical teachers, who all too often know nothing about statistics, would read this text; they would find it written simply in their own language, and could not advance the same objections that they are inclined to raise with books written for biologists without mathematical proofs. The reason why this course is recommended is that the subject has yet to find an established place in the curriculum of many mathematical schools, and a wider knowledge of the field is likely to lead to the realization that more must be done to cater for the needs of students who are interested in the subject, either for its own sake or, more often, because interesting and lucrative avenues for employment are open to those who have been trained as statisticians. It should be emphasized, however, that the book is only an introduction, and if the teaching is entrusted, as it should be, to the specialist in statistics, such a man will be required to go beyond what is written here.

Most of the content of the book should by now be standard knowledge. An innovation is the introduction of the Gamma and Beta distributions, which are used to define three classes of variate having general properties which are set out and proved. This is an attractive way of approaching distributional theory (on the usual assumption of normality in the parent population) because the standard results can be at once applied (*e.g.*, to χ^2 or the Variance Ratio), provided the groundwork is complete. The only major criticism the reviewer has is that the author should have gone further than he did. He proves (Theorem III) that if the sum of two independent positive variates is a Gamma variate with parameter $l + m$, and one of them is a Gamma variate with parameter l, then the other is a Gamma variate with parameter m. But there is no reference to Cochran, or to his theorem, of which a special case, in the present language, is: if the sum of two positive variates, which are themselves Gamma variates with parameters l and m respectively, is a Gamma variate with parameter $l + m$, then the two variates are independent. It is this theorem, extended to three and more variates, which is required in the analysis of variance. The author realizes on p. 212 that it is necessary to demonstrate the independence of the two estimates of variance in the "one criterion of classification" case, in order to use the Variance Ratio test, but says nothing on p. 216 (or on later pages) about the necessity for a similar demonstration in the two and higher criteria of classification cases, despite a reference at the end to Irwin's paper on the independence of the constituent items. The demonstration does not follow from Weatherburn's theorem, but from Cochran's, and this theorem is rightly regarded as important by Kendall and Wilks. The same lack of completeness in proof is shown in the treatment of the analysis of covariance. At this stage the book becomes almost completely descriptive of methods, without the mathematics, and while it is very useful to have the methods set out, the theory requires a great deal more attention to be paid to it than is possible in a short account, particularly along the lines of what circumstances justify the covariance analysis and what can be learnt from it. Only the case of a single concomitant variable is studied, and the following chapter on multivariate distributions does not return to the subject, nor does it refer in dealing with partial correlation coefficients to the value of their calculation in certain circumstances from the "error" line of an analysis of variance and covariance table.

J. W.

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7.—*International Implications of Full Employment in Great Britain.* By Allan G. B. Fisher. London and New York, Royal Institute of International Affairs, 1946. $7\frac{3}{4}'' \times 5\frac{1}{4}''$. vii + 201 pp. 15*s*.

Highly simplified versions of the far from lucid economic theory elaborated by the late Lord Keynes are commonly used as a basis for asserting that a solution has been found for the problem of "full employment" which baffled the economists of the past. Professor Fisher devotes a good deal of his book to showing up the specious nature of this claim—so much so that he seems to have upset some of the members of the group of economists in close association with whom, we are informed, the book was written. While the group as a whole found itself in general agreement with the book, certain of its members have felt it necessary to append a comment "to give a somewhat different emphasis to one particular topic"—namely, what they regard as Professor Fisher's undue emphasis that an improvement in our standard of living cannot be achieved in the future, any more than it could in the past, "unless we remain constantly ready to make changes." Now, a change implies not only that someone must begin producing something that was not produced before, "but also that someone, and indeed many thousands of people, and if we look at the world as a whole, perhaps many millions of people, must cease doing what they have been doing before." Under an open foreign trade system the need for such adjustments becomes evident at once; under a closed system it may be possible to defer them, but if they are deferred the standard of living certainly suffers.

Professor Fisher believes that current ideas and enthusiasms in regard to a domestic policy for "full" employment have dangers in this connection; for in practice they must both militate against the objective of wresting large numbers of people from their traditional work, and encourage the idea that people are entitled to expect to be kept in employment in their present occupation.

The reply of the economists who dissent from this line of argument resolves itself primarily into an appeal to politics and the emotions. They remind one "that all political parties have agreed that it is the responsibility of society to secure that large-scale unemployment does not develop." They point out that, "There is a widespread *feeling*, which we share, that we have now reached a level of affluence, of organization and of control over our destinies, at which the evil of large-scale unemployment should be regarded as intolerable" (the italics are the reviewer's). While paying the fullest tribute to these humanitarian sentiments, it is pertinent to insist that the job of the economist is not to emulate the revivalist preacher, but to provide the raw materials for practical economic policy. Now one looks in vain, in the writings of advocates of a "full" employment policy, for real facts, figures, lessons drawn from experience, or even estimates of probabilities. There is not even agreement on the meaning of the phrase "full employment."

To give but one example. The opponents of Professor Fisher's claim that the source of his distrust of a "full" employment policy "seems to be due in part to insufficient distinction between structural unemployment and unemployment due to a deficiency of effective demand. It is the latter problem which forms the main target of unemployment policy."

But it is precisely this kind of distinction which it is quite impossible to draw in real life from the limited facts at our disposal. Indeed, such statistical information as exists strongly suggests that, outside the imagination of the economists, there is no such thing as the trade cycle at all! So much so that when economists claim to have found a cure for it, they are, in effect, simply claiming to be able to cure a non-existent disease of their own invention!

What pass for trade cycles are mixtures of trade oscillations, the mean distances between whose more prominent peaks or troughs have no statistical significance at all. There are thus no such things as trade cycles whose recurrence can be closely predicted. This has been demonstrated not only by the voluminous data collected in the U.S.A. by Arthur F. Burns and Wesley C. Mitchell (see *Measuring Business Cycles*), but is also supported by the work of M. G. Kendall in this country in a paper on "Oscillatory Time Series," read before this Society in January 1945.

As Mr. Kendall explains, "the reason why people continually discover cycles in all kinds of time series is that they are looking for them. With a little goodwill, nothing is easier than to detect a cyclical movement in random material." What economists have not realized is that cyclical effects can be generated by random disturbances as well as by schemes of harmonic oscillation.

Now, until one understands the functions and purposes of *each* of the different types of oscillation that go to make up the so-called trade cycles, one cannot forecast what effect a particular *general* "remedy" will have on the continuing expansion and development of the economic system as a whole. If, as some of the advocates of a "full" employment policy appear to recommend, all oscillations were removed from our industrial system, it might become either so brittle as to snap at the first serious strain, or permanently ossified. There may be much to be said for damping-springs and for shock-absorbers, but not for the elimination of springs altogether.

When economists triumphantly assert that, to cure "trade cycles," there is no need to know *how* the economic system works they are guilty of the same fallacy as the mediæval philosophers who opposed the emergence of the scientific method in the sixteenth century. A. N. Whitehead summed up this difference in approach when he wrote: "Galileo keeps on harping on how things happen, when his adversaries had a complete theory as to why things happened." A homely

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modern illustration of the difference in approach of the economist who asserts that all we need to know is that trade cycles are due to a failure to maintain general purchasing power, and of the scientist who insists on knowing how the economic system works before committing himself to the application of a particular treatment, is that between a man who explains that the reason why a particular motor-car is no longer moving is because the engine is not running, the wheels are not going round, or the petrol supply is not circulating, and a mechanic who, having learnt how a motor-car works, proceeds to overhaul it before suggesting treatment. It is doubtless because Professor Fisher instinctively feels that the Keynsians incline to belong to the former category that he is sceptical of their advice to the practical man.

One may agree that unemployment is an evil, but unless one knows how the economic system works, attempts to cure it by applying some vague and largely theoretical nostrum may, in the long run, have a more adverse effect on the well-being of the 90 to 95 per cent. who at present manage to be kept in regular employment, than can possibly be compensated for by an improvement in the lot of the 5 to 10 per cent. who happen periodically to suffer from temporary unemployment.

Professor Fisher's book is more than timely in its appearance. R. G.

8.—*The Money Value of a Man.* (Revised Edition.) By Louis I. Dublin and Alfred J. Lotka. The Ronald Press Company, New York. 1946. 6″ × 9″. 214 pp. $6.

Since the publication in 1930 of the original edition of this work substantial changes have occurred in the factors involved in the authors' computations. They have accordingly undertaken an extensive revision, in the course of which, besides giving effect both to the most recent data on population mortality and to contemporary trends in the market rate of interest, they have widened the scope of their enquiry and added new material.

The authors define the money value of a man as his value to persons having a direct interest in his earnings. Their aim, *inter alia*, is to provide a standard by which claims for compensation for death or injury resulting from accident may be assessed. In the case of death the problem is to compute the present value of the dependants' share of the deceased person's potential earnings; in the case of injury the amount required is the present value of the future reduction in total earning power. By the combination of rates of mortality and interest with average earnings and average number of consumption units per family at each age (all based upon data from American sources), the authors have compiled a valuable set of tables from which the money value of a man may be ascertained according to age and income group at the time of death or injury. The necessity of making separate computations for each income group arises mainly from the sharply varying slopes which emerge in different occupational classes when income is graphed against age. Among the unskilled classes, wages remain at a fairly constant level throughout the working lifetime. Among the professional classes, however, the slope is steep, reaching a maximum at about age fifty-five.

That these tables fulfil an outstanding need in the legal profession is made abundantly clear by the wide variations in the amounts awarded in compensation cases. The English reader will inevitably recall the egregious claims by a deceased person's representatives for loss of expectation of life which, owing to a statutory anomaly unforeseen at the time when the relevant legislation was passed, have become common in this country in recent years. It should be hardly necessary to state that no guide for settling claims of this nature will be found in the present work and the English courts must struggle on in their valiant endeavour to apply common sense in a situation which is wholly senseless.

Although compensation claims were the primary stimulus which generated this book, the authors have extended its scope over a wider field. The money value of a man depends among other things upon probabilities of survival and consequently will increase if mortality declines. In so far as falling mortality may be a result of expenditure on public health, the authors suggest that such expenditure may be measured against the increased money value of the community, and on these lines endeavour to persuade American municipal authorities that money spent on public health is well invested.

In their researches into family expenditure, the authors are led into some detailed surveys of American family budgets and of the cost of rearing children. Unfortunately, recent inflation has already rendered most of the data obsolete. Nevertheless, readers who are interested may find it profitable to compare the information published in this book with English data for a similar period. In fact, the book may be confidently recommended to all students of the social sciences as a compendium of useful sociological data on the American nation immediately prior to the recent war. R. D. C.

9.—*A Century of American Life Insurance. A History of the Mutual Life Insurance Company of New York 1843–1943.* By Shepard B. Clough. New York: Columbia University Press, 1946. xiv + 402 pp. $4.50.

This book, by the Professor of Economic History at Columbia University, gives an account of the first American life assurance institution, open to all persons, which has had a continuous existence to the present time; it enables the reader to form an accurate idea of the history of life assurance (other than industrial life assurance) in the United States of America.

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The book is divided into four parts: the first gives a sketch of life assurance in America and some notes on the early history of life assurance, and deals with the founding of the Mutual by a Charter which provided that the corporate powers were to be exercised by a board of thirty-six Trustees—it seems a large board for a new company. The remaining three parts deal with the periods 1843–70, 1871–1906 and 1907–43. The earliest may be said to cover the beginnings of life assurance in the United States, the middle period deals with its growing pains when the thirst for large figures, the wild competition between the large companies, managerial control which amounted at times almost to dictatorship, the extension to other countries and the issue of deferred dividend policies (a semi-tontine) culminated in an investigation conducted by a committee of the New York State Legislature under Armstrong's chairmanship. The final period shows a wiser development of the business; the opening of new avenues such as group insurance, the gradual withdrawal from the foreign field and the development of a friendly attitude between the various companies in pleasant contrast with the kind of competition that had characterized the middle period.

Throughout the book and in the Appendix much statistical information is given, both for the Mutual and for American companies as a whole, showing the new business and the business in force; the reader will notice the growth in industrial insurance and group insurance; figures are also given of surrenders and lapses, investments, rates of interest and expenses. The expenses are related to total income, and not, as is usual in Great Britain, to premium income alone; this disguises the heavy expense rate in the worst period. A few of the tables and charts in the book are not completely defined, and there are a few printer's errors and one or two statements that are open to question, but such little weaknesses may be forgiven to an author who has been commendably frank (a virtue that must be ascribed also to the Mutual) and has not attempted to gloss over the less satisfactory facts in the hundred years' experience.

The book can be recommended to any one interested in the statistics and history of life assurance in the United States, and those who have experience of the business in the United Kingdom will find that many of the excitements in America had no counterpart—or none so recently—here. There has never, for instance, so far as the reviewer is aware, been a "run" on insurance companies on this side of the Atlantic, but a "run" on the Mutual occurred in the 1870's; and in the depression between the two world wars a "run" on insurance companies necessitated action on the part of the legislature so that the guarantee of surrender values became for the time inoperative. On the lighter side two quotations may be given; the first relates to the founding of the Equitable of the United States by H. B. Hyde, who was an employee of the Mutual and "conceived the idea of establishing an institution to write the large risks which the Mutual refused". At the age of twenty-four he broached his plan to the President of the Mutual, "who dismissed brusquely both him and his plan. Thereupon Hyde leased rooms in the same building as his erstwhile employer and hung out a sign that completely dwarfed the Mutual's shingle." The second relates to the use of the Press. Companies not only inserted advertisements, but paid the editors to insert editorials and articles: here is the story. "For the campaign against Jacob L. Greene of the Connecticut Mutual, the Mutual, along with the Equitable and the New York Life, hired the services of C. C. Hine, editor of the *Insurance Monitor*, Stephen English of the *Insurance Times*, and Charles J. ('Dollar-a-line') Smith of the *Insurance Record*. The editor of the *Insurance Monitor* had a fixed price for his Connecticut Mutual extras of $50 a thousand and boasted that he sold them 'by the ton.' The use of such methods had, however, serious disadvantages, for not only was the practice expensive, but it sustained a group of mercenaries who could be employed for retaliation in kind or who could make a living out of blackmail. English, in fact, served the Equitable's Hyde so successfully in a campaign against the Mutual that McCurdy [President of the Mutual] finally hired him. And a certain Joe Howard Jr. who worked for both the Mutual and the Equitable, was not above holding up either company."

In contrast to this kind of thing which has been long outgrown, the reader will find an account of good work on the actuarial, investment and practical side by many public-spirited servants of the Mutual over a hundred years; that still persists and will not be outgrown.

W. P. E.

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STATISTICAL NOTES

1. British Official Statistics

In Part II of the *Journal* we gave a table and notes summarizing movements in the overseas trade of the United Kingdom in the first nine months of 1946. The table given here shows the

£ million

	1938	1946	1946		1947
	Quarterly average		Third Quarter	Fourth Quarter	First Quarter
Retained Imports, c.i.f.					
Food, drink and tobacco	104·5	156·3	157·2	173·2	182·5
Raw materials and articles mainly unmanufactured	54·5	89·1	88·4	104·3	89·8
Articles wholly or mainly manufactured ...	53·8	59·5	63·6	72·3	67·2
Other articles	1·7	6·9	6·5	6·4	6·1
Total	214·5	311·8	315·7	356·2	345·6
United Kingdom Exports, f.o.b.					
Food, drink and tobacco	9·0	15·9	18·7	14·1	12·9
Raw materials and articles mainly unmanufactured	14·2	8·2	7·4	9·1	10·0
Articles wholly or mainly manufactured—					
Metal goods	43·9	100·6	106·4	123·0	115·0
Textiles	25·3	46·6	49·0	54·0	52·7
Other manufactures...	22·1	49·4	53·1	57·9	50·6
Other articles	3·2	7·2	6·4	8·1	8·5
Total	117·7	227·9	241·0	266·2	249·7
Exports of Imported Merchandise, f.o.b. ...	15·4	12·6	13·2	13·9	18·2

comparable figures for the fourth quarter of 1946 and the first quarter of 1947, together with the quarterly averages for the years 1938 and 1946.

The adverse balance shown by the official statistics amounted to £96 million in the first three months of this year, compared with £75 million and £90 million respectively in the third and fourth quarters of 1946. This increase on the preceding quarter was due in large measure to the fuel crisis and the severe weather, which both held up production and caused considerable interference with the movement of goods to the ports. The recorded exports still include some N.A.A.F.I. and U.N.R.R.A. shipments, which in this quarter totalled £8·6 million (about 3½ per cent. of total exports). At the present time, however, the balance of our trade with individual countries or groups of countries is probably of greater interest than the overall picture. The Trade and Navigation Accounts now include each month (one month in arrear) details of exports and re-exports consigned to and imports received from each country and a table published monthly in the *Board of Trade Journal* shows the proportion of our total imports received from and exports (including re-exports) consigned to hard and soft currency areas. In the first three months of this year the U.S. dollar and other hard currency areas supplied us with imports valued at £173 million (48 per cent. of the total compared with only 34 per cent. in 1938), whereas total exports to those areas were only £57 million (21 per cent.—the same proportion as in 1938). The deficit with these areas thus exceeded the total deficit by £20 million.

Total imports in the first three months of this year amounted to £363·8 million, compared with £370·1 million in the previous quarter and an average of £324·4 million for 1946 as a whole; the recorded value represents an increase of over a half compared with 1938, but average values (which have risen by one-fifth over the last year) are now 138 per cent. above those of 1938 and, in terms of volume, imports were lower by one-third than before the war. The index for retained

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imports was 67 against 69 for the whole year 1946. Re-exports, which throughout 1946 were at about two-fifths of the pre-war volume, increased to 58 per cent. in the first quarter; the recorded value of £18·2 million consisted mainly of shipments of rubber (£7·0 million, the quantity re-exported being more than six times the 1938 average), of raw wool (£4·8 million) and of tea (£0·9 million).

Compared with the year 1946, the increase of £33·8 million in retained imports was accounted for mainly by increased supplies of foodstuffs, though manufactured articles also showed a substantial increase; retained imports of materials were roughly equal to last year's average. Only for food, drink and tobacco, however, was the previous quarter's level exceeded, retained imports of materials being reduced by one-sixth and those of manufactures by about 7 per cent. Supplies of tobacco, though still above those in 1938, showed a heavy fall from the previous quarter's high level. Imports of foodstuffs were generally higher, the only exceptions being a seasonal fall in imports of tea and smaller supplies of dried egg. Imports of wheat increased to about three-quarters of the pre-war average and those of maize and of other animal feeding-stuffs exceeded the quantity received in the whole of 1946, though they still amounted to only a quarter of normal pre-war imports. Otherwise the most important features were the higher level of imports of dairy produce (other than dried egg), of fresh fruit and vegetables and of miscellaneous foodstuffs, each of these three groups being at roughly three-quarters of the 1938 volume.

The reduction in retained imports of raw materials was more than accounted for by the seasonal fall of over £11 million in supplies of timber, together with reduced imports and high re-exports of rubber. As in the first quarter of last year timber supplies were about half of those in the corresponding period of 1938; compared with a year ago, however, imports of pit-props, which increased rapidly during 1946, were more than double while those of softwoods were reduced by nearly one-third. Rubber imports were still about double those in 1938 but the heavy re-exports mentioned above amounted to over two-thirds of the quantity imported. Rubber was the only group for which the pre-war volume of imports was exceeded. Imports of hides and skins were within one-tenth of those in 1938, and those of iron ore and scrap and of raw cotton each at about four-fifths of the pre-war volume, but supplies of raw wool were at less than half and those of paper-making materials at only two-fifths of the 1938 level.

For manufactured goods changes in the individual groups were mostly small, the most important being the reduction in imports of aluminium, copper and lead which brought the total value of the non-ferrous metals group to less than three-quarters of that in the preceding three months, and the increase of £1·5 million in imports of machinery—the quantity imported (15,700 tons) being, however, only half of that in 1938.

The value of United Kingdom exports in the first quarter was £249·7 million, compared with £240·9 million in the third and £266·2 million in the fourth quarter of 1946. The full effects of the fuel crisis will probably not be seen until later in the year but, allowing for the different number of working days, the rate of exports in each of the first three months (88, 82½ and 82½ £ million compared with the peak figure of £92 million per month of 26 working days in November) suggests that both the February and the March figures were affected by the bad weather conditions and, to some extent, by the closing down of industry. Average values of exports have risen by over 3 per cent. in each of the last two quarters and are now 111 per cent. above those in 1938. The volume figure for total exports (1938 = 100) fell from 111 in the last quarter of 1946 to 101 in the first three months of this year. As mentioned above, N.A.A.F.I. and U.N.R.R.A. shipments are included in the total so that commercial exports were about 2 per cent. below the 1938 volume.

Exports of food, drink and tobacco (£12·9 million) were nearly one-fifth below the 1946 average; there were heavy grain shipments to Germany in the summer months of last year and this accounts for some of the reduction, but both whisky and tobacco exports were also below those in 1946. Among raw materials, coal exports amounted to only 324,000 tons, the lowest figure since the coal stoppage in 1926, but brass scrap to the value of £3·0 million was sent to the United States for refining and return of the copper to this country; this kind of transaction commenced in the last quarter of 1946 when scrap valued at £1·6 million was shipped to the United States and Canada.

The volume of exports of manufactured articles, which had risen continuously throughout

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1946, fell in the first quarter by 10 per cent., the quarterly index numbers (1938 = 100) since the beginning of 1946 being 91, 110, 117, 126 and 114. Exports of metal goods (£115 million), as in 1946, accounted for over half of the total for manufactured goods, the volume, though nearly one-tenth below that in the last quarter of 1946, being two-fifths above that in 1938. Machinery was the only group for which the previous quarter's level was exceeded; the quantity exported (166,000 tons—the highest quarterly figure on record) was 45 per cent. above the 1938 level, the small increase over the preceding three months being the result of substantially higher shipments of boilers and boiler-house plant and of electrical machinery, which were only partly offset by reduced exports of agricultural machinery and cranes and hoists. The effect of the slowing down of industrial production is shown most clearly by the figures for exports of vehicles, some works being affected seriously by shortage of coal even before the general stoppage for a week or more in February. The total value fell by £5 million, exports of motor cars, commercial vehicles, cycles and railway rolling-stock, each of which increased rapidly during 1946, all showing substantial reductions compared with the previous quarter. The volume figure for the group was, however, still about 50 per cent. higher than in 1938. Exports of iron and steel (445,000 tons) showed the third successive decline from the peak level of the second quarter of last year, and were slightly lower than in 1938. For the remaining three groups of metal manufactures exports, though substantially below the previous quarter's level were, by volume, still between one-half and two-thirds above the pre-war levels.

Exports of textile manufactures amounted to £52·7 million; compared with 1938 the increase in average values for textiles (186 per cent.) is nearly three-quarters above that for total manufactures and the volume index, after increasing slowly during 1946 to 78, fell in this quarter to 73. This fall was more than accounted for by a reduction of £1·6 million from the previous quarter's peak figure for exports of apparel, though by volume apparel exports still exceeded the 1938 level by three-fifths. For the other textile groups exports continued at much the same level as in 1946. Cotton yarns and manufactures were about two-fifths and woollen and worsted goods five-sixths of 1938, but silk and artificial silk manufactures exceeded those in 1938 by a half.

Exports of chemicals (£15·3 million) of pottery, glass, etc. (£7·4 million) and of miscellaneous manufactures (£15·3 million) were all well below the previous quarter's figures but, in terms of volume, the 1938 average was just exceeded for the last-named group, while for chemicals and pottery, etc., exports were two-fifths and one-half higher than in 1938.

The annual *index-number of agricultural prices* calculated by the Ministry of Agriculture for the calendar year 1946 shows a marked increase over the previous year, the general index-figure having risen to 184 as against 173½ in 1945. In the pre-war period 1937–39 the corresponding figure was 90½. The changes in the last five years are shown below for the three groups of commodities which make up the index-number. Allowance is made for Government subsidies, including payments in respect of the acreage under wheat and potatoes based on the estimated quantities entering into sale. These two latter payments were equivalent to 4*s.* 1*d.* per cwt. for wheat and 39*s.* per ton for potatoes.

(Base 1927–29 = 100)

	1942	1943	1944	1945	1946
Cereals and farm crops	187½	181	180½	184	187
Live-stock and live-stock products	150	151½	158½	164	178
Fruit, vegetables and glasshouse produce ...	186	204	204	193½	201
General index	161½	163½	169½	173½	184

The increase in the general index is chiefly due to live-stock and live-stock products, all the important items coming in this group having risen on the average in the course of the year. The index-number for milk, the largest single item, stood at 206 in 1946 as compared with 192 in 1945; its average price in 1946 was a little more than double what it was in 1939.

It is perhaps of some interest to note that after the 1914–18 war, the general index-number (adjusted for comparison with the present series) was 178 in 1919 and 201 in 1920, the latter figure being the highest average level of agricultural prices so far reached.

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An interesting paper has been issued by the Ministries of Agriculture and Food on the *Post-War Contribution of British Agriculture to the Saving of Foreign Exchange* (Cmd. 7072, price 2d.). It is a study made in 1944–45, the object being to estimate what savings in expenditure on imported food could be achieved after the war by increasing the output of British Agriculture. The value in 1936–38 and in 1943–44 of the *net* output (*i.e.*, deducting from the gross output the value of imports of feeding-stuffs, seeds and store animals) of home agriculture are compared with an estimated net output (*a*) in 1947–48 and (*b*) at some date after 1950–51. All the figures are in terms of pre-war values. In making these estimates of net output, the major emphasis is placed on the contribution that could be made by increasing the production of those foods, particularly animal products, which are "expensive" in terms of foreign exchange. In so far as these can be provided from our own soil we can save the whole of the foreign exchange which would otherwise be used to purchase the finished product. A proportionately smaller saving can be effected by importing feeding-stuffs and converting them into animal products in this country. The net result of these estimates, which are naturally extremely tentative, is that as compared with a net output in pre-war years of £239 millions the output in 1947–48 might be increased by £60 millions and in 1950–51 by possibly £80 millions. This expansion of home agriculture can, it is considered, be achieved only on certain conditions of which perhaps the most important are the provision of additional capital for both landlord and tenant amounting to £250 millions and an increase in the labour force of 5–10 per cent. over pre-war.

The memorandum points out, however, that the saving in exchange is bound to be affected by the increase in food consumption which is likely to arise from an increase in purchasing power, and proceeds to estimate what expenditure would be necessary on imports of food, after allowing for an estimated increase in agricultural output (*a*) assuming that rationing were to continue so as to maintain a national diet similar to that of 1944, and (*b*) assuming the restoration of pre-war levels of consumption (subject to some changes). After allowing for the various factors contributing to increased purchasing power and the natural increase in population, it is concluded that there would be a potential demand for food which, if freely exercised, would be likely to absorb the equivalent of the whole additional output from domestic agriculture. Import control and rationing so severe as to allow no increase in the war-time diet would save something of the order of £116 millions at pre-war prices. Without such control of consumption the net saving would be extremely small. It follows that if post-war consumption rates were allowed to rise normally, the foreign exchange required to pay for imported food would be increased by anything up to £80 million at pre-war prices unless home agriculture is expanded to an equivalent extent

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2. Other Statistics

The statistical side of the work of the *Food and Agriculture Organization of the United Nations* was the subject of two Reports * presented at the second session of the Conference in Copenhagen in September 1946. Various recommendations were made for its development. One of these—the decision to take over the International Institute of Agriculture and its statistical activities—has already been mentioned in this *Journal* (Part III, 1946, p. 307). Arising out of this transfer is a scheme for a World Census of Agriculture in 1950 in continuation of previous efforts made by the Institute in 1930 and to some extent in 1940. The actual collection of census information is, of course, carried out by the individual countries, but much can be done by a central body such as the F.A.O. by preparing an outline schedule of the information required and generally by endeavouring in consultation with the various Governments to ensure that the census is taken on a comparable basis.

Apart from the above, the establishment of the International Emergency Food Council to deal with problems of food allocation and distribution during the period of world shortages has greatly increased the scope of the statistical work. It has also increased the urgency of obtaining necessary statistics on national production and consumption as well as on food requirements to meet health standards.

A point of some interest is the recommendation that the statistical publications of the F.A.O.

* Report of Second Session of Conference; Standing Advisory Committee on Statistics, First Report to the Director General, Copenhagen, August, 1946.

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should aim not only at the presentation of accurate figures, but that there should be appropriate interpretations and analyses of the trends which the statistics reveal. A recommendation that does not seem likely to be very easily realizable is the inclusion of data relating to farm incomes. It is suggested that in co-operation with the statistical services of the United Nations, the F.A.O. should "work out agreements on nomenclature and exact definitions of the various items of national income, in order that fluctuations of agricultural income in relation to the incomes of other sections of the population may be kept under constant review."

Notwithstanding the prominence given to statistics and the many suggestions made for enlarging the scope of the service, the allocation of funds for the purpose in the budget of the F.A.O. seems relatively small. The Committee which dealt with this question observes that the proposed budget "does not appear adequate to meet the basic requirements of the programme outlined" and adds that taking into account the changes in price levels since before the war, "the amounts now budgeted for statistical work in the F.A.O. are not much different from those previously available for the very much more restricted programme of the International Institute of Agriculture."

The Forestry work of the F.A.O. also gives rise to statistical questions and a note on proposals in this connection has been given in this *Journal* (Part III, 1946, p. 307).

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STATISTICAL AND ECONOMIC ARTICLES IN RECENT PERIODICALS

UNITED KINGDOM—

Annals of Eugenics—

December 1946—Records of eye colours for British populations and a description of a new eye-colour scale: *J. Grieve* and *G. M. Morant*. The inheritance of premature baldness in men: *H. Harris*. Orthogonal partitions of the 6 × 6 Latin squares: *D. J. Finney*.

The Banker—

September 1946—Some consequences of the Borrowing Act: *Roland Bird*. Northern Ireland finances: *Major J. M. Sinclair*. Ulster's order-books now: *Sir Walter Smiles*. Encouragement for new industries: *Sir Roland T. Nugent*. The Port of Belfast: *John McCaughey*.

Institute of Bankers, Journal—

October 1946—Two conceptions of the International Monetary Fund: *Paul Bareau*.

Mathematical Tables and other Aids to Computation—

April 1946—Conferences on advanced computation techniques. The application of commercial calculating machines to scientific computing.

Oxford Institute of Statistics, Bulletin—

September 1946—Reconstruction and wage policy: *J. Steindl*. French reconstruction and the Franco-U.S. loan agreement, Part II: *T. Balogh*. Models of the future national income in Sweden: *E. Lundberg* and *I. Ohlsson*.

November 1946—Private and social cost in retail distribution: *R. Bellamy*. A new index number of building material prices: *I. Bowen*.

December 1946—An index of social conditions in 81 county boroughs: *E. J. Buckatzsch*. Low cost family diets in a period of cereal shortages: *T. Schulz*.

Public Administration—

Autumn 1946—The machinery of government: *The Rt. Hon. Sir John Anderson*. Post-war housing finance: *J. P. Macey*. The organisation of control in the British Zone of Germany: *D. R. Woodman*. The transition to the National Health Service: *Harold E. Martin*.

Royal Society of Edinburgh, Proceedings—

Vol. LXII. Part II—Studies in practical mathematics. IV. On linear approximation by least squares: *A. C. Aitken*. Tables of Chebyshev polynomials: *C. W. Jones, J. C. P. Miller, J. F. C. Conn*, and *R. C. Pankhurst*. Two numerical applications of Chebyshev polynomials: *J. C. P. Miller*.

INDIA—

Sankhyā—

July 1946—A sample survey of after-effects of the Bengal famine of 1943: *P. C. Mahalanobis, R. K. Mukherjea* and *A. Ghosh*. On a measure of divergence of two multinomial populations: *A. Bhattacharyya*. Discriminant functions in multivariate analysis: *C. R. Rao*. Confidence interval for the correlation coefficient: *K. C. S. Pillai*. A new solution in symmetrical balanced incomplete block design: *K. N. Bhattacharyya*. A study of expenditure pattern of Calcutta middle class family budgets: *S. J. Poti*. Distribution of Muslims in the population of India: *P. C. Mahalanobis*. Test of significance for intra-class correlation when the family sizes are not equal: *R. P. Bhargava*. On locally most powerful tests when alternatives are one-sided: *C. R. Rao* and *S. J. Poti*. On the most efficient designs in weighing: *C. R. Rao*. A note on concordance: *M. M. Mukherjee*. A note on concentration of agricultural wealth in Bengal: *R. K. Mukherjea*. A statistical study of the labour in the Assam tea plantation: *M. V. S. Rao*.

UNITED STATES—

American Academy of Political and Social Science, Annals—

September 1946—Belgium in Transition. (Whole number.)

American Economic Review—

September 1946—John Maynard Keynes: 1883–1946: *J. A. Schumpeter*. Marginal analysis and empirical research: *Fritz Machlup*. Prospective national income and capital formation in the United Kingdom: *P. S. Brown*. Monetary reform in liberated Europe: *F. H. Klopstock*. International aspects of American agricultural policy: *M. S. Gordon*.

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UNITED STATES—*Contd.*

American Statistical Association, Journal—

September 1946—One statistical world: *Earl Latham.* Objectives, uses and types of labor force data in relation to economic policy: *Louis J. Ducoff* and *Margaret J. Hagood.* Recent experiences with problems of labor force measurement: *Gertrude Bancroft* and *Emmett H. Welch.* Employment statistics in the planning of a full-employment program: *Charles Stewart* and *Loring Wood.* Measuring and forecasting consumption: *Frank R. Garfield.* The use of adjusting factors in the analysis of data with disproportionate subclass numbers: *R. E. Patterson.* A punched card method for presenting, analyzing, and comparing many series of statistics for areas: *Bertram J. Black* and *Edward B. Olds.* Linear regression functions with neglected variables: *Howard L. Jones.* The analysis of latin squares when some observations are missing: *D. B. de Lury.*

Annals of Mathematical Statistics—

September 1946—Sample criteria for testing equality of means, equality of variances, and equality of covariances in a normal multivariate distribution: *S. S. Wilks.* Contributions to the theory of sequential analysis II, III: *M. A. Girshick.* Sufficient statistical estimation functions for the parameters of the distribution of maximum values: *Bradford F. Kimball.* On functions of sequences of independent chance vectors with applications to the problem of the "random walk" in k dimensions: *D. Blackwell* and *M. A. Girshick.* Approximation of the distribution of the product of beta variables by a single beta variable: *John W. Tukey* and *S. S. Wilks.* Some fundamental curves for the solution of sampling problems: *Edward C. Molina.* Enlargement methods for computing the inverse matrix: *Louis Guttman.* The frequency distribution of deviates from means and regression lines in samples from a multivariate normal population: *D. J. Finney.* On the asymptotic distributions of certain statistics used in testing the independence between successive observations from a normal population: *P. L. Hsu.* Estimating the parameters of a rectangular distribution: *A. George Carlton.* On the power function of the sign test for slippage of means: *John E. Walsh.* An approximation to the probability integral: *J. D. Williams.* Distribution of the ratio of sample range to sample standard deviation for normal and combinations of normal distributions: *G. A. Baker.*

Bell System Technical Journal—

October 1946—A study of the delays encountered by toll operators in obtaining an idle trunk: *S. C. Rappleye.*

Harvard Business Review—

Autumn 1946—A case study in international organization: *Milton Katz.* The survival of free enterprise: *Paul G. Hoffmann.* Hollywood and international understanding: *Harry L. Hansen.* Objectives of business education: *Robert D. Calkins.* The transportation problem: *William J. Cunningham.* German contracting in Occupied Europe: *Arthur Leon Horniker.* Recovery in American claims abroad: *Willard B. Cowles.* The U.S.S.R.'s rôle in international finance: *Charles Prince.*

Journal of Experimental Education—

September 1946—The prediction of teaching efficiency: *Leo Joseph Lins.* An evaluation of certain types of personal data employed in the prediction of teaching efficiency: *Herbert I. von Haden.* The prediction of teaching efficiency from objective measures: *Ronald DeVall Jones.*

Journal of Political Economy—

October 1946—Monopolistic competition and the use and price of urban land service: *Joseph J. Spengler.* Wartime control of tin in Great Britain, 1939–41: *Leo Fishman.* Quantity and frequency of use of money in the United States, 1919–45: *Clark Warburton.*

Milbank Memorial Fund Quarterly—

October 1946—Food supply and nutrition in the Netherlands during and immediately after World War II: *M. J. L. Dols* and *D. J. A. M. van Arcken.* Preliminary observations on diurnal and other variations in hemoglobin levels: *Walter Wilkins* and *Ruth Blakely.* Hemoglobin variations for women on iron therapy for thirty-one months: *Dorothy G. Wiehl* and *H. D. Kruse.* The migration of native laborers in South Africa: *Wilbert E. Moore.*

Quarterly Journal of Economics—

August 1946—Reflections on price control: *J. K. Galbraith.* The locus of union control in collective bargaining: *Joseph Shister.* Gasoline rationing in the United States, I: *James A. Maxwell.*

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UNITED STATES—*Contd.*

Review of Economic Statistics—

August 1946—Some measurements of elasticities of substitution: *J. Tinbergen.* Consumers expenditures in war and transition: *V. Lewis Bassie.* Sources of business funds: selected statistics, 1930–44: *Charles Cortez Abbott.* Tertiary production as a postwar international economic problem: *Allan G. B. Fisher.* Wage diversity and its theoretical implications: *Richard A. Lester.* Bullion prices and the gold–silver ratio, 1929–45: *Dickson H. Leavens.* Dovetailing rural public works into employment policy: *E. Jay Howenstine, Jr.*

Social Research—

September 1946—Financing international institutions: *Theodore A. Sumberg.* The significance of foreign trade for domestic employment: *Hans Neisser.*

BELGIUM—

Bulletin de l'Institut de Recherches Economiques et Sociales—

September 1946—Chronique de la conjoncture économique de la Belgique: *Yves Urbain.*

FRANCE—

Journal de la Société de Statistique de Paris—

September–October 1946—Progrès techniques dans l'établissement des statistiques: *Alfred Sauvy.* Analyse statistique du langage: *M. L. Dufrenoy.* La statistique de morbidité à la Sécurité sociale: *Dr. Norbert Marx.* Sur une statistique d'extractions dentaires: *Jean Dufrenoy.*

ITALY—

Giornale degli Economisti e Annali de Economia—

July–August 1946—Della teoria dinamica: *E. Fossati.* Sul concetto in genere di rischio e di rischio statico e dinamico: *R. Maggi.* L'imposta sui redditi variabili e incerti: *P. Gennaro.*

MEXICO—

Estadística—

March 1946—Metodología en Estadísticas Agrícolas de las Américas: *Hernan Montoya.* Calculation of life tables for population of countries and regions: *Nathan Keyfitz.* Renta nacional, su significado y medición, parte II: *Loreto M. Dominguez.* El censo Industrial de Colombia de 1945: *C. Luis Ortiz.* Machine tabulation: *John J. Mahaney.* Prueba de Registro de Nacimientos en Paraguay: *Jorge Roig Ocampos* and *Charles G. Bennett.*

INTERNATIONAL—

Revue de l'Institut International de Statistique—

1945—ISI, IASI and UN: *S. A. Rice.* Sur les limites de la dispersion de certaines estimations: *G. Darmois.* Nouveaux essais d'explication de la répartition des revenus: *R. M. Fréchet.* Sur les fondements des évaluations de la richesse nationale: *C. Gini.*

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LIST OF ADDITIONS TO THE LIBRARY

Since the ssue of Part III, 1946, the Society has received the publications enumerated below.

I.—OFFICIAL PUBLICATIONS

(a) United Kingdom.

Cabinet, Offices of the. Chief of Staff Committee. Diagrammatic representation of certain phases of the war. London: H.M.S.O., 1946. 16½″. 39 fols.

Electricity Commission. Electricity (Supply) Acts, 1882 to 1936. Electricity Commission electricity supply 1938–39 to 1942–43. Return of engineering and financial statistics relating to authorized undertakings in Great Britain for the five years. . . . London: H.M.S.O. 10½″. lxxiv + 518 pp. 40*s*.

Food, Ministry of. Second review of the world food shortage, July 1946. London: H.M.S.O., 1946. Cmd. 6879. 9½″. 35 pp. 6*d*.

Fuel and Power, Ministry of. Statistical digest, 1945. London: H.M.S.O., 1946. Cmd. 6920. 9½″. 114 pp. 2*s*.

Fuel and Power, Ministry of, & Department of Scientific and Industrial Research. Domestic heating in America: a study of heating, cooking and hot-water supply in small houses in U.S.A. and Canada. Report of a joint party of the Ministry of Fuel and Power and the Department of Scientific and Industrial Research. . . . London: H.M.S.O., 1946. 9½″. x + 152 pp. 3*s*.

Labour and National Service, Ministry of.

Final report of the Jute Industry Factory Advisory Committee. London: H.M.S.O., 1946. 9½″. 55 pp. 1*s*.

Time rates of wages and hours of labour, 1st August, 1946. London: H.M.S.O., 1946. 9½″. vi + 133 pp. 2*s*.

Privy Council Office. Treasury. Report of the Committee on the Provision of Social and Economic Research. London: H.M.S.O., 1946. 9½″. 15 pp. 3*d*.

Royal Commission on Equal Pay 1944–46. Report. London: H.M.S.O., 1946. Cmd. 6937. 9½″. xi + 220 pp. 4*s*.

Scotland, Department of Agriculture. Scottish farming. Eleventh economic report 1938–39—1940–41. Edinburgh: H.M.S.O., 1946. 13″. 49 pp. 1*s*. 6*d*.

Town and Country Planning, Ministry of. Advisory Committee for Regional Planning report . . . and report of the technical sub-committee. London: H.M.S.O., 1946. 13″. 39 pp. Map. 2*s*. 6*d*.

Trade, Board of.

Report of the Textile Mission to Japan, January–March 1946. London: H.M.S.O., 1946. 9½″. v + 28 pp. 6*d*.

U.K.–Dominion Wool Disposals Ltd. (Joint Organization). London: H.M.S.O., 1946. Cmd. 6855. 9½″. 54 pp. 9*d*.

Working Party reports. Furniture, 1946. 209 pp. 3*s*. 6*d*. Jewellery and silverware, 1946. v + 105 pp. 3*s*. London: H.M.S.O. 9½″. 3*s*.

(b) Other National and International Publications.

Australia.

Ministry of Munitions. Lectures on statistical methods of inspecting and controlling quality, delivered during a course of instruction arranged by the Department of Munitions for members of the technical staff, held at the University of Melbourne August 1944. Edited by W. N. W. Wallace, Directorate of Explosives Supply. Melbourne, 1944. 9¾″. 236 fols.

Belgium.

Office Central de Statistique.

Annuaire statistique de la Belgique et du Congo Belge. lxii, 1940. lxiii, 1941. lxiv, 1942. 10¾″. lxv, 1943. lxvi, 1944. 11¼″. 5 vols.

Bulletin de statistique. **30**, 1944, 1–12. **31**, 1945, 1–6. 11¼″. 18 parts.

Démographie de la Belgique de 1921 à 1939. 1943. 11¼″. 458 pp.

Évolution des finances de l'état: aperçu statistique de la période décennale 1931–40. 11¼″. 64 pp.

Recensement des personnes émargeant au budget de l'état et relevant du pouvoir exécutif au 1er janvier 1939. 11¼″. 105 pp.

Recensement économique et social au 27 février 1937, I–V. 11¼″. 5 vols. Recensement des inoccupés: Le Royaume. 12¼″. 180 pp.

Statistique des budgets de l'exercice . . . pour les communes de 5000 habitants et plus et pour les provinces. 11¼″. 1938, 117 pp. 1939, 109 pp.

Statistique des étrangers 15 septembre, 1939. 11¼″. 69 pp.

Statistique des finances communales d'après les comptes de 1939 et les budgets de 1941, avec un aperçu des comptes provinciaux de 1939 et des budgets de 1941. 11¼″. 292 pp.

Statistique judiciaire de la Belgique. Années 1931–40. 1941–42. Brussels. 11¼″. 3 vols.

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(b) Other National and International Publications—*Contd.*

Brazil.

Instituto Brasileiro de Geografia e Estatistica. Comissão Censitaria Nacional. Recenseamento geral do Brasil (1 de setembro de 1940) Série Nacional Vol. I. Introdução, Tomo I. A cultura brasileira . . . Rio de janeiro, 1943. 10½″. 535 pp.
Sinôpse do censo demografico dados gerais. 1946. 10½″. vii + 47 pp.

Canada.

Department of Reconstruction and Supply.
Capital, repair and maintenance expenditures of business enterprises in Canada: forecast 1946. Ottawa, 1946. 9¾″. 31 pp.
Manpower and material requirements for a housing program in Canada. Ottawa, 1946. 9¾″. 137 pp.

International Labour Office.

Wartime labour conditions and reconstruction planning in India. (Studies and Reports, New Series 2.) Montreal, 1946. 9″. 113 pp. 2*s.*

Italy.

Istituto Centrale di Statistica.
Annali di statistica. Serie VII, Vol. VI, 554 pp. 1940. Vol. VI, 287 pp. 1943. Rome, 1940–43. 2 vols. 10½″.
Annuario statistico Italiano. Quarta serie. VII, 1940. VII, 1941. IX, 1942. X, 1943. Rome, 1940–43. 4 vols. 10½″.
Censimento della popolazione del Regno d'Italia al 1° dicembre, 1921. Vols. VII, VIII, X, XVIII. Rome, 1927. 11 vols. 10¼″.
Censimento industriale 1937. Industria degli olii vegetali . . . (Monografia 5.) Rome, 1940. 12″. vii + 239 pp.
Censimento industriale e commerciale 1937–40. Industria della pesca in acque marine. Risultati sommari, Parte I. 1939. 12″. 43 pp. Risultati generali. Vol. I Industrie (1–3). 1942–43. 3 vols. 12″.
Movimento della popolazione e cause di morte. Serie II, vol. II, 1939. 10½″. vol. III, 1940. 11½″. Rome, 1941, 1943. 2 vols.
Statistica del commercio speciale di importazione e esportazione del Regno d'Italia e dell' Unione Doganale Italo–Albanese . . . Jan. 1940–Nov. 1942 inclusive. Jan.–June 1943 inclusive. Rome, 1940–43. 41 parts. 11½″.
Statistica del commercio con l'estero. Serie II. No. 1, giugno 1946, 2, luglio 1946. Rome, 1946. 2 parts. 10½″.
Statistica del movimento della navigazione per l'anno 1938. Rome, 1939. 12″. xi + 439 pp.
Supplementi all' *Annuario statistico Italiano*. 1. Agricoltura e foreste. 1944. 57 pp. 2. Industria. 1944. 47 pp. 3. Comunicazioni e trasporti, prezzi, costo della vita, consumi. 1944. 58 pp. 4. Commercio con l'estero. 1944. 58 pp. 5. Mercato monetario e credito, finanze pubbliche. 1944. 57 pp. Rome, 1944. 5 parts. 10½″.

League of Nations.

Economic, Financial and Transit Department.
The course and control of inflation: a review of monetary experience in Europe after World War I. (1946, II. A. 6.) 9″. v + 136 pp.
Economic and Financial Organization. Conditions of private foreign investment: report by the Special Joint Committee. (1946, II. A. 1.) 9″. 45 pp. 2*s.*
Food, famine and relief 1940–46. (1946, II. A. 5.) 9″. 4 + 162 pp. 6*s.*

Palestine.

National income of Palestine 1944, by P. J. Loftus . . . Government Statistician. Jerusalem, 1946. 9″. 48 pp. 100 *Mils.*

Switzerland.

Bureau Fédéral de Statistique.
Finanzen und Steuern von Bund, Kantonen und Städten . . . 1944–45. (Beiträge zur schweizerischen Statistik 15). Bern, 1946. 9½″. 167 pp.
Recensement fédéral de la population 1er decembre, 1941. 16. Canton du Valais. (Statistiques de la Suisse, 169.) Bern, 1946. 11¾″. 132 pp.

Union of South Africa.

Social and Economic Planning Council. Report No. 7. Taxation and fiscal policy. 1945. 13¼″. 74 pp. 8*s.* 6*d.* Summary of Report No. 7. Pretoria, 1935. 13¼″. 8 pp. 6*d.*

United Nations Food and Agricultural Organization.

First annual report of the Director-General to the FOA Conference. Washington, 1946. 10¼″. 45 pp.
Forestry and forest products: world situation 1937–46. Washington, 1946. 10¼″. 93 pp.
1946 Handbook of agricultural statistics. . . . Washington, 1946. 10¼″. 134 pp.

United Nations Relief and Rehabilitation Administration.

Economic recovery in the countries assisted by UNRRA. Washington, 1946. 9″. iii + 182 pp.

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II.—AUTHORS AND MISCELLANEOUS

BARKWORTH (H.) AND IRWIN (J. O.). Comparative detection of coliform organisms in milk and water by the presumptive coliform test, with an appendix on the possible bactericidal effect of bile salt. *Journal of Hygiene*, **43** (1943), pp. 129–135. 10¼″. (Presented by Dr. J. O. Irwin.)

BOULDING (KENNETH E.). Economic analysis. New York and London, Harper, 1941. 8¼″. xviii + 809 pp. 25*s.* 6*d.*

BOWLEY (ARTHUR L.). Elements of statistics. 6th ed. London, Staples Press, 1946. 8½″. xv + 503 pp. 30*s.*

BURTON (CLEMENT). A Concise manual of statistics (with special reference to the requirements of students for municipal examinations), 2nd ed. London, Gee & Co., 1946. 8¼″. 182 pp. 15*s.*

COCKE (SIR HUGH). A Summary of the principal legal decisions affecting auditors . . ., 4th ed. London, Gee & Co., 1946. 8½″. 109 pp. 12*s.*

COX (P. R.). A Post-war plan for population statistics. *Journal of the Institute of Actuaries Students' Society*, **6** (1945). 13 pp. (Presented by the Author.)

COWLES COMMISSION FOR RESEARCH IN ECONOMICS. Cowles Commission papers. New Series 13. Games and economic behaviour: two review articles by L. Hurwicz and J. Marschak. 1946. 9″. 46 pp. 17. Estimating relations from non-experimental observations: abstracts of papers presented at Cleveland, Jan. 25, 1946, by J. Marschak, L. Hurwicz, T. Koopmans, R. B. Leipnik. 10″. 6 pp. Chicago, Cowles Commission.

DUBLIN (LOUIS I.) AND LOTKA (ALFRED J.). The Money value of a man, by L. I. Dublin . . . and A. J. Lotka . . . in collaboration with M. Spiegelman. Revised edition. New York, Ronald Press, 1946. 9″. xvii + 214 pp. $6.

FINNEY (D. J.). Field sampling for the estimation of wireworm populations. *Biometrics Bulletin*, **2** (1946), pp. 1–7. 9″.

—— The Frequency distribution of deviates from means and regression lines in samples from a multivariate normal population. *Annals of Mathematical Statistics*, **17** (1946), pp. 344–349. 10″.

—— Latin squares of the sixth order. *Experimentia*, **2** (1946), 5 pp. 8¼″. (Presented by the Author.)

GINI (CORRADO). Teorie della popolazione, con appendici a cura di C. Gini e G. Livada. Rome, Casa Editrice Castellani, 1945. 3, 160 pp. 300*l.*

—— Les facteurs de la decroissance de la natalité a l'époque contemporaine (2ème edition). 9¼″. 42 pp.

—— L'assimilazione fisica dei discendenti degli immigrati. 9¼″. 15 pp.

—— Ressemblance parentale et ressemblance fraternelle. 9¼″. 15 pp.

—— I pericoli della statistica (*Società Italiana di Statistica, Atti della Riunione*, **1** (1939), 44 pp. 9¾″.

—— Sul concetto di caso. *Ibid.*, **3** (1941), 16 pp. 9½″.

—— Per la determinazione della probabilità di transvariazione tra più gruppi. *Ibid.*, **5** (1942), 3 pp. 9½″.

—— Sull'uso delle curve integrali ai fini dell'interpolazione grafica. *Ibid.*, **5** (1942), 4 pp. 9½″.

—— Sul fabbisogno alimentare alle varie età. *Ibid.*, **5** (1942), 8 pp. 9½″.

—— La dipendenza per materie prime dei singoli stati prebellici europei. *Ibid.*, **5** (1942), 11 pp. 9½″.

—— AND LIVADA (G.). Nuovi contributi alla teoria della transvariazione. *Ibid.*, **7** (1943), 26 pp. 9½″.

—— —— Transvariazione a più dimensioni. *Ibid.*, **7** (1943), 38 pp. 9½″.

—— —— Sulla probabilità inversa nel caso di grandezze intensive ed in particolare sulle sue applicazioni a collaudi per masse a mezzo di campioni. *Ibid.*, **7** (1943), 7 pp. 9½″.

—— Sulla probabilità inversa nel caso di grandezze a distribuzione costante. *Ibid.*, **7** (1943), 17 pp. 9½″.

—— Osservazioni sulla comunicazione della Dott. Geppert "Sul valore dei cosiddetti testi di significatività." *Ibid.*, **7** (1943), 3 pp. 9½″.

—— I testi di significatività. *Ibid.*, **7** (1943), 39 pp. 9½″.

—— Degli indici sintetici di correlazione e delle loro relazioni con l'indice interno di correlazione (Intraclass correlation cefficient) e con gli indici di correlazione tra serie di gruppi. *Metron*, **14** (1941), 21 pp. 9½″.

—— Dell'influenza che il raggruppamento delle singole modalità esercita sul valore di alcuni indici statistici nel caso di serie. *Ibid.*, **12** (1936), 34 pp. 9½″.

—— Alle basi del metodo statistico: il principio della compensazione degli errori accidentali e la legge dei grandi numeri. *Ibid.*, **14** (1941), 70 pp. 9½″.

—— On the measure of fertility in women. *Rural Sociology*, **7** (1942), pp. 84–86. 9½″.

—— La teoria europea e la teoria americana delle migrazioni internazionali. *Economia e Commercio* (1942), 28 pp. 9½″.

—— Prolificità e frequenza dei parti plurimi. *Atti della III Riunione della Società Italiana di Genetica ed Eugenica* (1938), 19 pp. 9½″.

—— Il principio della compensazione degli errori accidentali. *Atti del secondo Congresso dell'Unione Matematica Italiana* (1940), pp. 673–676. 11½″.

—— Delle relazione tra serie di gruppi. *Statistica* (1941), 12 pp. 9½″.

—— Les bases de la biotypologie et les criteriums du recrutement militaire. *Revue anthropologique*, **49** (1939), 16 pp. 9½″.

—— Methods of eliminating the influence of several groups of factors. *Econometrica*, **5** (1937), pp. 57–73. 10″.

—— Über die differenzierte Fruchtbarkeit und einige verwandte Fragen. *Archiv für Rassen- und Gesellschaftsbiologie*, **35** (1941), pp. 177–208. 10″.

—— A co-ordination of the different population theories. *Revue de l'Institut International de Statistique*, **11** (1943), 35 pp. 10¼″.

—— Sur la mesure de l'efficacité des pratiques anticonceptionnelles. *Ibid.*, **10** (1942), 39 pp. 10¼″.

—— Sur la mesure de la fécondité naturelle de la femme mariée. *Ibid.*, **9** (1941), 20 pp. 10¼″.

—— Sur la "méthod des profils" et sur d'autres diagrammes à ordonnées jointes, dans le cas de séries non ordonnées. *Ibid.*, **7** (1939), 53 pp. 10¼″.

—— The relative importance of hereditary and non-hereditary factors in determining the heterogeneity of a generation. *Annals of Eugenics*, **10** (1940), pp. 42–47. 10¾″. (31 items, presented by Prof. C. Gini.)

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Authors and Miscellaneous—*Contd.*

GLAISYER (JANET) *and others*. County town: a civic survey for the planning of Worcester, by Janet Glaisyer, T. Brennan, W. Ritchie and P. Sargant Florence, Commerce Department, University of Birmingham, prepared for the Worcester City Council. London, John Murray, 1946. 9½″. xii + 320 pp. 21*s*.

GRANT (EUGENE L.). Statistical quality control. New York and London, McGraw-Hill, 1946. 9″. xii + 563 pp. 25*s*.

GREENWOOD (MAJOR). Tenth report on data of anti-rabies treatments supplied by Pasteur Institutes. *Bulletin of the Health Organization of the League of Nations*, **12** (1946), pp. 301–364. 9½″.

HARTKEMEIER (HARRY PELLE). Principles of punch-card machine operation: how to operate punch-card tabulating and alphabetic accounting machines. . . New York, Thomas Y. Crowell Co., 1942. 11″. xv + 269 pp.

HAWTREY (R. G.). Bretton Woods, for better or worse. London, Longmans, Green, 1946. 7½″. vii + 142 pp. 7*s*. 6*d*.

HOBSON (E. W.). A Treatise on plane trigonometry. . . . 2nd ed. Cambridge University Press, 1897. 8¾″. xv + 360 pp.

HOEL (PAUL G.). Introduction to mathematical statistics. New York, John Wiley, 1947. (London, Chapman and Hall). 8½″. x + 258 pp. $3.50.

HOLIDAY (E. R.), IRWIN (J. O.) *and others*. The Assay of crystalline preparations of aneurine by means of the ultra-violet absorption spectrum. *Quarterly Journal of Pharmacy*, **19** (1946), pp. 155–172. 8½″.

IRWIN (J. O.). The Error of the biological assay of insulin by the mouse-convulsion test. *Quarterly Journal of Pharmacy*, **16** (1943), pp. 352–362. 8½″.

—— On the calculation of the errror of biological assays. *Journal of Hygiene*, **43** (1943), pp. 121–128. 10¼″.

—— A Statistical examination of the accuracy of vitamin A assays: an analysis of three co-operative experiments designed to ascertain the value of the conversion factor for transforming spectrophotometric values into international units. *Journal of Hygiene*, **43** (1944), pp. 291–314. 10¼″.

—— On the characteristic function of the distribution of the product of two normal variates. *Proceedings of the Cambridge Philosophical Society*, **42** (1945), pp. 82–84. 10¼″.

—— AND GOODMAN (NANCY). The Statistical treatment of measurements of the carcinogenic properties of tars (Part I) and mineral oils (Part II). *Journal of Hygiene*, **44** (1946), pp. 362–420. 10¼″. (Presented by the Author.)

JORDAN (CHARLES). Remarques sur la loi des erreurs. *Acta Scientiarum Mathematicarum*. Szeged, Hungary, **10** (1941), pp. 111–133. 9½″.

—— Complément au théorème de Simmons sur les probabilités. *Acta Scientiarum Mathematicarum*, **11** (1946), pp. 19–27. 9½″. (Presented by the Author.)

KUZNETS (SIMON). National income: a summary of findings. (Twenty-fifth Anniversary Series 1.) New York, National Bureau of Economic Research, 1946, [8], 144 pp. 9″. $1.50.

LAKDAWALA (D. T.). Justice in taxation in India, with special reference to British Gujarat. Bombay, Popular Book Depot, 1946. 8½″. xviii + 312 pp. *Rs*. 12.8.

LEAKE (P. D.). Leake's register of industrial plant. [30 ruled sheets 13″ × 16½″ in loose-leaf binder.] Introductory notes on the use of Leake's Register of Industrial plant for the classification of capital outlay and the measurement of depreciation, either monthly or yearly, by means of a regular record of expired cost values, based on estimates and revised estimates of the useful life of each class of plant. London, Gee & Co., 1946. 8½″. 18 pp. 35*s*.

LUNDBERG (OVE). On Random processes and their application to sickness and accident statistics. Uppsala, Almqvist and Wiksells Boktryckeri, 1940. 9¼″. vii + 172 pp. (Presented by the Author.)

NIXON (EDNA). John Hilton: the story of his life. London, Allen and Unwin, 1946. 344 pp. Plates. 8¼″. 15*s*.

RUSSELL (RONALD). Britain's commercial treaty position. London, Empire Economic Union, 1946. 8½″. 87 pp. 1*s*.

SCHWEIZERISCHEN NATIONALBANK. Das schweizerische Bankwesen im Jahre 1945. (Mitteilungen der Volkswirtschaftlichen und Statistischen Abteilung der Schweizerischen Nationalbank, 29.) Zürich, 1946. 9½″. 186 pp.

SIMPKINS (JOHN G.). Accounts from incomplete records. London, Gee & Co., 1946. 8½″. vii + 64 pp. 12*s*. 6*d*.

TATE (W. E.). A Hand-list of Buckinghamshire Enclosure Acts and awards. Aylesbury, Bucks County Council, 1946. 8½″. vi + 46 pp. 2*s*. 6*d*.

THOMPSON (CATHERINE M.) AND MERRINGTON (MAXINE). Tables for testing the homogeneity of a set of estimated variances . . . with a prefatory note by E. S. Pearson and H. O. Hartley. *Biometrika*, **33** (1946), pp. 293–304. London, 1946. 10¾″. 2*s*. (Presented by Prof. E. S. Pearson.)

TIPPETT (L. H. C.). The Methods of statistics: an introduction mainly for experimentalists. 3rd ed. London, Williams and Norgate, 1945. 8½″. 284 pp. 17*s*. 6*d*.

TOMASEVICH (JOZO). International agreements on conservation of marine resources, with special reference to the North Pacific. California, Food Research Inst., Stanford University, 1943. 9″. xi + 297 pp. $3.

VINCI (FELICE). Unità Mediterranea. 2nd ed. Milan, A. Giuffre, 1946. 8¾″. 194 pp. 200*l*.

WEATHERBURN (C. E.). A First course in mathematical statistics. Cambridge University Press, 1946. 8½″. xv + 271 pp. 15*s*.

WILKS (S. S.). Mathematical statistics. New Jersey, Princeton University Press, 1946 (London, Sir Humphrey Milford) [1], xi + 284 pp. 26*s*. 6*d*.

WORKING (HOLBROOK). Statistical quality control in war production. *Journal of the Amer. Statistical Assoc.*, **40** (1945), pp. 425–447. 9½″.

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